UNIT- I

THREE PHASE CIRCUITS

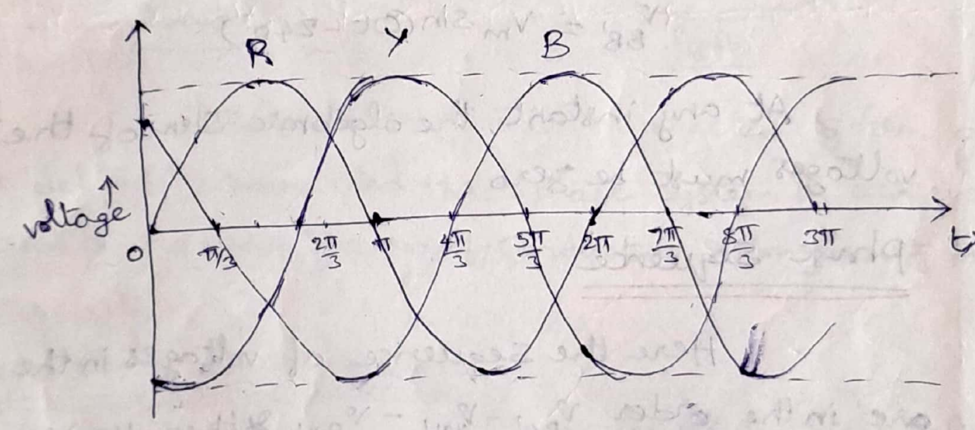
Introduction :-

In an ac system, it is possible to connect two or more number of individual circuits to a common polyphase source.

The power generated by an AC machine increases 41.4% from single phase to ~~two~~ two phase & the increase in the power is 50% from single phase to three phase.

In general, a three phase system of voltages or currents is merely a combination of three single phase systems of voltages of which the three voltages or currents differ in phase by 120° (electrical) from each other in a particular sequence.

Following figure represents the three phase system of sinusoidal voltages.



Advantages of Three-phase System

1. Power in single phase circuit is pulsating, whereas the three phase circuits the total power supplied by all the three phases is constant at every instant of time.
2. To transmit a given amount of power over a given length, a three phase transmission circuit requires less conductor material than a single phase circuit.
3. In a given frame size, a three phase motor or a three phase generator produces more output than single phase generator.

4. Three phase motors are more easily started than single phase motors. Single phase motors are not self starting whereas three phase motors are self starting.

Generation of Three Phase voltages

The voltages generated by a three-phase alternator is shown in earlier figure. Here, the three voltages of the same magnitude and frequency. But, these displaced by 120° in space.

Assuming the voltages to be Sinusoidal, we can write the equations for the instantaneous values of the voltages of the three phases. These are,

$$v_{RR'} = V_m \sin \omega t$$
$$v_{YY'} = V_m \sin (\omega t - 120^\circ)$$
$$v_{BB'} = V_m \sin (\omega t - 240^\circ)$$

At any instant, the algebraic sum of the three voltages must be zero.

Phase Sequence

Here the sequence of voltages in the three phases are in the order $v_{RR'} - v_{YY'} - v_{BB'}$ & they undergo changes one after the other in the above mentioned order. This is called "phase sequence".

It depends upon the rotation of the field. If the field System is rotated in anticlockwise direction then the sequence of the voltages in the three phases is $v_{RR'} - v_{BB'} - v_{YY'}$. Now, the voltage equations can be written

$$v_{RR'} = V_m \sin \omega t$$
$$v_{BB'} = V_m \sin (\omega t - 120^\circ)$$
$$v_{YY'} = V_m \sin (\omega t - 240^\circ).$$

Inter connection of three phase sources & Loads

1. Inter Connection of three-phase sources

In a 3-phase alternator, there are three independent phase windings or coils. Each phase or coil has two terminals, viz. start and finish.

The end connections of the three sets of the coils may be brought out of the machine, to form three separate single phase sources to feed three individual circuits as shown in following figures (a) & (b).

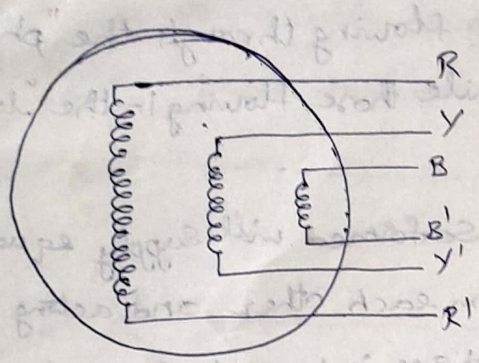


fig (a)

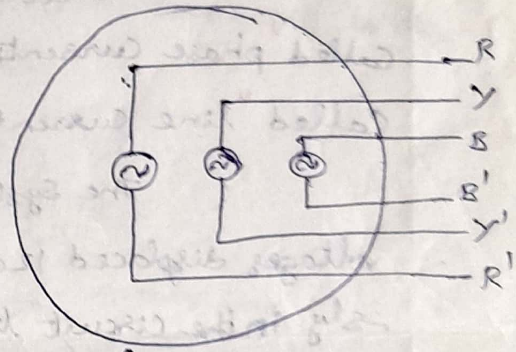
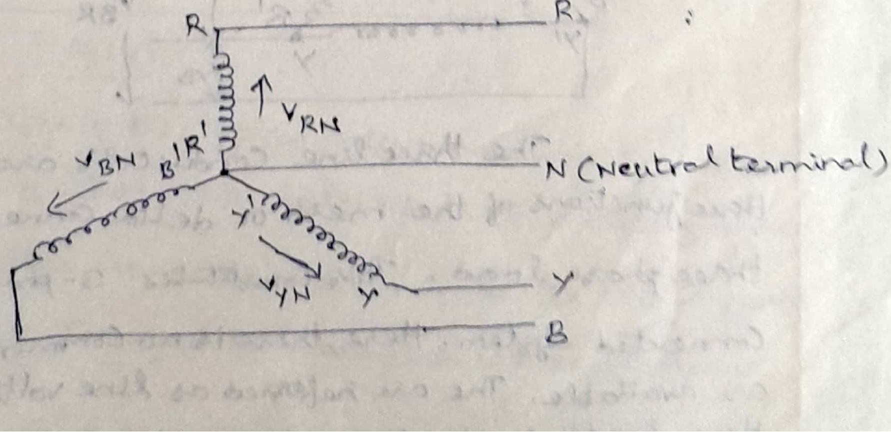


fig (b)

The coils are inter-connected to form a wye (Y) or delta (Δ) connected three phase system to achieve economy and to reduce the no. of conductors & complexity in the circuit.

WYE (or) Star-connection

In this connection, similar ends (start or finish) of the three phases are joined together within the alternator as shown in following figure.



The Common terminal so formed is referred to a neutral point (N), or neutral terminal.

Three lines are run from the other free end (R, Y, B) to feed power to the three phase load. The above figure represents a "3-phase, 4-wire, Star Connected Sy

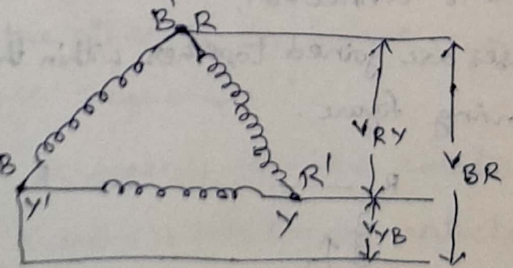
The voltage between any line and the neutral point is called the "phase voltage" (V_{RN}, V_{YN} and V_{BN}), while voltage between any two lines is called the "line voltage" V_{YB} and V_{BR}).

The currents flowing through the "phases" are called "phase currents" while those flowing in the "lines" are called "line currents".

The system so formed will supply equal line voltages displaced 120° from each other and acting simult -sly in the circuit like three independent single phase sources in the same frame of a three phase alternat

Delta (or) Mesh Connection

In this method of connection, the dissimilar e of the windings are joined together. i.e. R' is connect to Y, Y' to B and B' to R as shown below.



The three line conductors are taken from the three junctions of the mesh or delta connection to feed the three phase load. This constitutes 3-phase, 3-wire delta connected system. Here, there is no common terminal. V_{RY}, are available. The are referred as line voltages. and phase Here, loads can be connected across R, Y, B line terminals o

Interconnection of Loads

~~The Phasory~~

An impedance, or load connected across any two terminals of an active network will draw power from the source, and is called a single phase load.

Similar to an alternator phase windings, load can also be connected across any two terminals, ~~of~~ or between line and neutral terminal (star connection).

Usually, the three phase load impedances are connected in star or delta formation & then connected to the three phase source as shown below.

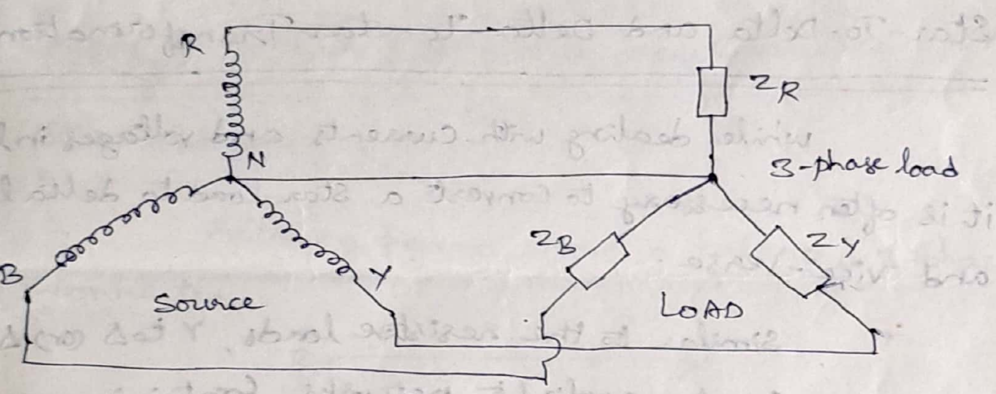


fig.(a)

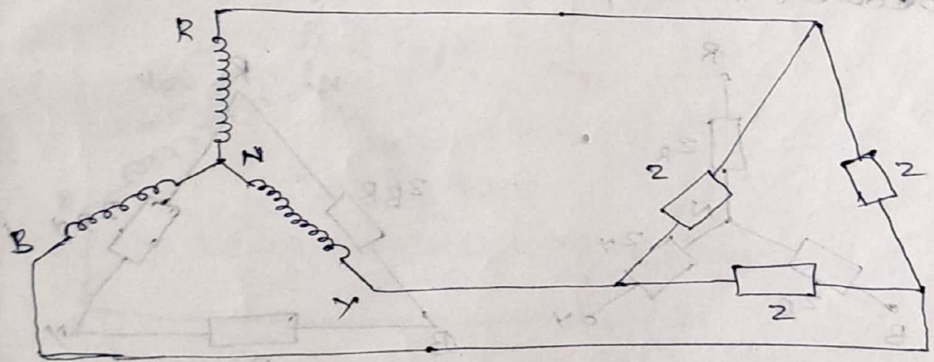


fig.(b)

Fig (a) represents the typical interconnections of loads and sources in a 3-phase star system & is of 3-phase 4-wire system. A 3-phase star connected load is connected to a 3-phase star connected source, terminal to terminal & both neutrals are joined.

Fig (b) represents the 3-phase 3-wire system. Here, a

delta connected load is connected to a 3-phase sta
-cted source, terminal to terminal.

As in the case of 3-phase source, a 3-pha
can be either balanced or unbalanced.

A balanced 3-phase load is one in which
branches have identical impedances, i.e. each impe
has the same magnitude & phase angle. The resis
reactive components of each phase are equal.

Any load which does not satisfy above req
is said to be an unbalanced load.

Star-To-Delta and Delta-To-star Transformation

while dealing with currents and voltages in lo
it is often necessary to convert a star load to delta lo
and vice-versa.

Similar to the resistive loads, Y to Δ (or) Δ t
conversion can be applied to networks containing gene
impedances in complex form.

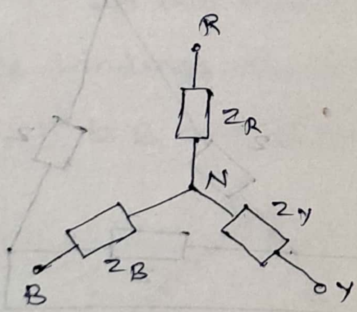


fig (a)

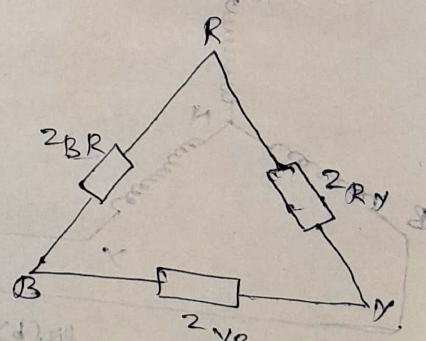


fig (b)

Delta impedances, in terms of star impedances as

$$Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}$$

Similarly, star equivalent impedances of a Δ Connected load are

$$Z_R = \frac{Z_{RY}\,Z_{BR}}{Z_{RY}+Z_{BR}+Z_{YB}}$$

$$Z_Y = \frac{Z_{RY}\,Z_{YB}}{Z_{RY}+Z_{YB}+Z_{BR}}$$

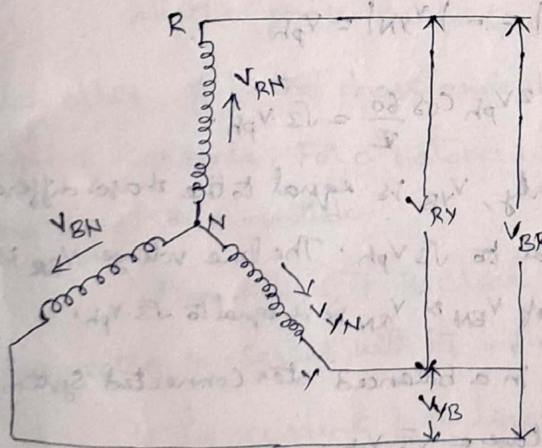
$$Z_B = \frac{Z_{YB}\,Z_{BR}}{Z_{RY}+Z_{BR}+Z_{YB}}$$

Here, all impedances are to be expressed in their complex form.

Voltage, Current And Power in a star Connected System

Star-Connected System:

Following figure shows a balanced 3-phase system. (Y-Connected):



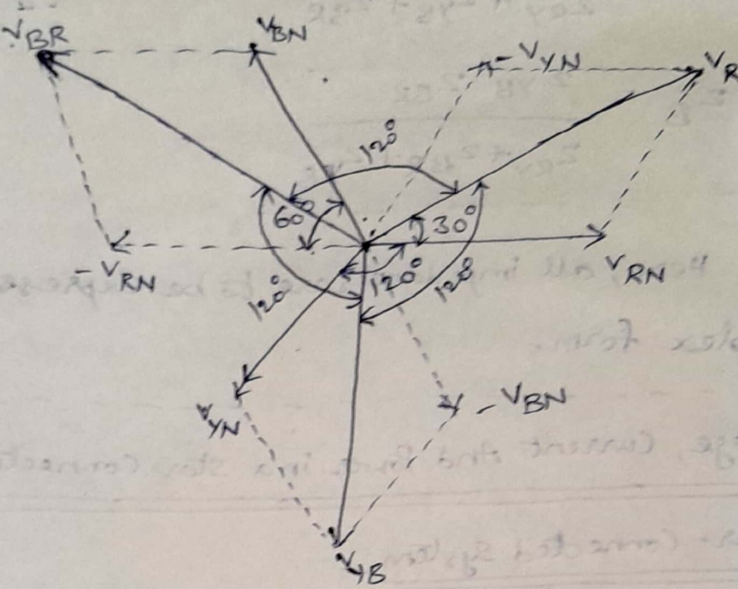
Here, V_{RN}, V_{YN} & V_{BN} represents the RMS values of the induced voltages in each phase.

The voltages V_{RY}, V_{YB}, V_{BR} are known as "line voltages". V_{RY} represents a voltage between points 'R' and 'Y', with R being positive w.r.t. point 'Y' during its positive half cycle.

Similarly, V_{YB} represents voltage b/w Y & B, with 'Y' being positive with respect to 'B' during its +ve half Cycle.

Voltage Relation

The phasors corresponding to the phase v... constituting a three-phase system can be represented ... following phasor diagram.



From figure, considering the lines R, Y & B, t... voltage V_{RY} is equal to the phasor sum of V_{RN} & V_{RY}

$$|V_{RN}| = -|V_{YN}| = V_{ph}.$$

$$\therefore \quad V_{RY} = 2V_{ph} \cos\frac{60}{2} = \sqrt{3}\, V_{ph}.$$

Similarly, V_{YB} is equal to the phasor difference and V_{BN} & is equal to $\sqrt{3}\, V_{ph}$. The line voltage V_{BR} is equa... phasor difference of V_{BN} & V_{RN} & is equal to $\sqrt{3}\, V_{ph}$.

Hence in a balanced star connected system,

(i) Line voltage = $\sqrt{3}\, V_{ph}$.

(ii) All line voltages are equal in magnitude & are di... by $120°$ &

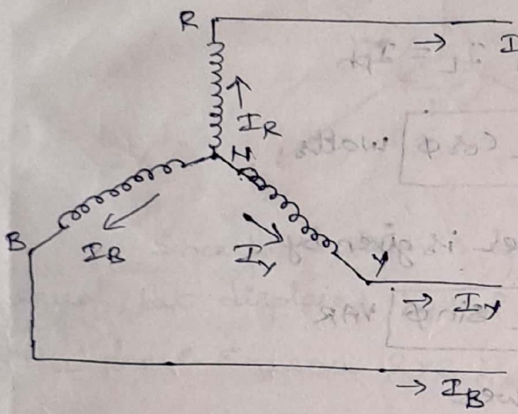
(iii) All line voltages are $30°$ ahead of their respective phase voltages.

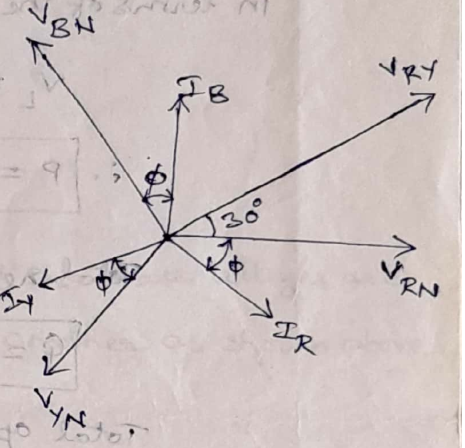
Current Relations

Following figure shows a balanced three phase, Y connected system indicating phase currents and line currents.

Here, I_R, I_Y & I_B flowing in the three phases indicate directions of currents when they are assumed to be positive & not the directions at that particular instant.

The phasor diagram for phase currents w.r.t. their phase voltages is shows below.





All the phase currents are displaced by 120° w.r.t. each other, 'ϕ' is the phase angle between phase ~~angle~~ voltage & phase currents. For a balanced load, all the phase currents are equal in magnitude.

From figure, it is clear that each line conductor is connected in series with its individual phase winding.

$\therefore$ The current in a line conductor is the same as that in the phase to which the line conductor is connected.

i.e. $I_L = I_{ph} = I_R = I_Y = I_B$ (in magnitude)

It can also be observed that the angle between the line (phase) current & the corresponding line voltage is $(30+\phi)$ for a lagging p.f. load & $(30-\phi)$ for a leading p.f. load.

-nts the assigned

then referred to as a "direc ... ph."

Power in the Star-Connected Network

The total active power or true power in the three phase load is the sum of the powers in the three phase.

For a balanced load, the power in each load is same. Hence total power = 3 × power in each phase

(or) $P = 3 V_{ph} I_{ph} \cos\phi$

It is usual practice to express the three phase in terms of the line quantities as follows,

$$V_L = \sqrt{3} V_{ph}, \quad I_L = I_{ph}$$

$$\therefore \boxed{P = \sqrt{3} V_L I_L \cos\phi} \text{ watts}$$

Total reactive power is given by

$$\boxed{Q = \sqrt{3} V_L I_L \sin\phi} \text{ VAR}$$

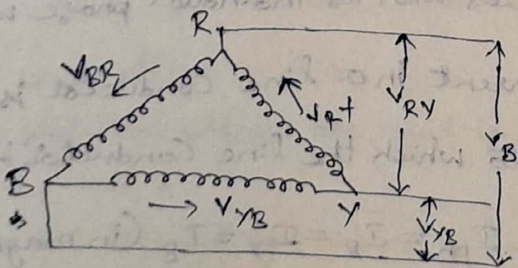
Total apparent power

$$\boxed{S = \sqrt{3} V_L I_L} \text{ VA}$$

Voltage, Current And Power in a Delta Connected System

Delta Connected System

Following figure shows a balanced 3-phase, 3-wire Δ-connected system.



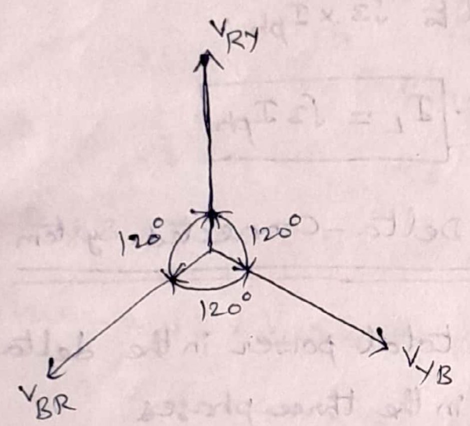
In this connection, it may appear that the three are short circuited among themselves. However, this not since the system is balanced, the sum of the three voltages round the closed mesh is zero; consequently, no current flow around the mesh when the terminals are open.

Voltage Relation

In the following figure, we can notice that only one phase is connected between any two lines.

Hence, the voltage between any two lines 'V_L' is equal to the phase voltages 'V_{ph}'.

$$\therefore V_{RY} = V_L = V_{ph}$$

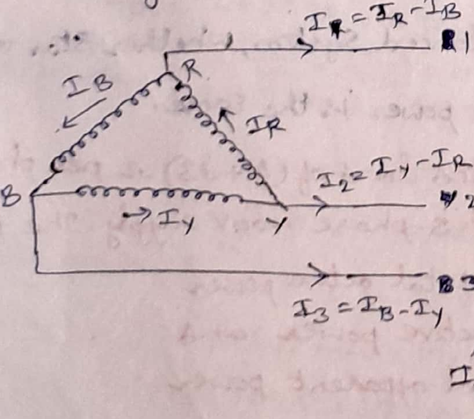


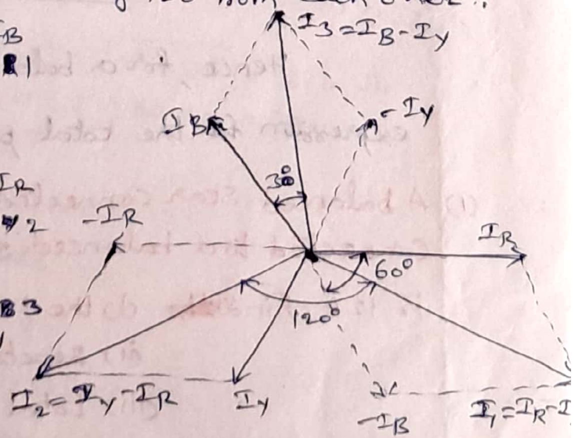
Since the system is balanced, all the phase voltages are equal, but displaced by 120° from one another as shown above. The phase sequence RYB is assumed.

$$|V_{RY}| = |V_{YB}| = |V_{BR}| = V_L = V_{ph}$$

Current Relation

From following figure, we can notice that, since the system is balanced, the three phase currents (I_{ph}), i.e. I_R, I_Y, I_B are equal in magnitude but displaced by 120° from each other.





Current in line '1', $I_1 = I_R - I_B$,

" " " '2', $I_2 = I_Y - I_R$ and

" " " '3', $I_3 = I_B - I_Y$

The phasor addition of these currents is

$$\bar{I}_1 = \bar{I}_R - \bar{I}_B$$

$$I_1 = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cos 60^\circ}$$

$$I_1 = \sqrt{3}\, I_{ph} \quad [\because I_R = I_B = I_{ph}]$$

Similarly, the remaining two line currents are also equal to $\sqrt{3} \times I_{ph}$.

$$\therefore \boxed{I_L = \sqrt{3}\, I_{ph}}$$

Power in the Delta-Connected System

The total power in the delta circuit is the sum of the powers in the three phases.

Since the load is balanced, the power consumed in each phase is the same. Total power is equal to three times the power in each phase.

$$\text{Power per phase} = V_{ph} I_{ph} \cos\phi$$

where 'ϕ' is the phase angle between voltage and current.

$$\text{Total power, } P = 3 V_{ph} I_{ph} \cos\phi$$

In terms of line quantities,

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi} \text{ Watts } [\because I_L =$$

Hence, for a balanced system, whether star or delta, the expression for the total power is the same.

(1) A balanced star connected load of $(4 + j3)\ \Omega$ per phase is connected to a balanced 3-phase 400V supply. The phase current is 12A. Find (i) the total active power

(ii) reactive power and

(iii) total apparent power

Sol:-

Supply voltage, $V_L = 400V$

Supply current, $I_L = 12A$

Per phase impedance, $Z_{ph} = (4+j3)\,\Omega$

$$|Z_{ph}| = \sqrt{4^2+3^2} = 5\Omega$$

power factor, $\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{4}{5} = 0.8$

$$\sin\phi = \sin[\cos^{-1}(0.8)] = 0.6$$

(i) Active power $= \sqrt{3} \times V_L \times I_L . \cos\phi$

$$P = \sqrt{3} \times 400 \times 12 \times 0.8 = 6651 \text{ watts}$$

(ii) Reactive Power $= \sqrt{3} \times V_L \times I_L \sin\phi$

$$\phi = \sqrt{3} \times 400 \times 12 \times 0.6 = 4988.36 \text{ VAR}$$

(iii) Apparent Power, $S = \sqrt{3}\, V_L I_L$

$$= \sqrt{3} \times 400 \times 12 = 8313.84 \text{ VA}$$

(2) A balanced delta connected ~~system~~ load of $(2+j3)\,\Omega$ per phase is connected to a balanced three phase 440V supply. The phase current is 10A. Find the (i) total active power (ii) reactive power and (iii) apparent power in the circuit.

Sol:-

$V_L = 440V$

$I_{ph} = 10A \Rightarrow I_L = \sqrt{3} \times I_{ph} = \sqrt{3} \times 10 = 17.32A.$

$Z_{ph} = (2+j3)\,\Omega \Rightarrow |Z_{ph}| = \sqrt{2^2+3^2} = 3.6\,\Omega$

$\cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{2}{3.6} = 0.55$; $\sin\phi = 0.83$

(i) Active power, $P = \sqrt{3}\, V_L I_L \cos\phi$

$$= \sqrt{3} \times 440 \times 17.32 \times 0.55$$

$$P = 7259.78 \text{ Watts}$$

(ii) Reactive power, $\phi = \sqrt{3}\, V_L I_L \sin\phi$

$$= \sqrt{3} \times 440 \times 17.32 \times 0.83$$

$$= 10955.67 \text{ VAR}$$

(iii) Apparent power, $S = \sqrt{3}\, V_L I_L$

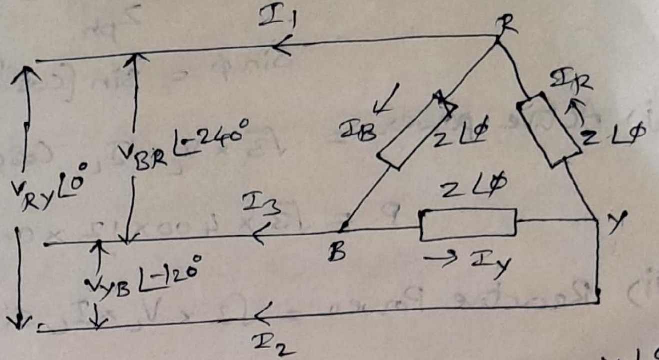
$$= \sqrt{3} \times 440 \times 17.32$$

$$= 13199.61 \text{ VA}$$

Three Phase Balanced circuits

(i) Delta Load

Following figure shows 3-phase, 3-wire bala[nced] system supplying power to a balanced 3-phase delta lo[ad]. The phase sequence is RYB.



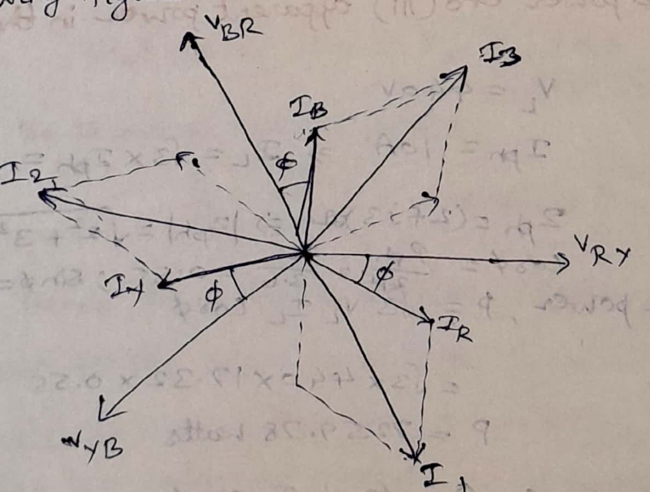
Let us assume the line voltage, $V_{RY} = V\angle 0^\circ$ ... reference phasor. Then the three source voltages are gi[ven]

$$V_{RY} = V\angle 0^\circ \ V$$

$$V_{YB} = V\angle{-120^\circ} \ V$$

$$V_{BR} = V\angle{-240^\circ} \ V$$

These voltages are represented by phasors a[s] in following figure.



Since the load is delta connected, the line [voltage] of the source is equal to the phase voltage of the load.

The current in phase RY, I_R will lag (or lea[d]) behind (or ahead of) the phase voltage V_{RY} by an angle 'ϕ' dictated by the nature of the load impedance.

If the load impedance is $Z\angle\phi$, the current flowing in the three load impedances are then

$$I_R = \frac{V_{RY}\angle 0^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$$

$$I_y = \frac{V_{YB}\angle -120^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -120^\circ - \phi^\circ$$

$$I_B = \frac{V_{BR}\angle -240^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -240^\circ - \phi^\circ$$

Current in line '1' is given by

$$I_1 = \sqrt{3}\cdot\left|\frac{V}{Z}\right|\angle(-\phi - 30^\circ), \text{ (or) } (I_R - I_B)$$

Similarly, $I_2 = \sqrt{3}\left|\frac{V}{Z}\right|\angle -\cancel{120^\circ} - \phi - 30^\circ$ (or) $(I_y - I_R)$

and $I_3 = \sqrt{3}\left|\frac{V}{Z}\right|\angle(-240^\circ - \phi - 30^\circ)$ (or) $(I_B - I_y)$

(3) A 3-phase, balanced delta connected load of $(4+j8)\,\Omega$ is connected across a 400V, 3-phase balanced supply. Determine the phase & line currents. Assume the phase sequence to be RYB. Also calculate the power drawn by the load.

Sol:- Taking $V_{RY} = V\angle 0^\circ$ as reference,

$V_{RY} = 400\angle 0^\circ$ V, $V_{YB} = 400\angle -120^\circ$ V and

$V_{BR} = 400\angle -240^\circ$ V

Z per phase $= (4+j8)\,\Omega = 8.94\angle 63.4^\circ\,\Omega$

phase currents are

$$I_R = \frac{400\angle 0^\circ}{8.94\angle 63.4^\circ} = 44.74\angle -63.4^\circ \text{ A}$$

$$I_y = \frac{400\angle -120^\circ}{8.94\angle 63.4^\circ} = 44.74\angle -183.4^\circ \text{ A}$$

$$I_B = \frac{400\angle -240^\circ}{8.94\angle 63.4^\circ} = 44.74\angle -303.4^\circ \text{ A}$$

The three line currents are

$$I_1 = \vec{I_R} - \vec{I_B} = 44.74\angle -63.4^\circ - 44.74\angle -303.4^\circ$$

$$= 20.03 - j40 - 24.62 + j37.35$$

$$I_1 = (-4.59 - j77.35)\text{A} = 77.49\angle 266.6^\circ \text{A}$$

$$I_1 = \sqrt{3}\times 44.74\angle -63.4^\circ = 77.49\angle -93.4^\circ \text{A}$$

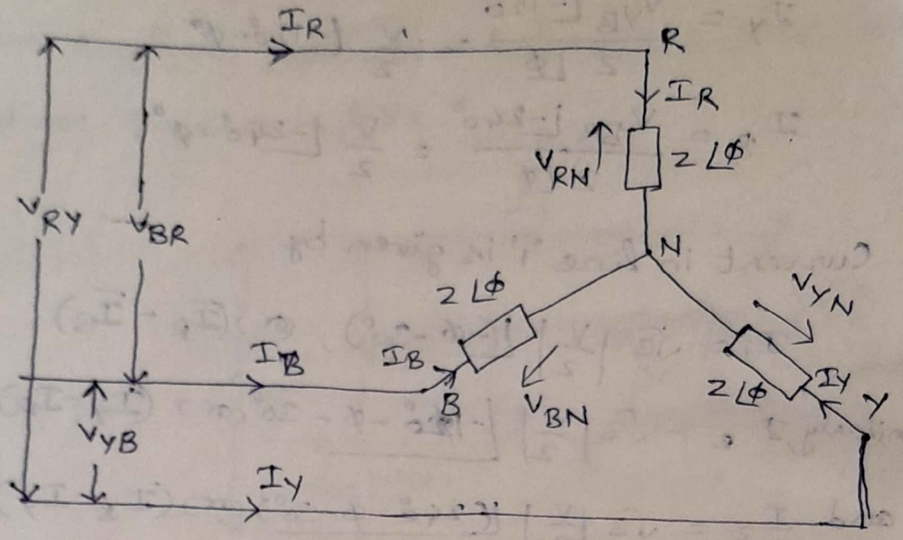
Similarly, $I_2 = \vec{I_y} - \vec{I_R} = 77.49\angle 146.6^\circ$ A

and $I_3 = \vec{I_B} - \vec{I_y} = 77.49\angle 26.6^\circ$ A

Power drawn by the load, $P = 3 V_{ph} I_{ph} \cos\phi$

$$P = \sqrt{3} V_L I_L \cos\phi = 24.039 \text{ kW}$$

Balanced 3-phase system - star connected Load



Above figure shows a 3-phase, 3 wire sys supplying power to a balanced 3-phase star connecte

In star connection, phase currents are sam the line currents. I_R, I_Y and I_B.

Assuming $V_{RN} = V\angle 0^\circ$ as the reference pha three phase voltages are

$$V_{RN} = V\angle 0^\circ, \quad V_{YN} = V\angle -120^\circ \text{ and}$$

$$V_{BN} = V\angle -240^\circ.$$

$$\therefore I_R = \frac{V_{RN}}{Z\angle\phi} = \frac{V\angle 0^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -\phi^\circ$$

$$I_Y = \frac{V_{YN}}{Z\angle\phi} = \frac{V\angle -120^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -120^\circ - \phi$$

$$\text{and} \quad I_B = \frac{V_{BN}}{Z\angle\phi} = \frac{V\angle -240^\circ}{Z\angle\phi} = \frac{V}{Z}\angle -240^\circ - \phi.$$

It makes no difference to the current flo in the load phases, as well as to the line currents, the sources have been connected in sta

Dt: 24/7/17

UNIT-II: TRANSIENT ANALYSIS

Introduction

A circuit having constant sources is said to be in "steady state" if the 'currents' and 'voltages' do not change with 'time'. It means that, the amplitude and frequency of a sinusoid or dc voltage or current never changes in a steady state circuit.

In a network containing energy storage elements, with change in excitation, the currents and voltages change from one state to other state. This behaviour of voltage or current when it is changed from one state to another is called the "transient state".

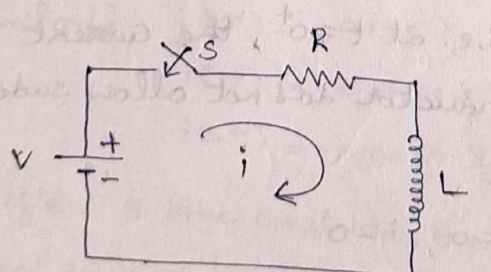
Note:- (i) Example for a steady state circuit is pure resistive circuit.

(ii) Example for a transient circuit is R-L, R-C (or) R-L-C network.

The time taken for the circuit to change from one steady state to another steady state is called "transient time".

DC Response of an R-L circuit

Consider a circuit consisting of a resistance and inductance as shown below.



The inductor in the circuit is initially uncharged and is in series with the resistor. When the switch 's' is closed, we can find the complete solution for the current.

then reference ...

Application of kirchoff's voltage law to the circ[illegible] results in the following differential equation.

$$V = Ri + L\frac{di}{dt} \quad \text{—— (1)}$$

(or) $$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \quad \text{—— (2)}$$

In the above equation, the current 'i' is t[illegible] solution to be found and 'v' is the applied constant vol[illegible] The voltage 'v' is applied to the circuit only when the [illegible] is closed.

The above equation is a linear different[illegible] equation of first order. Comparing it with a non-hom[illegible] differential equation,

$$\frac{dx}{dt} + Px = k \quad \text{—— (3)}$$

whose solution is

$$x = e^{-pt}\int k e^{pt}.dt + ce^{-pt} \quad \text{—— (4)}$$

where 'c' is an arbitrary constant. In a sim[illegible] way, we can write the current equation as

$$i = ce^{-(R/L)t} + e^{-(R/L)t}\int \frac{V}{L} e^{(R/L)t}.dt$$

$$\therefore \quad i = ce^{-(R/L)t} + \frac{V}{R} \quad \text{—— (5)}$$

To determine the value of 'c', we use the in[illegible] conditions. i.e. when the switch 's' is closed at t=0 after closing 's', i.e. at $t=0^+$, the current in the ind[illegible] is zero (since inductor does not allow sudden change [illegible] current).

$\therefore$ at $t=0$, $i=0$

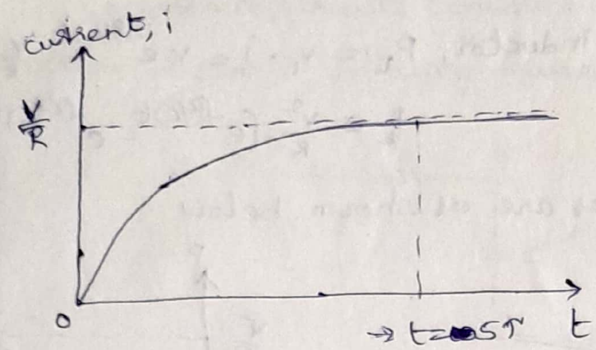
$$\Rightarrow \quad 0 = c.e^{0} + \frac{V}{R}$$

$$\Rightarrow \quad \boxed{c = -\frac{V}{R}}$$

Substituting 'c' in equation (5), we get

$$\boxed{i = \frac{V}{R}\left[1 - \exp\left(\left(-\frac{R}{L}\right)t\right)\right]} \quad \text{—— (6)}$$

Variation of current 'i' with respect to time 't' can be drawn as shown in following figure:



Equation (6) consists of two parts, the steady state part $\frac{V}{R}$, and the transient part $\frac{V}{R} e^{(-R/L)t}$.

When switch 'S' is closed, the response reaches a steady state value after a time interval as shown in above figure.

Here, the "transition period" is defined as the time taken for the current to reach its "final or steady value" from its "initial value".

In equation (6), the term $\frac{L}{R}$ is called the "time constant" and is denoted by 'τ'.

$$\therefore \tau = \frac{L}{R} \text{ sec.}$$

It is defined as the time at which the exponent of e is unity. i.e. $t = \tau$.

At one time constant, the transient term reaches 36.8% of its final value.

Similarly, $i(2\tau) = -0.135\,\frac{V}{R}$

$i(3\tau) = -0.0498\,\frac{V}{R}$

$i(5\tau) = -0.0067\,\frac{V}{R}$

∴ After 5 time constants, the transient part reaches more than 99% of its final value.

Here, voltage across resistor is

$$V_R = Ri = R \times \frac{V}{R}\left[1 - e^{-(\frac{R}{L})t}\right] = V\left[1 - e^{-(R/L)t}\right]$$

Similarly, $V_L = L\frac{di}{dt} = L \times \frac{V}{R} \times \frac{R}{L} e^{-(R/L)t} = V e^{-(R/L)t}$

then referred to ---

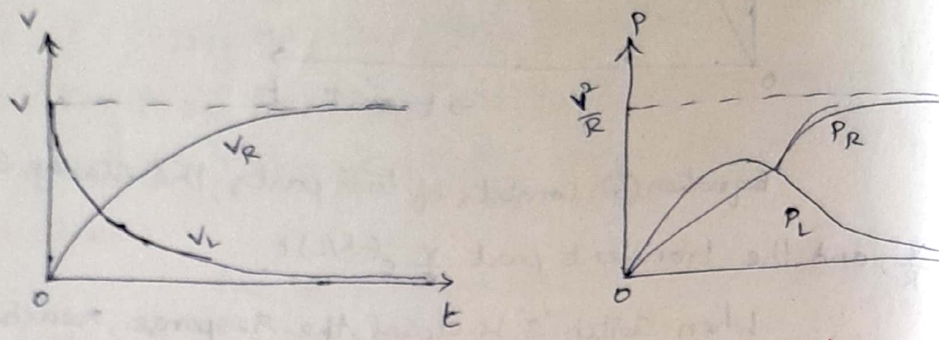
Power in the resistor, $P_R = V_R \cdot i = V[1-e^{-(R/L)t}] \frac{V}{R}[1-$

$$P_R = \frac{V^2}{R}(1-e^{-(R/L)t})^2$$

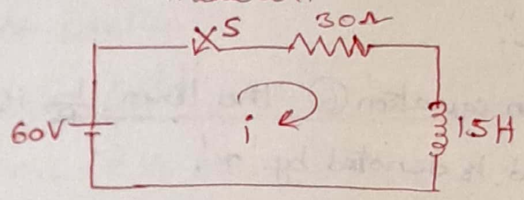
Power in the inductor, $P_L = V_L \cdot i = Ve^{-(R/L)t} \cdot \frac{V}{R}[1-e^{-(R}$

$$P_L = \frac{V^2}{R} \cdot [e^{-(R/L)t} - e^{-(2\frac{R}{L})t}]$$

The responses are as shown below.



1. A Series RL circuit with $R = 30\Omega$ and $L = 15H$ has a voltage $V = 60V$ applied at $t = 0$ as shown in following fi[g]. Determine the current i, the voltage across resisto[r], voltage across the inductor.



Sol:- By applying kirchoff's voltage law, we get

$$15\frac{di}{dt} + 30i = 60$$

$$\therefore \frac{di}{dt} + 2i = 4$$

The general solution for a linear differenti[al] equation is

$$i = ce^{-pt} + e^{-pt}\int ke^{pt}\,dt$$

where $p = 2$, $k = 4$

$$\therefore i = ce^{-2t} + e^{-2t}\int 4e^{2t}\,dt = ce^{-2t} + 2$$

At $t = 0$, the switch 's' is closed.

Since the inductor never allows sudden chang[e in] currents. At $t = 0^+$, the current in the circuit is ze[ro]

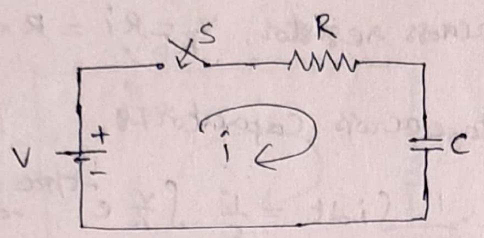
$\therefore$ at $t = 0^+$, $i = 0$

$\therefore$ $0 = c + 2 \Rightarrow c = -2$ $\therefore i = 2(1-e^{-2t})A$

$V_R = iR = 2(1-e^{-2t}) \times 30 = 60(1-e^{-2t})V$; $V_L = L\frac{di}{dt} = 15 \times \frac{d}{dt}2(1-e$

DC Response of An R-C Circuit

Consider a circuit consisting of resistance and capacitance as shown in following figure.



The capacitor in the circuit is initially uncharged; and is in series with a resistor.

When the switch 's' is closed at $t=0$, we can determine the complete solution for the current.

Applying KVL for the circuit, we get

$$V = Ri + \frac{1}{C}\int i\,dt \quad \text{—} (1)$$

Differentiating above equation w.r.t. 't', we get

$$0 = R\frac{di}{dt} + \frac{i}{C} \quad \text{—} (2)$$

$$\text{or } \frac{di}{dt} + \frac{1}{RC}\cdot i = 0 \quad \text{—} (3)$$

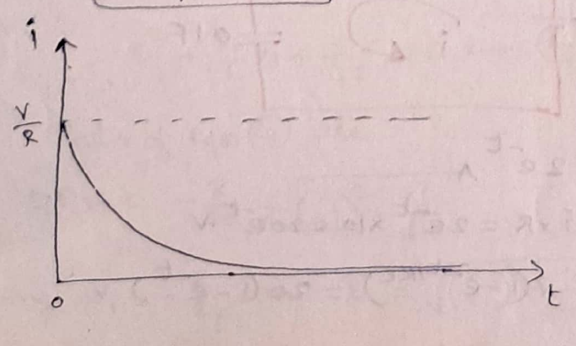
Equation (3) is a linear differential equation with only complementary function. The particular solution is zero. The solution for this type of differential equation is

$$i = c\,e^{-t/RC} \quad \text{—} (4)$$

To obtain 'c', substitute initial condition. i.e. at $t=0$, $i = \frac{V}{R}$

$$\therefore \frac{V}{R} = c\,e^{-0} \Rightarrow c = \frac{V}{R}$$

$$\Rightarrow \boxed{i = \frac{V}{R}e^{-t/RC}} \quad \text{—} (5)$$



then referred to ...

In the solution, 'RC' is the time constant deno

where, $\tau = RC$. After 5τ, the curve reaches 99% of i

value i.e. nearly zero.

voltage across resistor, $V_R = Ri = R \times \frac{V}{R} e^{-t/RC}$ -

Similarly, voltage across capacitor is

$$V_c = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{V}{R} e^{-t/RC}\,dt$$

$$= \frac{V}{RC}(-RC) \cdot e^{-t/RC} + c = -Ve^{-t/RC} + c$$

At $t=0$, voltage across capacitor is zero.

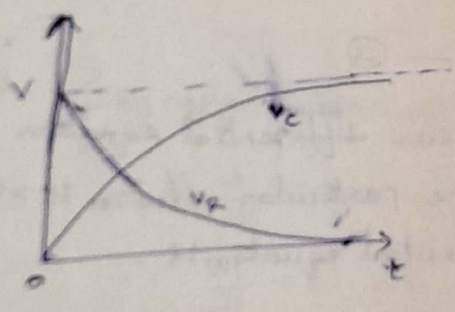
i.e. $c = V$

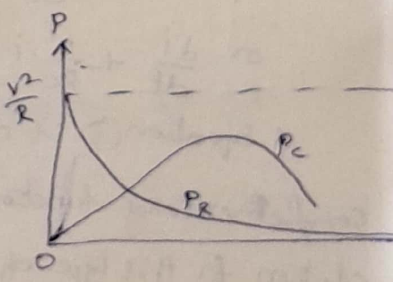
$$\Rightarrow V_c = V(1 - e^{-t/RC}) \quad -(6)$$

Power in the resistor, $P_R = V_R \cdot i = Ve^{-t/RC} \cdot \frac{V}{R} e^{-t/RC} =$

Power in the capacitor, $P_c = V_c \cdot i = V(1 - e^{-t/RC}) \cdot \frac{V}{R} e^{-t}$

$$P_c = \frac{V^2}{R}(e^{-t/RC} - e^{-2t/RC}) \quad -(8)$$



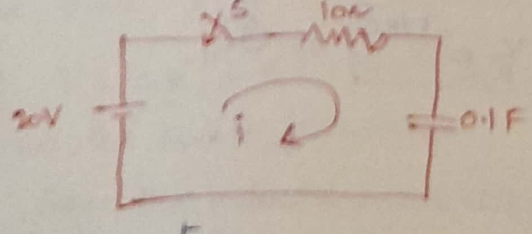


(-) A series RC circuit consists of resistor of 10Ω an

capacitor of 0.1 F as shown below. A constant voltage of 2

applied to the circuit at $t=0$. Obtain the current equ

determine the voltages across resistor and the capacitor



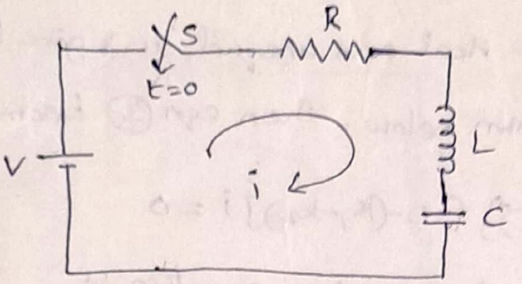
Ans:- $i = 2e^{-t}$ A

$$V_R = i \times R = 2e^{-t} \times 10 = 20e^{-t}\ V$$

$$V_c = V(1 - e^{-t/RC}) = 20(1 - e^{-t})\ V$$

DC Response of an R-L-C Circuit

Consider a circuit consisting of resistance, inductance and capacitance as shown below.



The capacitor and inductor are initially uncharged, and are in series with a resistor. When switch 'S' is closed at t=0, we can determine the complete solution for the current.

Application of KVL to the circuit results in the following differential equation.

$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{--- (1)}$$

Differentiating the above equation w.r.t. 't', we get

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}\cdot i \quad \text{--- (2)}$$

(or)

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \quad \text{--- (3)}$$

Above equation is a second order linear differential equation, with only complementary function. The particular solution is zero.

General characteristic equation for the above differential equation is

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0 \quad \text{--- (4)}$$

Roots of eqn (4) are

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Assuming $k_1 = -\frac{R}{2L}$ and $k_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$D_1 = k_1 + k_2$ and $D_2 = k_1 - k_2$

Here, k_2 may be positive, negative or zero.

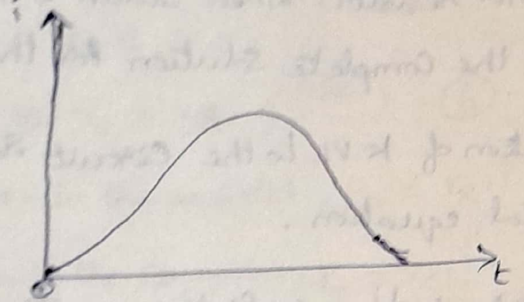
k_2 is positive when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The roots are real and unequal, and give the over[damped] response as shown below. Then eqn (2) becomes

$$[D-(k_1+k_2)][D-(k_1-k_2)]i = 0$$

The solution for the above equation is

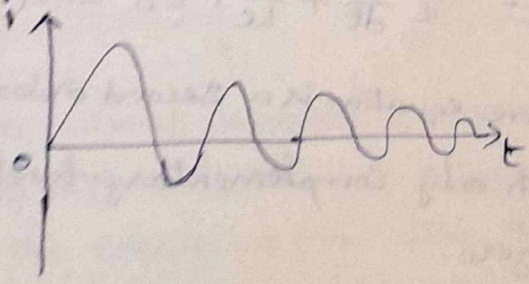
$$i = c_1 e^{-(k_1+k_2)t} + c_2 e^{-(k_1-k_2)t}$$



The current curve for the overdamped case is s[hown]

k_2 is negative, when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

The roots are complex conjugate & give unde[rdamped] response as shown below-

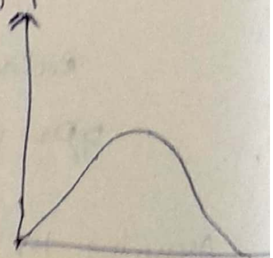


Then eqn (2) becomes

$$[D-(k_1+jk_2)][D-(k_1-jk_2)]i = 0$$

The solution for the underdamped case is ~~shown above~~

$$i = e^{k_1 t}[C_1 \cos k_2 t + C_2 \sin k_2 t]$$



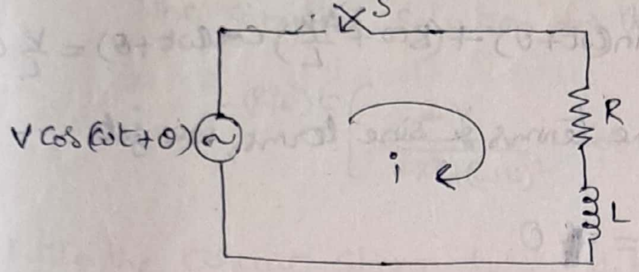
$k_2 = 0$ when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

The roots are equal and give the critically damped response. Now eqn (5) becomes, $(D-k_1)(D-k_1)i = 0$

Solution for above equation is $i = e^{k_1 t}(C_1 + C_2 t)$

Sinusoidal Response of R-L circuit

Consider a circuit consisting of resistance and inductance as shown below.



The switch 's' is closed at t=0. At t=0, a sinusoidal voltage $V\cos(\omega t+\theta)$ is applied to the series R-L circuit, where 'V' is the amplitude of the wave and 'θ' is the phase angle.

Application of kirchoff's voltage law to the circuit results in the following differential equation.

$$V\cos(\omega t+\theta) = Ri + L\frac{di}{dt} \quad \text{— (1)}$$

$$\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}\cos(\omega t+\theta)$$

The corresponding characteristic equation is

$$\left(D+\frac{R}{L}\right)i = \frac{V}{L}\cos(\omega t+\theta) \quad \text{— (2)}$$

For the above equation, the solution consists of two parts; "complementary function" and "particular integral."

The complementary function of the solution is

$$i_c = c\,e^{-t(R/L)} \quad \longleftarrow (3)$$

The particular solution can be obtained by using undetermined coefficients.

Assume $i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$ — (4)

Then, $i_p' = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)$ — (5)

Substituting equations (4) & (5) in equation (2);

$$\left[-A\omega \sin(\omega t+\theta) + B\omega \cos(\omega t+\theta)\right] + \frac{R}{L}\left[A\cos(\omega t+\theta) \ldots\right.$$
$$= \frac{V}{L}\cos(\omega t+\theta)$$

$$\Rightarrow \left(-A\omega + \frac{BR}{L}\right)\sin(\omega t+\theta) + \left(B\omega + \frac{AR}{L}\right)\cos(\omega t+\theta) = \frac{V}{L}$$

Comparing cosine terms & sine terms, we get

$$-A\omega + \frac{BR}{L} = 0$$

and $$B\omega + \frac{AR}{L} = \frac{V}{L}$$

Solving above equations for 'A' & 'B';

$$A = V \cdot \frac{R}{R^2+(\omega L)^2} \quad \text{and} \quad B = V \cdot \frac{\omega L}{R^2+(\omega L)^2}$$

Substituting the values of A and B in equation (4),

$$i_p = V \frac{R}{R^2+(\omega L)^2} \cdot \cos(\omega t+\theta) + V \frac{\omega L}{R^2+(\omega L)^2} \sin(\omega \ldots$$

putting $$M\cos\phi = \frac{V.R}{R^2+(\omega L)^2}$$

and $$M\sin\phi = \frac{V.\omega L}{R^2+(\omega L)^2}$$

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi = \frac{\omega L}{R}$$

Squaring both equations & adding, we get

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{V^2}{R^2+(\omega L)^2}$$

$$\Rightarrow M = \frac{V}{\sqrt{R^2+(\omega L)^2}}$$

∴ The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2+(\omega L)^2}} \cos\left[\omega t+\theta - \tan^{-1}\frac{\omega L}{R}\right] \quad \text{—— (7)}$$

The complete solution for the current $i = i_c + i_p$

$$\therefore i = c e^{-t(R/L)} + \frac{V}{\sqrt{R^2+(\omega L)^2}} \cos\left[\omega t+\theta - \tan^{-1}\frac{\omega L}{R}\right]$$

$\because$ Inductor does not allow sudden changes in currents,

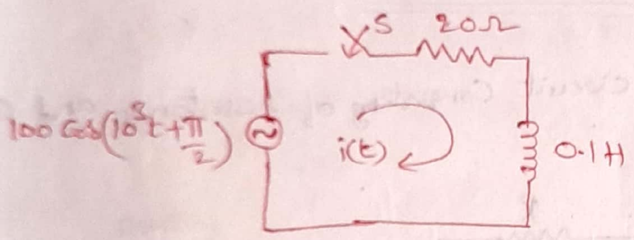
at $t=0$, $i=0$

$$\therefore \quad c = -\frac{V}{\sqrt{R^2+(\omega L)^2}} \cdot \cos\left[\theta - \tan^{-1}\frac{\omega L}{R}\right]$$

The complete solution for the current is

$$i = e^{-(R/L)t}\left[\frac{-V}{\sqrt{R^2+(\omega L)^2}} \cdot \cos\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)\right] + \frac{V}{\sqrt{R^2+(\omega L)^2}} \cos\left(\omega t+\theta - \tan^{-1}\frac{\omega L}{R}\right)$$

1. In the circuit shown below, determine the complete solution for the current, when switch 'S' is closed at $t=0$. Applied voltage is $v(t) = 100\cos(10^3 t + \frac{\pi}{2})$. Resistance $R = 20\Omega$ and inductance $L = 0.1H$.



Sol:- By applying KVL to the circuit, we have

$$20i + 0.1\frac{di}{dt} = 100\cos(10^3 t + \pi/2)$$

$$\frac{di}{dt} + 200i = 1000\cos(10^3 t + \pi/2)$$

$$(D + 200)i = 1000\cos(1000t + \pi/2)$$

The complementary function $i_c = c\,e^{-200t}$

By assuming particular integral as

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$$

we get, $$i_p = \frac{V}{\sqrt{R^2+(\omega L)^2}} \cdot \cos\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right)$$

where, $\omega = 1000$ rad/sec, $V = 100V$

$\theta = \frac{\pi}{2}$, $L = 0.1H$, $R = 20\Omega$

Substituting these values in the above equation, we get

$$i_p = \frac{100}{\sqrt{(20)^2 + (1000\times0.1)^2}} \cos\left(1000t + \frac{\pi}{2} - \tan^{-1}\frac{100}{20}\right)$$

$$i_p = \frac{100}{101.9}\cos\left(1000t + \frac{\pi}{2} - 78.6^\circ\right)$$

$$= 0.98\cos\left(1000t + \frac{\pi}{2} - 78.6\right)$$

The complete solution is

$$i = ce^{-200t} + 0.98\cos\left(1000t + \frac{\pi}{2} - 78.6\right)$$

At t=0, the current flow through the circuit is zero

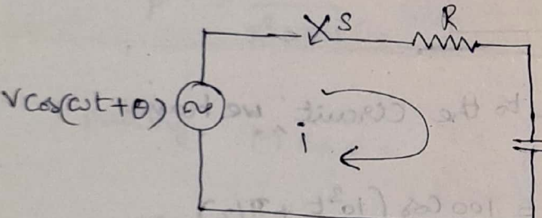
$$\therefore c = -0.98\cos\left(\frac{\pi}{2} - 78.6^\circ\right)$$

∴ The complete solution is

$$i = \left[-0.98\cos\left(\frac{\pi}{2} - 78.6^\circ\right)e^{-200t} + 0.98\cos\left(1000t + \frac{\pi}{2}\right.\right.$$

Sinusoidal Response for R-C Circuit

Consider a circuit consisting of resistance and c
as shown below.



The switch 's' is closed at t=0, so that a voltage $V\cos(\omega t+\theta)$ is applied to the R-C circuit, where 'V' amplitude of the wave and 'θ' is the phase angle.

Applying KVL to the circuit results in the follo differential equation

$$V\cos(\omega t+\theta) = Ri + \frac{1}{C}\int i\,dt \quad \text{— (1)}$$

$$R\frac{di}{dt} + \frac{i}{C} = -V\omega\sin(\omega t+\theta)$$

$$\left(D + \frac{1}{RC}\right)i = -\frac{V\omega}{R}\sin(\omega t+\theta) \quad \text{— (2)}$$

The complementary function $i_c = ce^{-t/RC}$ — (3)

The particular solution can be obtained by using undetermin coefficients

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta) \quad \text{— (4)}$$

$$i_p' = -A\omega\sin(\omega t+\theta) + B\omega$$

Substituting equations 4 & 5 in (

$$[-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta$$

$$+\frac{1}{RC}[A\cos(\omega t+\theta) + B\sin$$

Comparing sine & cosine term

$$-A\omega + \frac{B}{RC} = -\frac{V\omega}{R}$$

$$B\omega + \frac{A}{RC} = 0$$

From which,

$$A = \frac{VR}{R^2 + \left(\frac{1}{\omega C}\right)^2}, \quad B =$$

Substituting the values of

$$i_p = \frac{VR}{R^2 + \left(\frac{1}{\omega C}\right)^2}\cdot\cos(\omega t+\theta)$$

putting

$$M\cos\phi = \frac{VR}{[R^2 +}$$

$$M\sin\phi = \frac{V}{\omega C[R^2}$$

To find M and φ, we

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi =$$

Squaring both equation

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{}{R^2}$$

$$\therefore M = \frac{V}{\sqrt{R^2 + (}}$$

∴ The particular curr

$$i_p = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\ C$$

Scanned with OKEN Scanner

$$i_p' = -A\omega \sin(\omega t+\theta) + B\omega \cos(\omega t+\theta) \quad — (5)$$

Substituting equations 4 & 5 in (2), we get

$$[-A\omega \sin(\omega t+\theta) + B\omega \cos(\omega t+\theta)]$$

$$+\frac{1}{RC}[A\cos(\omega t+\theta) + B\sin(\omega t+\theta)] = -\frac{V\omega}{R}\sin(\omega t+\theta)$$

Comparing sine & Cosine terms on both sides,

$$-A\omega + \frac{B}{RC} = -\frac{V\omega}{R}$$

$$B\omega + \frac{A}{RC} = 0$$

From which,

$$A = \frac{VR}{R^2+\left(\frac{1}{\omega C}\right)^2}, \quad B = \frac{-V}{\omega C\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}$$

Substituting the values of A and B in eqn (6), we have

$$i_p = \frac{VR}{R^2+\left(\frac{1}{\omega C}\right)^2}\cos(\omega t+\theta) + \frac{-V}{\omega C\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}\sin(\omega t+\theta)$$

putting

$$M\cos\phi = \frac{VR}{\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}$$

$$M\sin\phi = \frac{V}{\omega C\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}$$

To find M and ϕ, we divide one equation by the other,

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi = \frac{1}{\omega CR} \Rightarrow \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Squaring both equations & adding, we get

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{V^2}{R^2+\left(\frac{1}{\omega C}\right)^2}$$

$$\therefore \quad M = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

$\therefore$ The particular current becomes,

$$i_p = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}\cos\left[\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right]$$

The complete solution for the current $i = i_c + i_p$

$$\therefore i = ce^{-(t/RC)} + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left[\omega t + \theta + \tan^{-1}\frac{1}{\omega CR}\right.$$

Since the capacitor does not allow sudden chan[ges in] voltages at $t=0$, $i = \frac{V}{R}\cos\theta$

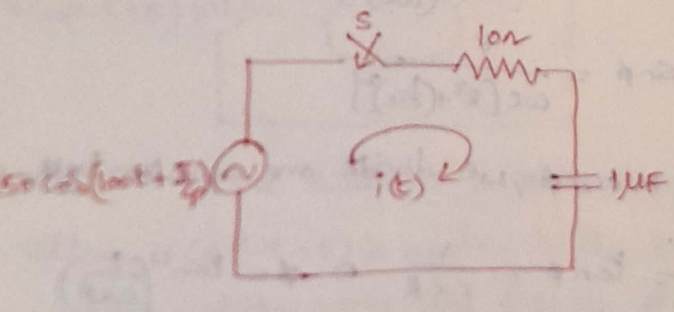
$$\therefore \frac{V}{R}\cos\theta = c + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left[\theta + \tan^{-1}\frac{1}{\omega CR}\right]$$

The complete solution for the current is

$$i = e^{-(t/RC)}\left[\frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cdot \cos\left(\theta + \tan^{-1}\frac{1}{\omega CR}\right)\right]$$

$$+ \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cdot \cos\left(\omega t + \theta + \tan^{-1}\frac{1}{\omega CR}\right)$$

(2) In the circuit shown below, determine the complete so[lution] for the current when switch 'S' is closed at $t=0$. Applied [voltage] is $v(t) = 50\cos\left(10^2 t + \frac{\pi}{4}\right)$. Resistance $R = 10\Omega$ and c[apacit]ance, $C = 1\mu F$.



sol:- By applying KVL to the circuit, we have

$$10i + \frac{1}{1\times10^{-6}}\int i\,dt = 50\cos\left(100t + \frac{\pi}{4}\right)$$

$$10\frac{di}{dt} + \frac{i}{1\times10^{-6}} = -5\times10^3 \sin\left(100t + \frac{\pi}{4}\right)$$

$$\left(D + \frac{1}{10^{-5}}\right)i = -500\sin\left(100t + \frac{\pi}{4}\right)$$

The complementary solution is $i_c = ce^{-t/10^{-5}}$.

By assuming particular integral as

$$i_p = A\cos(\omega t + \theta) + B\sin(\omega t + \theta),$$

We get

$$i_p = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cdot \cos\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

where $\omega = 100\,rad/sec$, $\theta = \frac{\pi}{4}$

$C = 1\mu F$ $R = 10\Omega$

Substituting the values in the above equation, we have

$$i_p = \frac{50}{\sqrt{(10)^2+\left(\frac{1}{100\times10^{-6}}\right)^2}} \cos\left(\omega t+\frac{\pi}{4}+\tan^{-1}\frac{1}{100\times10^{-6}\times10}\right)$$

$$i_p = 4.99\times10^{-3}\cos\left(100t+\frac{\pi}{4}+89.94^\circ\right)$$

At $t=0$, the current flowing through the circuit is

$$\frac{V}{R}\cos\theta = \frac{50}{10}\cos\frac{\pi}{4} = 3.53A$$

$$i = \frac{V}{R}\cos\theta = 3.53A$$

$$\therefore i = Ce^{-t/10^{-5}} + 4.99\times10^{-3}\cos\left(100t+\frac{\pi}{4}+89.94^\circ\right)$$

At $t=0$, $C = 3.53 - 4.99\times10^{-3}\cos\left(\frac{\pi}{4}+89.94^\circ\right)$

Hence the complete solution is

$$i = \left[3.53 - 4.99\times10^{-3}\cos\left(\frac{\pi}{4}+89.94^\circ\right)\right]e^{-(t/10^{-5})}$$
$$+ 4.99\times10^{-3}\cos\left(100t+\frac{\pi}{4}+89.94^\circ\right)$$

UNIT-III

NETWORK TOPOLOGY

Introduction

The purpose of network analysis is to find voltage across and current through all the elements. When the network is complicated, and has a large number of nodes and closed paths, network analysis can be done conveniently by using "Network Topology".

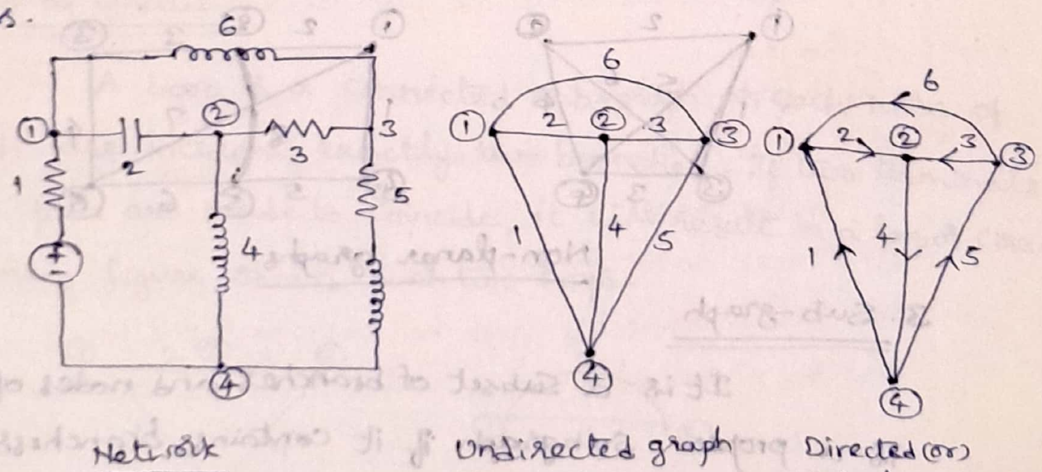
Graph of a Network

A "linear graph" is a collection of nodes and branches. The nodes are joined by branches.

All the voltage and current sources are replaced by lines which correspond to the network elements of each branch & their internal impedances.

The voltage sources are replaced by short circuits whereas current sources are replaced by open circuits. Nodes and branches are numbered.

Following figure shows a network & its associated graphs.



Network Undirected graph Directed (or) Oriented graph

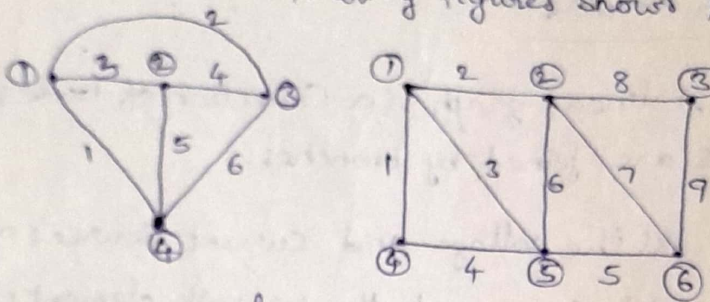
Each branch of a graph may be given an orientation or a direction with the help of an arrow head which represents the assigned direction for current. Such a graph is then referred to as a "directed or oriented graph."

Branches whose ends fall on a node are said be incident at that node. In the above graph, branches 2, 3 and 4 are incident at node 2. So, degree of node 2 is 3.

Definitions associated with a graph

1. Planar Graph

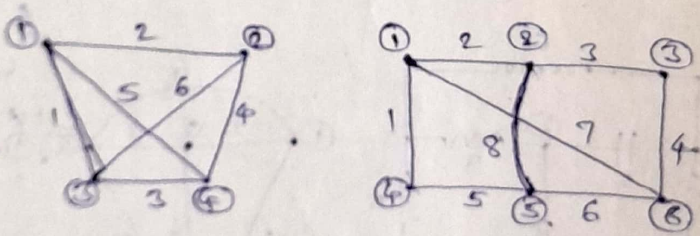
A graph on a two-dimensional plane is said to be planar if two branches donot intersect or cross at a point which is other than a node. Following figures shows planar graphs.



planar graphs

2. Non-planar Graph

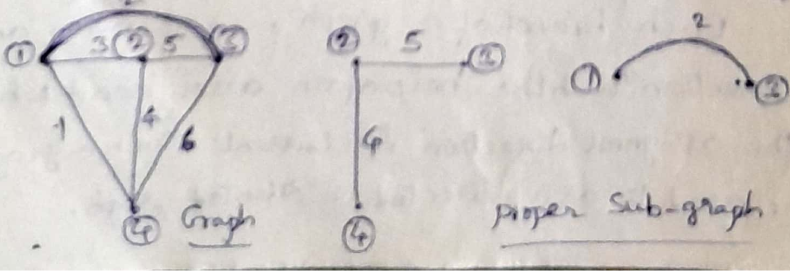
A graph drawn on a two-dimensional plane is said to be non-planar if there is intersection of two or more branches at another point which is not a node. Following figure shows non-planar graphs.



Non-planar graphs

3. Sub-graph

It is a subset of branches and nodes of a graph. It is a proper subgraph if it contains branches & nodes less than those on a graph. Following figure shows a graph & its proper subgraph



Graph

Proper Sub-graph

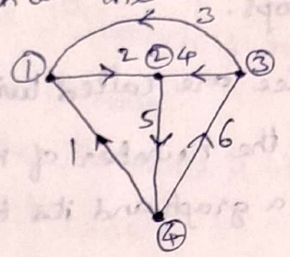
4. Path

It is an improper subgraph having the following properties:

(i) At two of its nodes called terminal nodes, there is incident only one branch of sub-graph.

(ii) At all remaining nodes called internal nodes, there are incident two branches of a graph.

In the ~~above~~ graph shown below, branches 2, 5 & 6 together with all the four nodes, constitute a path.



5. Connected Graph

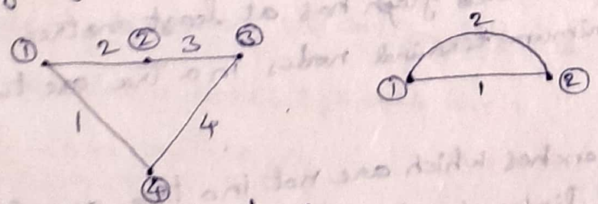
A graph is said to be connected if there exists a path between any pair of nodes. otherwise, the graph is disconnected.

6. Rank of a graph

If there are 'n' nodes in a graph, the rank of the graph is (n-1).

7. Loop or Circuit

A Loop is a connected sub-graph at each node of which are incident exactly two branches. If two terminals of a path are made to coincide, it will result in a loop or circuit. Following figure shows such two loops.



Loops

Loops of a graph have the following properties:

(i) There are at least two branches in a loop.

(ii) There are exactly two paths between any pair of nodes in a circuit.

iii) The maximum number of possible branches is equal to the number of nodes.

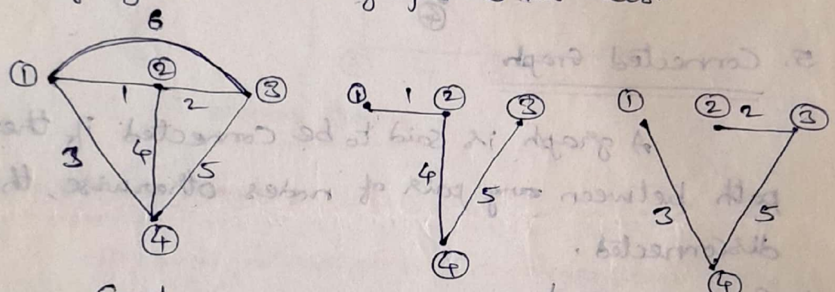
8. Tree

A tree is a set of branches with every node connected to every other node in such a way that, removal of any branch destroys this property.

(or)

A tree is defined as a connected subgraph of a connected graph containing all the nodes of the graph but not containing any loops.

Branches of a tree are called twigs. A tree contains (n-1) twigs where 'n' is the number of nodes in the graph. Following figure shows a graph and its trees.



Graph Twigs: 1,4,5 Twigs: 2,3,5

Trees have the following properties:

(i) There exists only one path between any pair of nodes in a tree.
(ii) A tree contains all nodes of the graph.
(iii) If 'n' is the no. of nodes of the graph, there are (n-1) branches in the tree.
(iv) Trees do not contain any loops.
(v) Every connected graph has at least one tree.
(vi) The minimum terminal nodes in a tree are two.

9. Co-tree

Branches which are not in a tree are called links or chords. All links of a tree together constitute the compliment of the corresponding tree & is called the co-tree.

A cotree contains b-(n-1) links where 'b' is the no. of branches of the graph. In the above figures, the links are (2,3,6) & 1,4,6 respectively.

2017

Incidence matrix (A)

The incidence c
graph is shown by the el
dimensions of the matrix A
and 'b' is no. of branches.

In matrix 'A' w
a_{ij} in the i^{th} row and j

$a_{ij} = 1$, if the j^{th} bra
from i^{th} nod

$a_{ij} = -1$, if the j^{th} b
the i^{th} nod

$a_{ij} = 0$, if the j^{th} b

For the following

Nodes \ Branches	a	
1	1	
2	-1	
3	0	
4	0	

A =

The entries
branches a, c and
oriented away from
+1. Other entries
connected to node 1,
matrix for the remo

Properties of Incide

1. Each column rep
entries +1 and –
2. The unit entries
a node. Their n

Incidence matrix (A)

The incidence of elements to nodes in a connected graph is shown by the element node incidence matrix (A). The dimensions of the matrix 'A' is $n \times b$ where 'n' is the no. of nodes and 'b' is no. of branches.

In matrix 'A' with 'n' rows and 'b' columns, an entry a_{ij} in the i^{th} row and j^{th} column has the following values.

$a_{ij} = 1$, if the j^{th} branch is incident to and oriented away from i^{th} node.

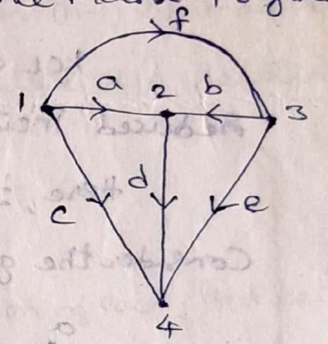
$a_{ij} = -1$, if the j^{th} branch is incident to and oriented towards the i^{th} node.

$a_{ij} = 0$, if the j^{th} branch is not incident to the i^{th} node.

For the following graph, the incidence matrix is given by

A =

Nodes \ Branches	a	b	c	d	e	f
1	1	0	1	0	0	1
2	-1	-1	0	1	0	0
3	0	1	0	0	1	-1
4	0	0	-1	-1	-1	0



The entries in the first row indicates that three branches a, c and f are incident to node '1' and they are oriented away from node '1' & hence entries a_{11}, a_{13} & a_{16} are +1. Other entries in the first row are zero as they are not connected to node 1. Likewise, we can complete the incidence matrix for the remaining nodes 2, 3 and 4.

Properties of Incidence matrix 'A'

1. Each column representing a branch contains two non-zero entries +1 and -1. The rest being zero.
2. The unit entries in a row identify the branches incident at a node. Their number is called the degree of the node.

3. A degree of 1 for a row means that there is one bra[nch]
incident at the node. This is commonly possible in a tree.

4. ~~If the degree of '1' for a row means that there is one~~
~~incident at the node,~~

4. If the degree of a node is two, then it means that
branches are incident at the node or these are in s[eries]

5. Columns of 'A' with unit entries in two identical rows,
to two branches with same end nodes and hence they a[re]
parallel.

6. Given the incidence matrix 'A', the corresponding graph c[an be]
easily constructed since 'A' is a complete mathematical
of the graph.

7. If one row of 'A' is deleted, the resulting $(n-1) \times b$ m[atrix]
is called the reduced incidence matrix 'A_1'.

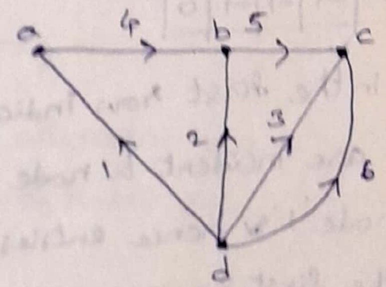
12/9/17

Incidence Matrix And KCL

KCL of a graph can be expressed in terms of
reduced incidence matrix is $A_1 I = 0$

Here, 'I' represents branch current vectors I_1, I_2

Consider the graph as shown below.



This graph consists of four nodes 'a', 'b', 'c', and 'd'
node 'd' is taken as the reference node.

The positive reference direction of the branch c[urrents]
corresponds to the orientation of the graph branches.

Let the branch currents be $i_1, i_2, i_3 \ldots i_6$. A[pplying]
KCL to nodes 'a', 'b' and 'c'.

$$-i_1 + i_4 = 0$$
$$-i_2 - i_4 + i_5 = 0$$
$$-i_3 - i_5 - i_6 = 0$$

These equations can be written in the matrix form as follows:

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{--- (1)}$$

$$A_1 I_b = 0 \quad \text{--- (2)}$$

Here, I_b represents column matrix or a vector of branch currents.

$$I_b = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_6 \end{bmatrix}$$

A_1 is the reduced incidence matrix of a graph which contains 'n' nodes and 'b' branches. It is $(n-1) \times b$ matrix obtained from the complete incidence matrix of 'A' deleting one of its rows.

Link Currents: Tie-set Matrix

For a given tree of a graph, addition of each link between any two nodes forms a loop called the fundamental loop.

In a loop, there, exists a closed path and a circulating current, which is called the link current. The current in any branch of a graph can be found by using link currents.

The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link. This loop is also called "f-loop" or a "tie-set".

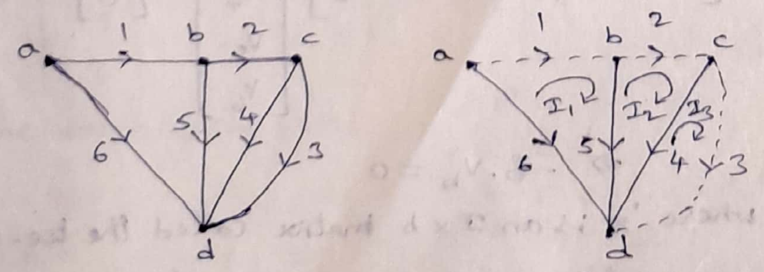


fig.(a) fig.(b)

Consider a connected graph as shown in fig (a). It has four nodes & six branches. One of its trees is arbitrarily chosen and is shown in fig (b).

The twigs of this tree are branches 4, 5 and 6. The links corresponding to this tree are branches 1, 2 and 3. Every link defines a fundamental loop of the network.

By adding links 1, 2, 3, three fundamental loops (or) f-loops with link currents I_1, I_2 and I_3 can be formed as shown in fig (b). Orientation of current is same as that of a link.

Tie-set matrix

Kirchoff's voltage law can be applied to the f-loops to get a set of linearly independent equations.

There are three fundamental loops I_1, I_2 and I_3 corresponding to the link branches 1, 2, & 3 respectively.

If $V_1, V_2 \dots V_6$ are the branch voltages, the KVL equations for the three f-loops can be written as

$$\left.\begin{aligned} V_1 + V_5 - V_6 &= 0 \\ V_2 + V_4 - V_5 &= 0 \\ V_3 - V_4 &= 0 \end{aligned}\right\} - (3)$$

In order to apply KVL to each fundamental loop, we take the reference direction of the loop which coincides with the reference direction of the link defining the loop.

Above equation can be written in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - (4)$$

$$\Rightarrow \; B \cdot V_b = 0$$

where 'B' is an $l \times b$ matrix called the tie-set matrix (or) fundamental loop matrix and V_b is a column vector of branch voltages.

19/11

Tie-set Matrix and Branch currents

It is possible to express branch currents as a linear combination of link current using matrix B.

If I_B and I_L represents the branch & loop current matrices respectively & 'B' is the tie set matrix, then

$$[I_b] = [B^T][I_L] \quad \text{—— } (6)$$

where $[B^T]$ is the transpose of the matrix $[B]$. Eqn (6) is known as link current transformation equation.

$$[B] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$[B^T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix};$$

The branch current vector $[I_b] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}$

$\Rightarrow$ The loop current vector $[I_L] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$

$\therefore$ The link current transformation equation is given by

$$[I_b] = [B^T][I_L]$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The branch currents are

$i_1 = I_1$

$i_2 = I_2$

$i_3 = I_3$

$i_4 = I_2 - I_3$

$i_5 = I_1 - I_2$

$i_6 = -I_1$

14-9-17

Cut set and Tree Branch voltages

A cut set is a minimal set of branches of a connected graph such that the removal of these branches causes the graph to be cut into exactly two parts.

The important property of a cutset is that by restoring any one of the branches of the cut set the graph should become connected.

A cutset consists of one and only one branch of the network tree, together with any links which must be cut to divide the network into two parts.

Consider the graph shown below. fig (a).

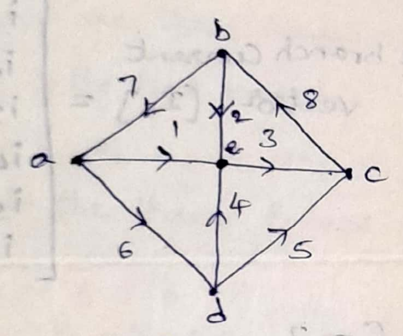


fig: (a)

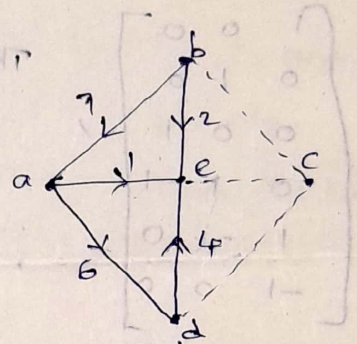


Fig (b)

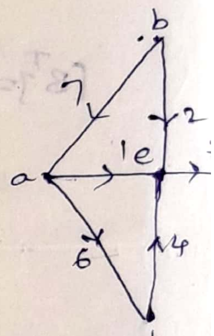


Fig (c)

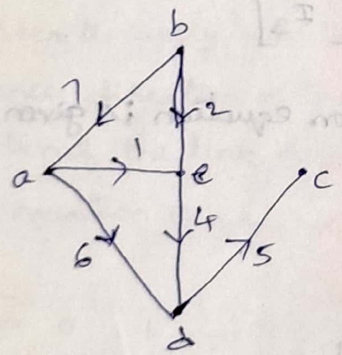


Fig: (d)

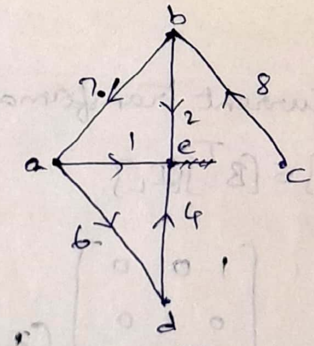


Fig: (e)

Figures (a), (b), (c), (d) & (e) are cut-sets for the g

Cut set orientation :

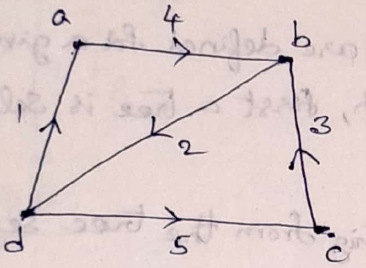
A cut set is oriented by arbitrarily selecting the
The orientation of some branches of the cut-set may coin
with the orientation of the cut-set while some branches of
cutset may not coincide.

Cut-set Matrix and KCL for Cut-sets

KCL is also applicable to a cut-set of a network. For any lumped electrical network, the algebraic sum of all the cut-set branch currents is equal to zero.

While writing the KCL equation for a cut-set, we assign positive sign for the current in a branch if its direction coincides with the orientation of the cut-set and a negative sign to the current in a branch whose direction is opposite to the orientation of the cutset.

Consider the graph shown below.



This graph has five branches & four nodes. The following six cutsets are possible.

Cut-set $C_1 : \{1, 4\}$; Cut-set $C_2 : \{4, 2, 3\}$

Cut-set $C_3 : \{3, 5\}$; Cut-set $C_4 : \{1, 2, 5\}$

Cut set $C_5 : \{4, 2, 5\}$; Cut-set $C_6 : \{1, 2, 3\}$

Applying KCL for each of the cut-set, we obtain the following equations. Let $i_1, i_2, \ldots i_6$ be the branch currents.

$$\left.\begin{aligned} &C_1 : i_1 - i_4 = 0 \;;\; C_2 : -i_2 + i_3 + i_4 = 0 \;;\; C_3 : -i_3 + i_5 = 0 \\ &C_4 : i_1 - i_2 + i_5 = 0 \;;\; C_5 : -i_2 + i_4 + i_5 = 0 \;;\; C_6 : i_1 - i_2 + i_3 = 0 \end{aligned}\right\} \quad (7)$$

These equations can be put into matrix form as

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(or)} \quad Q \cdot I_b = 0 \quad (8)$$

where 'Q' matrix is called augmented cut-set matrix of the graph or all cut-set matrix of the graph. The matrix I_b is the branch-current vector.

Fundamental cut-sets

Observing the set of equations (7) w.r.t. the graph, only first three equations are linearly independent, remaining equations can be obtained as a linear combination of the first three.

The concept of fundamental cutset (f-cutset) can be used to obtain a set of linearly independent equations in branch current variables.

The f-cutsets are defined for a given tree of the graph. From a connected graph, first a tree is selected, and then a twig is selected.

Removing this twig from the tree separates the tree into two parts. All the links which go from one part of the disconnected tree to the other, together with the twig of the selected tree will constitute a cut-set. This cut set is called a fundamental cut-set. (or) f-cutset of the graph.

Thus, a f-cut set of a graph w.r.t. a tree is a cut-set that is formed by one twig and a unique set of links.

For each branch of the tree, i.e. for each twig, there will be a f-cutset. So, for ~~each branch of the tree~~ a connected graph having 'n' nodes, there will be (n-1) twigs in a tree, the number of f-cutsets is also equal to (n-1).

Fundamental cut set matrix 'Q_f' is one in which each row represents a cut-set w.r.t. a given tree of the graph. The rows of Q_f correspond to the fundamental cut-sets and the columns correspond to the branches of the graph.

Tree Branch voltages and f-cutset matrix

From the cutset matrix, the branch voltages can be expressed in terms of tree branch voltages (twig voltages)

Since all the branches are connected to all the nodes in the graph, it is possible to trace a path from one node to any other node by traversing through the tree-branches.

Let us consider the following graph. It contains eight branches. Let the branch voltages be $V_1, V_2 \dots V_8$.

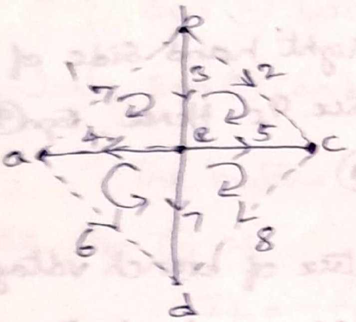


Fig:(a)

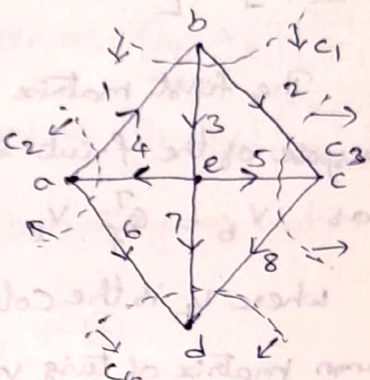


Fig:(b)

From the Cutset matrix, the branch voltages can be expressed in terms of tree branch voltages.

~~Since all tree branches~~

There are, four twigs, let the twigs voltages be V_{t3}, V_{t4}, V_{t5} and V_{t7} for twigs 3, 4, 5, 7 respectively.

We can express each branch voltage in terms of twig voltages as follows:

$$V_1 = -V_3 - V_4 = -V_{t3} - V_{t4}$$

$$V_2 = V_3 + V_5 = V_{t3} + V_{t5}$$

$$V_3 = V_{t3}$$

$$V_4 = V_{t4}$$

$$V_5 = V_{t5}$$

$$V_6 = V_7 - V_4 = V_{t7} - V_{t4}$$

$$V_7 = V_{t7}$$

$$V_8 = V_7 - V_5 = V_{t7} - V_{t5}$$

The above equations can be written [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Inter relation among the matrices

The three matrices which are concerned [illegible] incidence matrix (A), tie-set matrix (B_f) and f-cut set matrix (Q_f) are [illegible] interdependent [illegible]

[illegible] without actually drawing the graph.

(a) Inter relation between A & B_f

The columns of both the matrices A and B_f [illegible] to the branches of the graph. If the columns of these two matrices are arranged in the same order, then

$$A B_f^T = 0 \quad \text{--- (8)}$$

(ii) Inter-relation between Incidence Matrix 'A' & f-cut set matrix 'Q_f' ⑧

The KCL equations for a given network graph can be written in two ways as

$$A I_b = 0 \quad \text{—— (2)}$$

(or) $$Q_f I_b = 0 \quad \text{—— (3)}$$

we select a tree & partition the branch current vector as

$I_b = \begin{bmatrix} I_t \\ I_l \end{bmatrix}$, where I_t = set of twig currents
I_l = set of link currents.

∴ Eqn (2) can be written as $[A_t \; A_l]\begin{bmatrix} I_t \\ I_l \end{bmatrix} = 0$ — (4)

Similarly, Eqn (3) can be expressed as $[Q_t \; Q_l]\begin{bmatrix} I_t \\ I_l \end{bmatrix} = 0$ — (5)

The matrix A_t is a non singular matrix. Hence we can write eqn (4) as

$$I_t = -A_t^{-1} \cdot A_l \cdot I_l \quad \text{— (6)}$$

From eqn (5), we have $I_t = -Q_l \cdot I_l$ $[\because Q_t = U]$ — (7)

From eqns (6) & (7), we have

$$\boxed{Q_l = +A_t^{-1} \cdot A_l} \quad \text{— (8)}$$

(iii) Inter-relation Between f-loop Matrix and f-cut set Matrix

we know that, $B_t^T = -A_t^{-1} \cdot A_l$

and $Q_l = -A_t^{-1} \cdot A_l$

∴ we have $Q_l = -B_t^T$ — (9)

Hence, we can write

$$B_f = [B_t \; U] = [-Q_l^T \cdot U] \quad \text{— (10)}$$

and $$Q_f = [U \; Q_l] = [U \; -B_t^T] \quad \text{— (11)}$$

From (10) & (11), we have

$$B_f \cdot Q_f^T = [-Q_l^T \; U]\begin{bmatrix} U \\ Q_l^T \end{bmatrix} = -Q_l^T + Q_l^T = 0 \quad \text{— (12)}$$

Similarly, $Q_f \cdot B_f^T = [U \; Q_l]\begin{bmatrix} -Q_l \\ U \end{bmatrix} = -Q_l + Q_l = 0$ — (13)

Voltage & Current Transformations

For a network with 'b' elements, we need to find 'b' voltages and 'b' currents. In all, there are '2b' variables. Hence, '2b' linearly independent equations are needed in terms of '2b' variables.

The terminal relations of individual elements provide 'b' independent equations. The remaining 'b' equations can be obtained by applying KCL and KVL.

(i) Twig voltage Transformation

For any general network with 'n' nodes & 'b' branches, the KVL equations are written as

$$B_f \cdot V_b = 0$$

where, B_f is f-loop matrix of the order of $(b-n+1) \times b$ & 'V_b' is the column matrix of 'b' branch voltages.

Partitioning matrix 'B_f' corresponding to twig branches and links, we have

$$V_l = -B_t \cdot V_t$$

Substituting for B_t from eqn ⑨, in the above equation,

$$V_l = Q_l^T V_t \quad \text{—— (14)}$$

Now, we have, $V_b = \begin{bmatrix} V_t \\ V_l \end{bmatrix} = \begin{bmatrix} U V_t \\ Q_l^T V_t \end{bmatrix} = \begin{bmatrix} U \\ Q_l^T \end{bmatrix} V_t$ —— (15)

(or) $\boxed{V_b = Q_f^T \cdot V_t}$ —— (16)

Eqn (16) is called twig voltage transformation.

(ii) Node voltage Transformation

Instead of using the twig voltages as a set of independent voltages, we can also use the set of node voltages as an independent set to express the branch voltages.

In the network graph, there are two kinds of branches: those which are incident to reference node & those are not incident.

For the former, the branch voltage is the same as node voltage or negative depending on the orientation of the branch. For the later, branch voltage forms a loop with two node voltages.

Hence, in both cases, the branch voltages are linear combination of node voltages.

Let v_k be the branch voltage of the k^{th} branch and let v_i' be the node voltage of the ith node.

In case k^{th} branch connects the ith node to reference node, then, $v_k = v_i'$, if branch 'k' leaves i

and $v_k = -v_i'$, if branch 'k' enters node 'i'.

On the other hand, if the k^{th} branch leaves the node 'i' and enters the node 'j', then

$$v_k = v_i' - v_j' \quad \text{—— (1)}$$

The eqn (1), can be written in matrix form as,

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_b \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1,n-1} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2,n-1} \\ c_{31} & c_{32} & c_{33} & \cdots & c_{3,n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ c_{b1} & c_{b2} & c_{b3} & \cdots & c_{b,n-1} \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \\ v_3' \\ \vdots \\ v_{n-1}' \end{bmatrix} \quad \text{—— (2)}$$

where, $c_{ki} = 1$, if k^{th} branch leaves node i

$= -1$, " " " enters " "

$= 0$, " " " is not incident at node i.

From the above definition of c_{ki}, the matrix 'C' is recognized to be the transpose of incidence matrix,

i.e. $C = A^T$. Hence we get

$$V_b = A^T \cdot V_n \quad \text{—— (3)}$$

where V_n is the node voltage vector. This is known as node voltage transformation equation.

(iii) Link Current Transformation.

It is possible to express branch currents as linear combinations of link currents. We know that,

$$Q_f \cdot I_b = 0.$$

where 'Q_f' is f-cutset matrix and 'I_b' is the branch current vector.

Rearranging the branches of the network, we partition 'Q_f' into Q_t and Q_l and write

$$\begin{bmatrix} Q_t & Q_l \end{bmatrix} \begin{bmatrix} I_t \\ I_l \end{bmatrix} = 0 \quad — (3)$$

$$I_t = B_t^T \cdot I_l \quad — (4)$$

Combining equation (4) with the trivial equation, $I_l = UI_l$, we get

$$\begin{bmatrix} I_t \\ I_l \end{bmatrix} = \begin{bmatrix} B_t^T \\ U \end{bmatrix} [I_l]$$

(or) $$I_b = B_f^T \cdot I_l \quad — (5)$$

Eqn (5) shows that the branch currents are expressed as linear Combination of $(b-n+1)$ link currents. Eqn (5) is called the link Current transformation.

(iv) Mesh Current Transformation

In Case of a planar graph, we can select a set of independent mesh currents instead of link currents.

If B_m is the mesh matrix & I_m is the mesh current vector, can be easily shown that

$$I_b = B_m^T \cdot I_m \quad — (6)$$

where 'B_m' is $(b-n+1) \times b$ dimensional matrix and I_m is $(b-n+1)$ dimensional vector.

Eqn (6) is known as "mesh current transformation". It indicates that the 'b' branch currents can be expressed as linear Combination of $(b-n+1)$ mesh currents.

Duality And Dual Networks

Two networks are said to be the dual of each other when the mesh equations of one network are the same as the node equations of the other.

Kirchoff's voltage law and current law are same, with voltage substituted for current, independent loop for independent node pair etc.

Similarly, two graphs are said to be dual of each other if the incidence matrix of any one of them is equal to the circuit matrix of the other. Only planar networks have duals.

Conversion for dual electrical circuits

Loop Basis	Node Basis
Current	voltage
Resistance	Conductance
Inductance	Capacitance
Branch Current	Branch voltage
Mesh	Node
Short Circuit	Open circuit
Parallel Path	Series Path

Steps involved in constructing the dual of a network

1. Place a node inside each mesh of the given network. These internal nodes correspond to the independent nodes in the dual network.
2. Place a node outside the given network. The external node corresponds to the datum node in the dual network.
3. Connect all internal nodes in the adjacent mesh by dashed lines crossing the common branches. Elements which are the duals of the common branches will form the branches connecting the corresponding independent node in the dual network.

4. Connect all internal nodes to the external node by dashed lines corresponding to all external branches. Duals of these external branches will form the branches connecting independent nodes and the datum node.

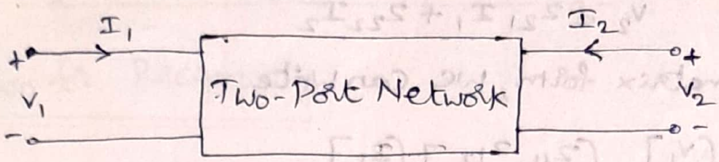
5. A clockwise current in a mesh corresponds to a positive polarity at the dual independent node.

6. A voltage rise in the direction of a clockwise mesh current corresponds to a current flowing towards the dual independent node.

UNIT-IV : TWO-PORT NETWORKS

Introduction

A two-port network has two pairs of terminals, one pair at the input known as "input port" and one pair at the output known as "output port". It is as shown below.



There are four variables V_1, V_2, I_1 and I_2 associated with a two-port network. Two of these variables can be expressed in terms of the other two variables.

Thus, there will be two dependent and two independent variables. The number of possible combinations generated by four variables taken two at a time is $^4C_2$, i.e. six. Hence, there are six possible sets of equations describing a two-port network.

Two-Port Parameters

Parameter	Variables		Equation
	Express	In terms of	
Open circuit Impedance	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
Short circuit Admittance	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
Transmission	V_1, I_1	V_2, I_2	$V_1 = A V_2 - B I_2$ $I_1 = C V_2 - D I_2$
Inverse Transmission	V_2, I_2	V_1, I_1	$V_2 = A' V_1 - B' I_1$ $I_2 = C' V_1 - D' I_1$
Hybrid	V_1, I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
Inverse Hybrid	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

Open-circuit Impedance parameters (Z-parameters

The Z-parameters of a two-port network may defined by expressing two-port voltages V_1 and V_2 in terms of two port currents I_1 and I_2.

$$(V_1, V_2) = f(I_1, I_2)$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The individual 'Z' parameters for a given network defined by setting each of the port currents equal to z

Case (1):- when the output is open circuited, i.e. $I_2 = 0$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

where 'Z_{11}' is the "driving point impedance" with the output port open circuited. It is also called as "open circuit input im

similarly, $Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

where 'Z_{21}' is the transfer impedance with the output circuited. It is also called as "open circuit forward tran impedance."

Case (2):- when input port is open circuited, i.e. $I_1 = 0$

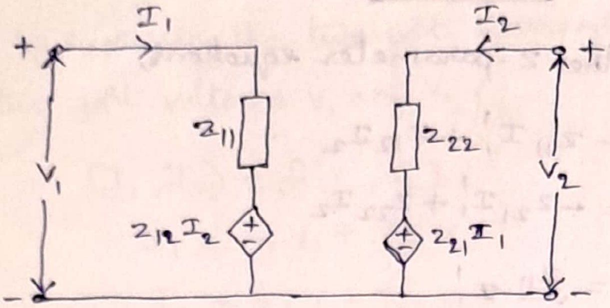
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

where Z_{12} is the transfer impedance with the input port circuited. It is also called as "open circuit reverse transfer

similarly, $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

where Z_{22} is the open circuit driving point impedance input port open circuited. It is also called as "open circuit impedance."

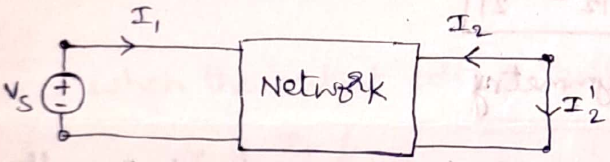
The equivalent circuit of the two port network in terms of z-parameters is as shown below.



Condition for Reciprocity

A network is said to be reciprocal, if "the ratio of excitation at one port to response at the other port is same" if excitation and response are interchanged.

Case (a)



As shown in figure, voltage V_s is applied to the input port with the output port short circuited.

i.e. $V_1 = V_s$ and $V_2 = 0$.

$I_2 = -I_2'$

From z-parameter equations, we have

$$V_s = z_{11} I_1 - z_{12} I_2'$$

$$0 = z_{21} I_1 - z_{22} I_2'$$

$$\Rightarrow I_1 = \frac{z_{22}}{z_{21}} I_2'$$

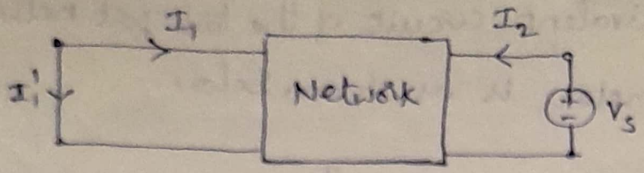
$$V_s = z_{11} \frac{z_{22}}{z_{21}} I_2' - z_{12} I_2'$$

$$\frac{V_s}{I_2'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{21}}$$

Case (b)

As shown in the figure below, voltage V_s is applied at the output port with the input port short circuited.

i.e. $V_2 = V_s$ and $V_1 = 0$, $I_1 = -I_1'$



From the z-parameter equations,

$$0 = -z_{11} I_1' + z_{12} I_2$$

$$V_S = -z_{21} I_1' + z_{22} I_2$$

$$\Rightarrow I_2 = \frac{z_{11}}{z_{12}} I_1'$$

$$\Rightarrow V_S = -z_{21} I_1' + z_{22} \frac{z_{11}}{z_{12}} I_1'$$

$$\frac{V_S}{I_1'} = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{12}}$$

Hence for the network to be reciprocal,

$$\frac{V_S}{I_1'} = \frac{V_S}{I_2}$$

i.e. $\boxed{z_{12} = z_{21}}$

Condition for Symmetry

For a network to be symmetrical, the voltage-to-current ratio at one port should be the same as the voltage-to-current ratio at the other port with one of the ports open-circuited.

Case (a)

When the output port is open circuited, i.e. $I_2 = 0$

From the z-parameter equation,

$$V_S = z_{11} I_1$$

or $$\frac{V_S}{I_1} = z_{11}$$

Case (b)

When the input port is open circuited, i.e. $I_1 = 0$

From the z-parameter equation,

$$V_S = z_{22} I_2$$

or $$\frac{V_S}{I_2} = z_{22}$$

Hence, for the network to be symmetrical,

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

i.e. $\boxed{z_{11} = z_{22}}$

4.7 Short-Circuit Admittance Parameters (Y-Parameters)

The Y-parameters of a two-port network may be defined by expressing the two port currents I_1 and I_2 in terms of the two-port voltages V_1 and V_2.

$$(I_1, I_2) = f(V_1, V_2)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

In matrix form, we can write

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual Y-parameters for a given network can be defined by setting each of the port voltages equal to zero.

Case (i)

when the output port is short circuited, i.e. $V_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

where Y_{11} is the "driving point admittance" with the output port short circuited. It is also called as "short circuit input admittance".

Similarly,

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

where Y_{21} is the "transfer admittance" with the output port short circuited. It is also called as "short circuit forward transfer admittance"

Case (ii)

when the input port is short circuited, $V_1 = 0$

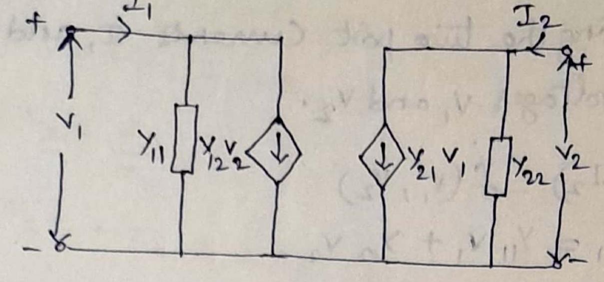
$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

where Y_{12} is the "transfer admittance" with the input port short circuited. It is also called as "short circuit Reverse transfer admittance"

Similarly, $Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

where Y_{22} is the "short circuit driving point admittance" with the input port short circuited. It is also called as "short circuit output admittance"

The equivalent circuit of the two port network in terms of Y-parameters is as shown below.

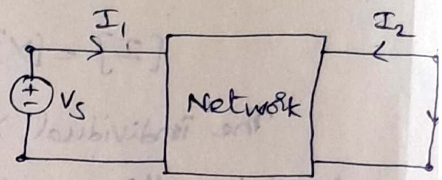


Condition for Reciprocity

Case (a):- As shown below, let a voltage of 'V_s' is applied at the input with the output port short circuited.

i.e.,

$$V_1 = V_s$$
$$V_2 = 0$$
$$I_2 = -I_2'$$



From the Y-parameter equation,

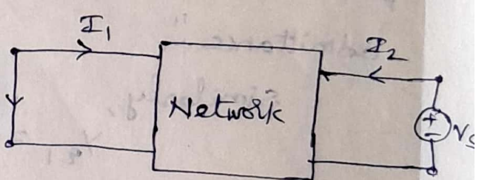
$$-I_2' = Y_{21} V_s$$

$$\therefore \frac{I_2'}{V_s} = -Y_{21}$$

Case (b):- As shown in figure below, voltage 'V_s' is applied at the output port with the input port short circuited.

i.e.,

$$V_2 = V_s$$
$$V_1 = 0$$
$$I_1 = -I_1'$$



From the Y-parameter equation,

$$-I_1' = Y_{12} V_2$$

$$\frac{I_1'}{V_s} = -Y_{12}$$

Hence, for the network to be reciprocal,

$$\frac{I_2'}{V_s} = \frac{I_1'}{V_s}$$

i.e. $\boxed{Y_{12} = Y_{21}}$

Condition for Symmetry

Case (a) when the output port is short circuited, i.e. $V_2 = 0$

From the Y-parameter equation,

$$I_1 = Y_{11} V_S$$

$$\frac{V_S}{I_1} = \frac{1}{Y_{11}}$$

Case (b) when the input port is short circuited, i.e. $V_1 = 0$

From the Y-parameter equation,

$$I_2 = Y_{22} V_S$$

$$\frac{V_S}{I_2} = \frac{1}{Y_{22}}$$

Hence, for the network to be symmetrical,

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

i.e. $\boxed{Y_{11} = Y_{22}}$

10-17

Transmission Parameters (ABCD Parameters)

The transmission parameters or chain parameters or ABCD parameters serve to relate the voltage and current at the input port to voltage and current at the output port. In equation form,

$$(V_1, I_1) = f(V_2, -I_2)$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Here, the negative sign is used with I_2 and not for parameters B and D. The reason the current I_2 carries a negative sign is that in a transmission field, the output current is assumed to be coming out of the output port instead of going into the port.

In matrix form, we can write,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where the matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission matrix.

For a given network, these parameters are determined as follows:

Case (i) :- when the output port is open circuited, $I_2 = 0$

$$\Rightarrow A = \frac{V_1}{V_2}\bigg|_{I_2=0}$$

where 'A' is the "reverse voltage gain" with the output port opend

Similarly, $$C = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

where 'C' is the "transfer admittance" with the output port open ckt

Case (ii) :- when the output port is short circuited, $V_2 = 0$

$$\Rightarrow B = -\frac{V_1}{I_2}\bigg|_{V_2=0}$$

where 'B' is the "transfer impedance" with the output port short cktd

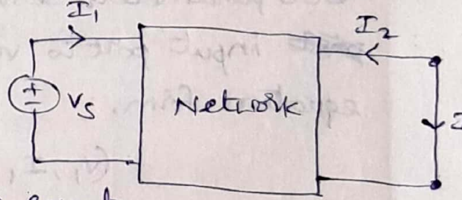
Similarly, $$D = -\frac{I_1}{I_2}\bigg|_{V_2=0}$$

where 'D' is the "reverse current gain" with the output port short cktd.

Condition for Reciprocity

Case (a) :- As shown in below figure, voltage 'V_s' is applied at the input port with the output port short circuited.

Then, $V_1 = V_s$
$V_2 = 0$
$I_2' = -I_2$



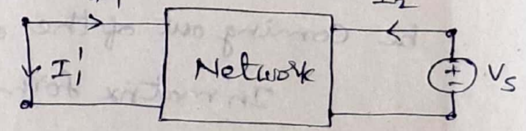
From the transmission parameter equations

$$V_s = B I_2'$$

$$\Rightarrow \frac{V_s}{I_2'} = B$$

Case (b) :- As shown in following figure, voltage 'V_s' is applied at the output port, with the input port short circuited.

Then, $V_2 = V_s$
$V_1 = 0$
$I_1' = -I_1$



From the transmission parameter equations,

$$0 = AV_s - BI_2$$

$$-I_1' = CV_s - DI_2 \Rightarrow \boxed{I_2 = \frac{A}{B} V_s}$$

$$\frac{V_s}{I_1'} = \frac{B}{AD-BC}$$

For the network to be reciprocal,

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'} \Rightarrow B = \frac{B}{AD-BC} \text{ or } \boxed{AD-BC = 1}$$

Condition for Symmetry

Case (a)

When the output port is open circuited, $I_2 = 0$.

From the transmission parameter equations,

$$V_S = A V_2$$

$$I_1 = C V_2$$

$$\frac{V_S}{I_1} = \frac{A}{C}$$

Case (b)

When the input port is open circuited, $I_1 = 0$.

From the transmission parameter equation,

$$C V_S = D \cdot I_2$$

$$\frac{V_S}{I_2} = \frac{D}{C}$$

Hence, for the network to be symmetrical,

$$\frac{V_S}{I_1} = \frac{V_S}{I_2}$$

i.e. $\boxed{A = D}$

Hybrid Parameters (h-parameters)

The hybrid parameters of a two-port network may be defined by expressing the ~~output~~ voltage of input port V_1 and current of output port I_2 in terms of current of input port I_1 and voltage of output port V_2.

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual h-parameters can be defined by setting $I_1 = 0$ and $V_2 = 0$.

For a given network ..

Case (i) :- when the output port is short circuited, i.e. $V_2 = 0$

$$h_{11} = \frac{V_1}{I_1}\Bigg|_{V_2=0}$$

where 'h_{11}' is the short circuit input impedance

$$h_{21} = \frac{I_2}{I_1}\Bigg|_{V_2=0}$$

where 'h_{21}' is the short circuit forward current gain.

Case (ii) :- when the input port is open circuited, i.e. $I_1 = 0$

$$h_{12} = \frac{V_1}{V_2}\Bigg|_{I_1=0}$$

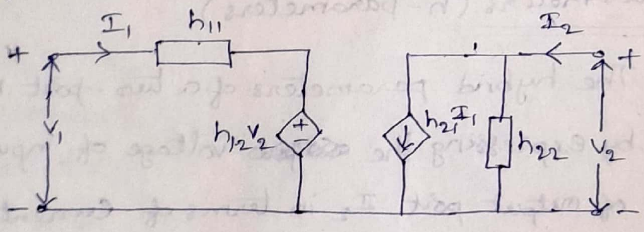
where 'h_{12}' is the open circuit reverse voltage gain.

$$h_{22} = \frac{I_2}{V_2}\Bigg|_{I_1=0}$$

where 'h_{22}' is the open circuit output admittance.

Since h-parameters represent dimensionally an impedance, an admittance, a voltage gain and a current gain, these are called as "hybrid parameters".

The equivalent circuit of a two port network in terms of hybrid parameters is as shown below.



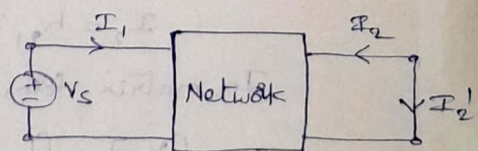
Condition for Reciprocity

Case (a) :- As shown in following figure, voltage 'V_s' is applied at the input port and the output port is short circuited.

$V_1 = V_s$

$V_2 = 0$

$I_2' = -I_2$



From the h-parameter equations,

$$V_s = h_{11} I_1$$

$$-I_2' = h_{21} I_1$$

$$\frac{V_s}{I_2'} = -\frac{h_{11}}{h_{21}}$$

Case (b) :- As shown in the

at the output port with

$V_1 = 0$

$V_2 = V_s$

$I_1 = -I_1'$

From the h-parameter

$0 = h_{11} I_1 + h_1$

$h_{12} V_s = -h_{11} I_1$

$$\frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}}$$

Hence, for the net

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

i.e. $\boxed{h_{21} =}$

Condition for Symmetry

The condition

$Z_{11} = \frac{V_1}{I_1}$

But with $I_2 =$

$0 =$

$\frac{V_2}{I_1}$

$Z_{11} = h$

where $\Delta h =$

Similarly,

with $I_1 = 0$

For a Sym

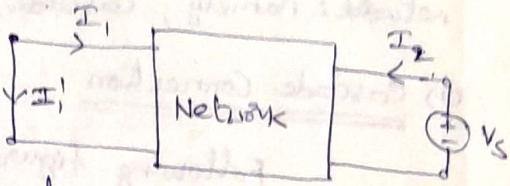
i.e.

Case (b) :- As shown in the following figure, voltage V_s is applied at the output port with the input port short circuited.

$V_1 = 0$

$V_2 = V_s$

$I_1 = -I_1'$



From the h-parameter equations,

$$0 = h_{11} I_1 + h_{12} V_s$$

$$h_{12} V_s = -h_{11} I_1 = h_{11} I_1'$$

$$\frac{V_s}{I_1'} = \frac{h_{11}}{h_{12}}$$

Hence, for the network to be reciprocal,

$$\frac{V_s}{I_2'} = \frac{V_s}{I_1'}$$

i.e. $\boxed{h_{21} = -h_{12}}$

Condition for Symmetry

The condition for symmetry is obtained from the z-parameters

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{h_{11} I_1 + h_{12} V_2}{I_1}\right|_{I_2=0} = h_{11} + h_{12}\frac{V_2}{I_1}$$

But with $I_2 = 0$,

$$0 = h_{21} I_1 + h_{22} V_2$$

$$\frac{V_2}{I_1} = -\frac{h_{21}}{h_{22}}$$

$$z_{11} = h_{11} - \frac{h_{12} h_{21}}{h_{22}} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} = \frac{\Delta h}{h_{22}}$$

where $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

Similarly, $z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

with $I_1 = 0$, $I_2 = h_{22} V_2$.

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$$

For a symmetrical network, $z_{11} = z_{22}$

i.e. $\frac{\Delta h}{h_{22}} = \frac{1}{h_{22}} \Rightarrow \Delta h = 1$

(or) $\boxed{h_{11} h_{22} - h_{12} h_{21} = 1}$

Inter-Connection of Two-port Networks

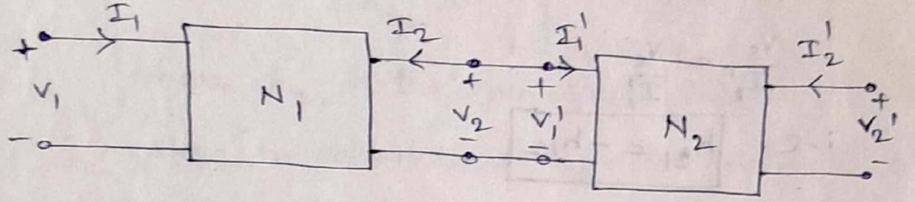
There are various types of inter Connections of a two port network; namely, cascade, parallel, series etc.

i) Cascade Connection

Following figure shows a two-port networks connected in cascade. In the cascade connection, the output port of the first network becomes the input ~~port~~ of the second network.

Since it is assumed that, input and output currents are positive when they enter the network, we have

$$I_1' = -I_2.$$



Let A_1, B_1, C_1 and D_1 be the transmission parameters of the the network N_2.

For the network N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad — ①$$

For the network N_2,

$$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \quad — ②$$

Since $V_1' = V_2$ and $I_1' = -I_2$, we can write

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \quad — ③$$

Combining equations ① and ③,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

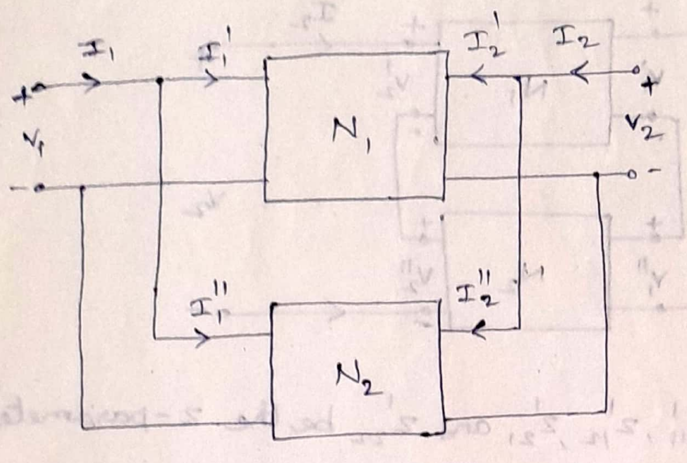
Hence,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \quad — ④$$

Equation ④ shows that the resultant ABCD matrix of the cascade connection is the product of the individual ABCD matrices.

(ii) Parallel Connection

Following figure shows two-port networks connected in parallel. In the parallel connection, the two networks have the same input voltages and the same output voltages.



Let $y_{11}', y_{12}', y_{21}'$ and y_{22}' be the Y-parameters of the network N_1 and $y_{11}'', y_{12}'', y_{21}''$ and y_{22}'' be the Y-parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} y_{11}' & y_{12}' \\ y_{21}' & y_{22}' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For the network N_2,

$$\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} y_{11}'' & y_{12}'' \\ y_{21}'' & y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

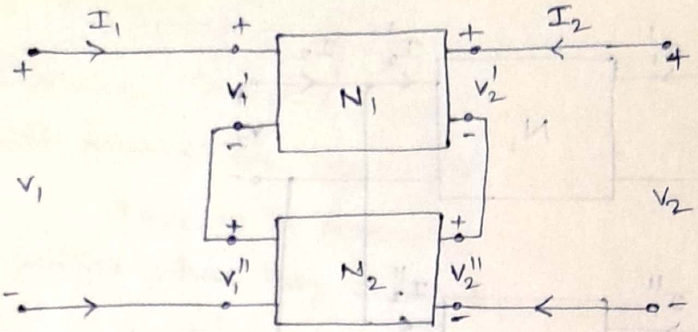
For the combined network, $I_1 = I_1' + I_1''$ and $I_2 = I_2' + I_2''$

Hence,
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' + I_1'' \\ I_2' + I_2'' \end{bmatrix} = \begin{bmatrix} y_{11}' + y_{11}'' & y_{12}' + y_{12}'' \\ y_{21}' + y_{21}'' & y_{22}' + y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus the resultant Y-parameter matrix for parallel connected networks is the sum of Y-matrices of each individual two-port networks.

(iii) Series Connection

Following figure shows two-port networks connected in s[eries].
In series connection, both the networks carry the same input cu[rrent].
Their output are also equal.



Let $z_{11}', z_{12}', z_{21}'$ and z_{22}' be the z-parameters of the network N_1 and $z_{11}'', z_{12}'', z_{21}''$ and z_{22}'' be the z-parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}'' \\ z_{21}' & z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the network N_2,

$$\begin{bmatrix} V_1'' \\ V_2'' \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}'' \\ z_{21}' & z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the combined network, $V_1 = V_1' + V_1''$ and $V_2 = V_2' + V_2''$.

Hence,
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1' + V_1'' \\ V_2' + V_2'' \end{bmatrix} = \begin{bmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus, the resultant z-parameter matrix for the series-connected networks is the sum of z-p matrices of each individual two-port network.

Unit - ~~III~~ V: Filters

Classification of filters

→ A "filter" is a reactive network that freely "passes" the desired band of frequencies while almost totally "suppressing" all other bands.

→ A "filter" is constructed from purely reactive elements, otherwise the attenuation would never becomes zero in the pass band of the filter network.

→ Ideal filter should not produce attenuation in the desired band called the "transmission band" (or) "Pass band", and should provide total ~~or~~ or infinite attenuation for all other frequencies called "attenuation band" (or) "Stop band".

→ Filter networks are widely used in communication networks to separate various voice channels in carrier frequency telephone circuits.

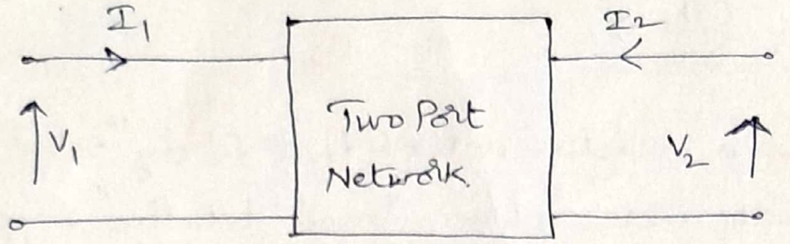
→ Filters also find applications in instrumentation, telemetering equipment etc. where it is necessary to transmit (or) attenuate a limited range of frequencies.

Decibel and Neper

→ The attenuation of a filter can be expressed in "decibels" (or) "Nepers"

→ "Neper" is defined as the natural logarithm of the ratio of input voltage (or current) to the output voltage (or current) provided that the network is properly terminated in its characteristic impedance 'Z_0'.

Consider a two-port network shown below.



→ From figure, the number of nepers, $N = \log_e\left|\frac{V_1}{V_2}\right|$ (or) $\log_e\left|\frac{I_1}{I_2}\right|$.

→ A "neper" can also be expressed in terms of input power, P, and the output power P_2 as $N = \frac{1}{2}\log_e\left|\frac{P_1}{P_2}\right|$.

→ A "decibel" is defined as ten times the common logarithms of the ratio of the input power to the output power.

∴ Decibel, $D = 10\log_{10}\left|\frac{P_1}{P_2}\right|$

→ The decibel can be expressed in terms of the ratio of input voltage (or current) and the output voltage (or current).

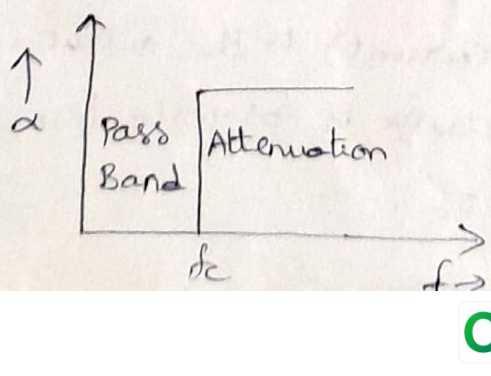
$$D = 20\log_{10}\left|\frac{V_1}{V_2}\right| = 20\log_{10}\left|\frac{I_1}{I_2}\right|$$

→ ∴ One decibel is equal to 0.115 N.

Low Pass Filter

→ A "Low Pass filter" is one which passes without attenuation all frequencies up to the "cut-off frequency, f_c"; and attenuates all other frequencies greater than 'f_c'.

→ The attenuation characteristic of an ideal LP filter is as shown in the following fig:



→ This filter transmits currents of all frequencies from zero upto the cut-off frequency. The band is called pass band (or) transmission band.

→ Thus the pass band for the LP filter is the frequency range 'o' to 'f_c'

→ The stop band for the LP filter is the frequency range above 'f_c'.

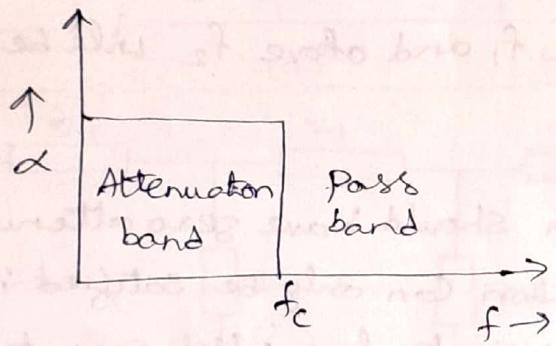
High Pass Filter

→ A high pass filter attenuates all frequencies below a designated cut-off frequency 'f_c', and passes all frequencies above 'f_c'.

→ Thus the Pass band of this filter is the frequency range above f_c.

→ Stop band is the frequency range below 'f_c'.

→ The attenuation characteristic of a HP filter is as shown below;

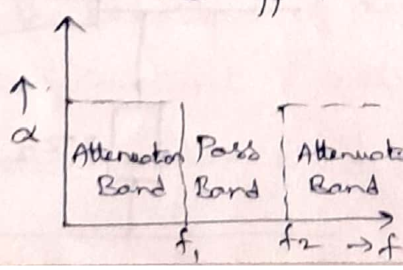


Band Pass Filter

→ A band pass filter passes frequencies between two designated cut-off frequencies and attenuates all other frequencies.

→ It is abbreviated as BP filter.

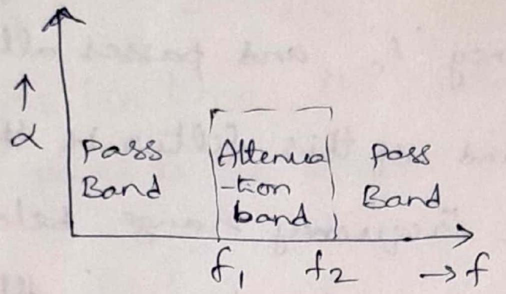
→ As shown in fig, a BP filter has two cut-off frequencies and will have the pass band $f_2 - f_1$; 'f_1' is called lower cut off frequency while 'f_2' is called the upper cut-off frequency.



Band Elimination filter

→ A band elimination filter passes all frequencies lying outside certain range, while it attenuates all frequencies between two designated frequencies. It is also referred to as "Band stop filter".

→ The characteristic of an ideal band elimination filter is as shown below:



→ All frequencies between f_1 and f_2 will be attenuated while frequencies below f_1 and above f_2 will be passed.

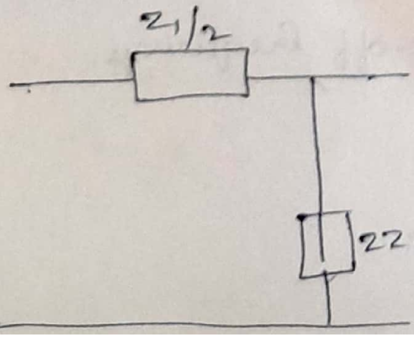
Filter Networks

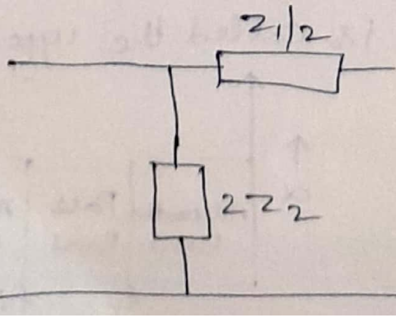
→ Ideally a filter should have zero attenuation in the pass band. This condition can only be satisfied if the elements of the filter are dissipationless, which cannot be realized in practice.

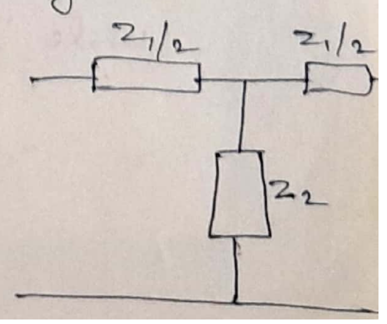
→ Filters are designed with an assumption that the elements of the filters are purely reactive.

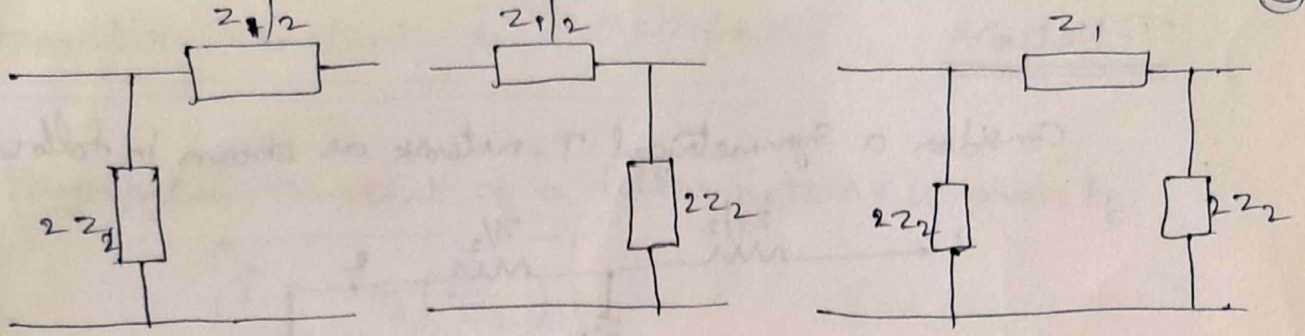
→ Filters are made of symmetrical 'T' (or) 'π' sections.

→ T and π sections can be considered as combinations of unsymmetrical 'L' sections as shown in following figures.



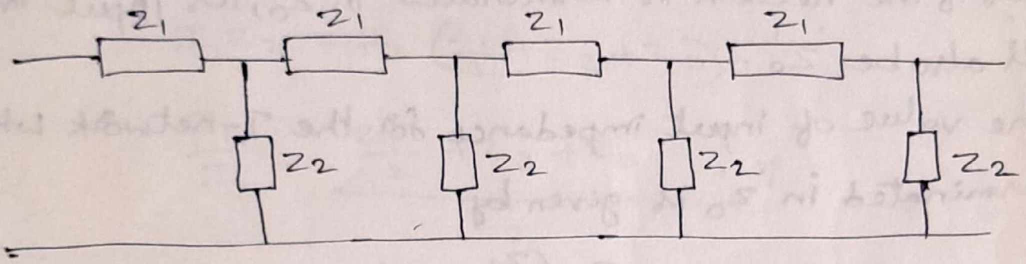




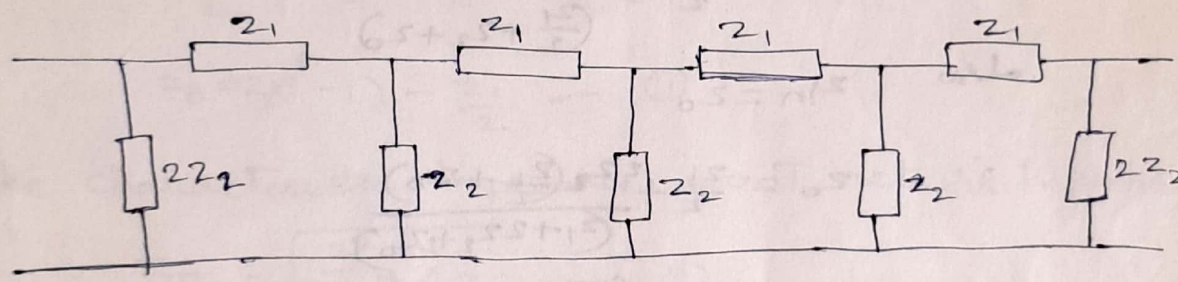


→ The "ladder" structure is one of the commonest forms of filter network.

→ A Cascade Connection of Several T and π Sections Constitutes a ladder networks. A Common form of the ladder network is Shown in figure.



(a)



(b)

→ Figure (a) represents a T Section ladder network whereas fig (b) represents the π-section ladder network.

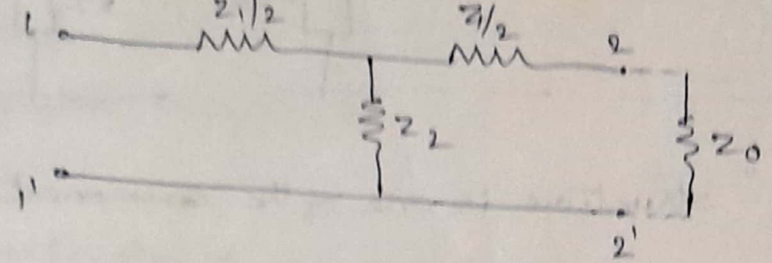
→ It Can be observed that both networks are identical except at the ends.

Equations of Filter Networks

→ The study of the behaviour of any filter requires the calculation of its propagation constant 'γ', attenuation 'α', phase shift 'β' and its characteristic impedance 'Z₀'.

T-Network

Consider a Symmetrical T-network as shown in following.



→ If image impedances at port 1-1' and port 2-2' are equal to each other, the image impedance is then called the "characteristic (or) the iterative impedance", Z_0.

→ Thus if the network is terminated in Z_0, its input impedance will also be 'Z_0'.

→ The value of input impedance for the T-network when it is terminated in 'Z_0' is given by

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_0\right)}{\left(\frac{Z_1}{2} + Z_2 + Z_0\right)}$$

also $\quad Z_{in} = Z_0$

$$\therefore \quad Z_0 = \frac{Z_1}{2} + \frac{2Z_2\left(\frac{Z_1}{2} + Z_0\right)}{(Z_1 + 2Z_2 + 2Z_0)}$$

$$Z_0 = \frac{Z_1}{2} + \frac{(Z_1Z_2 + 2Z_2Z_0)}{(Z_1 + 2Z_2 + 2Z_0)}$$

$$Z_0 = \frac{Z_1^2 + 2Z_1Z_2 + 2Z_1Z_0 + 2Z_1Z_2 + 4Z_2Z_0}{2(Z_1 + 2Z_2 + 2Z_0)}$$

$$4Z_0^2 = Z_1^2 + 4Z_1Z_2$$

$$Z_0^2 = \frac{Z_1^2}{4} + Z_1Z_2$$

The characteristic impedance of a symmetrical T-section is

$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1Z_2}$$

Also, $Z_{oc} = \frac{Z_1}{2} + Z_2$, $Z_{sc} = \frac{Z_1}{2} + \frac{\frac{Z_1}{2} \times Z_2}{\frac{Z_1}{2} + Z_2} = \frac{Z_1^2 + 4Z_1Z_2}{2Z_1 + 4Z_2}$

$$Z_{oc} \times Z_{sc} = \frac{Z_1^2}{4} + Z_1Z_2 = Z_{0T}^2 \quad (\text{or}) \quad \boxed{Z_{0T} = \sqrt{Z_{oc} \times Z_{sc}}}$$

$Z_1 Z_0^2$

$$\therefore \sinh\gamma = \sqrt{\cosh^2\gamma - 1} = \sqrt{\left(1+\frac{Z_1}{2Z_2}\right)^2 - 1} = \sqrt{\frac{Z_1}{Z_2} + \frac{Z_1^2}{4Z_2^2}}$$

$$\sinh\gamma = \frac{1}{Z_2}\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \frac{Z_{0T}}{Z_2} \quad \text{— (4)}$$

Dividing (4) by (3), we get

$$\tanh\gamma = \frac{Z_{0T}}{Z_2 + \frac{Z_1}{2}} = \frac{Z_{0T}}{Z_{0C}}$$

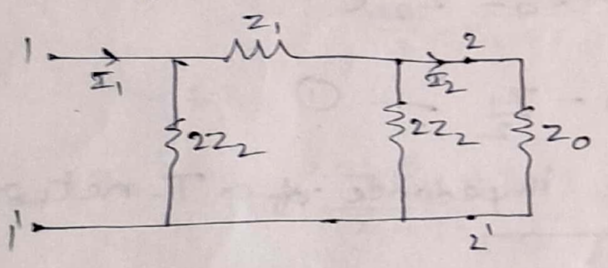
$$\left[\because \frac{Z_1}{2} + Z_2 = Z_{0C}\right]$$

also $\tanh\gamma = \sqrt{\dfrac{Z_{SC}}{Z_{0C}}}$

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{1}{2}(\cosh\gamma - 1)} = \sqrt{\frac{1}{2}\left(\frac{Z_1}{2Z_2} + 1 - 1\right)} = \sqrt{\frac{Z_1}{4Z_2}} \quad \text{— (5)}$$

π – Network

Consider a Symmetrical π-section shown in following figure.



→ When the network is terminated by Z_0 at port 2-2', its input impedance is given by

$$Z_{in} = \frac{2Z_2\left[Z_1 + \dfrac{2Z_2 Z_0}{2Z_2 + Z_0}\right]}{Z_1 + \dfrac{2Z_2 Z_0}{2Z_2 + Z_0} + 2Z_2}$$

→ By definition of characteristic impedance, $Z_{in} = Z_0$

$$\therefore Z_0 = \frac{2Z_2\left(Z_1 + \dfrac{2Z_2 Z_0}{2Z_2 + Z_0}\right)}{Z_1 + \dfrac{2Z_2 Z_0}{2Z_2 + Z_0} + 2Z_2}$$

$$Z_0 Z_1 + \frac{2Z_2 Z_0^2}{2Z_2 + Z_0} + 2Z_2 Z_0 = 2Z_1 Z_2 + \frac{4Z_2^2 Z_0}{2Z_2 + Z_0}$$

$2L_2 = 2 \times 3.57 = 7.14\,mH$

Band Elimination Filter

→ A band elimination filter is one which passes without attenuation all frequencies less than the lower cut-off frequency f_1, and greater than the upper cut-off frequency f_2.

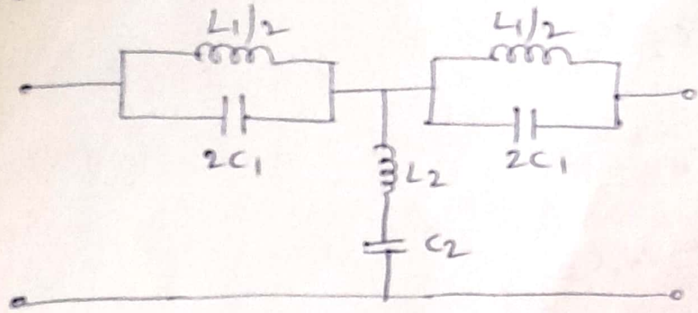
→ Frequency lying between f_1 and f_2 are attenuated. It is also known as "band stop" filter.

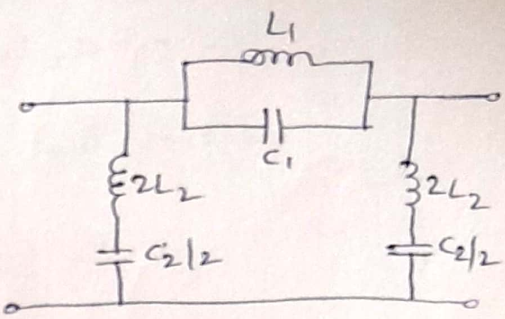
→ Therefore, a band stop filter can be realized by connecting a low pass filter in parallel with a high pass section, in which cut-off frequency of low pass filter is below that of a high pass filter.

→ A band elimination filter is ~~also~~ designed in the same manner as in the band pass filter.

Scanned with OKEN Scanner

The configurations of T and π Constant-k band stop sections are as shown below:





→ For the Condition of equal resonant frequencies

$$\frac{\omega_0 L_1}{2} = \frac{1}{2\omega_0 C_1} \quad \text{for the Series arm}$$

(or) $$\omega_0^2 = \frac{1}{L_1 C_1}$$

$$\omega_0 L_2 = \frac{1}{\omega_0 C_2} \quad \text{for the Shunt arm}$$

$$\omega_0^2 = \frac{1}{L_2 C_2}$$

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = k$$

$$L_1 C_1 = L_2 C_2$$

→ It can also be verified that

$$Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1} = k^2$$

and $f_0 = \sqrt{f_1 f_2}$

→ At cut-off frequencies, $Z_1 = -4Z_2$

→ Multiplying both sides with Z_2, we get

$$Z_1 Z_2 = -4Z_2^2 = k^2$$

$$Z_2 = \pm j\frac{k}{2}$$

→ If the load is terminated in a load resistance, $R = k$, then at lower cut-off frequency

$$Z_2 = j\left(\frac{1}{\omega_1 C_2} - \omega_1 L_2\right) = j\frac{k}{2}$$

$$\frac{1}{\omega_1 C_2} - \omega_1 L_2 = \frac{k}{2}$$

$$1 - \omega_1^2 C_2 L_2 = \omega_1 C_2 \frac{k}{2}$$

we know that $L_2 C_2 = \frac{1}{\omega_0^2}$

$$1 - \frac{\omega_1^2}{\omega_0^2} = \frac{k}{2} \omega_1 C_2$$

$$1 - \left(\frac{f_1}{f_0}\right)^2 = k \pi f_1 C_2$$

$$C_2 = \frac{1}{k\pi f_1}\left[1 - \left(\frac{f_1}{f_0}\right)^2\right]$$

$\because$ $f_0 = \sqrt{f_1 f_2}$

$$C_2 = \frac{1}{k\pi}\left[\frac{1}{f_1} - \frac{1}{f_2}\right] = \frac{1}{k\pi}\left[\frac{f_2 - f_1}{f_1 f_2}\right] \quad \text{— (5)}$$

$$\therefore L_2 = \frac{1}{\omega_0^2 C_2} = \frac{1}{\omega_0^2} \cdot \frac{k\pi f_1 f_2}{(f_2 - f_1)} \quad \text{— (6)}$$

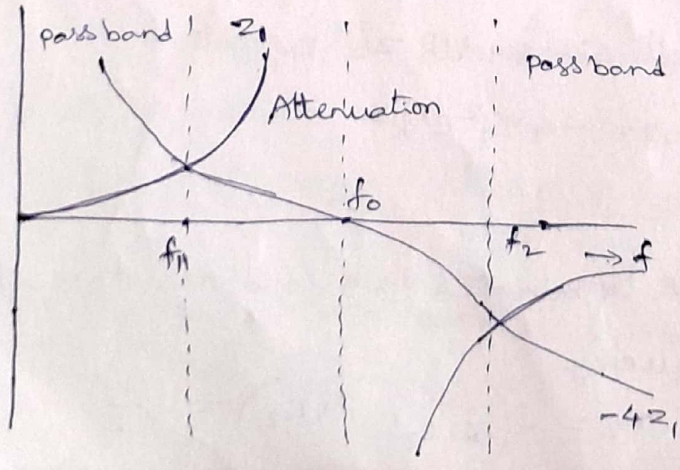
$\because$ $f_0 = \sqrt{f_1 f_2}$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)}$$

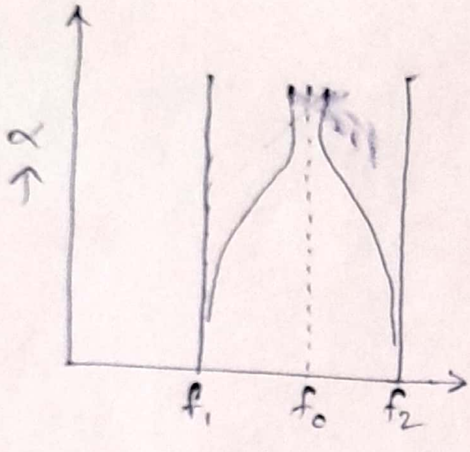
$$L_1 = k^2 C_2 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right) \quad \text{— (7)}$$

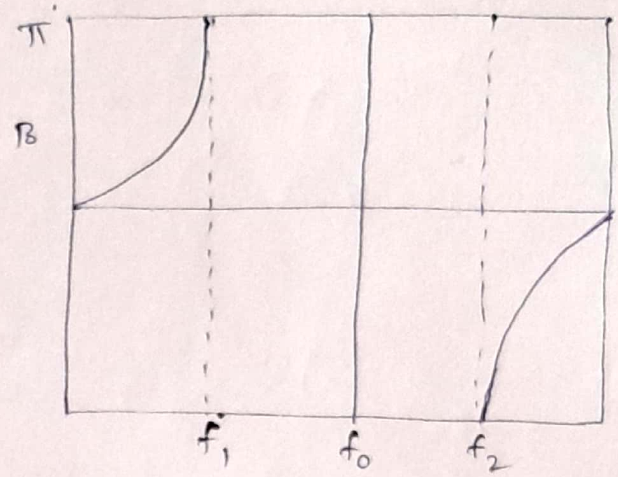
$$C_1 = \frac{L_2}{k^2} = \frac{1}{4\pi k(f_2 - f_1)} \quad \text{— (8)}$$

The variation of the reactances w.r.t. frequency is as shown below:



→ Equation (5) to (8) are the design equations of a prototype band elimination filter. The variation of α, β w.r.t. frequency is as shown below:





Ex:- (6) Design a band elimination filter having a design impedance of 600Ω and cut-off frequencies $f_1 = 2\,kHz$ and $f_2 = 6\,kHz$.

Sol:-

$$f_2 - f_1 = 4\,kHz$$

Making use of the equations (5) to (8), we have

$$L_1 = \frac{k}{\pi}\left(\frac{f_2 - f_1}{f_2 f_1}\right) = \frac{600 \times 4000}{\pi \times 12\,000\,000} = 63\,mH$$

$$C_1 = \frac{1}{4\pi k (f_2 - f_1)} = \frac{1}{4\pi \times 600 \times 4000} = 0.033\,\mu F$$

$$L_2 = \frac{k}{4\pi (f_2 - f_1)} = \frac{600}{4\pi \times 4000} = 12\,mH$$

$$C_2 = \frac{1}{k\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right) = \frac{1}{600 \times \pi}\left(\frac{4000}{2000 \times 6000}\right) = 0.176\,\mu F$$

→ Each of the two series arms of the constant-k, T-section filter is given by

$$\frac{L_1}{2} = 31.5\,mH \; ; \; 2C_1 = 0.066\,\mu F$$

→ The shunt arm elements of the network are

$$L_2 = 12\,mH \text{ and } C_2 = 0.176\,\mu F$$

→ For the constant k, π- section filter the elements of the series arm are

$L_1 = 63mH$, $C_1 = 0.033 \mu F$

→ The elements of the shunt arms are

$2L_2 = 24 mH$ and $\frac{C_2}{2} = 0.088 \mu F$