# **Relational Algebra**

## • Basic operations:

- <u>Selection</u> (  $\succ$  Selects a subset of rows from relation.
- <u>Projection</u> (  $\pi$  Deletes unwanted columns from relation.
- <u>Cross-product</u> ( ) X lows us to combine two relations.
- <u>Set-difference</u> ( ) Tuples in reln. 1, but not in reln. 2.
- <u>Union</u> (  $\bigcup$  Tuples in reln. 1 and in reln. 2.
- Additional operations:
  - Intersection, join, division, renaming: Not essential, but (very!) useful.
- Since each operation returns a relation, operations can be composed! (Algebra is "closed".)

Slide No:L6-4

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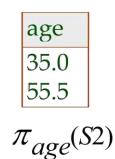
# Projection

- Deletes attributes that are not in projection list.
- Schema of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

 $\pi_{sname,rating}(S2)$ • Projection operator has to eliminate

- duplicates! (Why??)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)



### Slide No:L6-5

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Projection operator has to eliminate *duplicates*! (Why??)

Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)

# Selection

- Selects rows that satisfy *selection* condition.
- No duplicates in result! (Why?)
- Schema of result identical to schema of (only) input relation.
- *Result* relation can be the *input* for another relational algebra operation! (*Operator composition*.)

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0
	$\sigma$	$\sim (S)$	2)

$$rating > 8^{(S2)}$$

sname	rating
yuppy	9
rusty	10

 $\pi_{sname, rating}(\sigma_{rating>8}(S2))$ 

#### Slide No:L6-6

Selects rows that satisfy selection condition.

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Schema of result identical to schema of (only) input relation.

*Result* relation can be the *input* for another relational algebra operation! (*Operator* composition.)

## **Set Operations:**

#### Union, Intersection, Set-Difference

All of these operations take two input relations, which must be *union-compatible*:

– Same number of fields.

- Corresponding' fields have the same type.

What is the schema of result?

# **Union, Intersection, Set-Difference**

- All of these operations take two input relations, which must be <u>union-</u> <u>compatible</u>:
  - Same number of fields.
  - Corresponding' fields have the same type.
- What is the *schema* of result?

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

sid	sname	rating	age
22	dustin	7	45.0

S1 - S2

 $S1 \cup S2$ 

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

# $S1 \cap S2$

Slide No:L6-7

### **Cross-Product**

Each row of S1 is paired with each row of R1.

*Result schema* has one field per field of S1 and R1, with field names `inherited' if possible.

\_

Conflict: Both S1 and R1 have a field called sid.

#### Condition Join:

Result schema same as that of cross-product.

Fewer tuples than cross-product, might be able to compute more efficiently

Sometimes called a *theta-join*.

<u>Equi-Join</u>: A special case of condition join where the condition c contains only equalities.

*Result schema* similar to cross-product, but only one copy of fields for which equality is specified.

Natural Join: Equijoin on all common fields.

# Division

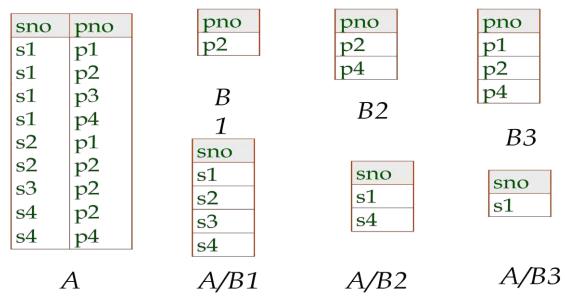
 Not supported as a primitive operator, but useful for expressing queries like:

Find sailors who have reserved <u>all</u> boats.

- Let A have 2 fields, x and y; B have only field y: -  $A/B = \{\langle x \rangle | \exists \langle x, y \rangle \in A \forall \langle y \rangle \in B\}$ 
  - i.e., A/B contains all x tuples (sailors) such that for <u>every</u> y tuple (boat) in B, there is an xy tuple in A.
  - Or: If the set of y values (boats) associated with an x value (sailor) in A contains all y values in B, the x value is in A/B.
- In general, x and y can be any lists of fields; y is the list of fields in B, and x y is the list of fields of A.

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# **Examples of Division A/B**



Slide No:L6-12

#### Find names of sailors who've reserved boat #103

Solution 1:

Find names of sailors who've reserved a red boat

Information about boat color only available in Boats; so need an extra join:

#### Find sailors who've reserved a red or a green boat

Can identify all red or green boats, then find sailors who've reserved one of these boats:

### Find sailors who've reserved a red and a green boat

Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that *sid* is a key for Sailors):

## **Relational Calculus:**

Comes in two flavors: <u>*Tuple relational calculus*</u> (TRC) and <u>*Domain relational calculus*</u> (DRC).

Calculus has variables, constants, comparison ops, logical connectives and quantifiers.

_	TRC: Variables range over (i.e., get bound to) tuples.
_	<u>DRC</u> : Variables range over <i>domain elements</i> (= field values).
_	Both TRC and DRC are simple subsets of first-order logic.

Expressions in the calculus are called *formulas*. An answer tuple is essentially an assignment of constants to variables that make the formula evaluate to *true*.

# **Tuple Relational Calculus:**

TRC - a declarative query language

### **TRC Formulas**

Atomic expressions are the following:

r (t) -- true if t is a tuple in the relation instance r

t1. Ai t2 .Aj compOp is one of  $\{, \geq, =, \neq\}$ 

t.Ai c c is a constant of appropriate type

Composite expressions:

Any atomic expression F1  $\land$  F2 ,, F1  $\lor$  F2 ,  $\neg$  F1 where F1 and F2 are expressions

 $(\forall t)$  (F), (∃t) (F) where F is an expression and t is a tuple variable Free Variables

Bound Variables - quantified variables

Obtain the rollNo, name of all girl students in the Maths Dept

 $\{s.rollNo, s.name \mid student(s) \land s.sex=`F' \land (\exists d)(department(d) \land d.name=`Maths' \land d.deptId = s.deptNo)\}$ 

s: free tuple variable

d: existentially bound tuple variable

#### Determine the departments that do not have any girl students

student (rollNo, name, degree, year, sex, deptNo, advisor) department (deptId, name, hod, phone)

 $\{d.na\ me|\ depart\,ment(d)\ \land\ \neg(\exists\ s)(stude\ nt(s)\ \land\ s.sex\ =`F'\ \land\ s.de\ ptN\ o=\ d.deptId)$ 

#### Obtain the names of courses enrolled by student named Mahesh

 ${c.name \mid course(c) \land (\exists s) (\exists e) (student(s) \land enrollment(e) \land s.name = "Mahesh" \land s.rollNo = e.rollNo \land c.courseId = e.courseId }$ 

Get the names of students who have scored 'S' in all subjects they have enrolled. Assume that every student is enrolled in at least one course.

 $\{s.name \ | \ stude \ nt(s) \ ^{\wedge} ( \ \forall e)(( \ enr \ ollment(e) \ ^{\wedge} e.rollN \ o = s.rollN \ o) \rightarrow e.gra \ de = `S') \ \}$ 

Get the names of students who have taken at least one course taught by their advisor

 $\{s.name \mid student(s) \land (\exists e)(\exists t)(enrollment(e) \land teaching(t) \land e.courseId = t.courseId \land e.rollNo = s.rollNo \land t.empId = s.advisor\}$ 

## **Domain Relational Calculus:**

Query has the form:

#### **DRC Formulas**

Atomic formula:

—

, or X op Y, or X op constant

- *op* is one of

#### Formula:

-	an atomic formul	la, or	
_		, where p	and q are formulas, or
_	, where variable X is <i>free</i> in $p(X)$ , or		
_	, where variable X is <i>free</i> in p(X)		
•	The use of quantifiers	and	is said to <u>bind</u> X.
– A variable that is not bound is free.			
Free and Bound Variables			
•	The use of quantifiers	and	in a formula is said to <u>bind</u> X.

– A variable that is not bound is <u>free</u>.

Let us revisit the definition of a query:

### Find all sailors with a rating above 7

The condition ensures that the domain variables *I*, *N*, *T* and *A* are bound to fields of the same Sailors tuple.

• The term to the left of `|' (which should be read as *such that*) says that every tuple that satisfies T>7 is in the answer.

Modify this query to answer:

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Find sailors who are older than 18 or have a rating under 9, and are called 'Joe'.

### Find sailors rated > 7 who have reserved boat #103

We have used as a shorthand for

Note the use of to find a tuple in Reserves that `joins with' the Sailors tuple under consideration.

#### Find sailors rated > 7 who've reserved a red boat

Observe how the parentheses control the scope of each quantifier's binding.

This may look cumbersome, but with a good user interface, it is very intuitive. (MS Access, QBE)

#### Find sailors who've reserved all boats

• Find all sailors *I* such that for each 3-tuple either it is not a tuple in Boats or there is a tuple in Reserves showing that sailor *I* has reserved it.

#### Find sailors who've reserved all boats

(again!) Simpler notation, same query.

(Much clearer!) To find sailors who've

reserved all red boats:

#### **Expressive Power of Algebra and Calculus**

It is possible to write syntactically correct calculus queries that have an infinite number of answers! Such queries are called *unsafe*.

e.g.,

It is known that every query that can be expressed in relational algebra can be expressed as a safe query in DRC / TRC; the converse is also true.

<u>Relational Completeness</u>: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.