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20BSC243: OPERATIONS RESEARCH

QUESTION BANK

II B.Tech. - IV Semester

20BSC243

OPERATIONS RESEARCH

LTPC 21 - 3

PRE-REQUISITES: A Course on Probability Theory.

COURSE EDUCATIONAL OBJECTIVES:

- 1. To select the constraints on the availability of resources and developing a model for rendering an optimal solution.
- 2. To evaluate the challenges in the transportation and assignment problems and furnishing a rational solution to maximize the benefits.
- 3. To investigate the nature of the project/ failure and offering methodical assistance towards sequencing
- 4. To analyze the decision criteria and strategies in game theory.
- 5. To expand the basic knowledge on queuing theory.

UNIT -1: LINEAR PROGRAMMING MODELS

(9)

Characteristics, phases and scope of operations research – Linear programming formulation – Graphical method of solution – Simplex method – Artificial variables – Big-M method – Two-phase method – Dual simplex method – Duality theorem – Principle of duality – Economic interpretation of the duality – Complementary slackness theorem – Revised simplex method.

UNIT -2: TRANSPORTATION AND ASSIGNMENT PROBLEMS

(9)

Transportation Problem: Formulation – Feasible solutions – North west corner rule, least cost method and Vogel's approximation method – Optimal solution by MODI method – Unbalanced transportation problem – Degeneracy – Maximization type. **Assignment Problem:** Formulation – Minimization and maximization problems – Unbalanced assignment problem – Traveling salesman problem.

UNIT -3: NETWORK AND SEQUENCING MODELS

(9)

Network models – Minimal spanning tree algorithm – Shortest route problem – Maximal flow models – Minimum cost flow problem – Network simplex method – Project network – CPM and PERT – Critical path and float calculations – Determining minimum time required to complete the project. Sequencing: 'n' jobs through two machines – 'n' jobs through 'm' machines.

UNIT -4: DECISION THEORY AND GAME THEORY

(9)

Decision Theory: Decision criteria and trees – Maximin and maximax – Hurwicz, laplace, savage and EOL criterion – Decision making under risk. **Game Theory:** Zero sum games – Games with and without saddle points – 2×2 games – Games with mixed strategies – Dominance principle and property – Graphical method.

UNIT –5: QUEUING THEORY

(9)

Queuing Theory: Queuing models and networks – Pure birth and death models Poisson queuing model – Poisson queuing model – Balking, Reneging, Jockeying – Kendall notation – Single, multi and machine server model – Exponential distribution – Constant rate service – Infinite and finite population – Exponential service – Monte Carlo simulation technique.



COURSE OUTCOMES:

On su	ccessful completion of the course, students will be able to	POs
CO1	Select the constraints on the availability of resources, develop a model and render an optimal solution during the given circumstances.	PO1, PO2, PO4, PO12
CO2	Appraise the challenges in the transportation and assignment problems and furnish a rational solution to maximize the benefits.	PO1, PO2, PO4, PO12
CO3	Construct the network diagram and estimate the time required to complete the project and determine optimum processing job order and investigate the nature of the project/ failure and offering methodical assistance towards sequencing.	PO1, PO2, PO4, PO12
CO4	Analyze the decision criteria's and strategies in game theory.	PO1, PO2, PO4, PO12
CO5	Expand the basic knowledge on queuing theory.	PO1, PO2, PO4, PO12

TEXT BOOKS:

- 1. P. Sankara Iyer, "Operations Research", McGraw Hill Education (India) Private Ltd. 1/e.
- 2. Hamdy A. Taha, "Operations Research an Introduction", Pearson Education Limited. 10/e, 2017.

REFERENCE BOOKS:

- 1. Frederick S. Hillier, Gerald J. Lieberman, Bodhibrata Nag and Preetam Basu, "Introduction to Operations Research", Tata McGraw Hill Education Pvt. Ltd. 10/e, 2017.
- 2. Wayne L Winston, "Operations Research: Applications and Algorithms", Cengage Learning, India. 4/e, 2020.
- 3. R. Panneerselvam, "Operations Research", PHI, Learning (P) Ltd. 2/e.
- 4. M. Sreenivasa Reddy, "Operations Research", Cengage Learning, India, 4/e, 2019.
- 5. N D Vohra, "Quantitative Techniques in Management", McGraw Hill Education(India) Private Ltd, 5/e, 2017.

REFERENCE WEBSITE:

- 1. https://nptel.ac.in/courses/112/106/112106134/
- 2. https://nptel.ac.in/courses/112/106/112106131/
- 3. https://nptel.ac.in/courses/110/106/110106059/



CO\PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO.1	3	2	-	1	-	-	-	-	-	-	-	1
CO.2	3	2	-	1	-	-	-	-	-	-	-	1
CO.3	3	2	-	1	_	-	-	-	-	-	-	1
CO.4	3	2	-	1	-	-	-	-	-	-	-	1
CO.5	3	2	-	1	-	-	-	-	-	-	-	1
CO*	3	2	-	1	-	-	-	-	-	-	-	1

UNIT -1: LINEAR PROGRAMMING MODELS

1. A Company is producing two products A and B. The resource requirements per unit of each of the products and total Availability are given below. Formulate the above Information as a LPP Model and solve it by graphical method.

Product	A	В	Total Availability
Man Hours/Unit	6	3	200 Man Hours
Machine Hours/Unit	2	5	350 Machine Hours
Material/Unit	1kg	2kg	100kgs
Profit	20	25	

2. Solve the following LPP by graphical method

Max
$$Z = 2x_1+3x_2$$

Subjected to
 $x_1+x_2 \le 30$
 $x_1 \le 20$
 $x_2 \le 12$
 $x_2 \ge 3$ and $x_1,x_2 \ge 0$

3. Solve the following LPP by graphical method

Maximize
$$Z= x_1+4x_2$$

Subjected to $3x_1+x_2 \le 3$
 $2x_1+3x_2 \le 6$
 $4x_1+5x_2 \ge 20$ and $x_1,x_2 \ge 0$

4. Solve the following LPP by Graphical method.

Minimize Z=
$$4x+5y$$

Subjected to
 $x+y\ge10$;
 $2x+5y\ge35$ and $x, y\ge0$.

5. Solve the following LPP by graphical method.

$$Max Z = 3X1 + 2X2$$

Subjected to

$$2X1 + X2 \le 2$$

$$3X1 + 4X2 \ge 12$$
 and $X1, X2 \ge 0$

6. Solve the following LPP by simplex method,

Maximize
$$Z = 6x_1 + 9x_2$$

Subjected to

$$x_1 + x_2 \le 12$$

$$x_1 + 5x_2 \le 45$$

$$3x_1+x_2 \le 30$$
 and $x_1, x_2 \ge 0$.

7. Solve the following LPP by simplex method.

Maximize
$$Z=3X_1+2X_2$$

Subjected to

$$X_1 + X_2 \le 4$$

$$X_1-X_2 \le 2$$
 and $X_1, X_2 \ge 0$.

8. Solve the following LPP by simplex method,

Maximize
$$Z = 6x_1 + 9x_2$$

Subjected to

$$x_1 + x_2 \le 12$$

$$x_1 + 5x_2 \le 45$$

$$3x_1+x_2 \le 30$$
 and $x_1, x_2 \ge 0$.

9. Solve the following LPP by simplex method.

$$Max Z = 3X1 + 2X2$$

Subjected to
$$2X1 + X2 \le 2$$

$$3X1 + 4X2 \ge 12$$

and X1,
$$X2 \ge 0$$

10. Solve the following problem by Big M method

Maximize
$$Z= x_1+4x_2$$

Subjected to

$$3x_1 + x_2 \le 3$$

$$2x_1 + 3x_2 \le 6$$

$$4x_1+5x_2 \ge 20$$
 and $x_1,x_2 \ge 0$



11. Solve the following by Big M method.

Maximize
$$Z=12X_1+20X_2$$
.

Subjected to
$$6X_1+8X_2 \ge 100$$
,

$$7X_1 + 12X_2 \ge 120$$
 and $X_1, X_2 \ge 0$.

12. Solve the following LPP by Big M method.

$$Max Z = 3X1 + 2X2$$

Subjected to

$$2X1 + X2 \le 2$$

$$3X1 + 4X2 \ge 12$$
 and $X1, X2 \ge 0$

13. Solve the following problem by two-phase method

Maximize
$$Z= x_1+4x_2$$

Subjected to

$$3x_1 + x_2 \le 3$$

$$2x_1 + 3x_2 \le 6$$

$$4x_1+5x_2 \ge 20$$
 and $x_1,x_2 \ge 0$

14. Solve the following LPP by Two phase method

Min
$$Z = 12x_1 + 20x_2$$

Subjected to

$$6x_1 + 8x_2 \ge 100$$

$$7x_1+12x_2 \ge 120$$
 and $x_1,x_2,x_3 \ge 0$

15. Solve the following LPP by Two phase method

Maximize $Z=X_1+5X_2+3X_3$.

Subjected to
$$X_1+2X_2+X_3=3$$

$$2X_1-X_2=4$$
 and $X_1, X_2, X_3 \ge 0$.



<u>Unit – 2: TRANSPORTATION AND ASSIGNMENT PROBLEMS</u>

1. Find the optimum solution to the following transportation problem.

Factory		Ware	Capacity		
	D	E			
A	1	2	1	4	30
В	3	3	2	1	50
С	4	2	5	9	20
Demand	20	40	30	10	100

2. Solve the following transportation problem

Factory		Ware	Capacity		
	\mathbf{W}_1	\mathbf{W}_2			
F ₁	11	20	7	8	50
F_2	21	16	10	12	40
F ₃	8	12	18	9	70
Demand	30	25	35	40	

3. Find the optimal solution to the following transportation problem.

		Ware			
Factory	W1	W2	W3	W4	Capacity
F1	6	8	8	5	30
F2	5	11	9	7	40
F3	8	9	7	13	60
Demand	35	28	32	25	

4. Find the optimal solution to the following problem

	D_1	D_2	D_3	Availability
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
Requirements	7	9	18	



5. Find the optimal solution to the following transportation problem

	D_1	D_2	D_3	D_4	Availability
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	

6. Find the optimal solution to the following transportation problem

	D_1	D_2	D_3	Availability
S_1	6	10	15	2
S_2	4	6	16	5
S_3	12	5	8	9
Requirements	1	8	7	

7. Find the optimal solution to the following transportation problem

	D_1	D_2	D_3	Availability
S_1	1	2	6	7
S_2	8	4	2	12
S_3	3	7	5	11
Requirements	10	10	10	

8. Solve the following assignment problem.

		Jobs					
		J_1	J_2	J_3	J_4		
ers	\mathbf{W}_1	10	15	24	30		
Workers	\mathbf{W}_2	16	20	28	10		
>	W_3	12	18	30	16		
	W_4	9	24	32	18		

9. The assignment costs of four operators to four machines are given in the following table. What are the operator assignments which minimizes the total cost?

		Operators					
		I	II	III	IV		
SS	A	10	5	13	15		
ine	В	3	9	18	3		
Machines	C	10	7	3	2		
\geq	D	5	11	9	7		



10. A Salesman has to visit five cities A, B, C, D, and E. The distances (in hundred kilometers) between the five cities are shown in table below. If the salesman starts from city A and has to come back to city A, which route should he select so that total distance travelled is minimum?

	A	В	С	D	Е
A	-	3	6	2	3
В	3	-	5	2	3
С	6	5	-	6	4
D	2	2	6	-	6
Е	3	3	4	6	-

11. Find the optimal solution to the following Assignment problem.

	J1	J2	J3	J4
Α	10	15	24	30
В	16	20	28	10
С	12	18	30	16
D	9	24	32	18

12. Solve the travelling salesman problem with the following cost matrix

			Cities			
		A	В	С	D	
	A	8	46	16	40	
Cities	В	41	∞	50	40	
Cit	С	82	32	8	60	
	D	40	40	36	8	

13. Solve the following Assignment problem.

	J1	J2	J3	J4	J5
A	-	4	10	14	2
В	12	-	6	10	4
С	16	14	-	8	14
D	24	8	12	-	10
Е	2	6	4	16	-



14. Solve the following Assignment problem

	J1	J2	J3	J4	J5
A	-	3	6	2	3
В	3	-	5	2	3
С	6	5	-	6	4
D	2	2	6	-	6
Е	3	3	4	6	-

15. A sales man has to visit five cities A,B,C,D, and E. The distances (in 100 km) between any two cities are given in the following table. The sales man starts from A and has to come back to A after visiting all other cities in a cycle. Which route he has to select in order that the total distance traveled by him is minimum?

	A	В	C	D	E
A	-	4	7	D 3 3	4
A B C D E	4	-`	6	3	4
C	7	6	-	7	5
D	3	3	7	-	7
E	4	4	5	7	-



<u>Unit – 3: Network Models and Sequencing Models</u>

1. For a certain project the data is given below. Draw the network diagram, identify the critical path and compute the minimum time required to complete the project.

Activity	1-2	1-4	1-7	2-3	3-6	4-5	4-8	5-6	6-9	7-8	8-9
Expected time	2	2	1	4	1	3	8	4	3	3	5
(in months)											

2. A small project is composed of seven activities whose time estimates in weeks are given below. Draw the network and find the critical path. What is the probability that the project will be completed at least four weeks earlier than expected?

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Optimistic time	1	1	2	1	2	2	3
Most likely time	1	4	2	1	5	5	6
Pessimistic time	7	7	8	1	14	8	15

3. A Project consists of the following activities and time estimation

Activity	Estimated Duration in Weeks						
	Optimistic	Most Likely	Pessimistic				
(1,2)	1	1	7				
(1,3)	1	4	7				
(1,4)	2	2	8				
(2,5)	1	1	1				
(3,5)	2	5	14				
(4,6)	2	5	8				
(5,6)	3	6	15				

- a) Draw the network.
- b) Find the expected time and variance for each activity.
- c) What is the probability that the project will be completed 4 weeks earlier than the expected time.



4. For a certain project the data is given below. Draw the network diagram, identify the Critical path and compute the project duration (in months) and also find the total float.

Activity	1-2	1-4	1-7	2-3	3-6	4-5	4-8	5-6	6-9	7-8	8-9
Time	2	2	1	4	1	3	8	4	3	3	5

5. A small project is composed of seven activities whose time estimates in weeks are given below. Find the critical path. What is the probability that the project will be completed at least four weeks earlier than expected?

activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Optimistic time	1	1	2	1	2	2	3
Most likely time	1	4	2	1	5	5	6
Pessimistic time	7	7	8	1	14	8	15

6. A Project consists of the following activities and time estimation. Draw the network and Find the minimum time required to complete the project.

	duration	Activity	duration
Activity	weeks		weeks
(1,2)	2	(5,8)	5
(2,3)	7	(6,7)	8
(2,4)	3	(6,10)	4
(3,4)	3	(7,9)	4
(3,5)	5	(8,9)	1
(4,6)	3	(9,10)	7

7. A project is composed of the following activities whose time estimates in weeks are given below. Draw the network, identify the critical path and compute the minimum time required to complete the project.

Activity	A	В	C	D	Е	F	G	Н
Immediate predecessor(s)	-	A	A	В	C	C	D,E	F,G
Duration in weeks	3	4	2	5	1	2	4	3

8. A project is composed of the following activities whose time estimates in days are given below. Draw the network, identify the critical path and compute the minimum time required to complete the project.

Activity	A	В	С	D	Е	F
Immediate predecessor(s)	-	A	A	В	C	D,E
Duration in weeks	2	5	1	1	6	1



9. A book binder has one printing press, one binding machine and manuscripts of 7 different books. The times required for performing printing and binding operations for different books are shown in table below. Decide the optimum sequence of processing of books in order to minimize the total time required to bring out all the books.

Book	1	2	3	4	5	6	7
Printing time in hours	20	90	80	20	120	15	65
binding time in hours	25	60	75	30	90	35	50

10. Determine the optimal sequencing of the following 7 jobs on two machines, Machine 1 and Machine 2. Also find the makespan, idle time of Machine 1& 2.

Job	1	2	3	4	5	6	7
Machine 1	3	12	15	6	10	11	9
Machine 2	8	10	10	6	12	1	3



UNIT – 4 Decision Theory & Game Theory

1. Mr Girish wants to invest Rs 10,000 in one of the three options A, B and C. The pay-off for his investment depends on the nature of the economy (inflation (E1), recession (E2) or no change (E3)). The possible returns under each economic situation are given below:

Strategies	Nature of economy							
	E1	E1 E2 E3						
A	2000	1200	1500					
В	3000	800	1000					
С	2500	1000	1800					

What course of action has he to take according to:

- (i) Maximin criterion (ii) Maximax (iii) Hurwicz criterion with alpha = 0.6 and (iv) Laplace criterion?
- 2. A company wants to introduce a new product in place of an old one. It is to be decided whether the price is to be fixed as very high, moderate or slightly increased (H, M or S). Three possible outcomes are expected, viz. increase in sales, no change in sales or decrease in sales (I, N or D). The expected sales are given in the following table (in lakhs of rupees)

Strategies	Events						
	I N D						
Н	70	70 30 1					
M	50	45	0				
S	30	30	30				

Which alternative should be chosen according to (i) Maximin criterion (ii) Maximax (iii) Hurwicz criterion with alpha = 0.7 and (iv) Laplace criterion?

3. Solve the following game by dominance principle.

		Strategies (Player B					
		B1 B2 B3 B4					
	A1	3	2	4	0		
Strategies	A2	3	4	2	4		
(Player A)	A3	4	2	4	0		
	A4	0	4	0	8		



4. Solve the following game graphically..

	B1	B2
A1	-6	7
A2	4	-5
A3	-1	-2
A4	-2	5
A5	7	-6

5. Solve the following game graphically.

	B1	B2	В3	B4
A1	2	1	0	-2
A2	1	0	3	2

6. Solve the following game by dominance principle.

		Strategies (Player B					
		B1 B2 B3 B4					
	A1	5	-10	9	0		
Strategies	A2	6	7	8	1		
(Player A)	A3	8	7	15	1		
	A4	3	4	-1	4		

7. Solve the following game by dominance principle.

		Strategies (Player B					
		B1 B2 B3 B4					
	A1	19	6	7	5		
Strategies	A2	7	3	14	6		
(Player A)	A3	12	8	18	4		
	A4	8	7	13	-1		



8. Solve the following game by dominance principle.

		Strategies (Player B					
		B1	B2	В3	B4		
Stratagias	A1	1	7	3	4		
Strategies (Player A)	A2	0	2	7	8		
(Flayel A)	A3	5	1	6	7		

- 9. Describe the following terms: saddle point, minimax criteria, dominance principle, strategy, zero sum game.
- 10. Solve the following game by dominance principle.

		Strategies (Player B			
		B1	B2	В3	B4
Strategies (Player A)	A1	2	-2	4	1
	A2	6	1	12	3
	A3	-3	2	0	6
	A4	2	-3	7	1



UNIT – 5: QUEUEING THEORY

- 1. At a cycle repair shop on an average, a customer arrives every 5 minutes and on an average the service time is 4 minutes per customer. Suppose that the inter arrival time follows Poisson distribution and the service times are exponentially distributed. Find L s, L q, Ws, W q, shop busy and idle. Assuming that there is only one server.
- 2. In railway yard goods trains arrive at the rate of 30 trains per day. Assuming that the service time follows exponential distribution with an average of 36 minutes, find
 - a) The probability that the number of trains in the yard exceeds 10.
 - b) The average number of trains in the yard.
- 3. In a store with one server,9 customers arrive on an average of 5 minutes. Service is done for 10 customers in 5 minutes. Find
 - a) The average number of customers in the system.
 - b) The average queue length.
 - c) The average time a customer spends in the store.
 - d) The average time a customer waits before being served.
- 4. In a telephone booth, the arrivals follow Poisson distribution with an average of 9 minutes between two consecutive arrivals. The duration of a telephone call is exponential with an average of 3 min.
 - a) Find the probability that a person arriving at the booth has to wait.
 - b) Find the average queue length
 - c) Find the fraction of the day, the phone will be in use
 - d) The company will install a second booth if a customer has to wait for phone, for at least 4 minutes. If so, find the increase in the flow of arrivals in order that another booth will be installed.
- 5. A T.V repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If the repairs sets in the order in which they come, and if the arrival of sets is approximately poisson with an average rate of 10 per 8 hour day. The is repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?
- 6. A foreign bank is considering opening a drive-in window for customer service. Management estimates that customers will arrive for service at the rate of 12 per hour. The teller whom it is considering to staff the window can serve customers at the rate of one every three minutes. Assuming Poisson arrival and exponential service, find: (a) Utilization of teller (b) average number of customers in the system (c) average waiting time in the line (d) average waiting time in the system.
- 7. At a railway station only one train is handled at a time. The yard can accommodate only two trains to wait. Arrival rate of trains is 6 per hour and the railway station can handle



- them at the rate of 12 per hour. Find the steady state probabilities for the various number of trains in the system. Also find the average waiting time of a newly arriving train.
- 8. The capacity of a queuing system is 4. Inter-arrival time of the units is 20 minutes and the service time is 36 minutes per unit. Find the probability that a new arrival enters into service without waiting. Also find the average number of units in the system.
- 9. Patients arrive at a clinic at the rate of 30 patients per hour. The waiting hall cannot accommodate more than 14 patients. It takes 3 minutes on the average to examine a patient.
 - a) Find that probability that an arriving patient need not wait.
 - b) Find the probability that an arriving patient finds a vacant seat in the hall.
 - c) What is the expected duration of time a patient spends in the clinic?