16MEC321 - Heat and Mass Transfer

Lecture Notes

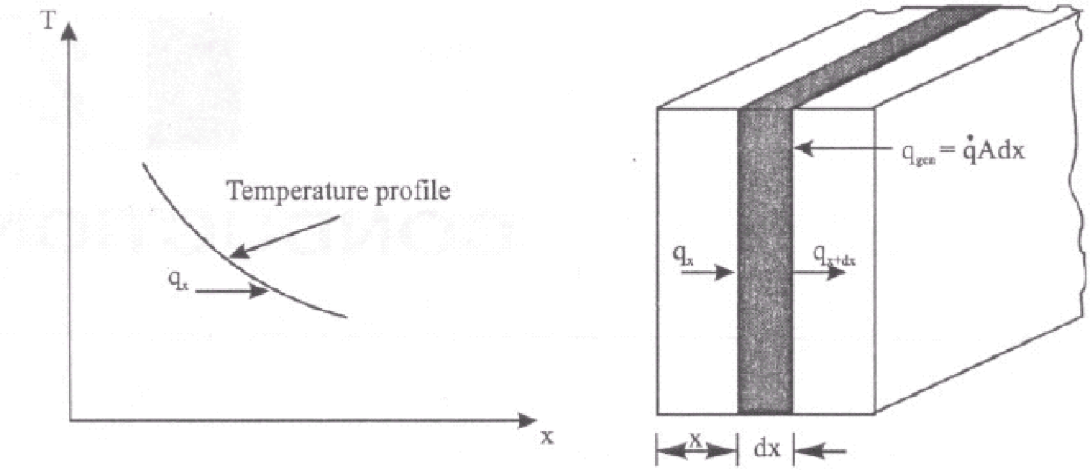
CONDUCTION

INTRODUCTION

- The term *heat conduction* is applied to the mechanism of internal energy exchange from one Body to another, or from one part of the body to another part, by the exchange of kinetic energy of motion of the molecules by direct communication or by the drift of free electrons in the case of I heat conduction in metals.
- This energy transfer takes place from the higher energy molecules to the lower energy molecules. Conduction usually takes place within the boundaries of a body, or across the boundary of a body into another body placed in contact with the first without any appreciable displacement of the matter comprising the body.

ONE AND THREE DIMENSIONAL HEAT CONDUCTION EQUATIONS

- Consider a one dimensional system as shown in Fig 1.
- In the steady state system, the Temperature doesn't change with time.
- If the temperature changes with time the system is known as *unsteady state* system.
- This is the general case where the temperature is not constant.



(a) Variation of temperature *(b) Elemental volume*

Fig 1: One dimensional heat conduction

q_x = Energy conducted in LHS of the element = $-\,kA\,\dfrac{\partial T}{\partial x}$

q_{gen} = Energy generated within the elemental strip = $q\,A\,dx$

q_{x+dx} = Energy conducted out of the RHS of the element = $-\,kA\,\left.\dfrac{\partial T}{\partial x}\right|_{x+dx}$

$\frac{dE}{dt}$ = Change in internal energy

$$\frac{dE}{dt} = \rho\, c\, A\, \frac{\partial T}{\partial t}$$

where

q = density

c = specific heat of material.

q = energy generated per unit volume.

Making energy balance for an elemental strip dx,

$$q_x + q_{gen} = \frac{dE}{dt} + q_{x+dx}$$

$$-kA\frac{\partial T}{\partial x} + qAdx = \rho cA\frac{\partial T}{\partial t}dx - kA\left.\frac{\partial T}{\partial x}\right|_{x+dx}$$

$$= \rho CA\frac{\partial T}{\partial t}dx - A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

Writing in differential form

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + q = \rho C\frac{\partial T}{\partial t}$$

- The above equation is known as *one dimensional heat conduction equation.*
- Generally the heat conduction problem consists of finding the temperature at any time and at any point within a specified solid that has been heated to a known initial temperature distribution.
- Whose surface has been subjected to a known set of *boundary condition.*

Consider a solid as shown in Fig 2 with heat conducting in and out of a unit volume in all Three coordinate directions x, y and z.

$$q_x + q_y + q_z + q_{gen} = q_{x+dx} + q_{y+dy} + q_{z+dz} + \frac{dE}{dt}$$

Making energy balance

$$q_x = -kdydz\frac{dT}{dx};\ q_{x+dx} = -\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]dydz$$

$$q_y = -kdxdz\frac{dT}{dy};\ q_{y+dy} = -\left[k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)dy\right]dxdz$$

$$q_z = -k\,dxdy\,\frac{dT}{dz}\;;\; q_{z+dz} = -\left[k\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)dz\right]dxdy$$

$$\frac{dE}{dt} = \rho c\,dxdydz\,\frac{\partial T}{\partial t}\;;\; q_{gen} = q\,dxdydz$$

Substituting all the values in equations above, general three dimensional heat conduction equation becomes

$$= \left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]dydz - \left[k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)dy\right]dxdz - \left[k\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)dz\right]dydx + \rho c\,dxdydz\,\frac{\partial T}{\partial t}$$

Re arranging and simplifying the above equation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c\frac{\partial T}{\partial t}$$

If thermal conductivity k is constant, the above equation becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{\rho C}{k} \times \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \times \frac{\partial T}{\partial t}$$

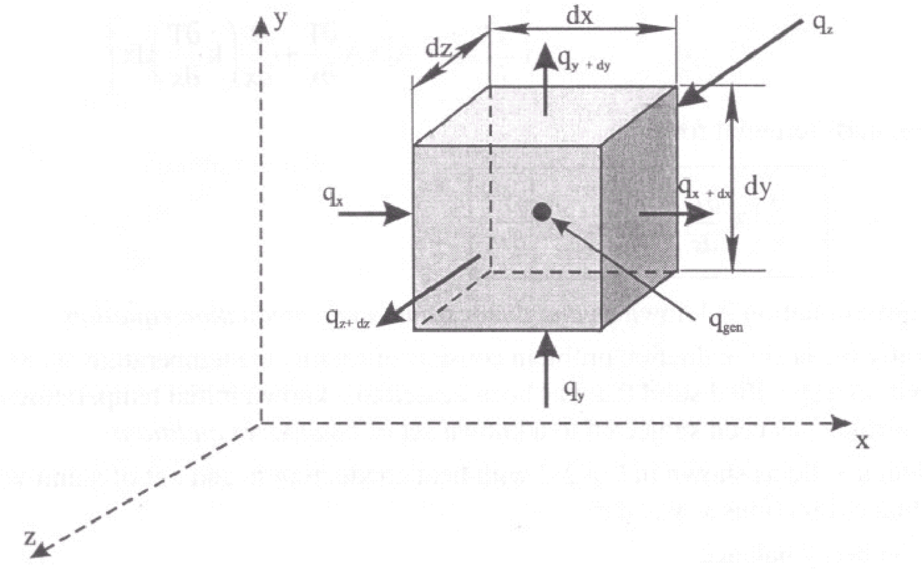


Fig* 2: Three *dimensional heat conduction in Cartesian coordinates

- In the above equation the quantity α is known as *thermal diffusivity* of the material. Rate of heat diffusion through the material is faster if α is higher.
- The term *q c* is known as *thermal heat capacity.* Higher value of a may be either due to higher value of thermal conductivity or lower value of thermal heat capacity.
- Lower value of thermal heat capacity means the energy moving through the material would be absorbed to a lesser degree and used to raise the temperature of the material. This means more energy is available for further transfer.

Cylindrical coordinates

Cylindrical coordinates are expressed in radius (r), axis (z) and longitude (0) as shown Fig 3.

Three dimensional heat conduction equations in cylindrical coordinates is given by

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \times \frac{\partial T}{\partial t}$$

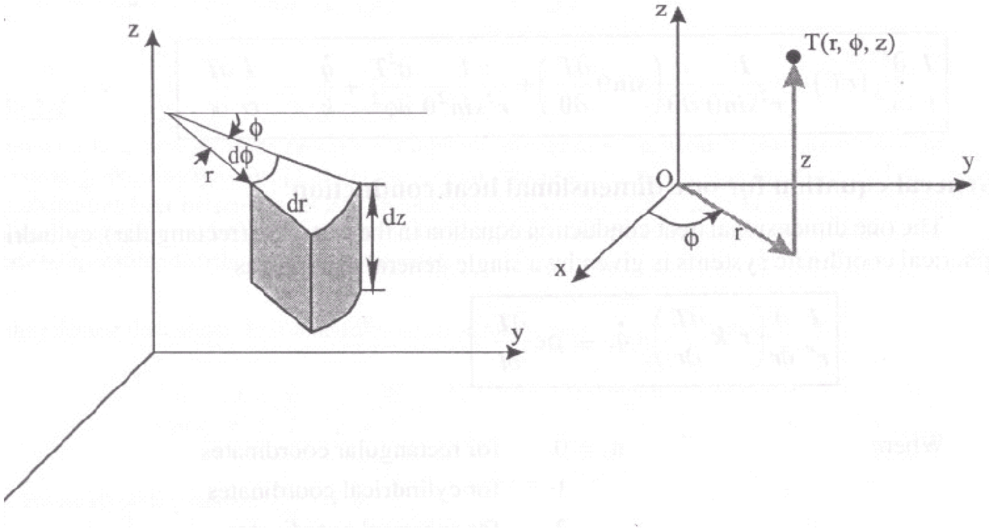


Fig 3: Three dimensional heat conduction cylindrical coordinates z

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Spherical coordinates

Spherical coordinates system expressed in (r, 0, z) is shown in Fig 4.

Three dimensional heat conduction equations in spherical coordinates is given by,

$$\boxed{\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

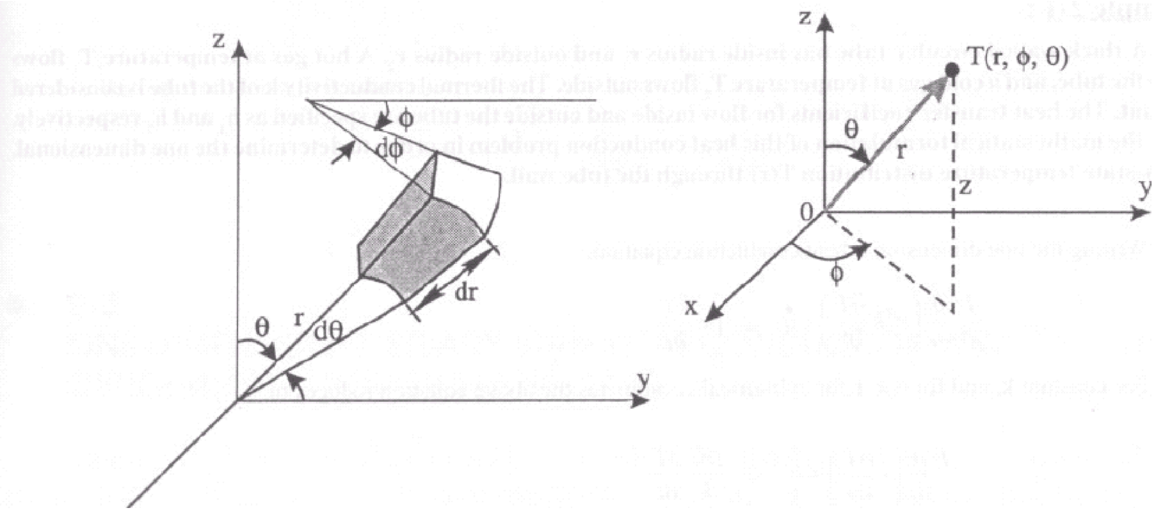


Fig 4: Three dimensional heat conduction spherical coordinates

General equation for one dimensional heat conduction

The one dimensional heat conduction equation in the Cartesian (rectangular), cylindrical and spherical coordinate systems is given by a single general equation as

$$\boxed{\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n k\frac{\partial T}{\partial r}\right)+\dot{q} \;=\; \rho c\frac{\partial T}{\partial t}}$$

Where n = 0 for rectangular coordinates

n = 1 for cylindrical coordinates

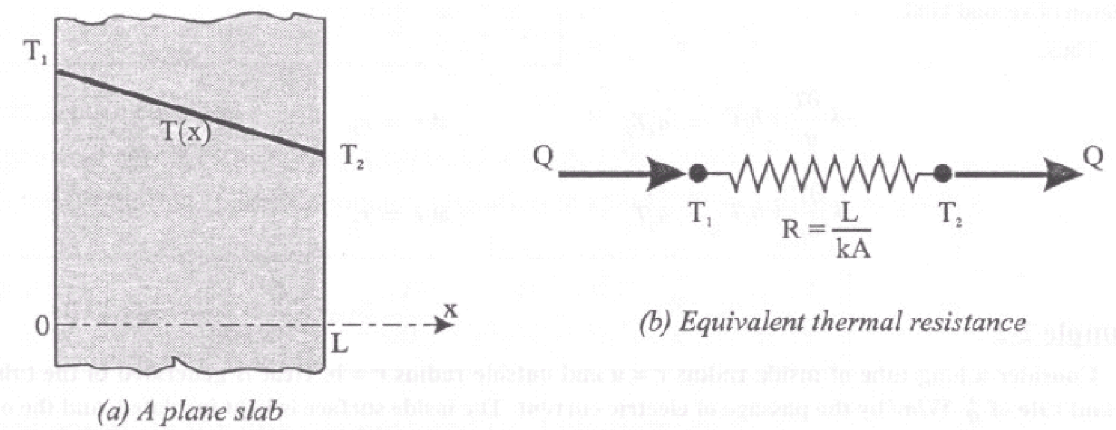
n = 2 for spherical coordinates

ONE DIMENSIONAL STEADY STATE HEAT CONDUCTION WITHOUT HEAT GENERATION THROUGH PLANE SLABS:

- Consider a plane slab of thickness L as shown in Fig 5.
- The plate is assumed to be large Enough along the y and z directions as compared to its thickness L. Hence heat transfer along y and z directions is negligible.

Using the three dimensional heat conduction equation,

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q}{k}=\frac{1}{\alpha}\times\frac{\partial T}{\partial t}$$



(a) A plane slab

(b) Equivalent thermal resistance

Fig 5: Steady state heat conduction without heat generation

Since the conduction takes place under steady state, without heat generation above equation reduces to the following differential equation.

$$\frac{d^2T}{dx^2} = 0$$

The boundary conditions are

At x=0; $T(x) = T_1$; At x=L; $T(x) = T_2$

The solution for the above differential equation is $T(x) = C_1x + C_2$

Where C_1 and C_2 are arbitrary constants

Applying the above boundary conditions,

At x=0; $C_2 = T_1$; At x=L; $T_2 = C_1L + C_2 = C_1L + T_1$

$$C_1 = \frac{T_2 - T_1}{L}$$

Substituting the constants in above equation,

$$T(x) = \left(\frac{T_2 - T_1}{L}\right)x + T_2$$

The heat flow through the slab of area A is given by *Fourier's conduction equation.*

$$Q = -\,\mathrm{kA}\left.\frac{\mathrm{dT}}{\mathrm{dx}}\right|_{x=0} = -\mathrm{kA}\,\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{T_2 - T_1}{L}\right)x + T_2 = -\mathrm{kA}\left(\frac{T_2 - T_1}{L}\right)$$

$$Q = \mathrm{kA}\left(\frac{T_1 - T_2}{L}\right)$$

Rearranging the above results, $Q = \dfrac{T_1 - T_2}{R}$ here $\mathrm{R} = \dfrac{\mathrm{L}}{\mathrm{kA}}$

Where R is called *thermal resistance of the slab.* This concept is analogous to electric resistance.

CONCEPT OF THERMAL RESISTANCE AND ELECTRICAL ANALOGY

Consider a slab of thickness as shown in Fig 6. A fluid at temperature T_{a1} having a heat transfer coefficient h_1 flows over the slab at x =0. Another fluid at a temperature T_{a2}having a heat transfer coefficient h_2 flows over the slab at x =L.

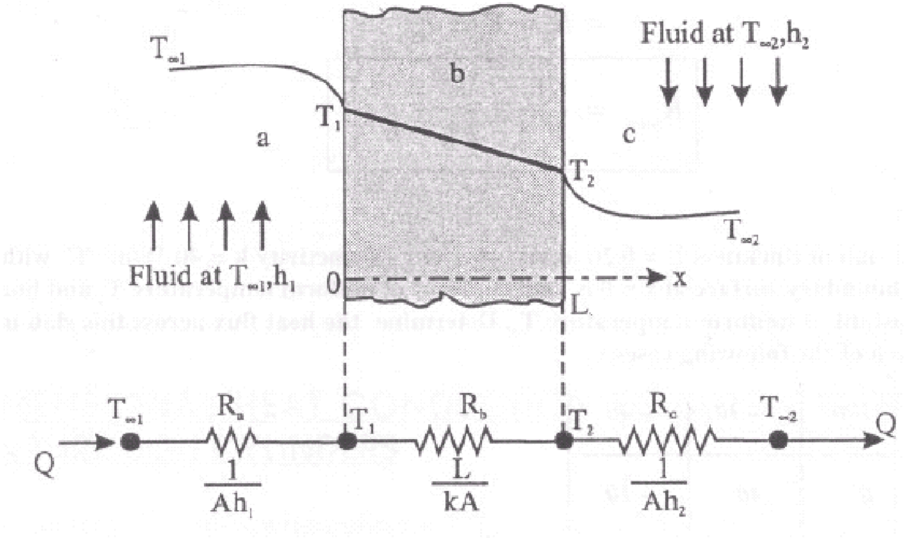


Fig.* 6 *Concept of Thermal Resistance

- As there is *no heat generation* within the medium, heat flow rate through the slab can be determined by using thermal resistance concept.
- Heat flow rate Q is by convection from the fluid at T_{a1} to the surface of the slab at x =0, by conduction through the slab and by convection from the surface at x = L to the fluid at T_{a2}.

$$Q = \mathrm{h}_1\mathrm{A}\left(\mathrm{T}_{\infty1} - T_1\right) = kA\left(\frac{T_1 - T_2}{L}\right) = \mathrm{h}_2\,\mathrm{A}\left(T_2 - T_{\infty2}\right)$$

$$Q = \frac{\left(\mathrm{T}_{\infty1} - T_1\right)}{1/\mathrm{h}_1\mathrm{A}} = \frac{\left(\mathrm{T}_1 - T_2\right)}{L/\mathrm{kA}} = \frac{\left(\mathrm{T}_2 - T_{\infty2}\right)}{1/\mathrm{h}_2\,\mathrm{A}}$$

This is analogous to Ohm's law with each term in the denominator representing the thermal resistance of heat flow through an electric circuit.

Adding the denominators and numerators of the above equation,

$$Q = \frac{\mathrm{T}_{\infty1} - T_{\infty2}}{1/\mathrm{h}_1\mathrm{A} + L/\mathrm{kA} + 1/\mathrm{h}_2\mathrm{A}} = \frac{\mathrm{T}_{\infty1} - T_{\infty2}}{R_a + R_b + R_c} = \frac{\mathrm{T}_{\infty1} - T_{\infty2}}{R_{total}}$$

R_{total}= Total thermal resistance to heat flow total = $R_a + R_b + R_c$

$$R_{total} = 1/\mathrm{h}_1\mathrm{A} + L/\mathrm{kA} + 1/\mathrm{h}_2\mathrm{A}$$

ONE DIMENSIONAL HEAT CONDUCTION EQUATION WITHOUT HEAT GENERATION THROUGH CYLINDERS

- The radial heat flow in solids with cylindrical geometry is of great engineering, importance. A few *examples are heat flow across thick-walled circular tubes, heat loss from a current carrying wire* etc.
- Consider a long cylinder of length L with inside radius r_1 and outside radius r_2 as shown in Fig 7. Let T_1 and T_2 be the corresponding temperatures at r_1 and r_2 respectively.
- As the length of the cylinder is very large compared to diameter, it may be assumed that the heat flows only in a radial direction.

The three dimensional heat conduction equations in cylindrical co-ordinates is given by,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \times \frac{\partial T}{\partial t}$$

For one dimensional steady -state heat conduction without heat generation, above equation can be written in differential form as follows.

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

The boundary conditions are,

At $r = r_1$; $T(r) = T_1$; At $r = r_2$; $T(r) = T_2$

Equation can be rewritten as

$$r\frac{d^2 T}{dr^2} + \frac{dT}{dr} = 0 \; ; \quad \frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrating the above, equation two times

$$r\frac{dT}{dr} = C_1 \text{ or } \frac{dT}{dr} = \frac{C_1}{r} \; ; T = C_1 \ln r + C_2$$

Where C_1 and C_2 are arbitrary constants

Applying the boundary conditions at $r = r_1$ and $r = r_2$, equation becomes

$$T_1 = C_1 \ln r_1 + C_2 \textit{ and } T_2 = C_1 \ln r_2 + C_2$$

Solving the above two equations for C_1 and C_2

$$C_1 = \frac{T_2 - T_1}{\ln\left(r_2/r_1\right)} \quad = T_1 - \left(T_2 - T_1\right)\frac{\ln r_1}{\ln\left(r_2/r_1\right)} \quad ; \quad C_2 = T_1 - C_1 \ln r_1$$

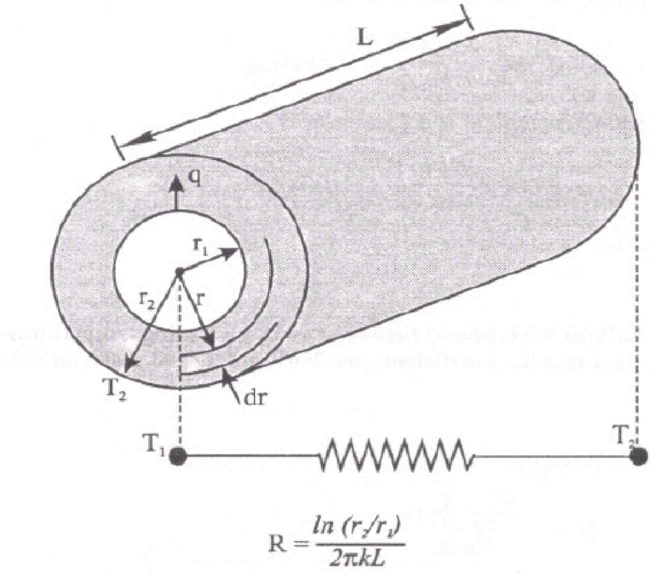


Fig 7: One dimensional steady state heat conduction without heat generation in a cylinder

Substituting the values of C_1 and C_2 in equations

$$T = \frac{(T_2 - T_1)}{\ln\left(r_2/r_1\right)}\left[\ln r + T_1 - \ln r_1\right]$$

The heat flow rate through the cylinder over the surface area A is given by *Fourier*'s *conduction equation.*

$$Q = \frac{2\pi kL(T_1 - T_2)}{\ln\left(r_2/r_1\right)};$$ Rearranging the above equation,

$$Q = \frac{(T_1 - T_2)}{\left(\frac{\ln\left(r_2/r_1\right)}{2\pi kL}\right)} \qquad Q = \frac{(T_1 - T_2)}{R} \qquad \text{and} \qquad R = \frac{\ln\left(r_2/r_1\right)}{2\pi kL}; \quad R = \frac{(r_2 - r_1)\ln\left(\frac{2\pi r_2 L}{2}\right)}{(r_2 - r_1)2\pi kL}$$

$$R = \frac{t}{kA_m}; \qquad A_m = \frac{A_2 - A_1}{\ln\left(A_2/A_1\right)}$$

A_m =logarithmic mean area

$A_1 = 2\pi r_1 L$

A_2 = Outer surface area = $2\pi r_2 L$

ONE DIMENSIONAL STEADY STATE HEAT CONDUCTION WITHOUT HEAT GENERATION THROUGH SPHERES

Consider a hollow sphere of inside radius r_l at temperature T_1 and outside radius r_2 at temperature T_2. The one dimensional heat conduction equation in spherical coordinates is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

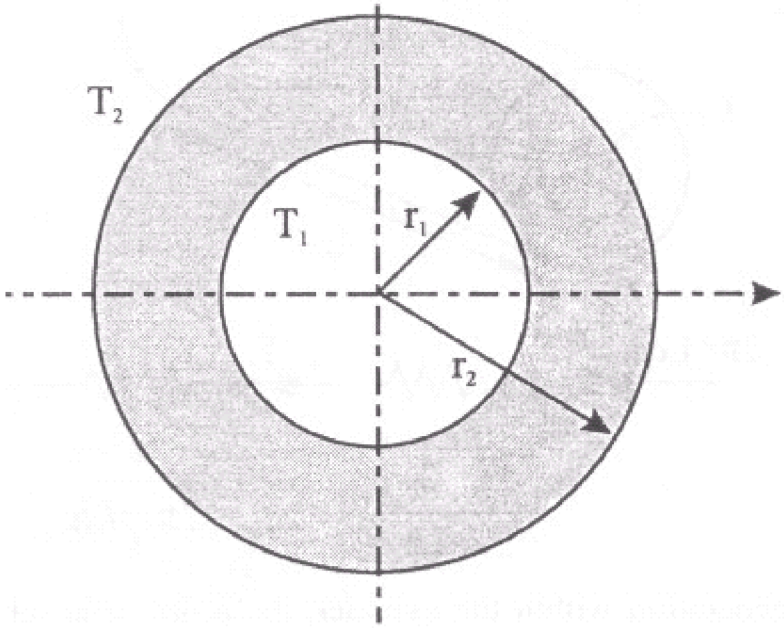


Fig. 8: One dimensional steady state heat conduction in a sphere

For one dimensional steady state heat conduction without heat generation, the above equation reduces to the following differential form.

$\frac{d}{dr}\left(r^2\frac{\partial T}{\partial r}\right)=0$; The boundary conditions are

At $r = r_1$; $T(r) = T_1$; At $r = r_2$; $T(r) = T_2$

Integrating the above equation

$$r^2\frac{dT}{dr}=C_1\ ;\ \frac{dT}{dr}=\frac{C_1}{r^2}$$

Integrating again,

$$T=-\frac{C_1}{r}+C_2$$

Where C_1 and C_2 are arbitrary constants. Using the boundary conditions C_1 and C_2 can be determined as follows.

$T_1 = -\frac{C_1}{r_1} + C_2$; $T_2 = -\frac{C_1}{r_2} + C_2$; Solving the above two equations,

$$C_1 = -\frac{r_1 r_2}{r_2 - r_1}\left(T_1 - T_2\right) ; C_2 = -\frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

Substituting in above equation

$T = \frac{1}{r}\left(\frac{r_1 r_2}{r_2 - r_1}\right)\left(T_1 - T_2\right) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$; On rearranging the temperature distribution,

$$T = \frac{r_1}{r}\left(\frac{r_2 - r}{r_2 - r_1}\right)T_1 + \frac{r_2}{r}\left(\frac{r - r_1}{r_2 - r_1}\right)T_2$$

The heat flow through the hollow sphere is given by,

$$Q = -kA\left.\frac{dT}{dr}\right|_{r=r_1} = -k4\pi r^2 \frac{C_1}{r^2}$$

Substituting C_1 in the above equation,

$Q = 4\pi k \frac{r_1 r_2}{r_2 - r_1}\left(T_1 - T_2\right)$; Rearranging the above equation,

$$Q = \frac{\left(T_1 - T_2\right)}{R} ; R = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

Where R is the *thermal resistance for a hollow sphere.*

HEAT TRANSFER THROUGH COMPOSITE SLAB

- If a medium is composed of several different layers each having different thermal conductivity, then the medium is known as *composite medium.*
- Consider a composite slab made of three parallel layers as shown in Fig 9.
- If Q is the heat flow rate through an area A of the slab and if h_a and h_b are the heat transfer
- Co-efficient at temperatures T_a and T_b respectively, then

$$\frac{Q}{A} = \frac{T_a - T_1}{\left(1/h_a\right)} = \frac{T_1 - T_2}{\left(L_1/k_1\right)} = \frac{T_2 - T_3}{\left(L_2/k_2\right)} = \frac{T_3 - T_4}{\left(L_3/k_3\right)} = \frac{T_4 - T_b}{\left(1/h_b\right)}$$

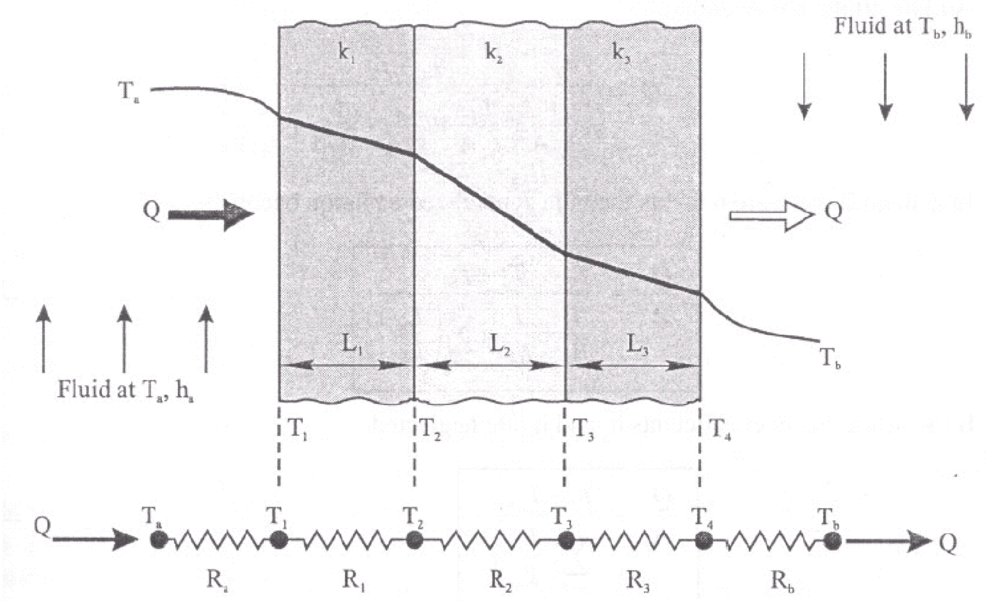


Fig 9: composite slab with equivalent thermal resistance network

Rearranging the above equation,

$$T_a - T_1 = \frac{Q}{Ah_a}\ ;\ \mathrm{T}_1 - T_2 = \frac{QL_1}{Ak_1}\ ;\ \mathrm{T}_2 - T_3 = \frac{QL_2}{Ak_2}\ ;\ \mathrm{T}_3 - T_4 = \frac{QL_3}{Ak_3}\ ;\ T_4 - T_b = \frac{Q}{Ah_b}$$

Adding all the above equations

$$Q = \frac{\mathrm{T}_a - T_b}{\left(1/\mathrm{h}_a\mathrm{A} + L_1/\mathrm{k}_1\mathrm{A} + L_2/\mathrm{k}_2\mathrm{A} + L_3/\mathrm{k}_4\mathrm{A} + 1/\mathrm{h}_b\mathrm{A} \right)}$$

In general if there are n layers then the generalized equation becomes,

$$\frac{Q}{A} = \frac{T_a - T_b}{\frac{1}{h_a} + \frac{1}{h_b} + \sum_{n=1}^{n} \frac{L_n}{k_n}}$$

; If the heat transfer coefficients h$_a$ and h$_b$ are neglected,

$$\frac{Q}{A} = \frac{T_1 - T_{n+1}}{\sum_{n=1}^{n} \frac{L_n}{k_n}}$$

Equation can be modified in terms of equivalent thermal resistance. Thus

$$Q = \frac{T_a - T_b}{R_a + R_1 + R_2 + R_3 + R_b}$$

Where various thermal resistances are represented by R with corresponding subscripts.

Special Cases

i) A composite of two materials in parallel paths

Consider a composite of two materials in parallel paths with their ends maintained at uniform temperatures T_a and T_b. An equivalent thermal resistance network is shown in Fig.10.

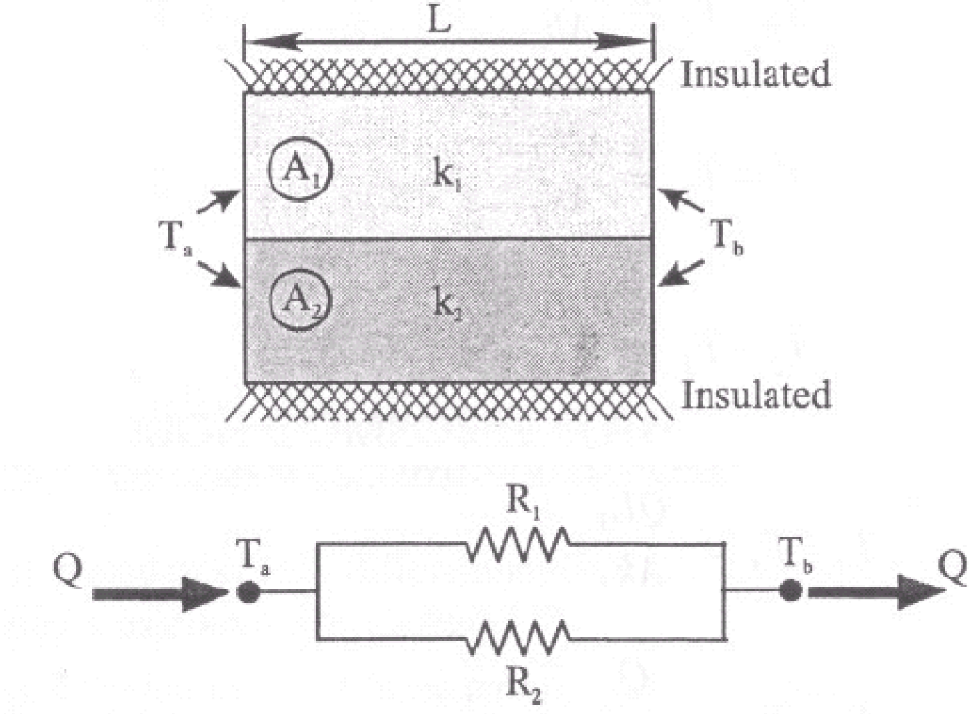


Fig. 10: Composite slab in parallel paths

If A_l and A_2 are the areas of slabs with thermal conductivities k_l and k_2, using the concept of thermal resistance, the heat flow rate Q is given by,

$$Q = \frac{T_a - T_b}{\frac{L}{k_1A_1} + \frac{L}{k_2A_2}} = \frac{T_a - T_b}{\frac{R_1 R_2}{R_1 + R_2}} = \frac{T_a - T_b}{R}$$

;Where $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ = *Equivalent* parallel resistance

ii) A composite wall with materials arranged in series and parallel

Consider a composite wall with materials arranged in parallel and series paths. Approximating heat flow as one directional, the equivalent thermal resistance network is developed as shown in Fig.11. Assuming thermal conductivity of all materials same, the arrangement is analyzed for One dimensional heat conduction.

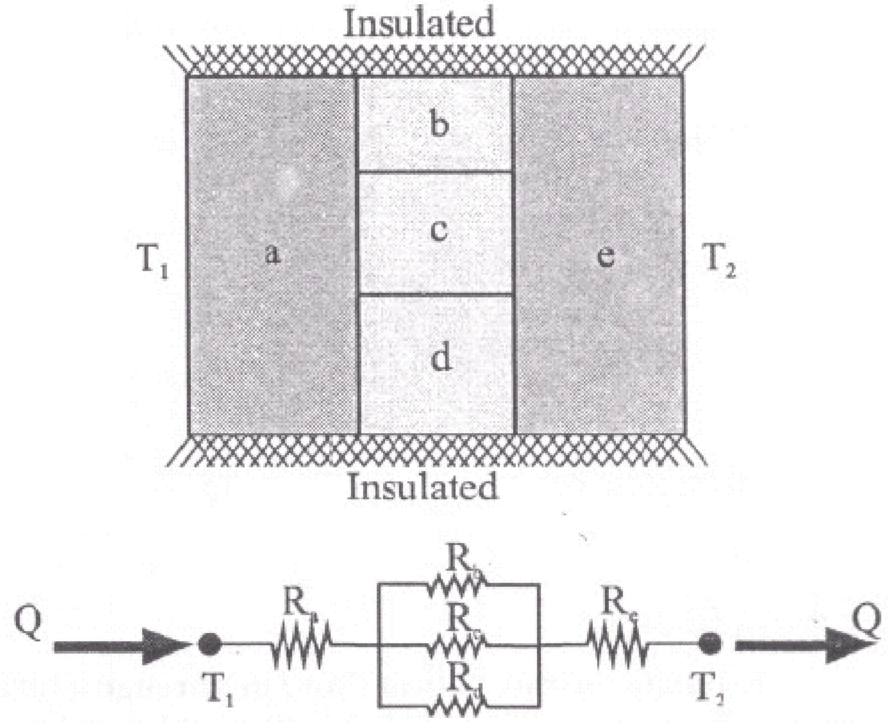


Fig. 11: Composite slab in series and parallel paths

Using the concept of thermal resistance the heat flow rate Q is expressed as,

$$Q = \frac{T_1 - T_2}{R_a + \frac{1}{R} + R_e}$$

$$\frac{1}{R} = \frac{1}{R_b} + \frac{1}{R_c} + \frac{1}{R_d}$$ = Equivalent parallel resistance

HEAT TRANSFER THROUGH COMPOSITE CYLINDERS

- Consider composite coaxial cylinders in perfect thermal contact as shown in Fig.12.
- Let temperature of the hot fluid flowing inside and outside the cylinder be T_a and T_b with respective heat transfer coefficients h_a and h_b

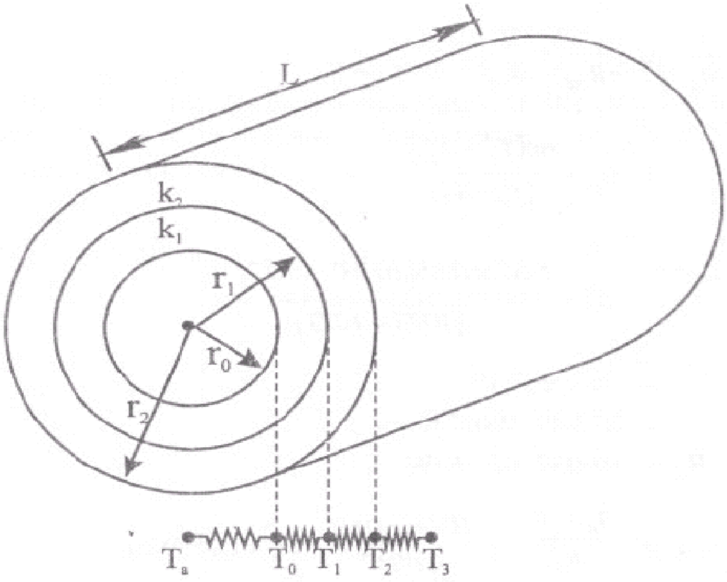


Fig 12: Composite cylinder with equivalent thermal resistance network

If Q is the heat flow rate through surface area A of the cylinder, then,

$$\frac{Q}{A} = \frac{T_a - T_0}{\left(1/h_a \right)} = \frac{k_1 \left(T_0 - T_1 \right)}{\ln\left(\frac{r_1}{r_0} \right)} = \frac{k_2 \left(T_1 - T_2 \right)}{\ln\left(\frac{r_2}{r_1} \right)} = \frac{T_2 - T_b}{\left(1/h_b \right)}$$

Rearranging the above equations,

$$T_a - T_0 = \frac{Q}{A \times h_a}; \; T_0 - T_1 = \frac{Q}{A} \times \frac{\ln\left(\frac{r_1}{r_0} \right)}{k_1}; \; T_1 - T_2 = \frac{Q}{A} \times \frac{\ln\left(\frac{r_2}{r_1} \right)}{k_2}; \; T_2 - T_b = \frac{Q}{A \times h_b}$$

Adding all the above equations,

$$Q = \frac{2\pi L \left(T_a - T_b \right)}{\left[\frac{1}{h_a r_0} + \frac{1}{k_1} \ln\left(\frac{r_1}{r_o} \right) + \frac{1}{k_2} \ln\left(\frac{r_2}{r_1} \right) + \frac{1}{h_b r_2} \right]}$$

In general if there are n concentric cylinders, then the generalized equation becomes,

$$Q = \frac{2\pi L \left(T_1 - T_{n+1} \right)}{\left[\frac{1}{h_a r_0} + \sum_{n=1}^{n} \frac{1}{k_n} \ln\left(\frac{r_{n+1}}{r_n} \right) + \frac{1}{h_b r_{n+1}} \right]}$$

If the heat transfer coefficients h_a and h_b are ignored,

$$Q = \frac{2\pi L \left(T_1 - T_{n+1} \right)}{\left[\sum_{n=1}^{n} \frac{1}{k_n} \ln\left(\frac{r_{n+1}}{r_n} \right) \right]}$$

Above equation can be modified in terms of equivalent thermal resistance,

$$Q = \frac{(T_a - T_b)}{[R_a + R_1 + R_2 + R_b]}$$

HEAT TRANSFER THROUGH COMPOSITE SPHERES

- Consider a composite sphere as shown in Fig.13.
- Let the interior and exterior surfaces be Subjected to two different fluids at temperatures T_a and T_b and heat transfer coefficients h_a and h_b respectively.
- Heat exchange takes place by convection and conduction.

It Q is the rate of heat transfer, then

$$Q = h_a A_a (T_a - T_0) = \frac{4\pi k_1 r_1 r_0 (T_0 - T_1)}{r_1 - r_0} = \frac{4\pi k_2 r_1 r_2 (T_1 - T_2)}{r_2 - r_1} = h_b A_b (T_2 - T_b)$$

Rewriting the above equation,

$$Q = \frac{(T_a - T_0)}{\left[\frac{1}{4\pi h_a r_0^2}\right]} = \frac{(T_0 - T_1)}{\left(\frac{r_1 - r_0}{4\pi k_1 r_1 r_0}\right)} = \frac{(T_1 - T_2)}{\left(\frac{r_2 - r_1}{4\pi k_2 r_2 r_1}\right)} = \frac{(T_2 - T_3)}{\left[\frac{1}{4\pi h_b r_2^2}\right]}$$

Adding the Numerators and Denominators of the equation,

$$Q = \frac{(T_a - T_b)}{\left[\frac{1}{4\pi h_a r_0^2} + \frac{r_1 - r_0}{4\pi k_1 r_1 r_0} + \frac{r_2 - r_1}{4\pi k_2 r_2 r_1} + \frac{1}{4\pi h_b r_2^2}\right]}; \quad Q = \frac{4\pi (T_a - T_b)}{\left[\frac{1}{h_a r_0^2} + \frac{r_1 - r_0}{k_1 r_1 r_0} + \frac{r_2 - r_1}{k_2 r_2 r_1} + \frac{1}{h_b r_2^2}\right]}$$

In general, if there are *n* concentric spheres the above equation modifies to,

$$Q = \frac{4\pi (T_a - T_b)}{\left[\frac{1}{h_a r_0^2} + \frac{1}{h_b r_{n+1}^2} + \sum_{n=1} \frac{r_{n+1} - r_n}{k_n r_n r_{n+1}}\right]}$$

If heat transfer coefficients are ignored, then

$$Q = \frac{4\pi (T_1 - T_{n+1})}{\left[\sum_{n=1}^{n} \frac{r_{n+1} - r_n}{k_n r_n r_{n+1}}\right]}$$; In terms of equivalent electrical circuit, equation can be written as,

$$Q = \frac{(T_a - T_b)}{[R_a + R_1 + R_2 + R_b]}$$

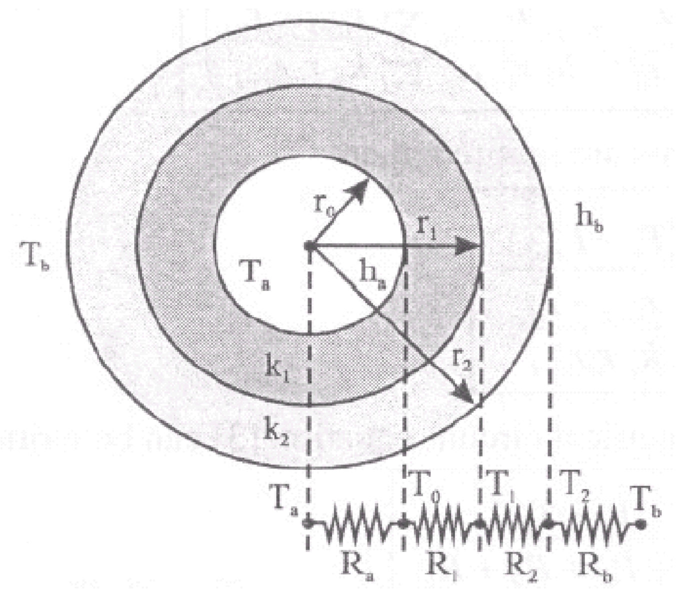


Fig. 13: Heat transfer through composite spheres with equivalent electrical circuit

OVERALL HEAT TRANSFER COEFFICIENT

In many instances it is customary to express the heat flow rate in the cases of single or multilayered plane walls and cylinders with convection at the boundaries in terms of an *overall conductance* or *overall heat transfer, U*

1. PLANE WALL

Consider a plane wall exposed to a hot fluid A on one side and a cold fluid B on the other side. The heat transfer is expressed as,

$$Q = h_a A\left(T_a - T_1\right) = \frac{kA}{L}\left(T_1 - T_2\right) = h_a A\left(T_2 - T_b\right); \; Q = \frac{T_a - T_1}{\left(\frac{1}{h_a A}\right)} = \frac{T_1 - T_2}{\left(\frac{L}{kA}\right)} = \frac{T_2 - T_b}{\left(\frac{1}{h_b A}\right)}$$

Adding the numerators and denominators of the above equation,

$$Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L}{kA} + \frac{1}{h_b A}}; \; Q = \frac{T_a - T_b}{R_a + R_1 + R_b}$$

The overall heat transfer coefficient due to combined heat transfer by convection and conduction is given as,

$$Q = \mathrm{U\,A}\,\Delta T_{overall} = \frac{\Delta T_{overall}}{\frac{1}{\mathrm{UA}}}$$

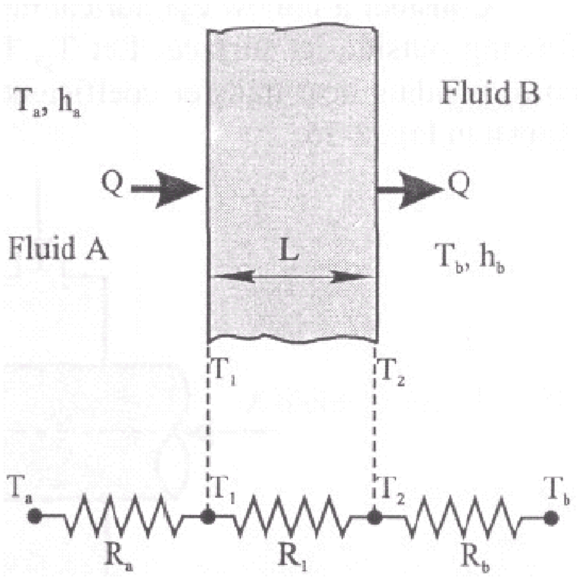


Fig.14: Overall heat transfer coefficient through a plane wall

Comparing equations

$$\frac{1}{U} = \frac{1}{h_a} + \frac{L}{k} + \frac{1}{h_b} = R_a + R_1 + R_b$$

$$U = \frac{1}{\frac{1}{h_a} + \frac{L}{k} + \frac{1}{h_b}} = \frac{1}{\left[R_a + R_1 + R_b\right]}$$

CRITICALTHICKNESSOF INSULATION

- Consider a small diameter pipe, cable or wire exposed to constant outer surface temperature and dissipating heat by convection into surrounding medium (air).
- If the surface is covered by some insulation, it is observed that sometimes increasing the thickness of insulation increases the heat loss until a critical value of thickness of insulation, and further increase in thickness of insulation results in drop in heat loss.
- *The thickness which gives the maximum heat loss* is *known as critical thickness of insulation.*

2. CYLINDER

- Consider a pipe of radius r_i maintained at uniform temperature T_i .covered with a layer of insulation of radius r_o as shown in Fig. 15.
- Let T_a be the temperature of ambient air with heat transfer coefficient h_o. Heat transfer from the outside surface of insulation occurs due to convection into the surrounding air.
- Using the concept of thermal resistance, rate of heat transfer,

R_{ins} = Thermal resistance of insulation $= \frac{1}{2\pi kL} \ln\left(\frac{r_0}{r_i}\right)$

R_o = Thermal resistance of convection at the outer surface $= \frac{1}{2\pi\, r_0 L h_0}$

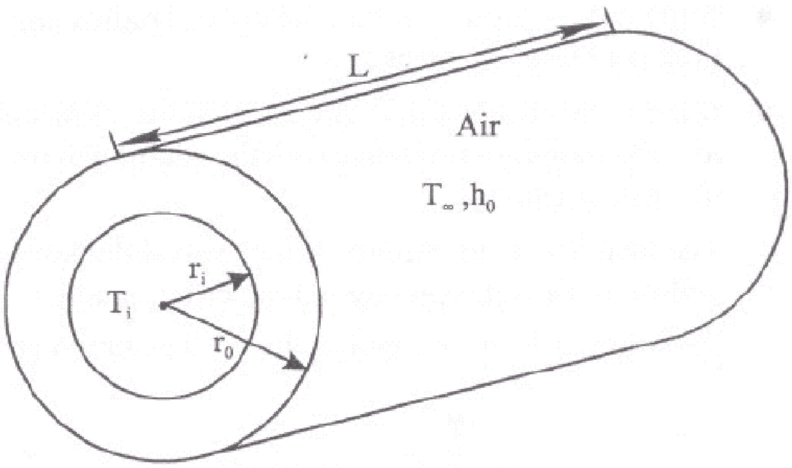


Fig.* 15: Critical *thickness of insulation

In the above equation all the factors T_i , T_a, k, L, h_o and r_j remain constant and only r_o is allowed to vary to find the *maximum heat loss* and hence to find the critical value of radius r_c equation is differentiated W.r.to r_o.

$$\frac{dQ}{dr_0} = \frac{-2\pi\, k\, L(T_i - T_\infty)}{\left[ln\frac{r_0}{r_i} + \frac{k}{(h_0 r_0)}\right]^2}\left(\frac{1}{r_0} - \frac{k}{h_0 r_0^2}\right) = 0$$

$$\frac{2\pi\, k\, L(T_i - T_\infty)}{\left[ln\left(\frac{r_0}{r_i}\right) + \frac{k}{(h_0 r_0)}\right]^2} \times \frac{1}{r_0} = \frac{2\pi\, k\, L(T_i - T_\infty)}{\left[ln\left(\frac{r_0}{r_i}\right) + \frac{k}{(h_0 r_0)}\right]^2} \times \frac{k}{h_0 r_0^2}$$

Simplifying the above equation,

$$r_c = \frac{k}{h_o} \text{ where } r_o = r_c\,;\ r_c = \frac{k}{h_o}$$

Considering the effect of radiation, heat transfer coefficient in the above equation h_o becomes the sum of convection and radiation heat transfer coefficients.

$$h_o = h_c + h_r$$

Physical significance

- If the radius is greater than the critical radius any addition of insulation on the surface of the tube decreases the heat loss.
- If the radius is less than the-critical radius as in small diameter *tubes, wires* or *cables,* the heat loss increases continuously with the addition of insulation until the outer surface radius reaches its critical value.
- The heat loss is maximum at the critical thickness of insulation and becomes lesser with the addition of insulation beyond the critical radius.
- Variation of heat loss with radius of insulation is shown in the *Fig.* 16

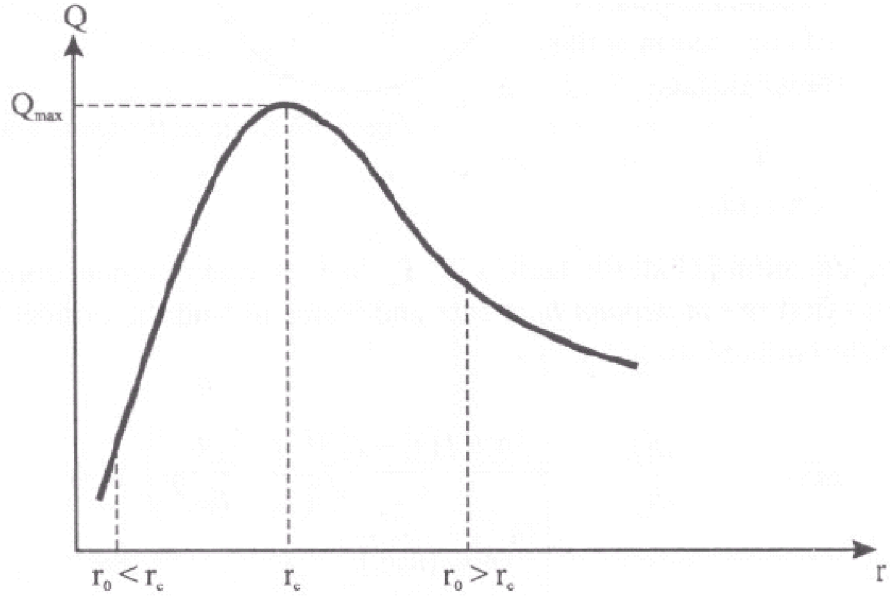


Fig.16. Variation of Q with r

B. SPHERE

$$\boxed{r_c = \frac{2k}{h_0}}$$

FINNED SURFACES

- The rate of heat transfer by convection between a surface and fluid surrounding can be increased by attaching thin strips of metal to the surface. These strips are known *as fins.*

- Fins of a variety of geometries are manufactured for the heat transfer applications as shown in Fig. 17.
- The uses of extended surfaces in practical applications are very large.
- Fins are used for cooling of air cooled engines, on the radiator tubes and heat exchangers. In all the above cases fins are used to increase the rate of heat transfer.

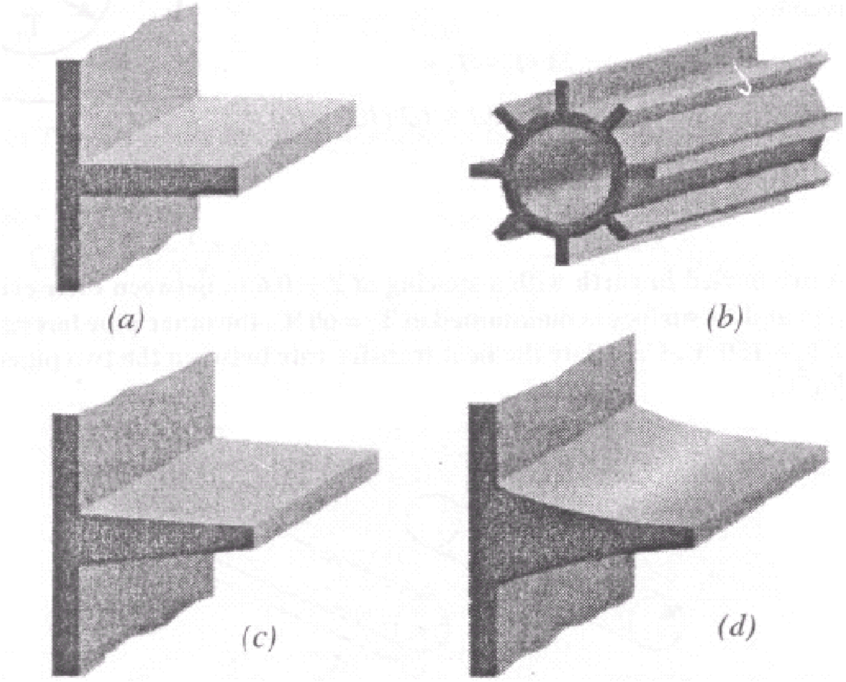


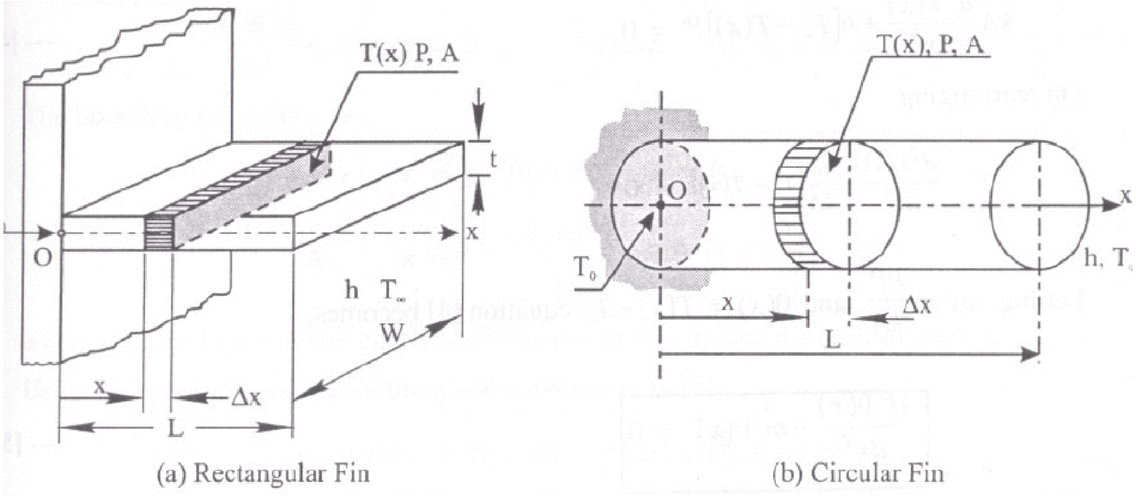
Fig.17: Examples of extended surfaces: (a) and (b), straight fins of uniform thickness; (c) and (d), straight fins of non-uniform thickness.

ONE DIMENSIONAL FIN EQUATION

- The problem of determining the heat flow through a fin requires knowledge of temperature distribution in the fin. This distribution can be found by developing a governing *energy equation.*
- Fig 18 shows nomenclature for the derivation of one dimensional fin equation under steady state conditions.

Consider a fin either *rectangular* or *circular* of uniform cross section subjected to a base temperature T_o and surrounding ambient air at $T_{\oe}$ having a heat transfer coefficient h.

x = Small volume element
A = Cross section area, m^2
P = Perimeter, m
t = Thickness of rectangular fin of width W, m



(a) Rectangular Fin (b) Circular Fin

Fig.* 18: *One – dimensional fin equation

Making steady state energy balance,

$$\begin{bmatrix} \text{Net heat gain along x - direction into volume} \\ \text{element } \Delta x \text{ by conduction} \\ \\ \text{I} \end{bmatrix} + \begin{bmatrix} \text{Net rate of heat gain through lateral surface into} \\ \text{volume element } \Delta x \text{ by convection} \\ \\ \text{II} \end{bmatrix} = 0$$

Net heat gain along x-direction into volume element Ax

$$I = -\frac{d}{dx}\left(qA\right)\Delta x = kA\frac{d^2T}{dx^2}\Delta x$$

Net heat gain through lateral surfaces into volume element Ax

$$\text{II} = h\left[T_\infty - T(x)\right]P\,\Delta x$$

$$kA\frac{d^2T(x)}{dx^2} + h\left[T_\infty - T(x)\right]P = 0$$

On rearranging

$$\frac{d^2T(x)}{dx^2} + \frac{hP}{kA}\left[T_\infty - T(x)\right] = 0 \text{ letting } m^2 = \frac{hP}{kA} \text{ and } \theta(x) = T(x) - T_\infty$$

$$\frac{d^2\theta(x)}{dx^2} + m^2\theta(x) = 0$$

The above equation is known as *one-dimensional fin equation for fins of uniform cross section.*

Equation is a linear, homogeneous, second-order ordinary differential equation with constant coefficients. The general solution for that equation is,

$$\theta(\mathrm{x}) = C_1 e^{-mx} + C_2 e^{mx}$$

The constants C_1 and C_2 are determined from the two boundary conditions specified for the Fin problem.

Equation can be written in terms of hyperbolic sine and cosine as follows.

$$\theta(\mathrm{x}) = C_1 \cosh mx + C_2 \sinh mx = C_1 \cosh m(L-x) + C_2 \sinh m(L-x)$$

The boundary condition for the problem one *at fin base* and the other *at fin tip* are required to eliminate the constants C_1 and C_2 and hence to determine the temperature distribution θ_x) in a fin of uniform cross section.

Usually the temperature at the fin base at x = 0 is known

$$T(x) = T_0 \text{ at } \mathrm{x} = 0 \,;\, \theta(\mathrm{x}) = \theta(0) = T_0 - T_\infty \equiv \theta_0$$

However at the fin tip several different conditions may exist. Three different possible conditions are discussed below.

1. Fins with Convection at the Tip:

Heat transfer by convection between the fin tip and the surrounding fluid is the more realistic boundary condition. The mathematical formulation for the problem becomes,

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \qquad \text{where} \qquad 0 \le \mathrm{x} \le \mathrm{L}$$

The boundary conditions are,

$$At, \mathrm{x=0};\ \theta(\mathrm{x}) = T_0 - T_\infty = \theta_0 \quad ; \quad \text{At x=L ;}\ \frac{\mathrm{k d\theta(x)}}{\mathrm{dx}} + h_f\theta(x) = 0$$

where h_f is the heat transfer coefficient between the fin tip and the surrounding fluid.

Using the general solution for the given equation from [7],

$$\theta(\mathrm{x}) = C_1 \cosh m(L-x) + C_2 \sinh m(L-x)$$

Applying the boundary conditions

At x = 0,

$$\theta_0 = \theta(0) = C_1 \cosh mL + C_2 \sinh mL$$

At x =L; $\theta(\mathrm{L}) = C_1 \cosh m(0) + C_2 \sinh m(0);\ \theta(\mathrm{L}) = C_1 = T_\infty$

$$\frac{d\theta}{dx}(\mathrm{L}) = C_1 m \sinh m(L-L) + C_2 m \cosh m(L-L) = -C_2 m$$

The second boundary condition becomes,

$$-k\,C_2\,m + h_f\,C_1 = 0$$

$$-k\,C_2\,m + h_f\,T_\infty = 0$$

Substituting the values of C_1 and C_2 into the equation

$$\boxed{\frac{\theta(x)}{\theta_0} = \frac{T(x)-T_\infty}{T_0-T_\infty} = \frac{\cosh m(L-x)+(h_f/mk)\sinh m(L-x)}{\cosh mL+(h_f/mk)\sinh mL}}$$

The heat flow through the fin in determined by evaluating the conduction heat transfer over The fin base using the relation.

$$Q = -kA\left.\frac{d\theta(x)}{dx}\right|_{x=0}$$

$$\boxed{Q = \theta_0\sqrt{PhkA}\,\frac{\sinh mL+(h_f/mk)\cosh mL}{\cosh mL+(h_f/mk)\sinh mL}}$$

Long Fin:

For a sufficiently long fin it can be assumed that the temperature at the fin tip approaches the surrounding fluid.

The mathematical formulation for the problem becomes,

Hence heat flow rate,

$$Q = kA\,m\theta_0 = \theta_0\sqrt{PhkA}$$

3. Fins with Negligible Heat loss at the Tip:

In this case the heat transfer area at the fin tip is small compared to the lateral area of the fin.

Hence loss of heat from the fin tip is negligible and the fin tip is assumed to be insulated.

The mathematical equation for this problem becomes,

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \qquad \text{for } 0 \le x \le L$$

$$\boxed{\frac{\theta(x)}{\theta_0} = \frac{T(x)-T_\infty}{T_0-T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}}$$

$$Q = kAm\theta_0 \tanh mL$$

$$= \theta_0\sqrt{PhkA}\tanh mL$$

Important note regarding parameter m :

1. For circular fin:

P = Perimeter =wD

$$m^2 = \frac{hP}{Ak} = \frac{h\pi D}{\left(\frac{\pi}{4}\right)D^2k} = \frac{4h}{kD}$$

2. For rectangular fin:

P = Perimeter = 2 (t + W)

$$\mathrm{m}^2 = \frac{hP}{Ak} = \frac{2h(t+W)}{(W\times t)k}$$

FIN EFFICIENCY

- The temperature of the fin surface away from the base goes on decreasing due to the thermal resistance of the fin material.
- For efficient heat transfer the lateral area of the fin at its base is used than that at the fin tip.
- Heat transfer analysis has been performed for a variety of fin geometries and is presented in terms of a parameter known as *fin efficiency.*

"Efficiency of fin is defined as the ratio of actual heat transfer through fin to the ideal heat transfer through fin if entire fin surface were at base temperature of fin".

$$\eta = \frac{\text{Actual heat transfer}}{\text{Ideal heat transfer from fin at base temperature } T_o} \quad ; \eta = \left(\frac{Q_a}{Q_i}\right)$$

The ideal heat transfer is given by,

$Q_i = a_f\, h\, \theta_o$

a_f = Surface area of fin

h = Heat transfer coefficient

$\theta_o = T_o - T_a$

In practical applications a finned heat transfer surface is composed of *fin surfaces* and the un finned portion. Hence total heat transfer is given by,

$Q_f = Q_a + Q_{unfinned}$

$$\boxed{Q_t = \eta\, a_f h\theta_0 + (a - a_f)\, h\theta_0}$$

Where a = Total heat transfer area which includes both finned and un finned surface.

Equation can be written as,

$Q_t = \eta'\, ah\theta_0$; η'= Area weighed fin efficiency = þη+1- þ

$$\beta = \frac{a_f}{a}$$

It can be noted that eventhough the additions of fins on a surface increases the surface area for heat transfer, it will also increase the thermal resistance over the portion of the surface where the fins are attached. ***To justify the use of fins, the ratio (Pk / Ah) should be much larger than unity.*** Fig. 19 shows the variation of the fin efficiency with parameter $L\sqrt{\frac{2h}{kt}}$

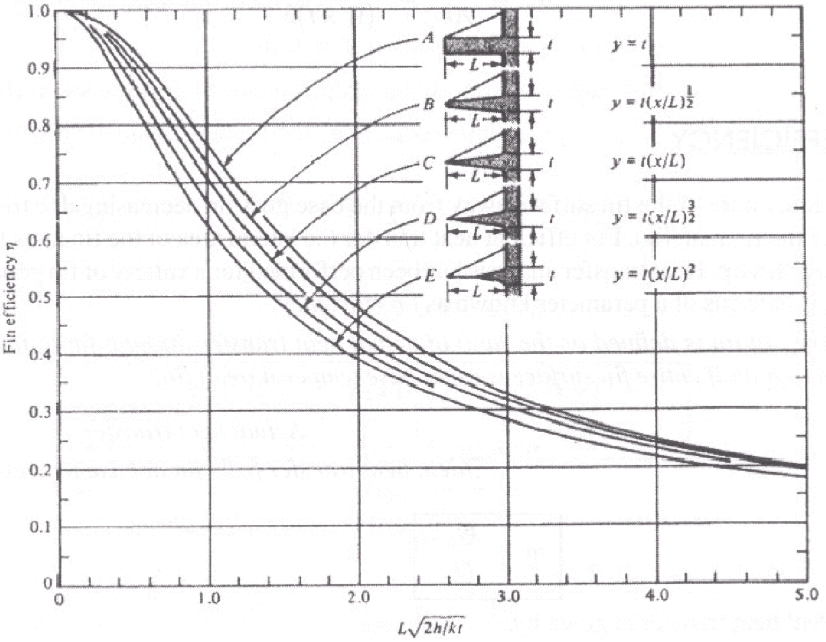


Fig. 19(a): Efficiency of axial fins where the fin thickness y varies with the distance x from the root of the fin where y = t

EFFECTIVENESS OF FIN

- The heat changing capacity of an extended surface relative to that of the primary surface with no fins is useful in defining the *effectiveness* of a fin.
- It is assumed that no contact resistance exists at the fin base in order that the fin base temperature and the primary surface temperature may be taken to be the same.

"The effectiveness of fin is defined as the ratio of heat lost with fin to the heat lost without fin".

$$\varepsilon = \frac{Heat\ lost\ with\ fin}{Heat\ lost\ without\ fin}$$

Value of s should be always greater than 1 if the fin were to be more effective.

For a fin with convection at the tip equation [1] above becomes,

$$\varepsilon = \frac{\theta_0 \sqrt{PhkA}\left(\dfrac{\sinh mL + \left(h_f/mk\right)\cosh mL}{\cosh mL + \left(h_f/mk\right)\sinh mL}\right)}{hA\theta_0}$$

ANALYTICAL SOLUTION FOR TRANSIENT HEAT CONDUCTION IN A SLAB

- There are some instances in which the internal temperature within the solid varies with the position instead of being uniform within the solid.
- For such cases the transient. Temperature charts are not available.
- Also it is not practical to construct charts for all special variations for initial temperature distribution. Hence it is necessary to find the analytical solution for such problems.
- Simple problems involving one-dimensional transient heat conduction in a slab with no generation of internal energy can be solved using the method of *separation of variables.*
- Consider a slab of thickness L confined to the region $0 \le x \le L$.

***The following assumptions are made to find the temperature distribution*.**

- F(x) is the initial temperature distribution which is a prescribed function of position within the solid.
- At t =0, the temperatures at boundary surfaces x = 0 and x = L are suddenly reduced to zero and maintained at that temperature for a duration t > 0.

Mathematical formulation of this problem is,

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \quad \text{in } 0 < x < L, t > 0$$

The boundary conditions are,

At x = 0; t >0; T(x, t) = 0

At x = L; t >0; T(x, t) = 0

The initial conditions is, T(x, t) = F(x) for t =0, $0 \leq x \leq L$

Method of solution

- The solution for the equations is given by the method of separation of variables.
- Thus assume that temperature T(x, t) can be represented by the product of two functions.

$$T(x,t) = \psi(x)\Gamma(t)$$

Where $\psi(x)$ is a function of x only and $\Gamma(t)$ is a function of t only.

- It is easier to solve the ordinary differential equations resulting by splitting the partial differential equation of heat conduction
- By knowing the functions $\psi(x)$ and $\Gamma(t)$ separately, the solution for T(x, t) can be determined by summing the solutions of these functions.

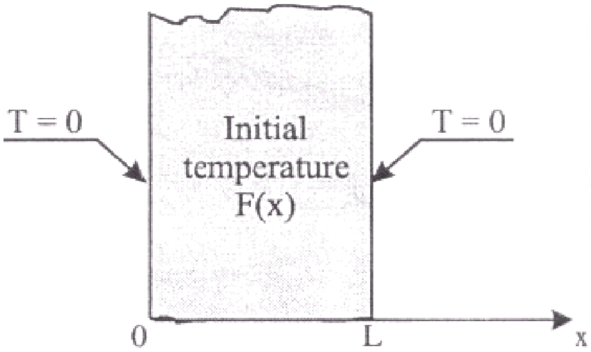


Fig. 20: Transient heat conduction in a slab -Analytical solution

$$\frac{\partial^2 \psi(x)\Gamma(t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \psi(x)\Gamma(t)}{\partial t} \quad \text{i.e., } \Gamma\frac{\partial^2\psi}{\partial x^2} = \frac{1}{\alpha}\psi\frac{\partial\Gamma}{\partial t}$$

$$T(x,t) = \sum_{n=1}^{\infty}\frac{1}{N}e^{-\lambda_n^2\alpha t}.\sin\lambda_n x\int_0^L F(x')\sin\lambda_n x'\,dx'$$

$$where, \quad \frac{1}{N} = \frac{2}{L} \text{ and } \lambda_n = \frac{n\pi}{L}$$

USE OF TRANSIENT TEMPERATURE CHARTS

- In many cases, the temperature gradient within a solid can not be ignored and hence the lumped system analysis doesn't hold good.
- The analysis of heat conduction problems in which both time and position vary is very complicated.
- However for one dimensional case the distribution of temperature is calculated and the results are represented in the form of transient temperature charts.
- In this section use of transient temperature charts for slab, long, cylinder and sphere are discussed.

SLAB

Consider a slab of thickness 2L between the region $-L \leq x \leq L$ .The slab with an initial temperature T_i is suddenly exposed to an ambient temperature of T_a with heat transfer coefficient h at t=0. The slab is maintained for some time t > 0 as shown in Fig. 21.

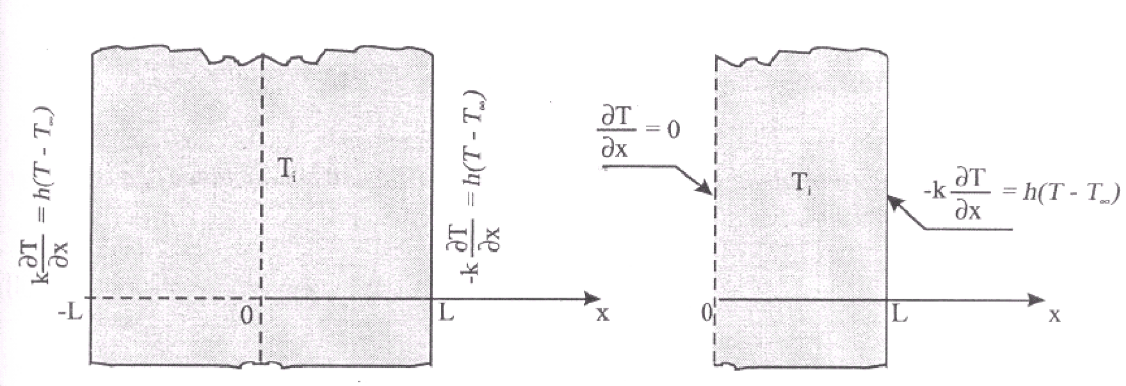


Fig.* 21: *Transient heat conduction in a slab

- Due to geometrical and thermal symmetry about the x-axis at x = 0, we can consider only half portion of the slab.
- The mathematical formulation is based by considering a slab of thickness L in the region 0 < x < L. Thus,

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \textit{confined} \text{ to } 0 < x < L, \text{ for } t > 0$$

Subjected to boundary conditions,

$$At\ x = 0;\ \frac{\partial T}{\partial x} = 0;\ \text{At } x = L;\ k\frac{\partial T}{\partial x} + hT = hT_\infty \quad \text{for } t > 0$$

The transient heat conduction problem can be expressed in the form of dimensionless equations using non-dimensional parameters. Two of them are,

1) Fourier Number (F_o)

- *Fourier number is a measure of the rate of heat conduction in comparison with the rate of heat storage in a given volume element.*
- Hence larger the Fourier number deeper will be the penetration of heat into solid during a given time.

$$F_0 = \frac{\alpha t}{L^2} = \frac{k\left(\frac{1}{L}\right)L^2}{\frac{\rho c_p L^3}{t}}$$

$$F_o = \frac{\text{Rate of heat conduction across L in volume } L^3}{\text{Rate of heat storage in volume } L^3}$$

2) Biot Number (B_i)

Biot number is the ratio of the heat transfer coefficient to the unit conductance of a solid over the characteristic dimension.

$$B_i = \frac{hL}{k} = \frac{h}{\left(\frac{k}{L}\right)} = \frac{\text{Heat transfer coefficient at the solid suiface}}{\text{Internal conductance of solid across length L}}$$

$$\theta = \text{Dimensionless temperature} = \frac{T\left(x,t\right) - T_\infty}{T_i - T_\infty}$$

X = Dimensionless coordinate = x/L

B_o=*Biot number*

F_o = Dimension less time or *Fourier number.*

Then equations can be written as,

$$\frac{\partial^2\theta}{\partial X^2} = \frac{\partial\theta}{\partial F_0} \text{ confined to } 0 < X < 1 \text{ for } F_0 > 0$$

Subjected to boundary conditions,

$$At\ X=0;\ \frac{\partial\theta}{\partial X} = 0 \text{ for } F_0 > 0\ ;\ \text{At } X=1\ ;\ \frac{\partial\theta}{\partial X} + B_i\theta = 0 \text{ for } F_0 > 0$$

$$and\ \theta = 1\ ;\ \text{for } F_0 > 0 \text{ in the region } 0 \le X \le 1$$

ASSIGNMENT QUESTIONS

1. Derive expressions for radial heat transfer and temperature distribution along the radius of a hollow cylinder whose inside and outside surfaces are maintained at steady state temperatures T_1 and T_2 respectively and constant thermal conductivity. Also obtain expression for overall heat transfer coefficient based on inner radius.

2. Define conduction shape factor and thermal diffusivity.

3. Derive an expression for critical thickness of insulation for a hollow sphere and explain its significance.

4. Derive an expression for temperature distribution and heat transfer from an extended rectangular surface of finite length with end insulated.

5. Differentiate between effectiveness and efficiency of heat transfer of extended surface.

6. What is conduction shape factor? Explain.

7. Write a note on thermal contact resistance between two surfaces.

8. The effectiveness of fin should be greater than unity. Explain.

9. Write a note on conduction shape factor.

10. Explain the term effectiveness of fin and express the same in terms of Biot number.

11. A concrete wall of thickness 12 cm has thermal conductivity 0.8 W/m^{o}C. The inside surface is exposed to air at 20°C and the outside surface to air at -18°C. The heat transfer co-efficient for the inside and outside surfaces are 8 W/m^{2}C and 40 W/m^{2}C respectively. Determine the rate of heat loss per square meter of wall surface.

12. A 10 cm OD steam pipe maintained at 130°C is covered with asbestos insulation 3 cm thick (k =0.1W/mC). The ambient air temperature is 30°C and the heat transfer co-efficient for convection at the outer surface of the asbestos insulation is 25

W/m^{2}C. By using thermal resistance concept calculate the rate of heat loss from the pipe per one meter length of pipe.

13. A cylindrical storage tank of radius 0.5 m and length 2.5 m is buried in the earth with its axis parallel to the earth's surface. The distance between the earths surface and the tank axis is 2m. If the tank's surface is maintained at 70°C and the earth's surface is at 20°C, determine the rate of heat loss from the tank. The earth's thermal conductivity may be taken as 1.2 W/m C.

14. A 16 cm diameter pipe carrying saturated steam is covered by a layer of lagging of thickness 40mm (k = 0.8 W/m C). Later an extra layer of lagging 10 mm thick (k = 1.2 W/mC is added. If the sounding temperature remains constant and heat transfer coefficient for both the lagging materials is 10 W/m^{2}C, determine the percentage change in the rate of heat loss due to extra lagging layers Perimeter of fin is 4 cm Surrounding air temperature 30°C

RADIATION

INTRODUCTION

- **If the radiation energy is emitted by bodies because of their temperature it is known as *thermal radiation.***
- The mechanism of radiation is not a simple phenomenon and several theories are proposed to explain the propagation of radiation.
- According to *Maxwell's* theory, "**Radiation is considered as electromagnetic waves, whereas *Max Planck's* concept treats radiation as photons or quanta of energy. However, both accepts are used to predict the emission and propagation of radiation"**.

PHYSICAL MECHANISM

Regardless of the type of radiation, it is propagated at the speed of light. This speed is equal to the product of the wave length and frequency of radiation. Thus

$C = \lambda\nu$

C = Speed of light = 3×10^8m/s

λ = Wave length.

ν =Frequency.

BLACK BODY EMISSIVE POWER

- *The total emissive power or the emissive power is defined as the total emitted thermal radiation leaving a surface, per unit time, per unit area of emitting surface.*
- It is of practical interest to determine the emissive power of a blackbody at an absolute temperature *T in* all directions into hemispherical space.
- Consider an elemental area *dA* maintained, at a temperature *T* as shown in Fig 7.3 (a).

  n = Normal surface of area *dA*

  0 = Polar angle measured from *n*

  ij= Azimuthal angle
- The surface is emitting radiation of spectral intensity $I_{b\lambda}$ in all the directions which is independent of all directions.
- Hence the spectral radiation energy emitted by the surface element *dA* through a solid angle dm in any given direction Ω is given by,

$$Energy = I_{b\lambda}\left(T\right) dA\cos\theta\, d\omega$$

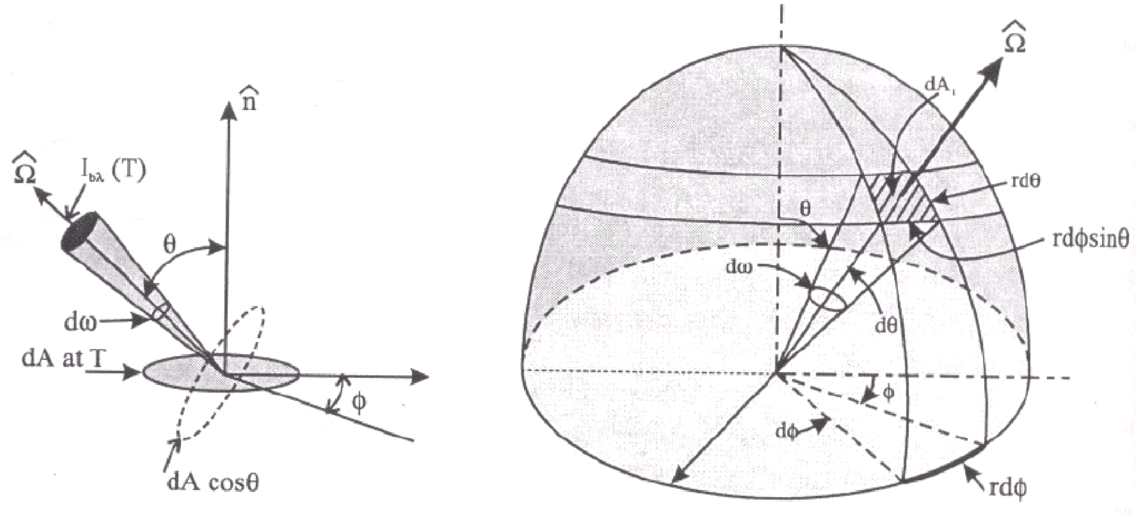
In the above equation dAcos0 is the projection *of* dA on a plane normal to the direction

Q as per the definition *of* I_{bA} (T).

Dividing equation by dA,

$$\frac{Energy}{dA} = \mathrm{I}_{b\lambda}\left(T\right)\cos\theta\, d\omega \ .$$

= Spectral black body radiation energy emitted by a unit area in any direction K and with a solid angle dm.



a) Emission of radiation from surface dA ***b) Solid angle dm in terms of (8,ij)***

Fig.1: Black body emissive power

From figure 1(b) *differential solid angle* dm in terms *of* polar angle 0 and azimuth angle ij is given by,

$$d\omega = \frac{dA}{r^2} = \frac{\left(rd\theta\right)\left(rd\phi\sin\theta\right)}{r^2} = \sin\theta\, d\theta\, d\phi$$

$$\text{Spectral blackbody radiation energy} = \mathrm{I}_{b\lambda}\left(T\right)\cos\theta\,\sin\theta\, d\theta\, d\phi \ ,$$

The spectral blackbody radiation energy emitted in all directions is given by integrating equation over $0 \leq \phi \leq 2\pi$ and $0 \leq \theta \leq \pi/2$.

$$E_{b\lambda} = I_{b\lambda}\left(T\right)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\cos\theta\,\sin\theta\, d\theta . d\phi = 2\pi I_{b\lambda}\left(T\right)\int_{\theta=0}^{\pi/2}\cos\theta\,\sin\theta\, d\theta$$

$$E_{b\lambda} = 2\pi I_{b\lambda}\left(T\right)\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2} = 2\pi I_{b\lambda}\left(T\right)\times\frac{1}{2} = \pi I_{b\lambda}\left(T\right)\ \mathrm{W}/\mathrm{m}^2.\mu m$$

$E_{b\lambda}$ = Spectral blackbody emissive power.

"Spectral blackbody emissive power is defined as the radiation energy emitted by a blackbody at an absolute temperature T per unit area per unit time per unit wavelength about λ in all directions into the hemispherical space".

Introducing the value of $I_{b\lambda}$ *(T)* from *Planck's* function in equation

$$E_{b\lambda} = \frac{2\pi hc^2}{\lambda^5\left\{\exp\left[\frac{hc}{\chi kT}\right]\right\} - 1} \; ; \; E_{b\lambda}(T) = \frac{c_1}{\lambda^5\left\{\exp\left[\frac{c_2}{\lambda T}\right]\right\} - 1}$$

$$c_1 = 2\pi hc^2 = 3.743 \times 10^8 \; \mathrm{W\mu}\, m^4 / m^2 \; ; \; \mathrm{c}_2 = hc/k = 1.4387 \times 10^4 \; \mathrm{\mu\, m.K}$$

λ = wavelength *µm.*

Equation is used to compute blackbody emissive power at any given λ and T.

Wien's Displacement law

- Fig. 2 shows the variation of blackbody emissive power as a function of A at different values of T.
- From the figure it is clear that increasing the temperature, emission of radiation increases for a given wavelength and at any given temperature the emitted radiation varies with wavelength and reaches a peak.
- All the peaks tend to shift towards smaller wavelengths as the temperature increases.
- The locus of these peaks is given by *Wien's displacement law. It states that the product of maximum wavelength and absolute temperature is a constant and is equal to* 2897.6 *µm.K.*

Thus the displacement of the maximum monochromatic emissive power is given by

$$(\mathrm{LT})_{\max} = 2897.6$$

"Monochromatic emissive power of a black body is defined as the rate of energy emission per unit area at a particular wave length λ".

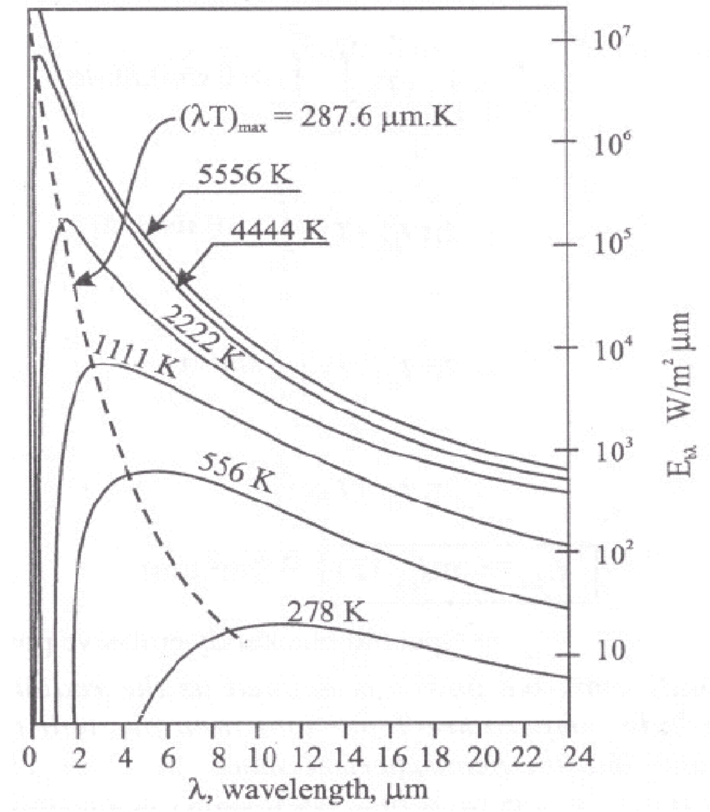


Fig 2. Spectral black body emissive power at different temperatures.

LAMBERTSCOSINE LAW

- The radiation *emanating* from a point on a surface is termed *diffuse* if the intensity $I_b\ (T)$ is constant.
- Consider a small black body surface *dA* located arbitrarily at a point in a space and emitting radiation in all directions. A blackbody radiation collector through which the radiations pass is located at an angle 0 (zenith angle) towards the surface normal *(n)* and the azimuth angle 0 of a spherical coordinate system.
- The collector subtends a solid angle dm when viewed from a point on the emitter.
- The intensity of radiation $I_b\ (T)$ is the energy emitted over all wavelengths in a particular direction per unit surface area through a solid angle dm.
- Again the area is the projected area of the surface on a plane perpendicular to the direction K.
- Depending on the angular position of the collector, the radiation measured by the collector varies and the maximum amount of radiation is measured when the collector is normal to the emitter.

- The maximum intensity of radiation measured normal to the emitter surface at an angle 0 is given by *Lambert's cosine law.*

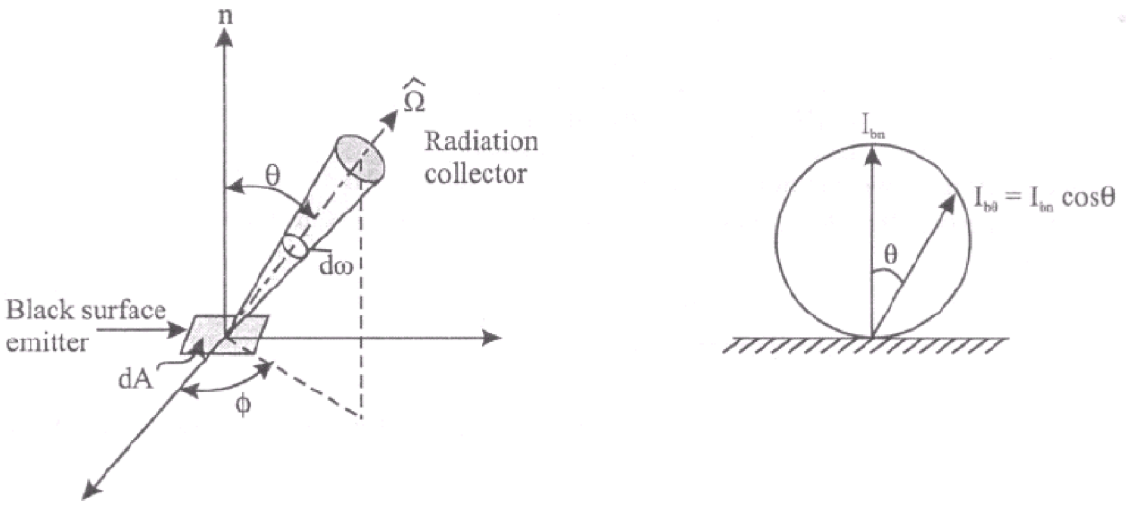


Fig 3: Spatial distribution of radiations emitted from a surface element.

“Lambert's cosine law states that the intensity of radiation in a direction & from the normal to a black emitter is proportional to the cosine of angle &”.

Thus mathematically we can write,

Intensity of radiation along the normal

I_{bn} a cos 0; where 0=0

Intensity of radiation at an angle 0 from the normal,

$$\frac{I_{bn}}{I_{b\theta}} = \frac{\cos 0}{\cos\theta};\ I_{b\theta} = I_{bn}\cos\theta$$

When the collector is oriented at an angle 8*J* from the normal to the emitter then the radiation by the collector is

$$\left(dE_b\right)_{\theta 1} = I_{b\theta 1}.d\omega_1.dA = I_{bn}.\cos\theta_1.d\omega_1.dA$$

Since the collector could be located at different angular positions, then for a solid angle dm$_2$ subtended by the collector at the surface of the emitter is given by,

$$\left(dE_b\right)_{\theta 2} = I_{b\theta 2}.d\omega_2.dA = I_{bn}.\cos\theta_2.d\omega_2.dA$$

From equations it is clear that for any surface located at an angle 0 from the normal and subtending a solid angle dm at the emitter,

$$\left(dE_b\right)_{\theta} = I_{bn}.\cos\theta.d\omega.dA$$

From above equation we can notice that the energy radiated out decreases with increase in angle 0 and becomes zero at 0 =*90*° *.* Also maximum energy is radiated out when 0 =*0* i.e., along the normal direction. Thus, for 0 = 0, I_{b0} = I_{bn}

$$I_{b\theta} = I_{bn}$$

STEFAN-BOLTZMAN LAW

"Stefan-Boltzman law states that the amount of radiant energy emitted per unit time from unit area of black surface is proportional to the fourth power of its absolute temperature".

$$E_b(T) = \sigma T^4$$

o = Stefan-Boltzman constant

= 5.67×10^{-8} W/m^{2}K^4

From *Planck's law,* monochromatic emissive power of a black body is

$$E_{b\lambda}(T) = \frac{c}{\lambda^5 \left\{\exp\left[c_2/\lambda T\right]\right\} - 1}$$

Hence the radiation energy emitted by a black body at an absolute temperature *T* over all wavelengths per unit time per unit area can be determined by integrating the above equation from L =*0* to L= œ.

$$E_b(T) = \int_{\lambda=0}^{\infty} \frac{c_1}{\lambda^5 \left\{\exp\left[c_2/\lambda T\right]\right\} - 1} d\lambda$$

$$\frac{c_2}{\lambda T} = x;\ \lambda = \frac{c_2}{xT}\ \text{or}\ d\lambda = \left(\frac{-c_2}{x^2 T}\right) dx$$

The limit changes as follows: At L =*0; x* = œ and at L = œ; *x*=*o*

$$E_b(T) = -c_1 \int_{\lambda=0}^{\infty} \frac{x^5 T^5 c_2}{c_2^5 \left\{\exp(x) - 1\right\} x^2 T} dx = \frac{c_1 T^4}{c_2^4} \int_0^{\infty} x^3 \left\{\exp(x) - 1\right\}^{-1} dx$$

$$E_b(T) = \frac{c_1 T^4}{c_2^4} \int_0^{\infty} x^3 \left\{\exp(-x) + \exp(-2x) + \exp(-3x) + ...\right\} dx$$

The above integral is *of* the *form*

$$\int_0^{\infty} x^n \left\{\exp(-ax)\right\} dx = \frac{n!}{a^{n+1}}$$

Substituting the values *of* c_1 and c_2

$$E_b(T) = \frac{c_1 T^4}{c_2^4}\left[\frac{3!}{1^4} + \frac{3!}{2^4} + \ldots\ldots\right] = \frac{c_1 T^4}{c_2^4} \times 6.48 = 5.67 \times 10^{-8}\,\mathrm{T}^4 \quad \mathrm{W}/\mathrm{m}^2K^4$$

$$E_b(T) = 5.67\left(\frac{T}{100}\right)^4 = \sigma T^4$$

A relation can be established between *E_b (T)* and *I_b (T).* We have already proved that

$$E_{b\lambda}(T) = \pi\, I_{b\lambda}(T)$$

Integrating the above equation *for* all wavelength *from* L=*0* to L = œ, we can write

$$E_b(T) = \pi\, I_b(T);\; I_b(T) = \frac{1}{\pi}\sigma T^4$$

BLACK BODY RADIATION FUNCTIONS

- Normally a body radiating heat is simultaneously receiving heat in the form of radiation from other bodies.
- If a surface maintained at temperature *T_1* is completely enclosed by another black surface at temperature *T_2,* then the net radiant heat flux is given by,

$$Q = \sigma\left(T_1^4 - T_2^4\right)$$

- In numerous applications one is always interested in the amount of energy radiated from a blackbody in a certain specified wavelength range.
- Say from L = 0 to L as a function of total emission from L = 0 to L = œ.This fraction of total energy radiated between *0* and L is given by,

$$\frac{\mathrm{E}_{b\lambda}(T)}{T^5} = \frac{c_1}{\left(\lambda T^5\right)\exp\left[c_2/(\lambda T)\right] - 1}$$ But,

Dividing above equation by *T^5* on both sides,

$$\frac{\mathrm{E}_{b\lambda}(T)}{T^5} = \frac{c_1}{\left(\lambda T^5\right)\exp\left[c_2/(\lambda T)\right] - 1}$$

Thus the integrals in equation can be expressed in terms of a single variable L*T* using above equation. These results are tabulated by *Dunkle.*

To determine the radiant energy emitted between wavelength L_1 and L_2 is required, then

$$E_{b(\lambda_1-\lambda_2)}(T) = E_{b_{0-\infty}}\left(\frac{E_{b_{0-\lambda_2}}}{E_{b_{0-\infty}}} - \frac{E_{b_{0-\lambda_1}}}{E_{b_{0-\infty}}}\right)$$

$E_{b_{0-\infty}} = \sigma T^4 =$ Total radiation emitted over all wavelengths

RADIATION PROPERTIES

When radiant energy strikes a material surface, part of the radiation is reflected, part is absorbed, and part is transmitted as shown in Fig 4.

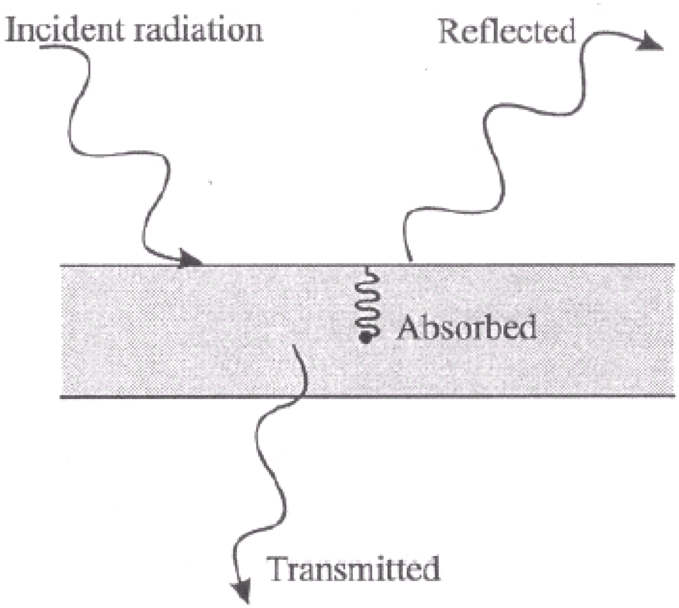


Fig 4: Effects of incident radiation

If Q_i is the incident radiant energy, Q_r is energy reflected, Q_a is the energy absorbed and Q_t is the energy transmitted, then

$$Q_i = Q_r + Q_a + Q_t$$

Dividing throughout by Q_i,

$$1 = \frac{Q_r}{Q_i} + \frac{Q_a}{Q_i} + \frac{Q_t}{Q_i}; 1 = \rho + \alpha + \tau$$

q = Reflectivity or fraction reflected

a = Absorptivity or fraction absorbed

T =Transmissivity or fraction transmitted

- Most of the solid bodies do not transmit thermal radiation and hence their transmissivity may be taken as zero. Such bodies are known as *opaque* bodies. For opaque bodies T = 0;q + a = 1

- *When a radiation strikes a surface, two type of reflection phenomenon are observed. Viz, specular reflection and diffuse reflection as shown in Fig 5.*

1. Specular reflection

- When radiation is incident on a real surface, a fraction of total incident energy is reflected by the surface.
- For a perfectly smooth surface where the roughness of the surface is much smaller than the wavelength of radiation, the incident and reflected rays lie symmetric with respect to the normal at the point of incidence.
- This mirror like reflection wherein the angle of incidence is equal to the angle of reflection, the reflection is called as *specular reflection.*

2. Diffuse reflection

- When radiation is incident on a real surface which is rough, the incident radiation is scattered in all directions.
- For such cases an ideal assumption is that the intensity of the reflected radiation is constant for all angles of reflection and independent of the direction of the incident radiation.
- Thus, when the incident beam is distributed uniformly in all directions after reflection, the reflection is called *diffuse reflection.*

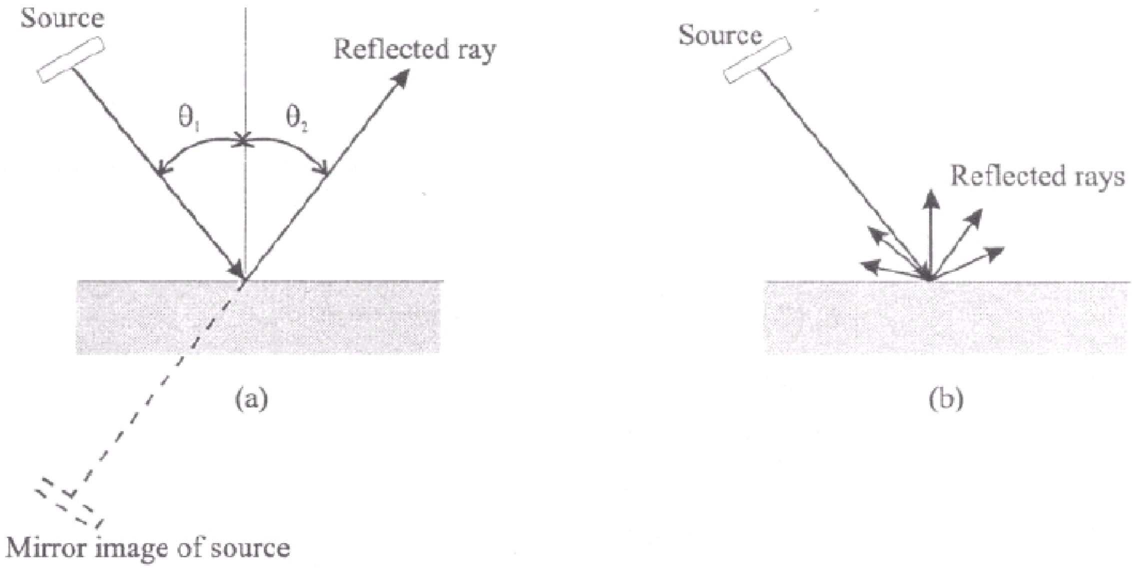


Fig 5: (a) Specular reflection ($8_1= 8_2$) (b) Diffuse reflection

KIRCHOFF'S LAW

- The absorptivity and emissivity of a body can be related by *Kirchoff's law of radiation.*
- Consider a perfectly black enclosure which absorbs all the incident radiation falling upon it as shown in Fig. 6.
- This enclosure will emit radiation according to the Stefan-Boltzman law. Let the radiant flux arriving at some area in the enclosure *be* q_i W/m^2.
- Suppose if the body is placed inside the enclosure and allowed to reach the equilibrium temperature with it. For this to happen there should not be an energy flow into or out of the body which would otherwise increase or lower its temperature.
- For equilibrium, the energy absorbed by the body must be equal to the energy emitted.

$$E(T)\,A = q_i A \times \alpha$$

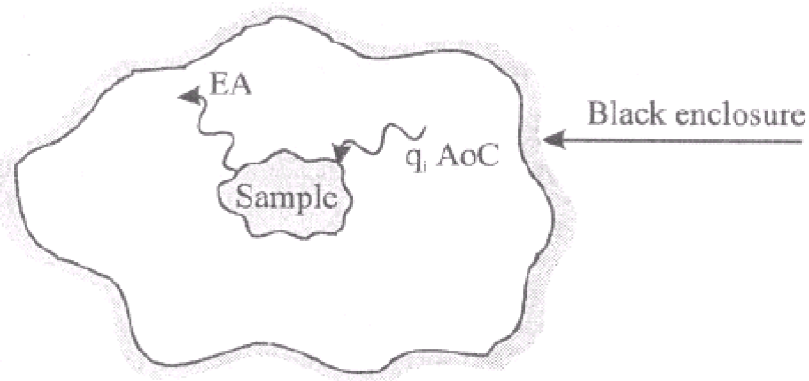


Fig 6: Kirchoff's law

When the body is replaced in an enclosure of a blackbody of the same size and shape, the enclosure will reach the equilibrium at the same temperature.

E_b (T) A = q_iA × 1; a=1 for a block body

$$\frac{E(T)}{E_b(T)} = \alpha(T);\ \varepsilon(T) = \alpha(T) \qquad \frac{E(T)}{E_b(T)} = \varepsilon(T)$$

"But, the ratio of the emissive power of a body to the emissive power of a blackbody is known as emissivity".

This is known as Kirchoff's law of radiation.

"Kirchoff's law states that spectral emissivity for the emission of radiation at temperature T is equal to the spectral absorptivity for radiation coming from a blackbody at the same temperature T".

$$\varepsilon_\lambda(T) = \alpha_\lambda(T)$$

GRAY BODY

- To simplify the analysis of radiative heat transfer, the radiative properties are assumed to be uniform over the entire wavelength spectrum.
- Thus *a gray body is defined as the body having monochromatic emissivity* is *independent of wavelength.*
- *The monochromatic emissivity is defined as the ratio of the monochromatic emissive power of the body to the monochromatic emissive power of a blackbody at the same temperature and wavelength.*

Thus,

$$\varepsilon_{\lambda}(T) = \frac{E_{\lambda}(T)}{E_{b\lambda}(T)}$$

Now, total emissivity of a body is given by

$$\varepsilon = \frac{E(T)}{E_b(T)} = \frac{\int_0^{\infty} \varepsilon_{\lambda} E_{b\lambda}\, d\lambda}{\int_0^{\infty} E_{b\lambda}\, d\lambda} = \frac{\int_0^{\infty} \varepsilon_{\lambda} E_{b\lambda}\, d\lambda}{\sigma T^4}$$

For a gray body, s_L= constant

$$\varepsilon_{\lambda} = \varepsilon = \frac{\int_0^{\infty} \varepsilon_{\lambda} E_{b\lambda}\, d\lambda}{\sigma T^4}$$

CONFIGURATION FACTOR (OR) SHAPE FACTOR (OR) VIEW FACTOR

- Most of the engineering applications involve radiation exchange between two or more surfaces. The radiation exchange among the surfaces is unaffected by the medium when the surfaces are separated by a non participating medium like vacuum.
- Air and other gases also closely approximate this condition. The orientation of the surfaces plays an important role in radiation heat exchange.
- The effects of orientation in the analysis of radiation heat exchange among the surfaces are formalized by using the concept of *view factor.* It is also known as *shape factor, angle factor, and configuration factor.*
- The view factor may be *specular* or *diffuse* view factor. *Diffuse view factor* is used for the surfaces which are *diffuse reflectors* and *diffuse emitters.*

- *Specular view factor* is used for the surfaces which are *diffuse emitters* and *specular reflectors.*
- The physical significance of the view factor is that *it represents the fraction of the radiative energy leaving one surface that strikes directly the other surface.*

VIEW FACTOR BETWEEN TWO ELEMENTAL SURFACES

Consider two elemental surfaces $dA_1$and dA_2 as shown in figure 7.

Let

r = Distance between the two surfaces.

0_1 = Polar angle between the normal N_1 to the element dA_1 and line r joining dA_1 *to* dA_2.

0_2 = Polar angle between the normal N_2 to the surface element dA_2 and the line r.

dv_{12} = Solid angle under which an observer at dA_1 sees the surface element dA_2.

I_1 = Intensity of radiation leaving the surface dA_1in all directions in hemispherical space.

The rate of radiative energy dQ_1 *leaving* dA_1 and ultimately striking dA_2 is,

$$dQ_1 = dA_1 I_1 \cos\theta_1 d\omega_{12} \; ; \; d\omega_{12} = \frac{dA_2 \cos\theta_2}{r^2}$$

Hence equation [1] becomes

$$dQ_1 = dA_1 I_1 \cos\theta_1 \frac{dA_2 \cos\theta_2}{r^2} = \frac{dA_1 dA_2 \cos\theta_1 \cos\theta_2}{r^2}$$

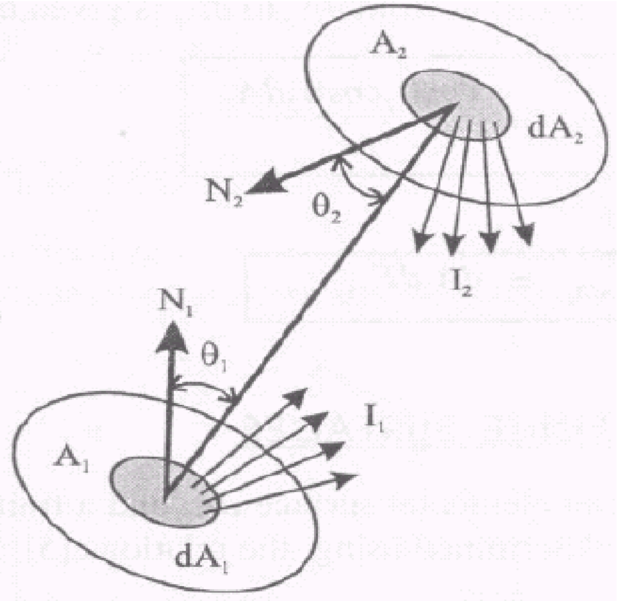


Fig 7: View factor between two elemental surfaces

The rate of radiation energy Q1 .leaving the surface element dA1 in all directions over hemispherical space is,

$$Q_1 = dA_1 \int_{\phi = 0}^{2\pi} \int_{\theta = 0_1}^{\pi/2} I_1 \cos\theta_1 \sin\theta_1 \, d\theta_1 \, d\phi$$

ij =Azimuthal angle.

For a diffusively reflecting and diffusively emitting surface, the radiation intensity leaving the surface is independent of direction.

Integrating the above equation:

$$dA_1 \, dF_{dA_1 - dA_2} = dA_2 \, dF_{dA_2 - dA_1}$$

RADIATION HEAT EXCHANGE BETWEEN TWO INFINITE PARALLEL PLANES

Following assumptions are made for the analysis of radiant heat exchange between two parallel planes.

- The distance between the two surfaces is small and the surfaces have equal area such that all radiations emitted by one surface fall on the other. The configuration factor of both surfaces thus becomes unity.
- The surfaces are diffuse and are maintained at uniform temperature. The reflective and emissive properties are constant over the entire surface.
- The surfaces are separated by a non absorbing medium such as air.

The amount of radiant energy leaving surface I/unit time is

$$Q_1 = E_1 - \left[\alpha_1 (1-\alpha_2) E_1 + \alpha_1 (1-\alpha_2)^2 E_1 + ...\right] = E_1 - \alpha_1 (1-\alpha_2) E_1 \left[1 + Z + Z^2 + \ ..\right]$$

$$\text{where } Z = (1-\alpha_1)(1-\alpha_2)$$

From the Stefan-Boltzman law for non-black surfaces,

$$E_1 = \varepsilon_1 \sigma T_1^4 \text{ and } E_2 = \varepsilon_2 \sigma T_2^4$$

$$Q_{12} = \frac{\sigma \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = f_{12} \, \sigma \left(T_1^4 - T_2^4\right) \quad f_{12} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \textit{Interchange} \text{ factor}$$

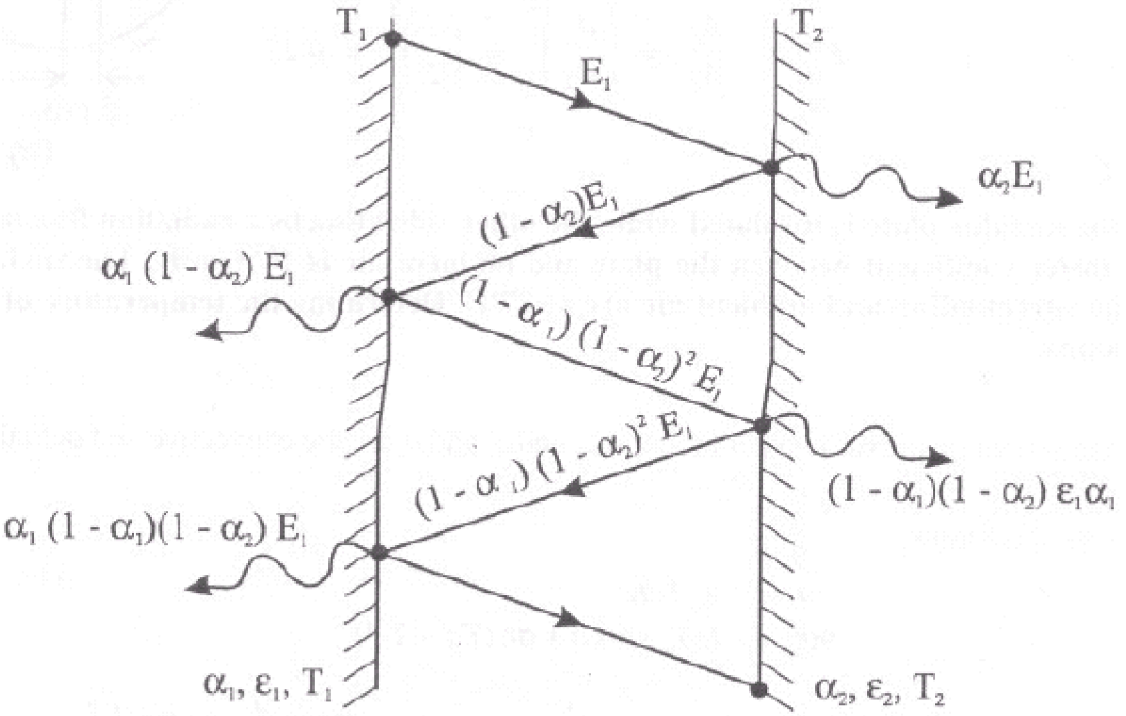


Fig 8: Radiant heat exchange between two non-black parallel surfaces

RADIATION SHIELDING

- Shielding between two surfaces reduces heat transfer significantly only if the shielding material is a low emissivity material.
- Shielding material placed between the two surfaces increases thermal resistance to radiation, reducing heat transfer rate.
- Thermal resistance increases if the emissivity of the material decreases.

PARALLEL PLATES

Consider two large opaque parallel plates at temperatures T_1 and T_2 and emissivities s_1 and s_2 respectively as shown in Fig 9. The heat transfer due to radiation between the plates across an area is given by

$$Q_0 = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

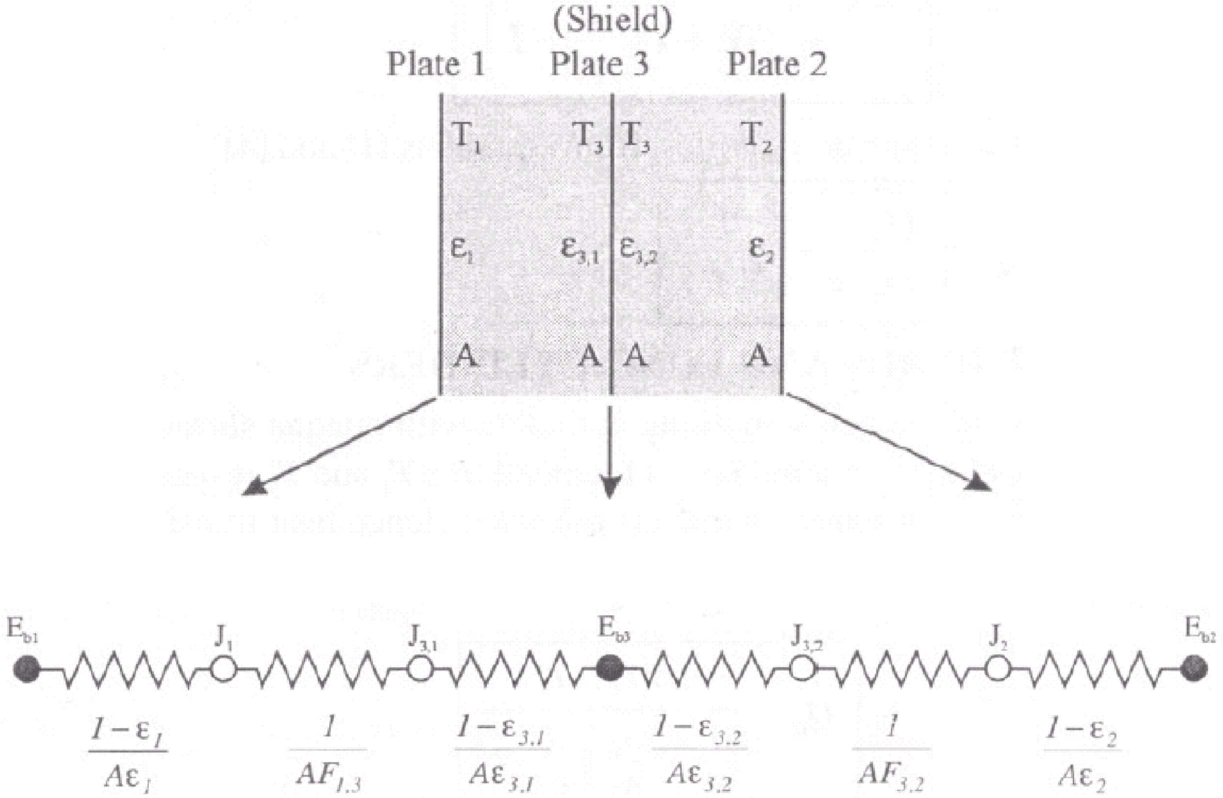


Fig.9: Radiation shielding with corresponding network

Consider a radiation shield placed between the plates. Let $s_{3,1}$ and $s_{3,2}$ be the emissivities of the shield at the surfaces 1 and 2 respectively. Noting that $F_{1,3} = F_{3,2} = 1$ for large parallel plates, the heat transfer across the system with one shield is given by,

$$Q_1 = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{\left(1-\varepsilon_{3,1}\right)}{\varepsilon_{3,1}} + \frac{\left(1-\varepsilon_{3,2}\right)}{\varepsilon_{3,2}} + \frac{1}{\varepsilon_2}} \; ; \; Q_1 = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

For parallel plates containing shields and equal emissivities

$$Q_N = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\left(N+1\right)\left(\frac{2}{\varepsilon} - 1\right)}$$

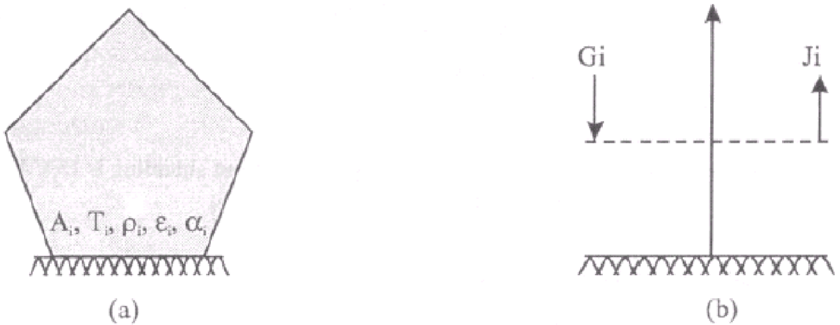
CONCENTRIC SPHERES AND LONG CYLINDERS

$$Q^1 = \frac{A_1\sigma\left(T_1^4 - T_1^4\right)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right)\left(\frac{1}{\varepsilon_2}\right) + \left(\frac{A_1}{A_3}\right)\left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

RADIATION HEATEXCHANGE IN AN ENCLOSURE

- Radiation heat exchange analysis in an enclosure becomes very much complicated especially when the surfaces are not black.
- This is because radiation leaving a surface may be reflected back and forth several times, the absorption being only partial at each reflection.
- The analysis can be done either by using *network method* or by using *radiosity matrix method.*
- The former is used for determining the radiation exchange in simple enclosures, whereas the latter is used for determining the radiation exchange having more number of zones.
- In this book only network method is used for the analysis. To make the analysis simple, the given enclosure is assumed to have several zones as shown in fig 10.

The following conditions are assumed to hold good for the zones *i* =1, 2………*N*



(a) An enclosure filled with a non participating medium, (b) Energy balance / unit area of zone i

Fig 10: Radiation exchange in an enclosure

Assumptions

1. *Surfaces are diffuse emitters and diffuse reflectors.*
2. *Radiative properties w, s and a are uniform and independent of frequency and direction.*
3. *Surfaces are opaque i.e.,* $\tau = O$.
4. *The radiative heat flux leaving the surface is uniform over the surface of each zone.*
5. *The irradiation is uniform over the surface.*
6. *The surface of each zone is either at uniform temperature or at uniform heat flux.*
7. *The enclosure contains a nonparticipating medium.*

It may be noted that assumptions 4 and 5 are not correct but are useful in simplifying the problem.

Before analysis of the problem it is necessary to develop the concept of surface resistance to radiation. The network method was introduced by Oppenheim.

ELECTRICAL NETWORK ANALOGY FOR THERMAL RADIATION SYSTEMS

An *electrical network analogy* is an alternative approach for analyzing radiation heat exchange between gray or black surfaces. In this approach the two terms commonly used are *irradiation* and *radiosity.*

Irradiation *(G):*

It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in W/m²

Radiosity (J):

This term is used to indicate the total radiation leaving a surface per unit time per unit area. It is also expressed in W/m².

The radiosity comprises the original emittance ij from the surface plus the reflected portion of any radiation incident upon it.

$$J = E + \rho\, G \quad = \varepsilon\, E_b + \rho\, G$$

E_b =emissive power of a perfect black body at the same temperature.

$\alpha + \rho + \tau = 1$; ($\tau = 0$, the surface being oblique)

$\alpha + \rho = 1; \rho = 1-\alpha$

$$J = \varepsilon E_b + (1-\alpha)G;\ \alpha = \varepsilon\ ;\ \mathrm{J} = \varepsilon E_b + (1-\varepsilon)G$$

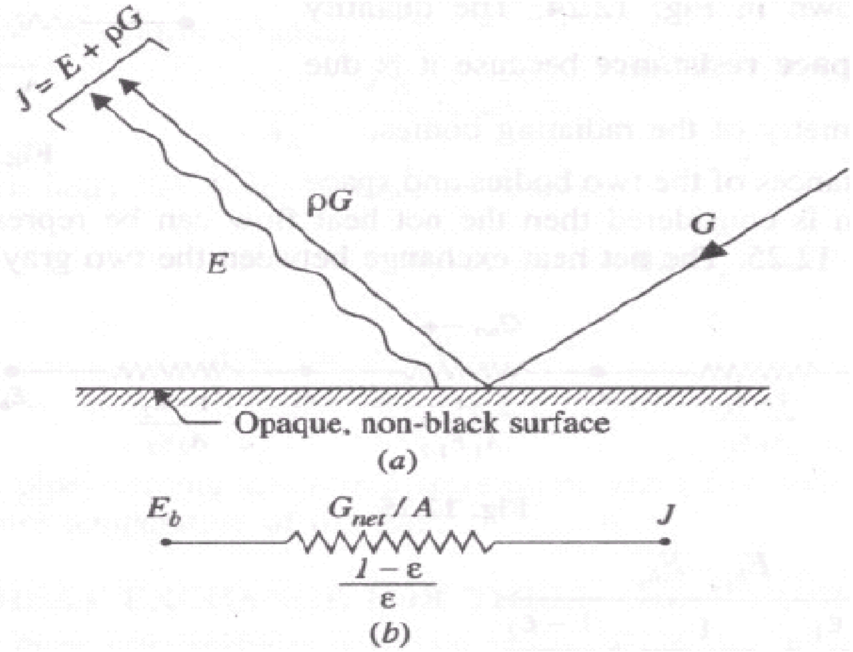


***Fig.11. Irradiation and radiosity*.**

$$J - \varepsilon E_b = (1-\varepsilon)G; \; G = \frac{J - \varepsilon E_b}{(1-\varepsilon)}$$

The net energy leaving a surface is the difference between its radiosity and irradiation. Thus,

$$\frac{Q_{net}}{A} = J - G = J - \frac{J - \varepsilon E_b}{(1-\varepsilon)} = \frac{J(1-\varepsilon) - (J - \varepsilon E_b)}{(1-\varepsilon)} = \frac{\varepsilon (E_b - J)}{1-\varepsilon}$$

$$Q_{net} = \frac{A\varepsilon (E_b - J)}{1-\varepsilon} = \frac{E_b - J}{1-\varepsilon \big/ A\varepsilon}$$

RADIATION FROM GASES, VAPOURS AND FLAMES

- The phenomenon of radiation exchange between gases and heat transfer surfaces is complex one.
- This is so because several gases such as H_2, O_2, N_2 etc. present in flames of the furnaces are transparent to radiation at low temperatures, whereas other gases like CO, CO_2, H_2O etc., emit and absorb radiation to an appreciable extent.

The radiation from gases differs from that from solids in the following ways:

1. Gases emit or absorb radiation only between *narrow ranges or bands of wavelength* where as *solids radiate at all wavelengths over the entire spectra.*
2. The intensity of radiation as it passes through an absorbing gas decreases with the length of passage through the gas volume whereas in case of solids the absorption of radiation takes place within a small distance from the surface.

$$dI_{\lambda x} = -k_{\lambda} . I_{\lambda x} . dx$$

I_{Lx} = Monochromatic intensity at a distance *x,* and

k_L = A proportionality constant or monochromatic absorption coefficient, depends on the state of gas (its temperature and pressure) and the wavelength.

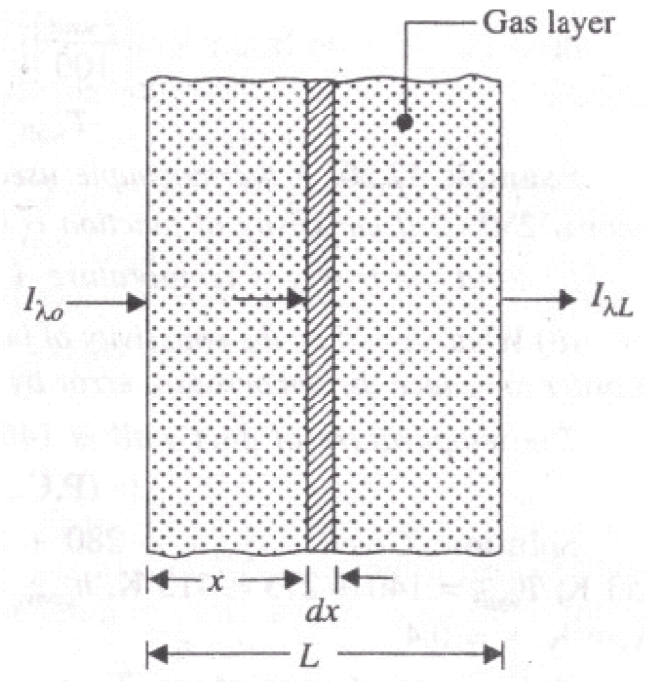


Fig. 12.Monochromatic radiation passing through an absorbing gas.

$$\int_{x=0}^{x=L} \frac{dI_{\lambda x}}{I_{\lambda x}} = \int_{x=0}^{x=L} -k_{\lambda}.dx; \ \ln\frac{I_{\lambda L}}{I_{\lambda o}} = -k_{\lambda}.L \quad ;$$

$I_{\lambda L} = I_{\lambda o} e^{-k_{\lambda}.L}$ where $I_{\lambda L} =$ radiation intensity at x = L

$$For\ CO_2 : E_{CO_2} = 3.5\,(pL)^{0.33}\left(\frac{T}{100}\right)^{3.5}$$

$$For\ H_2O : E_{H_2O} = 3.5\,p^{0.8}L^{0.6}\left(\frac{T}{100}\right)^{3}$$

Following formula is used to calculate the radiant heat exchange between a gas at a temperature T_g and a black surface of finite area *A* at temperature T_b

$$Q = \sigma A\left(\varepsilon_g T_g^4 - \alpha_g T_b^4\right)$$

The following formula is used to compute the net rate of heat transfer from the gas to the walls, when *inside of the enclosure* is *not black:*

$$Q = \sigma A \varepsilon_w \left(\varepsilon_g T_g^4 - \alpha_g T_b^4\right)$$

The net interchange of energy between a *flame* and its *enclosure* is given by:

$$Q_{net} = \sigma A_f F_{fw} \varepsilon_f \varepsilon_w \left(T_f^4 - T_w^4\right)$$

ASSIGNMENT QUESTIONS

THEORY QUESTIONS

1. Define a geometrical or shape factor.
2. Derive an expression for the shape factor in case of radiation exchange between two surfaces.
3. Derive the following relation for the radiant heat exchange between two gray surfaces, using definition of irradiation and radiosity.

$$(Q_{12})_{net} = \frac{A_1 \sigma \left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{1-2}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\frac{A_1}{A_2}}$$

4. What is a radiation shield?
5. Write a short note on radiation from gases, vapors and flames.

PROBLEMATIC QUESTIONS

1. Assuming the sun to radiate as a black body, calculate its temperature from the data given below:

   Solar constant $=1400 Wm^2$; Radius of the sun = 6.97×10^8m; Distance between the sun and the earth=14.96×10^{10}m.

2. A small sphere (outside diameter =50 mm) with a surface temperature of 277°C is located at the geometric centre of a large sphere (inside diameter =250 mm) with an inner surface temperature of 7°C. Calculate how much of emission from the inner surface of the large sphere is incident upon the outer surface of the small sphere assume that both sides approach black body behavior. What is the net interchange of heat between the two spheres?

3. A 60 mm thick plate with a circular hole of 30 mm diameter along the thickness is maintained at uniform temperature of 277°C. Find the loss of energy to the surroundings at 20°C, assuming that the two ends of the hole to be as parallel discs and the metallic surfaces and surroundings have black body characteristics.

4. Work out the shape factor of a hemispherical bowl of diameter D with respect to itself. Also calculate the radiative heat transfer from the cavity if inside temperature is 773 K and its emissivity is 0.6. The diameter of the cavity is 700 mm.

5. A double walled flask may be considered equivalent to two infinite parallel planes. The ernissivities of the walls are 0.3 and 0.8 respectively. The space between the walls of the flask is evacuated. Find the heat transfer per m^2 area when inner and outer surface temperatures are 300 K and 260 K. To reduce the heat flow, a shield of polished aluminum with s =0.05 is inserted between the walls. Find the reduction in heat transfer.

CONVECTIVE HEAT TRANSFER

INTRODUCTION

- Convection is the mode of heat transfer which involves the motion of the medium that is involved.
- Convection heat transfer requires an energy balance along with the analysis of the fluid dynamics of the problems considered.
- For basic understanding of convection heat transfer, some basic relations of fluid dynamics and boundary layer analysis are necessary. This chapter deals the concept of convection heat transfer in detail.

FLOW OVER A BODY

- The heat transfer by convection is strongly influenced by the velocity and temperature distribution of the immediate neighborhood of the surface of a body over which a fluid is flowing.
- For simple analysis of heat transfer involving convection, the velocity and temperature distribution at the boundary surface can be known by introducing the *boundary - layer* concept.
- Two different types of boundary layers are considered for this purpose viz., *velocity boundary layer and thermal boundary layer.*

VELOCITY BOUNDARY LAYER

- Consider a fluid flowing over a flat plate as shown in Figure 1. Let $u_{œ}$ be the velocity of the fluid parallel to the plate surface at the leading edge of the plate *at* $x = 0$.
- When there is no slip at the wall surface, the fluid moving, along the x direction that is in contact with the plate has no velocity. Thus the components of velocity $u(x, y) \div u$ retards along the x direction.
- Hence at the plate surface at $y = 0$ velocity u becomes zero. This retardation effect reduces considerably on the fluid moving at a sufficiently higher level *(y* - direction) and at one point the retardation effect is completely negligible.
- The velocity of the fluid at distance $y = ð(x)$ from the surface of the plate where the axial velocity component u is 99 percent of the free stream velocity $u_{œ}$.
- The locus of such points where $u = 0.99\ u_{œ}$ is known as *velocity boundary layer ð(x).*
- The flow over the plate results in separation of flow field into two distinct regions.

Boundary layer region:

In this region the velocity gradients and shear stress are large due to the rapid variation of the axial velocity component u(x, y) with the distance y from the plate.

Potential flow region:

In this region the velocity gradient and shear stress are negligible. This region is the region outside the boundary layer.

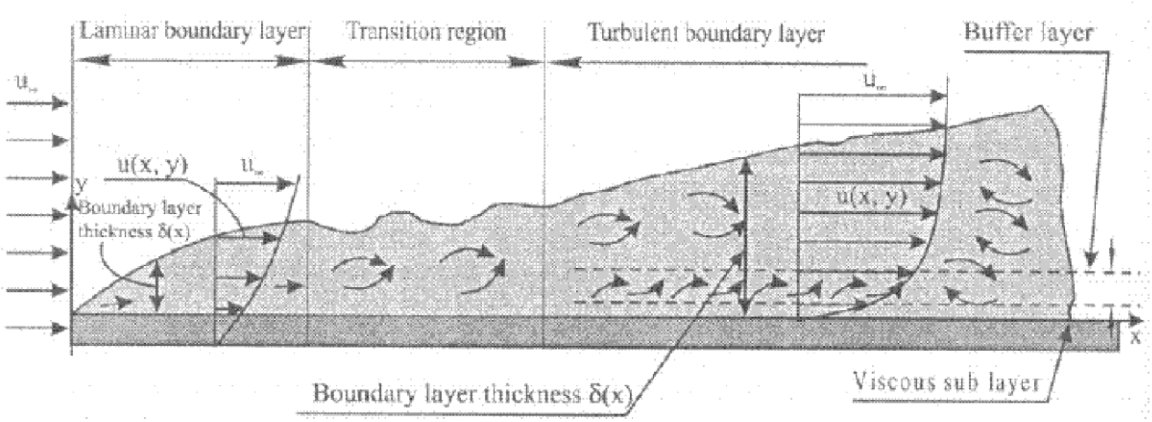


Fig 1: Different boundary layer flow regions on a flat plate

Behavior of flow in the boundary layer

Consider the boundary layer at a distance x from the leading edge of the plate. The flow characteristic is governed by *Reynolds's number.* For a flat plate it is expressed as,

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu}$$

u_∞ = Free-stream velocity

x = Distance from leading edge of plate,

ν = Kinematic viscosity of fluid

- Initially, the boundary layer development is *laminar* but at some critical distance from the leading edge of the plate, small disturbances in the flow begin to become amplified and a transition process takes place until the flow becomes turbulent.
- However, this process depends on flow field and fluid properties. For flow along a flat plate, the critical Reynolds number at which the transition from laminar to turbulent flow takes place is generally taken as,

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu} \cong 5 \times 10^5$$

- This value is dependent on the surface roughness and the turbulence level of the free stream. In the turbulent boundary layer next to the wall, there is a very thin layer called *viscous sub-layer* in which the viscous flow character is retained by the flow.
- The region adjacent to the viscous sub-layer is known as *buffer layer.* In this layer exists fine-grained turbulence and the mean axial velocity increases rapidly with the distance from the wall. The buffer layer is followed by *turbulent layer* with large scale turbulence.
- The change in relative velocity with the distance from the wall is very little in this layer. Curved body Consider a curved body on the surface of which the fluid flows.
- For a curved body the *x* co-ordinate is measured along the curved surface of the body starting from the stagnation point as shown in Fig. 2. The *y* co-ordinate is normal to the surface of the body.

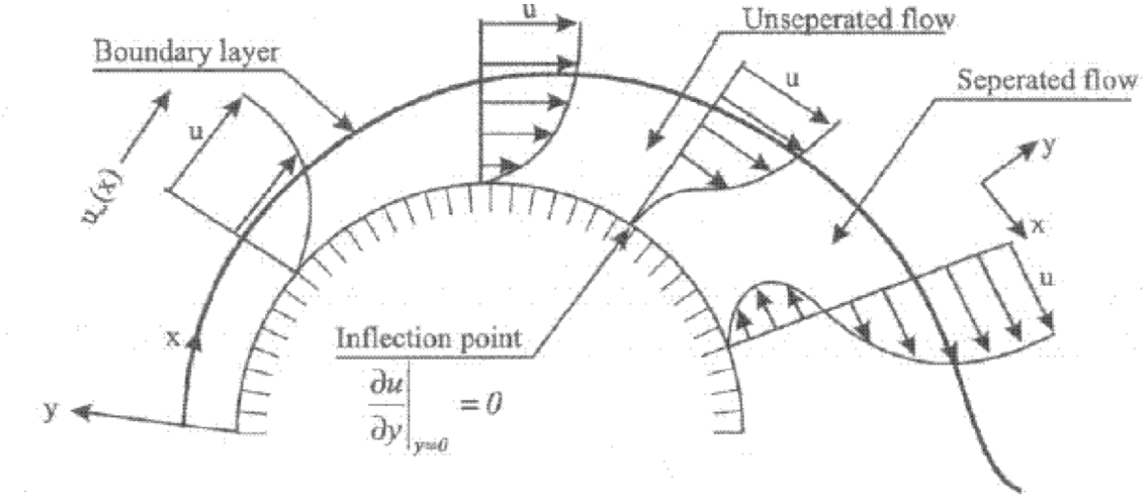


Fig.2: Flow along a curved body

- In the above case, the free stream velocity is not constant but varies with distance along the curved surface. The thickness of boundary layer 0(x) increases with distance *x* along the surface. After some distance *x,* the velocity profile *u(x, y)* exhibits a *point of inflection* in which a y =0 at the wall surface.
- This behavior is attributed purely to the curvature of the surface. Beyond this point flow reversal takes place and the boundary layer is detached from the surface. Beyond this point of flow reversal, boundary layer analysis is not applicable and flow patterns become very complicated.

DRAG COEFFICIENT

Consider a boundary layer having a velocity profile u(x, y). The viscous shear stress 1"x acting on the wall at any given position *x* is given by,

$$\tau_x = \mu \frac{\partial u(x,y)}{\partial y}\bigg|_{y=0}$$

Where μ = A constant known as viscosity of the fluid.

However for engineering applications the definition of shear stress given by the above equation is not applicable. In practice, it is represented in terms of *local drag coefficient,* c_x as follows.

$$\tau_x = c_x \frac{\rho u_\infty^2}{2}$$

q = Density of the fluid

u_α = Free stream velocity

The drag force exerted by the flowing fluid over the flat plate is determined by equating equations [1] and [2] as follows,

$$c_x = \frac{2\mu}{\rho u_\infty^2} \times \frac{\partial u(x,y)}{\partial y}\bigg|_{y=0}$$

$$= \frac{2\nu}{u_\infty^2} \times \frac{\partial u(x,y)}{\partial y}\bigg|_{y=0}$$

v = Kinematics viscosity of the fluid

The mean value of drag coefficient between the range x =0 to x =L is defined as,

$$c_m = \frac{1}{L}\int_{x=0}^{L} c_x dx$$

Hence the drag force acting on the plate between x =0 to x =L is given by,

$$F = w L c_m \frac{\rho u_\infty^2}{2}$$

THERMAL BOUNDARY LAYER

- *Thermal boundary layer* along the flat plate is associated with the temperature profile in the fluid. Consider a fluid at a uniform temperature $T_\alpha$flowing over a flat plate maintained at a constant temperature T_w was shown in Fig. 3.
- Let x and y be the co-ordinate axes along and perpendicular to the plate surface respectively. Then, the dimensionless temperature,

$$\theta(x, y) = \frac{T(x,y) - T_W}{T_\infty - T_W}$$

- Where T(x, y) is the local temperature in the fluid. At the wall surface, wall temperature and fluid temperature are equal. 0(*x, y)* = 0 at y = 0 .
- The fluid temperature remains the same at a distance sufficiently from the wall. At $\theta\ (x,y) \rightarrow 1$ at $y \rightarrow \infty$. Similar to velocity boundary layer, at each location x along the plate there exists a location y = δ_t(x) in the fluid where temperature 0(x, y) = 0.99.
- The locus of such point is known as *thermal boundary layer* δ_t(x).

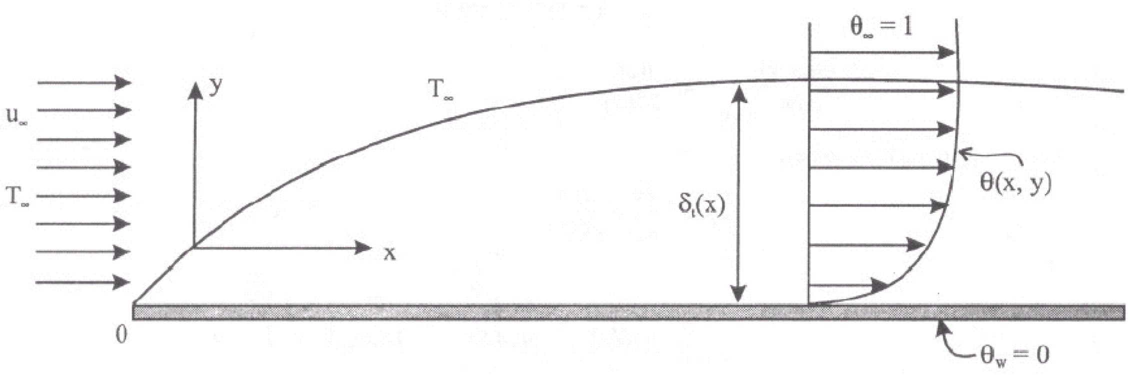


Fig.3: Thermal boundary layer

The thermal boundary layer thickness δ_t(x) and the velocity boundary layer thickness δ(x) depend on the *Prandtl* number of the fluid.

For fluids such as gases having Prandtl number $P_r = 1,\ \delta_t(x) = \delta(x)$
For fluids such as liquid metals having Prandtl number $P_r \le 1, \delta_t(x) \ge \delta(x)$
For fluids having Prandtl number $P_r \ge 1, \delta_t(x) \le \delta(x)$

HEAT TRANSFER COEFFICIENT

If the temperature distribution T(x, y) in the thermal boundary layer is known, then the heat flux from the fluid to the wall is given by,

$$q(x) = k \left. \frac{\partial T(x,y)}{\partial y} \right|_{y=0}$$

Where, k = Thermal conductivity of the fluid.

However for engineering applications the above definition of heat flux is not applicable. In practice it is represented by a *local heat transfer coefficient h(x).*

$$q(x) = h(x)\ (T_\infty - T_w)$$

$$h(x) = k\,\frac{\left[\partial T / \partial y\right]_{y=0}}{T_\infty - T_W}$$

In terms of dimensionless temperature,

$$h(x) = k\,\left.\frac{\partial \theta(x,y)}{\partial y}\right|_{y=0}$$

The *mean heat transfer coefficient h_m* over the distance x =0 to x =L along the plate surface is given as,

$$h_m = \frac{1}{L}\int_0^L h(x)\,dx$$

The heat transfer rate Q from the fluid to the wall from x =0 to x =L is given by,

$$Q = w\,L\,h_m\,(T_\infty - T_w)$$

FLOW INSIDE A DUCT

The flow analysis inside a duct is done by considering the velocity boundary layer and thermal boundary layer separately.

VELOCITY BOUNDARY LAYER

- Consider the flow inside a circular tube as shown in Fig. 4. The velocity of the fluid inside the tube is U_o and as the fluid enters the tube, *a velocity boundary layer* starts to develop along the surface of the wall.
- Due to retardation the velocity of fluid particles at the wall surface becomes zero and in order to maintain the continuity of flow, the velocity in the central portion of the tube increases. The thickness of the velocity boundary layer ð (z) grows continuously along the surface of the tube till it covers the entire tube.
- The region from the tube inlet a little beyond the point where the boundary layer reaches the tube centre is called *hydrodynamic entry region.* The region beyond this is known as *hydro dynamically developed region.*
- In the hydrodynamic entry region, the shape of the velocity profile changes in both axial and radial direction, whereas in the fully developed region the velocity profile is invariant along the length of the tube. Fully developed laminar flow exists in the developed region.

- If the boundary layer changes into turbulent before its thickness reaches the centre, then fully developed turbulent flow exists in the developed region. The velocity profile becomes flatter in case of a turbulent flow as shown in Fig. 4.

For flow inside a circular tube, Reynolds number is given by,

$$\mathrm{Re} = \frac{u_m D}{\nu}$$

The above equation is used as a criterion for change from laminar flow to turbulent flow. The turbulent flow is usually observed for Re > 2300. This value is dependent on the surface roughness, inlet conditions and the fluctuations in the flow. In general, transition occurs in the range 2000 < Re < 4000.

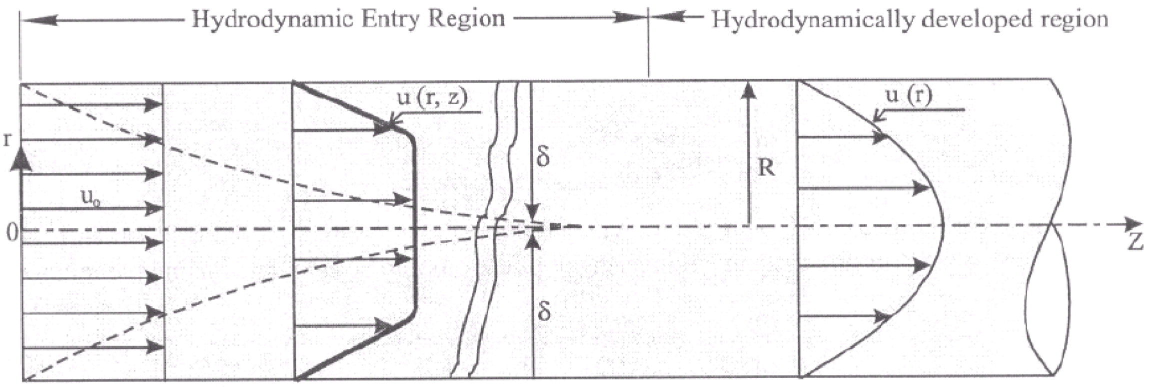


Fig. 4: Velocity boundary layer at the inlet of a circular tube

PRESSURE GRADIENT AND FRICTION FACTOR

The pressure drop along a given length of tube is determined by integrating dp/dz over the length. Consider a differential volume element of length dz as shown in Fig. 5. Making force

Pressure force = Shear force on the wall

$$(pA)_Z - (pA)_{Z+\Delta Z} = P\,\Delta z\,\tau_W \quad ; \quad \frac{\mathrm{dp}}{\mathrm{dz}} = -\frac{P}{A}\tau_W$$

$$= \frac{-\pi D}{\left(\pi/4\right) D^2}\tau_w = -\frac{4}{D}\tau_w$$

Where P and A are perimeter and cross-sectional area respectively. The shear stress acting at the wall,

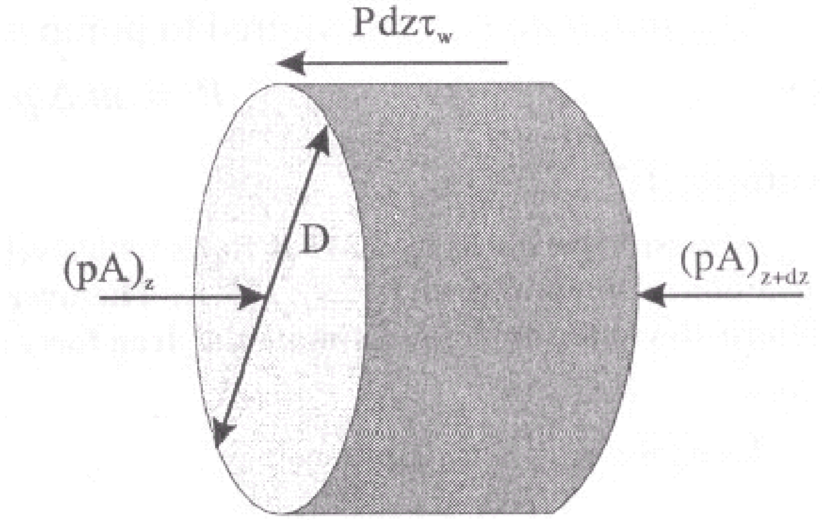


Fig. 5: Force acting on a differential volume element

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_w = -\mu \left.\frac{\partial u}{\partial r}\right|_w ;\ r = \frac{D}{2} - y ;\ \frac{dp}{dz} = \frac{4\mu}{D} \left.\frac{\partial u}{\partial r}\right|_W$$

In the above equation we need to evaluate the velocity gradient at the wall which is not practical. However, pressure drop in engineering applications can be calculated using the relation,

$$\frac{dp}{dz} = -f \frac{\rho u_m^2}{2D}$$

f = Friction factor

u_m= Mean velocity of flow inside the tube

q = Density of the fluid.

$$f = -\frac{8\mu}{\rho u_m^2} \left.\frac{\partial u}{\partial r}\right|_W$$

The pressure drop Ap = P_1 - P_2 over the length of the tube L = Z_2 – Z_1 is given by,

$$\int_{p_1}^{p_2} dp = -f \frac{\rho u_m^2}{2D} \int_{Z_1}^{Z_2} dZ ;\ \Delta p = f \frac{L}{D} \frac{\rho u_m^2}{2}$$

The *pumping power* required pumping m kg/s of fluid through the pipe is given by

P=m × Ap

THERMAL BOUNDARY LAYER

Consider a laminar flow inside a circular tube subjected to *uniform heat flux* at the wall. If r and z are radial and axial coordinates respectively, then the dimensionless temperature is given by,

$$\theta(r,z) = \frac{T(r,z) - T_W(z)}{T_m(z) - T_W(z)}$$

$T_W(z)$ = Tube wall temperature

$T_m(z)$ = Bulk mean fluid temperature

$T(r,z)$ = Local fluid temperature

At the wall surface 0(r, z) is zero and has some finite value at the tube center. The thermal boundary layer thickness $ð_t$ (z) grows continuously and completely fills the entire tube. *The region from the tube inlet to the point where the thermal boundary layer thickness reaches the tube centre is known as thermal entry region.* In the thermal entry region the shape of the temperature profile 0(r, z) changes in both axial and radial direction. The region beyond the thermal entry length is known as *thermally developed region.* In this region the shape of the temperature profile remains the same with respect to the distance along the tube.

For a fully developed thermal region,

$$\theta(r) = \frac{T(r,z) - T_W(z)}{T_m(z) - T_W(z)}$$

Mathematically it is proved that for sufficiently large values of z, the dimensionless parameter 0(r) depends only on r, provided either temperature or *constant heat flux* is maintained at the wall.

HEAT TRANSFER COEFFICIENT

Consider a fluid flowing inside a circular tube of inside radius R.

Let, *r* & z = Radial and axial coordinates respectively

k = Thermal conductivity of fluid

T(r, z) = Temperature distribution in the fluid

q (z) = Heat flux from the fluid to the tube wall

Then, Heat flux,

$$q(z) = -k \left. \frac{\partial T(r,z)}{\partial r} \right|_{wall}$$

In engineering applications the above equation is of little interest. A practical approach which uses local heat transfer coefficient h (z) is adopted. Hence heat flux is given by,

$$q(z) = h(z)\left[T_m(z) - T_w(z)\right]$$

Where $T_m(z)$ = Bulk mean temperature

$T_w(z)$ = Tube wall temperature

Relation between heat transfer coefficient and T(r, z) can be determined by

$$h(z) = -\frac{k}{T_m(z) - T_W(z)} \left.\frac{\partial T(r, z)}{\partial r}\right|_{r = R \text{ at wall}}$$

For a circular tube of radius R, the bulk mean temperature T_m (z) and the wall temperature T_w (z) are given by,

$$T_m(z) = \frac{\int_0^R u(r) T(r, z) 2\pi r dr}{\int_0^R u(r) 2\pi r dr} = \frac{\int_0^R u(r) T(r, z) 2\pi r dr}{u_m \pi R^2}; T_W(z) = T(r, z)_{r = R \text{ at wall}}$$

Writing above equation in terms of dimensionless temperature

$$h(z) = -k \left.\frac{\partial \theta(r, z)}{\partial r}\right|_{r = R \text{ at wall}}$$

For a fully developed thermal region, 0(r) is independent of z and hence,

$$h(z) = -k \left.\frac{\partial \theta(r)}{\partial r}\right|_{r = R \text{ at wall}} \quad ; \theta(r) = \frac{T(r, z) - T_W(z)}{T_m(z) - T_W(z)}$$

For a thermally developed region for constant temperature or heat flux at the wall, the heat transfer region doesn't vary with distance along the *tube.*

DIMENSIONLESS PARAMETERS

1. Reynolds number

"Reynolds number is defined as the ratio of inertia force to viscous force. When the Reynolds number is small the viscous forces are dominant, whereas when Reynolds number is large, the inertia forces are more dominant".

$$\mathrm{Re} = \frac{u_\infty L}{\nu} = \frac{u^2_\infty / L}{\nu u_\infty / L^2} = \frac{\text{Inertia force}}{\text{Viscous force}}$$

Reynolds number is used to determine the change from laminar to turbulent flow as higher Inertia forces result in small disturbances which amplify causing transition.

2. Prandtl number

"Prandtl number is defined as the ratio of molecular diffusivity of momentum to the molecular diffusivity of heat. It represents the momentum and energy transport by the diffusion process".

$$p_r = \frac{c_p \mu}{k} = \frac{\mu / \rho}{k / (\rho c_p)} = \frac{\nu}{\alpha} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}}$$

$$p_r \cong 1 \ \text{for gases}$$
$$p_r \geq 1 \ \text{for oils}$$
$$p_r \leq 1 \ \text{for liquid metals}$$

The development of velocity and thermal boundary layers for flow along a flat plate and their magnitudes depend on the magnitude of *Prandtl number.*

3. Nusselt Number

"Nusselt number is defined as the ratio of heat transfer by convection to conduction across the fluid layer of thickness L".

A larger value of *Nusselt number* means heat transfer by convection is more.

$$Nu = \frac{hL}{k} = \frac{h\Delta T}{\left(\frac{k\Delta T}{L}\right)} = \frac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$$

If *Nu* ÷ 1 then heat is transferred purely by conduction.

4. Stanton Number

"Stanton number is defined as the ratio of heat flux to the fluid to the heat transfer capacity of the fluid flow".

$$St = \frac{h}{\rho c_p u_m} = \frac{h\Delta T}{\rho c_p u_m \Delta T} = \frac{\text{Heat flux to the fluid}}{\text{Heat transfer capacity of the fluid}}$$

5. Graetz Number

"Graetz number is defined as the ratio of the heat capacity of the fluid flowing through the pipe per unit length of the pipe to the conductivity of the pipe".

It is significant only in heat flow to the fluid flowing through circular pipes. If D and L are diameter and length of the pipe respective] then

$$Gr = \frac{mc_p / L}{L / k} = \frac{\frac{\pi}{4} D^2 \rho u \frac{c_p}{L}}{k} = \frac{\pi D}{4L} \mathrm{Re}.\mathrm{Pr}$$

6. Grashoff Number

“Grashoff number is defined as the ratio of product of inertia force and buoyance force to the square of viscous force”.

$$Gr = \frac{\text{Inertia force} \times \text{Buoyance force}}{(\text{Viscous force})^2} = \frac{\rho^2 \beta g \Delta T L^3}{\mu^2}$$

Where V is the velocity of the fluid caused by buoyancy force (þgAT)

BUCKINGHAM w- THEOREM

Buckingham w -theorem states that “If there are n variables in a dimensionally homogeneous equation and if these variables contain m primary dimensions, then the variables can be group into (n-m) non dimensional parameters". The non-dimensional groups are called w –terms.

Let, x_1, x_2,x_3…….x_n be the physical variables in which x_1 is the dependent variable and the rest are independent variables on which x_1 depends, Expressing mathematically,

$$x_1 = f(x_2, x_{3 ....} x_n) \text{ rearranging the equation : } f_1(x_2, x_{3 ....n}) = 0$$

The above equation is dimensionally homogeneous and it can be represented in terms of dimensionless w-term containing w- variables and *m* fundamental dimensions.

$$f_1(\pi_1, \pi_2, \pi_3 ......\pi_{n-m}) = 0$$

In the above equation,

- Each w-term is dimensionless and is independent of the system.
- N-term will not change even by dividing or multiplying it by a constant,
- Each w-term contains m+ 1 variables and m fundamental dimensions known as *repeating variables.*

Let x_1, *x2, x3* be the repeating variables with m =3. Then we can represent each w-term as

$$\pi_1 = x_1^{a_1} . x_2^{b_1} . x_3^{c_1} . x_4$$

$$\pi_2 = x_1^{a_2} . x_2^{b_2} . x_3^{c_2} . x_5$$

$$\pi_{n-m} = x_1^{a_{n-m}} . x_2^{b_{n-m}} . x_3^{c_{n-m}} . x_n$$

Each one of the above equations is solved by the principle of dimensional homogeneity. The values of a_1, b_1, c_1 etc. thus obtained are substituted in the equation. All the w-values thus obtained are substituted in above equation. Finally anyone of the w-term is expressed as a function of others.

$$\pi_1 = \phi\left(\pi_2, \pi_3 \ldots\ldots \pi_{n-m}\right)$$
$$\pi_2 = \phi\left(\pi_1, \pi_3 \ldots\ldots \pi_{n-m}\right)$$

FORCED CONVECTION

If the heat transfer by convection is assisted by some external means it is known as *force convection.* The dimensional analysis for forced convection is correlated by

$$Nu = \phi\left(\mathrm{Re}, \mathrm{Pr}\right)$$

The different variables specifying the system behavior is shown in Fig 6, which represents forced convection of fluid flow over a flat plate.

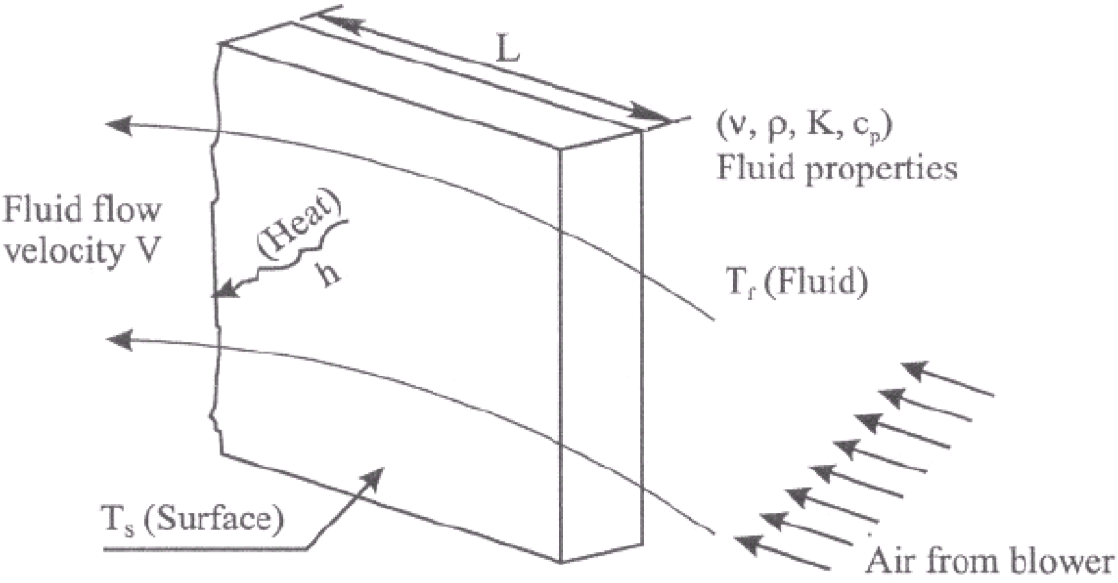


Fig 6: Dimensional analysis variables for forced convection

$$As\ \ we\ \ know\ \ \ Nu = \frac{hL}{k};\ \mathrm{Re} = \frac{\rho LV}{\mu};\ \mathrm{Pr} = \frac{\mu c_p}{k}$$

Heat transfer coefficient h can be represented as

$$h = f\left(\rho, L, V, \mu, c_p, k\right)\ \ or\ f\ \ \left(h,\ \rho,\ L, V,\ \mu, c_p,\ k\right) = 0$$

In the above equation

Number of variables= n = 7

Fundamental dimension=m=4

Number of w– terms = n – m = 7 – 4 = 3

$$\therefore\ f\left(\pi_1, \pi_2, \pi_3\right) = 0$$

Considering equation [1], one can notice that *μ,* k, L and V form non dimensional groups all four fundamental dimensions M,L,T and 0 are present.

***First** w–term*

$$\pi_1 = (\mu)^{a_1} (k)^{b_1} (L)^{c_1} (V)^{d_1}\ \rho = M^0 L^0 T^0 \theta^0$$

$$i.e., \left(ML^{-1}T^{-1}\right)^{a_1} \left(MLT^{-3}\theta^{-1}\right)^{b_1} (L)^{c_1} \left(LT^{-1}\right)^{d_1}\ ML^{-3} = M^0 L^0 T^0 \theta^0$$

Comparing the powers of M, L, T and e we have,

$$\begin{aligned} M &: \quad a_1 + b_1 + 1 = 0 \\ L &: -a_1 + b_1 + c_1 + d_1 - 3 = 0 \\ T &: \quad -a_1 - 3b_1 - d_1 = 0 \\ \theta &: \quad -b_1 = 0 \end{aligned}$$

Solving the above equations,

a_1 = -1; b_1 = 0, c_1 = 1, d_1 = 1

Substituting in above equation,

$$\pi_1 = \mu^{-1}.k^0.L^1.V^1.\rho = \frac{\rho LV}{\mu}$$

***Second** w-term*

$$\pi_2 = (\mu)^{a_2} (k)^{b_2} (L)^{c_2} (V)^{d_2}\ c_p = M^0 L^0 T^0 \theta^0$$

$$i.e., \left(ML^{-1}T^{-1}\right)^{a_2} \left(MLT^{-3}\theta^{-1}\right)^{b_2} (L)^{c_2} \left(LT^{-1}\right)^{d_2}\ L^2T^{-2}\theta^{-1} = M^0 L^0 T^0 \theta^0$$

Comparing the powers of M, L, T and 0 on both sides,

$$\begin{aligned} M &: \quad a_2 + b_2 = 0 \\ L &: \quad -a_2 + b_2 + c_2 + d_2 + 2 = 0 \\ T &: \quad -a_2 - 3b_2 - d_2 - 2 = 0 \\ \theta &: \quad -b_2 - 1 = 0 \end{aligned}$$

Solving the above equations,

a_2=+1; b_2= -1, C_2=0, d_2=0

Substituting in above equation

$$\pi_2 = \mu^1 . k^{-1} . L^0 . V^0 . c_p = \frac{\mu c_p}{k}$$

<u>Third</u> *<u>w-term:</u>*

$$\pi_3 = (\mu)^{a_3} (k)^{b_3} (L)^{c_3} (V)^{d_3} \; h = M^0 L^0 T^0 \theta^0$$

$$i.e., \left(ML^{-1}T^{-1}\right)^{a_2} \left(MLT^{-3}\theta^{-1}\right)^{b_2} (L)^{c_2} \left(LT^{-1}\right)^{d_2} \; MT^{-3}\theta^{-1} = M^0 L^0 T^0 \theta^0$$

Comparing the powers of M, L, T and e on both sides,

$$\begin{aligned} M &: \quad a_3 + b_3 + 1 = 0 \\ L &: \quad -a_3 + b_3 + c_3 + d_3 = 0 \\ T &: \quad -a_3 - 3b_3 - d_3 - 3 = 0 \\ \theta &: \quad -b_3 - 1 = 0 \end{aligned}$$

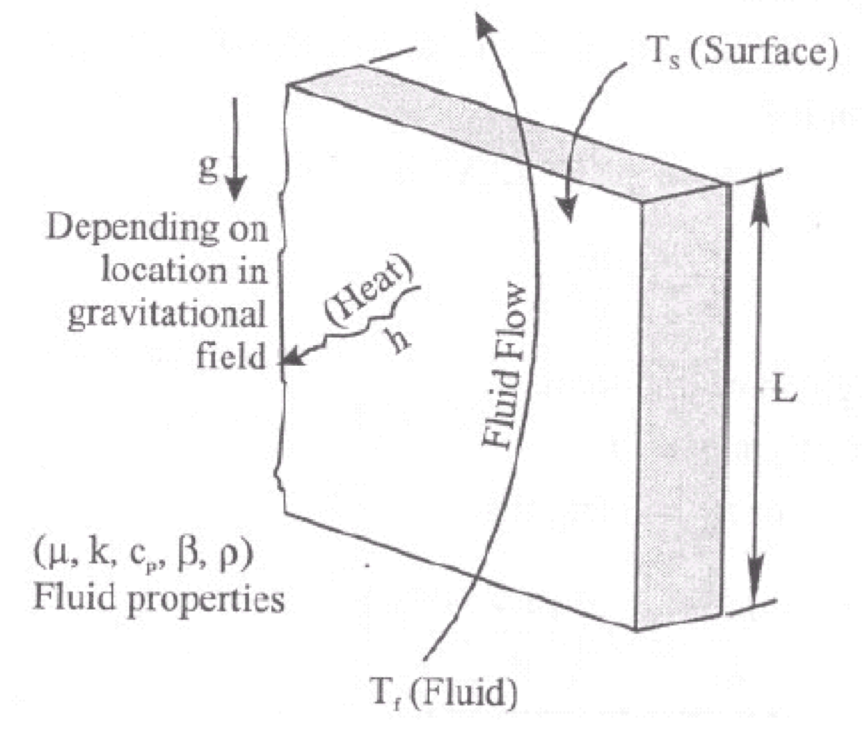


Fig. 7: Dimensional analysis variables in free convection

Using Buckingham w-theorem,

$$h = f \left(\mu, \rho, k, c_p, \beta\, g \Delta T, L, h\right) = 0$$

According to Buckingham 1t theorem,

$$\pi_3 = f(\pi_1, \pi_2); \frac{hL}{k} = f\left(\frac{\beta g \Delta T \rho^2 L^3}{\mu^2}, \frac{\mu . c_p}{k} \right); Nu = f(Gr, \mathrm{Pr}) = \phi(Gr, \mathrm{Pr})$$

In practice the above equation is represented as,

$$Nu = \mathrm{Constant}\,(Gr)^a\,(\mathrm{Pr})^b$$

VARIOUS CORRELATIONS USED IN FORCED CONVECTION HEAT TRANSFER

For forced convection heat transfer the following dimensionless numbers are extensively used.

Nusselt Number	$Nu = \frac{hL}{k}$
Reynolds Number	$\mathrm{Re} = \frac{\rho L V}{\mu}$
Prandtl Number	$\mathrm{Pr} = \frac{\mu c_p}{k}$
Stanton Number	$\mathrm{St} = \frac{\mathrm{h}}{\rho \mathrm{c}_\mathrm{p} V}$

In order to determine the value of convection heat transfer coefficient h, generalized basic relations are used.

$$Nu = f(\mathrm{Re.Pr}) = \mathrm{constant}\,(\mathrm{Re}^m.\mathrm{Pr}^n); St = (\mathrm{Re.Pr}) = \mathrm{constant}\,(\mathrm{Re})^a\,(\mathrm{Pr})^b$$

FLOW OVER A FLAT PLATE

Flow over flat plate remains laminar until the *Reynolds number* reaches the *critical* value. After this the *transition* begins. The *correlations* for the drag coefficient in the *laminar* and *turbulent* flow regimes are different.

LAMINAR BOUNDARY LAYER

Consider a two dimensional steady flow of an incompressible, constant property fluid along flat plate as shown in Fig 8. Let u (x, y) and v (x, y) be the velocity components in x and y direction. Let ð(x) is the thickness of the velocity boundary layer having a free stream velocity $u_{œ}$.

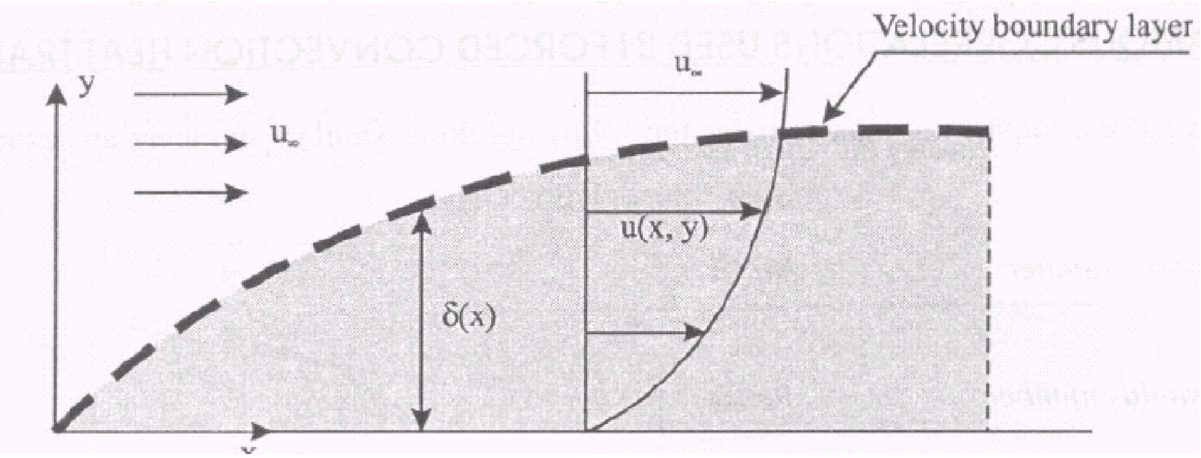


Fig 8: Forced laminar flow along a flat plate

The continuity & momentum equation for the given boundary layer is written as;

$$Continuity : \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \; Momentum : u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v\frac{\partial^2 u}{\partial y^2}$$

Subject to the boundary conditions:

$$At \;\; y = 0; \qquad u = 0; \quad v = 0$$

$$At \; y = \delta(x); \; u \rightarrow u_\infty$$

The local drag coefficient is given by,

$$c_x = \frac{2v}{u_\infty^2}\frac{\partial u(x,y)}{\partial y}\bigg|_{y=0} = \frac{0.646}{Re_x^{1/2}}$$

1. Drag coefficient

The *exact* value of the local drag coefficient is given by,

$$c_x = \frac{0.664}{Re_x^{1/2}}(exact) \;\; for \;\; Re \le 5\times10^5$$

The average drag coefficient c_m, over the length x = 0 to x = L is given by,

$$c_m = \frac{1}{L}\int_{x=0}^{L} c_x dx = \frac{1.328}{Re_L^{1/2}}(exact)$$

2. Boundary layer thickness

The exact solution of velocity boundary layer thickness is given by

$$\delta(x) = \frac{4.96\,x}{Re_x^{1/2}}(exact) \;\; for \;\; Re \le 5\times10^5$$

The turbulent boundary layer, thickness is given by,

$$\delta_t(x) = \frac{4.53\,x}{\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}}$$

3. Drag Force

The drag force acting on the plate over the length x =0 to x =L and width w is given by,

$$F = w\,L\,c_m \frac{\rho\,u_\infty^2}{2}$$

4. Local Heat transfer coefficient

The local heat transfer coefficient for flow over a flat plate with constant wall temperature is given by (Liquid Metal Fluid)

$$Nu_x = \left(\frac{hL}{k}\right)_x = 0.564\,Pe_x^{1/2}(exact) \quad for \quad \mathrm{Pr} \le 1;\, where\ Pe_x = Local\ Peclet\ number$$

$$Pe_x = \mathrm{Re}_x\,\mathrm{Pr} = \frac{u_\infty x}{\alpha}.2; \quad for\ \mathrm{Pr} \le 1$$

For flow over a flat plate for an ordinary fluid with constant wall temperature,

$$Nu_x = 0.332\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}(exact)$$

$$Nu_x = 0.339\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}(exact) \qquad for\ \mathrm{Re_x} \le 5\times10^5,\ \mathrm{Pr} \ge 1$$

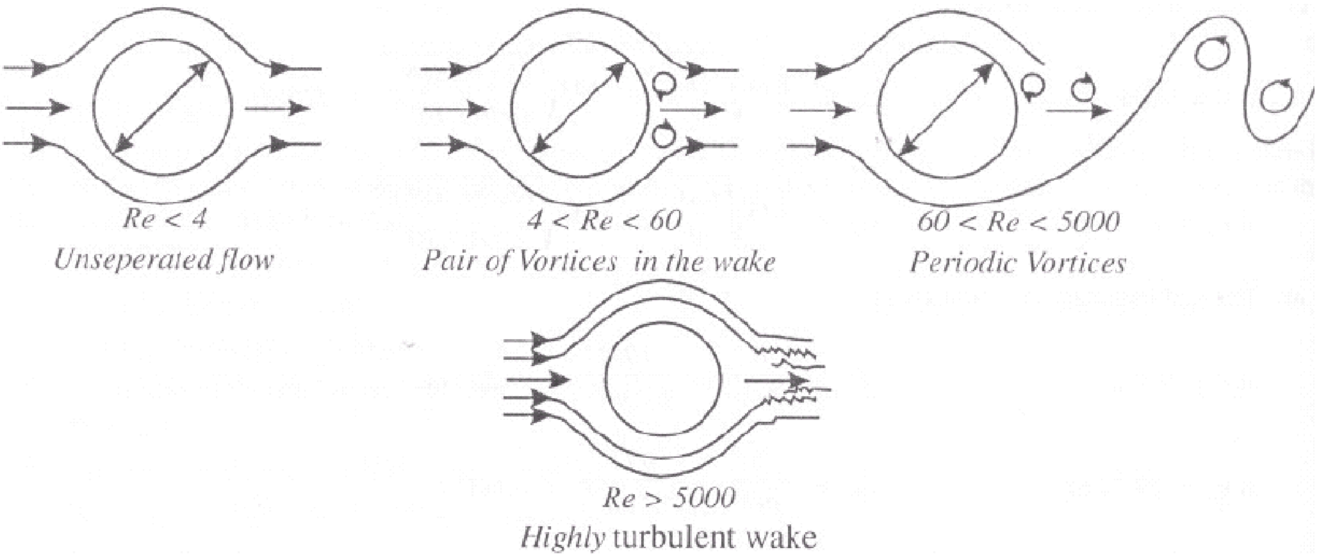
FLOW ACROSS A SINGLE CIRCULAR CYLINDER

- Because of the complexity of the flow patterns around the cylinder, determination of drag and heat transfer coefficients is a very complicated matter. Consider a fluid flowing around a circular cylinder of diameter D with a free stream velocity.
- The flow patterns at various Reynolds numbers are as shown in Fig 9.
- For Reynolds number lesser than 4, the flow remains unseperated and for Reynolds number more than 4, the vortices developing in the wake region make the velocity and temperature distribution analysis more complicated.

1. Drag coefficient

If F is the drag force acting on cylinder of diameter D and length L, then the drag coefficient C_D is given by,

$$\frac{F}{L\,D} = c_D \frac{\rho\,u_\infty^2}{2}$$



$Re < 4$ Unseperated flow

$4 < Re < 60$ Pair of Vortices in the wake

$60 < Re < 5000$ Periodic Vortices

$Re > 5000$ Highly turbulent wake

Fig. 9: Flow around a circular cylinder for different Reynolds numbers

2. Heat transfer coefficient

The average heat transfer coefficient h_m for the flow of gases or liquids across a single cylinder is given by,

$$Nu_m = \frac{h_m D}{k} = \left(0.4\,\mathrm{Re}^{0.5} + 0.06\,\mathrm{Re}^{2/3}\,\mathrm{Pr}^{0.4}\right)\left(\frac{\mu_\infty}{\mu_W}\right)^{0.25}$$

In the above equation all physical properties are evaluated at the free-stream temperature except for μ_W which is determined at wall temperature. This equation agrees with the experimental data within ± 25% in the range.

$$40 < \mathrm{Re} < 10^5;\ 0.67 < \mathrm{Pr} < 300;\ 0.25 < \frac{\mu_\infty}{\mu_W} < 5.2$$

A more elaborate but general correlation given by *Churchill et. al.* for the average heat transfer coefficient for flow across a single cylinder is given by,

$$Nu_m = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1 + \left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282,000}\right)^{5/8}\right]^{4/5}$$

FLOW ACROSS TUBE BUNDLES

- The design of heat exchanger and other industrial heat transfer equipments need the idea of heat transfer and pressure drop characteristics of tube bundles.
- The tube bundles used may be either *in-line* or *staggered* as shown in Fig 10. The geometry of the tube bundles comprises of defining *transverse pitch* P_T *and longitudinal pitch* P_L between the two centers.
- For staggered arrangement a diagonal pitch P_D represents the centers of tubes in the diagonal row. The *Reynolds number* for tube bundles is based on the flow velocity corresponding to minimum free flow area available for flow.
- This minimum flow area may occur in a transverse row or in a diagonal row.

$$If\ G_{\max} = \rho\, u_{\max} = Mass\ flow\ rate\ per\ unit\ area$$

$$u_{\max} = Maximum\ flow\ velocity$$

$$D = Outside\ diameter\ of\ the\ tube$$

Reynolds number is given by,

$$\mathrm{Re} = \frac{DG_{\max}}{\mu}$$

In the above equation $u_{\max}$ is measured based on the minimum free flow area available for fluid flow.

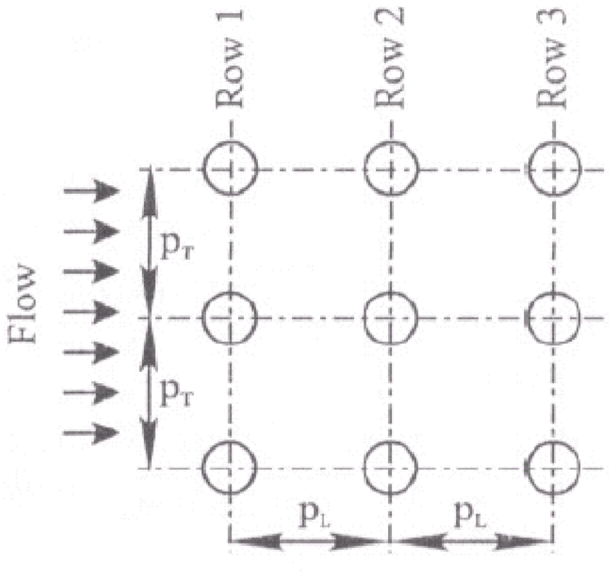


Fig 10: (a) In-line arrangement

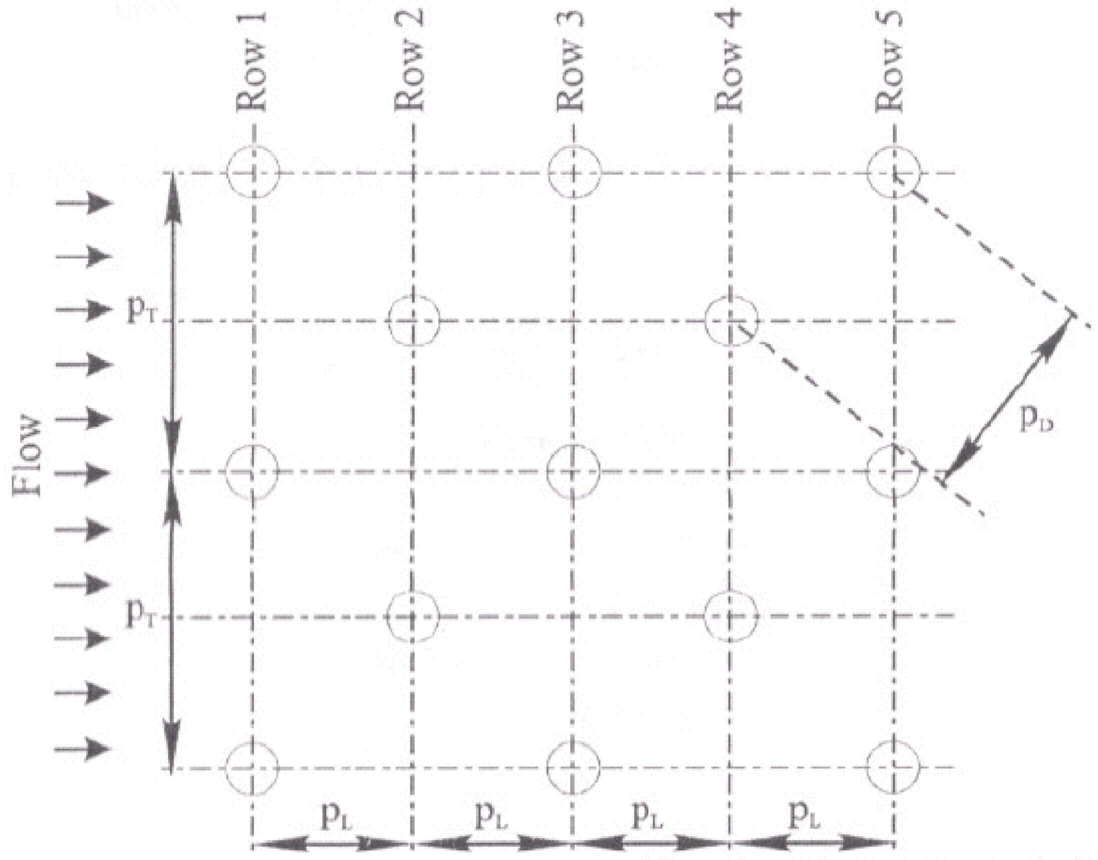


Fig 10: (b) Flow across tube bundles.

In-line Arrangement

For in-line arrangement, if u_∞ is the flow velocity in the heat exchanger before the fluid enters the tube banks,

$$u_{\max} = u_\infty \frac{p_T}{p_T - D} = u_\infty \frac{(p_T/D)}{(p_T/D) - 1}$$

Where, (P_T- D) =Minimum free flow area between the adjacent tubes in a transverse row per unit length of tube.

<u>Staggered Arrangement</u>

For, staggered arrangement,

$$u_{\max} = u_\infty \frac{p_T}{2(p_D - D)} = \frac{1}{2} u_\infty \frac{(p_T/D)}{(p_D/D) - 1}$$

Where (P_D - D) =Minimum free flow area that may occur between the adjacent tubes either in a transverse row or in a diagonal row per unit length of tube.

The flow patterns through tube bundles are very complicated and hence experimental analysis is the only approach to predict the heat transfer and pressure drop. Some of the correlations are given below.

Heat transfer coefficient

1. For flow across tube bundles having 10 or more transverse rows in direction of flow,

$$\frac{h_m D}{k} = 1.13 c_o \mathrm{Re}^n \mathrm{Pr}^{1/3} \; ; \; \mathit{for} \;\; 2000 < \mathrm{Re} < 40,000; \; \mathrm{Pr} > 0.7, \quad N \geq 10$$

$$\mathrm{Re} = \frac{D G_{\max}}{\mu} = \frac{D}{\mu}\left(\frac{M}{A_{\min}}\right)$$

2. Pressure Drop

Pressure drop for flow of gases over a bank of tubes may be calculated, by the following expression.

$$\Delta p = \frac{2 f^1 G_{\max}^2 N_T}{\rho}\left(\frac{\mu_W}{\mu_b}\right)^{0.14}$$

FLOW THROUGH CIRCULAR TUBES

A) LAMINAR FLOW

Most of the engineering equations involve steady state heat transfer and pressure drop in laminar forced convection inside circular tube regions away from the tube inlet where velocity and temperature profiles are fully developed. Hence the knowledge of friction factor and the heat transfer coefficient are essential to know the distributions of velocity and temperature.

1. a) Friction factor

The friction factor f for laminar flow inside a circular tube in the hydro dynamically developed region is given by,

$$f = \frac{64\,\mu}{\rho\, u_m D} = \frac{64}{\mathrm{Re}}$$

u_m = Mean flow velocity = ½ velocity at tube axis *(u_o)*

D = Inside radius of the tube

FREE CONVECTION

- When a hot plate is placed in a body of a fluid at rest and maintained at a uniform temperature lower than that of the plate, heat transfer takes place first by conduction, establishing a temperature gradient.
- The variation in temperature results in varied density which in a gravitational field will give rise to convective motion as a result of buoyancy forces. Fig 11 shows the development of boundary layer field in front of a hot vertical plate and cold vertical plate.
- In both the cases, the velocity boundary layer is developed. The peak velocity occurs somewhere within the boundary layer and the velocity is zero at both the plate surface and the edge of the boundary layer. In the region near the leading edge of the plate, the boundary layer development is *laminar* but becomes *turbulent* at a certain distance from the leading edge of the plate.
- Consider a fluid flowing between two parallel plates as shown in Fig 11. If the lower plate is better than the upper plate, a temperature gradient in the vertical direction is established. Due to the higher density of the fluid at the cold wall surface, the top layer is heavy.
- When the difference in temperature is increased beyond a certain critical value, the buoyancy forces override the viscous forces giving rise to convective motion.
- However, if the top plate is hotter, no natural convection currents are setup as the fluid is stable due to lighter top layer.
- The problem of energy transfer by natural convection arises in many engineering applications such as a *hot steam radiator for heating a room, refrigeration coils, electric transformers, transmission lines* etc.

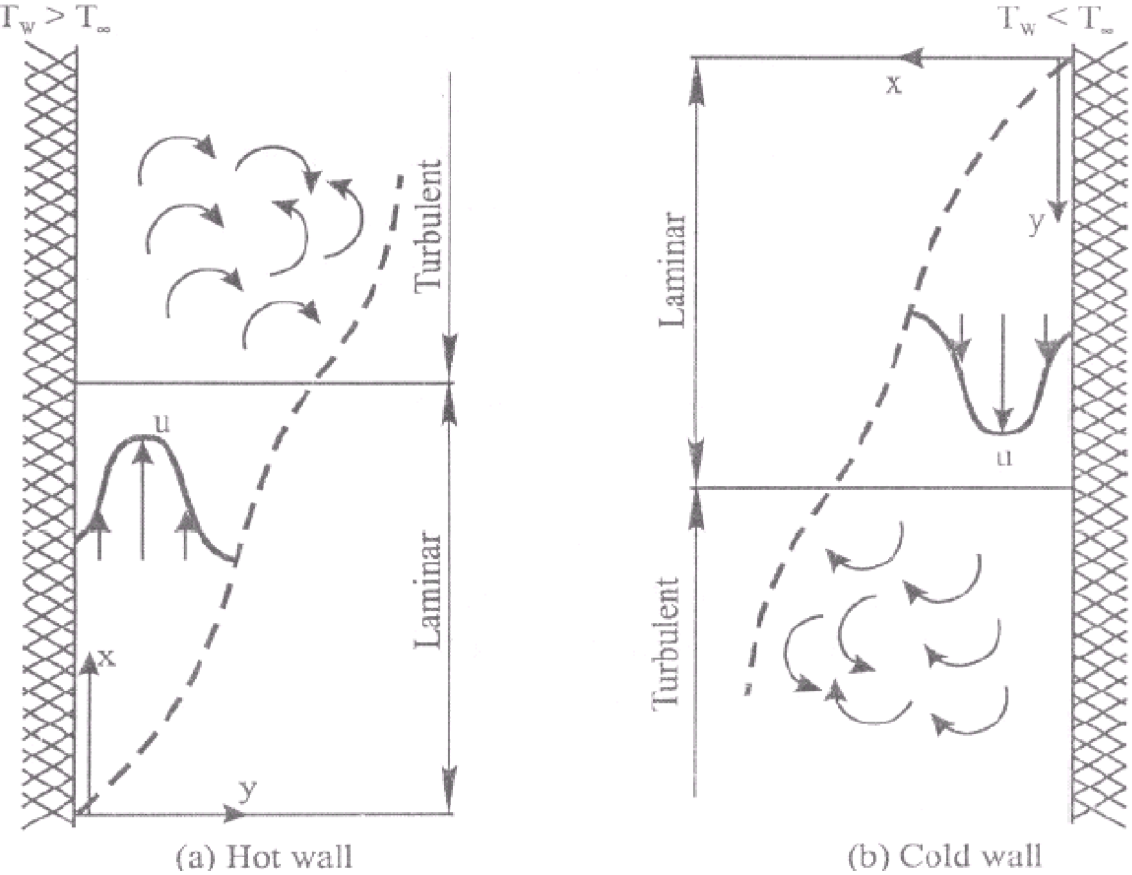


Fig. 4-15: Free convection on a Vertical plate

FREE CONVECTION CORRELATIONS

In this section different correlations used in determining free convection heat transfer are

VERTICAL PLATE

1. Uniform Wall Temperature

For constant wall temperature *McAdams* correlated the average *Nusselt number* with following expression.

$$Nu_m = c\left(Gr_L . \mathrm{Pr}\right)^n = cRa_L^n;$$

$$where\ \mathrm{L}{=}The\ vertical\ height\ of\ the\ plate;\ \ Gr = Grashoff\ number$$

$$\mathrm{Gr} = \frac{\beta\,\mathrm{gL}^3\left(T_W - T_\infty\right)}{\nu^2}$$

$$Nu_m = Nusselt\ number = \frac{h_m L}{k};\ \mathrm{Ra}_\mathrm{L} = Gr_L .\mathrm{Pr}$$

2. Uniform Wall Heat Flux

The following correlations are proposed for the local Nusselt number under uniform wall heat flux.

For laminar flow

$$Nu_x = 0.60\left(Gr_x^*.\mathrm{Pr}\right)^{1/5} \; for \; 10^5 < Gr_x^*\,\mathrm{Pr} < 10^{11}$$

For turbulent flow

$$Nu_x = 0.568\left(Gr_x^*.\mathrm{Pr}\right)^{0.22} \; for \; 2\times10^{13} < Gr_x^*\,\mathrm{Pr} < 10^{16}$$

$$Gr_x^* = Modified\ Grashof\ number = Gr_x\,Nu_x = \frac{\beta g\left(T_W - T_\infty\right)x^3}{\nu^2}\times\frac{q_W\,x}{k\left(T_W - T_\infty\right)} = \frac{\beta\, g q_W x^4}{k\nu^2}$$

VERTICAL CYLINDER

If the thickness of the thermal boundary layer is much smaller than the cylinder radius, then the average Nusselt number for free convection on a vertical cylinder is same as that of a vertical plate.

Hence *McAdams* correlation holds well here also i.e.

$$\mathrm{Nu}_m = c\left(Gr_L\,\mathrm{Pr}\right)^n = cRa_L^n$$

For fluids having *Prandtl number* equal to 0.7 and greater than 0.7, the vertical cylinder may be treated as a vertical flat plate when

$$\frac{L/D}{\left(Gr_L\right)^{1/4}} < 0.025 \; where\ D\ is\ the\ cylinder\ diameter$$

When the vertical cylinder is subjected to uniform wall heat flux, the local Nusselt numbers are given by the same empirical relations used for a vertical plate.

HORIZONTAL CYLINDER

For an isothermal horizontal cylinder, *Churchill* and *Chu* have proposed the following relation,

$$Nu_m = \frac{hD}{k} \;;\; Ra_D = Gr_D\,\mathrm{Pr} = \left(\frac{\beta g\left(T_W - T_\infty\right)D^3}{\nu^2}\right)\mathrm{Pr}$$

Morgan presented the fol1owing relation from the horizontal isothermal cylinder,

$$Nu_m = \frac{hD}{k} = cRa_D^n \; for \; 10^{-10} < Ra_D < 10^{12}$$

BOILING AND CONDENSATION

INTRODUCTION

- Boilers and condensers which are used as heat exchangers posses unique characteristics of heat transfer mechanism on the condensing and boiling side.
- When a vapour strikes a surface maintained at a temperature below the corresponding saturation temperature the vapour will immediately condense into the liquid phase.
- The process of condensation may take place into two different types.

1. *Film wise condensation*

If the condensation takes place continuously over the surface and the surface is kept cooled by some means the condensed liquid is removed from motion resulting from gravity, then the condensing surface is covered by means of a thin layer of liquid. This process is known *as* ***film wise condensation.***

2. *Drop wise condensation*

- If the traces of oil are present during the condensation of steam on a highly polished surface, the film of condensate formed is broken into droplets. This process is known as *drop wise condensation.*
- The rate of heat transfer in case of drop wise condensation is more as it offers much less resistance to heat flow on the vapour side than the film wise condensation. If the vapour contains some non condensable gas, this gas will collect on the condensing side and acts as resistance to heat flow on the condensing side.
- When a liquid is in contact with a surface that is maintained at a temperature above the saturation temperature of the liquid, *boiling* will occur.
- The boiling phenomenon is very complicated as it involves a large number of variables and complex hydrodynamic developments.

NUSSELT THEORY OF FILM CONDENSATION ON VERTICAL SURFACES

- Vapour condensation is the most commonly observed phenomenon in many engineering applications like steam condensation in condensers etc.
- When a liquid wets a surface, condensation occurs in the form of a smooth film, which flows down the surface by gravitational force. The liquid film thus formed offers resistance to heat flow reducing rate of heat transfer.

- Numerous experimental and theoretical investigations have been conducted to determine the heat transfer coefficients during film wise condensation of pure vapour over surfaces.

- Consider the condensation of a vapour on a vertical plane surface as shown in Fig. 1 Let x is the axial coordinate which is measured in the downward direction along the plate and y is the coordinate normal to the condensing surface.

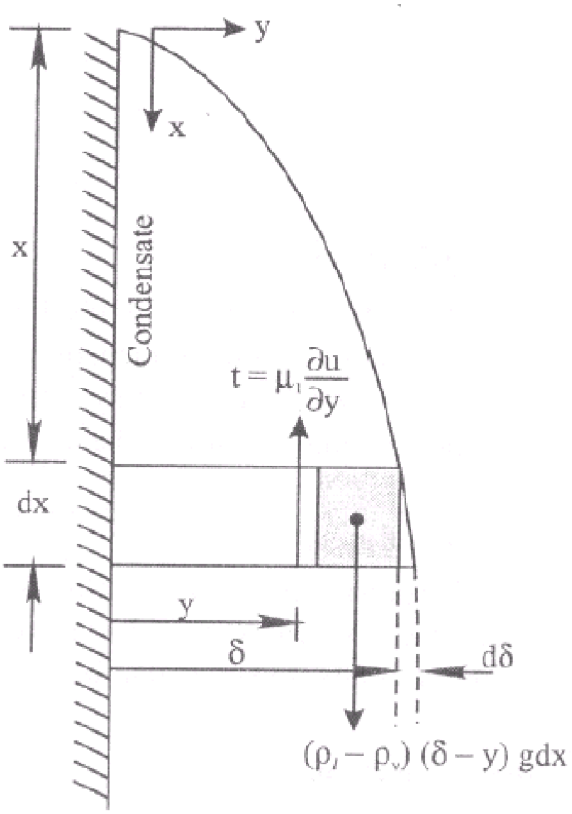


Fig.1: Film wise condensation on a vertical plane surface

Considering the force acting on a volume element we can equate the force acting upward to the buoyancy force acting downward.

$$\therefore \quad \mu_l \frac{du}{dy} dx = (\rho_l - \rho_v)(\delta - y)\, g dx$$

$$\frac{du}{dy} = \frac{g(\rho_l - \rho_v)}{\mu_l}(\delta - y)$$

Where δ ÷ δ (x) is the thickness of the condensate at x

μ ÷ Viscosity and subscripts *l* and v refer to liquid and vapour phases.

At the wall surface liquid velocity is zero. u = 0 and y = 0

Integrating the equation subject to boundary condition

$$u(y) = \frac{g(\rho_l - \rho_v)}{\mu_l}\left(\delta y - \frac{1}{2} y^2\right)$$

The mass flow rate / unit width of plate at any point x, is

$$m\,(x) = \int_0^\delta \rho_l u\,dy = \int_0^\delta \rho_l \frac{g(\rho_l - \rho_v)}{\mu_l}\left(\delta y - \frac{1}{2}y^2\right)$$

$$= \frac{g\rho_l(\rho_l - \rho_v)\delta^3}{3\mu_l}$$

Differentiating equation w.r.to δ,

$$dm = \frac{g\rho_l(\rho_l - \rho_v)\delta^2}{\mu_l}\,d\delta$$

The heat rate *dQ* during condensation of *dm* is

$$dQ = h_{f_g}\; dm$$

$$h_{f_g} = \text{Latent heat of condensation}$$

$$dQ = k_l\,(dx.1)\frac{T_V - T_W}{\delta}$$

k_l = Thermal conductivity of liquid

T_v = Vapour saturation temperature

T_w =Wall surface temperature.

Substituting equations

$$h_{f_g}\; dm = k_l\;(dx)\frac{T_V - T_W}{\delta}$$

$$h_{f_g}\left[\frac{g\rho_l(\rho_l - \rho_V)\delta^2}{\mu_l}\right]d\delta = k_l\;dx\,\frac{T_V - T_W}{\delta}$$

$$\frac{d\delta}{dx} = \frac{\mu_l k_l(T_V - T_W)}{g\rho_l(\rho_l - \rho_v)h_{f_g}\delta^3}$$

Integrating equation [0] with condition ð=0 for $x = 0$, thickness of the condensate layer as a function of position x is given by

$$\delta(\mathbf{x}) = \left[\frac{4\mu_l k_l(T_V - T_W)x}{g(\rho_l - \rho_v)\rho_l h_{f_g}}\right]^{1/4}$$

If h_x is the local heat transfer coefficient, then we can equate heat convected to heat conducted.

$$h_x\,(T_V - T_W) = k_l\,\frac{(T_V - T_W)}{\delta(x)} \qquad\qquad h_x = \frac{k_l}{\delta(x)}$$

$$\mathbf{h}_x = \left[\frac{g\rho_l(\rho_l - \rho_v)h_{f_g}k_l^3}{4\mu_l(T_V - T_W)x}\right]^{1/4}$$

Local Nusselt's number,

$$Nu_x = \frac{h_x x}{k_l} = \left[\frac{g\rho_l(\rho_l - \rho_v)h_{f_g}k_l^3}{4\mu_l(T_V - T_W)x}\right]^{1/4}$$

As the local heat transfer coefficient h_x varies with the distance x, the average heat transfer coefficient is given by,

$$h_m = \frac{1}{L}\int_0^L h_x\,dx = \frac{4}{3}h_x\Big|_{x=L} \quad \text{for } 0 \le x \le L$$

$$\boxed{\mathbf{h}_m = 0.943\left[\frac{g\rho_l(\rho_l - \rho_v)h_{f_g}k_l^3}{\mu_l(T_V - T_W)L}\right]^{1/4}}$$

Where the physical properties are evaluated at the film temperature,

$$T_f = \frac{1}{2}(T_W + T_V)$$

<u>Condensation on Inclined surfaces</u>

For an inclined surface having an inclination 0 with the horizontal, the *local heat transfer coefficient* is given by,

$$\boxed{h_x = \left(\frac{g\rho_l(\rho_l - \rho_V)h_{f_g}k_l^3\sin\phi}{4\mu_l(T_V - T_W)x}\right)^{1/4}}$$

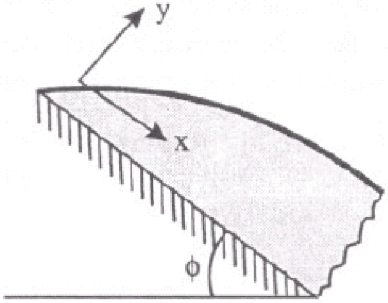


Fig. 2: Condensation on inclined surfaces

Condensation on a Horizontal tube

According to Nusselt's analysis for laminar film wise condensation on a horizontal tube surface, average heat transfer coefficient is given by,

$$\boxed{h_m = 0.725\left(\frac{g\rho_l(\rho_l-\rho_v)h_{f_g}k_l^3}{\mu_l(T_V-T_W)D}\right)^{1/4}}$$

$$\frac{h_m \text{ on vertical surface}}{h_m \text{ on horizontal tube}} = 1.30\left(\frac{D}{L}\right)^{1/4}$$

Where L and D are length of vertical surface and diameter of horizontal tube respectively.

Condensation on Horizontal Tube Banks

In the horizontal tube banks arranged in vertical tiers as shown in Fig 3, the condensate from one tube drains onto the tube just below it. Assuming smooth flow of drainage from one tube to the there, for a vertical tier of N tubes each of diameter D, the average heat transfer coefficient is given by

$$\boxed{\begin{aligned}[h_m]_{N\,tubes} &= 0.725\left(\frac{g\rho_l(\rho_l-\rho_v)h_{f_g}k_l^3}{\mu_l(T_V-T_W)ND}\right)^{1/4}\\ &= \frac{1}{N^{1/4}}[h_m]_{1\,tube}\end{aligned}}$$

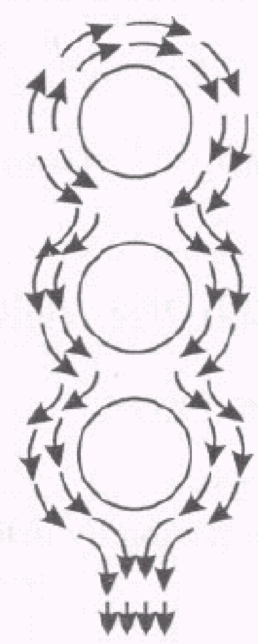


Fig. 3: Condensation on horizontal tube banks

REYNOLDS NUMBER FOR CONDENSATE FLOW

- Even though the chances of transition from laminar to turbulent flow in case of a single horizontal tube are very less, turbulence may start at the lower portions of a vertical tube.
- Due to turbulence, the average heat transfer coefficient increases. Hence the *Reynolds number for condensate flow* for transition from laminar to turbulent flow is to be defined.

If u_m = Average velocity of condensate film

D_h = Hydraulic diameter for condensate flow,

$$= \frac{4 \times \text{Cross sectional area of condensate flow}}{\text{Wetted perimeter}} = \frac{4A}{P}$$

$$Re = \frac{D_h u_m \rho_l}{\mu_l} = \frac{4(\rho_l u_m A)}{\mu_l P}$$

If m is the mass flow rate of the condensate, then Reynolds number at the lowest part of the condensing surface is expressed as

$$Re = \frac{4\dot{m}}{\mu_l P}$$

$$P = \begin{cases} \pi D & \text{for vertical tube of outside diameter } D \\ 2L & \text{for horizontal tube of length } L \\ W & \text{for vertical or inclined plate of width W} \end{cases}$$

Experimentally it is shown that the transition occurs at *Re* of about 1800.

CORRELATIONS USED IN FILM WISE CONDENSATION

LAMINAR FLOW

Vertical surface

McAdams equation for determining the average heat transfer coefficient is as follows.

$$h_m = 1.13\left(\frac{g\rho_l(\rho_l - \rho_v)h_{f_g}k_l^3}{\mu_l(T_V - T_W)L}\right)^{1/4}$$

$$\text{since } \rho_u \le \rho_l, \quad h_m = 1.13\left[\frac{g\rho_l^2 h_{f_g} k_l^3}{\mu_l(T_V - T_W)L}\right]^{1/4}$$

On rearranging the above equation,

$$h_m\left(\frac{\mu_l^2}{k_l^3\rho_l^2 g}\right)^{1/3} = 176\,Re^{-1/3}$$

For $Re < 1800$

Horizontal Tube

For a single horizontal tube, average heat transfer coefficient is given by,

$$h_m = 0.725\left[\frac{g\rho_l(\rho_l-\rho_v)h_{f_g}k_l^3}{\mu_l(T_V-T_W)D}\right]^{1/4}$$

TURBULENT FLOW

Kirk bride proposed the following empirical correlation for film condensation on a vertical plate after the start of turbulence

$$h_m\left(\frac{\mu_l^2}{k_l^3\rho_l^2 g}\right)^{1/3} = 0.0077\,Re^{0.4}$$

All the physical properties are evaluated at film temperature in all the above equations.

FILM CONDENSATION INSIDE HORIZONTAL TUBES

It is practically noticed that the vapour condensing inside horizontal tubes of condensers of refrigeration and air conditioning system have significant velocity. *Chato* has recommended the following correlation at low vapour velocities inside horizontal tubes

$$\boxed{\mathrm{h_m} = 0.555\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3h'_{f_g}}{\mu_l(T_{Sat}-T_S)D}\right]^{1/4}}$$

$$h'_{f_g} = h_{f_g}+\frac{3}{8}c_{pl}(T_V-T_W)$$

This equation holds good for inlet conditions and inside diameter D of the tube.

$$Re_v = \frac{\rho_v u_v D}{\mu_v} < 35{,}000$$

Akers, Dean's et.al. have recommended the following correlation at higher flow rates.

$$\boxed{\mathrm{h_m} = 0.026\,Pr_l^{1/3}\left[Re_l+Re_v\left(\frac{\rho_l}{\rho_v}\right)^{1/2}\right]^{0.8}}$$

For $Re_v > 20{,}000$ $Re_l > 5000$

m_l and m_v are the mass flow rate of liquid and vapour respectively.

DIFFERENT REGIMES OF BOILING MECHANISM

- When a liquid is in contact with a surface maintained at a temperature above the saturation temperature of the liquid, boiling occurs.
- The mechanism of heat transfer in boiling systems is better understood by considering *pool boiling.* Fig 4 shows the characteristics of pool boiling for water at atmospheric pressure. The boiling curve illustrates the variation of heat transfer coefficient as a function of temperature difference between wire and water saturation temperatures.
- Three different regimes can be explained from the curve by immersing an electric resistance wire into a body of saturated water and initiating boiling on the surface of the wire by passing current through it.

1. Free convection regime

- In this regime, the energy transfer from the heater surface to the saturated liquid takes place by free convection.
- Even though the surface is only a few degrees above the liquid saturation temperature, free convection currents produced in the liquid are sufficient enough to remove heat from the surface.
- As heat transfer takes place by free convection we can use all the correlations for free Convection in the form.

$$Nu = f(Gr\ Pr)$$

Hence heat flux in this regime

$$\boxed{q = h\,(T_W - T_{sat})}$$

2. Nucleate boiling regime

- In this regime bubbles are formed on the surface of the heater. This regime can be separated into two distinct regions.
- In the *region II,* bubbles start to form on heater surfaces at specific point and as soon as they detach from the surface they are dissipated in the liquid.
- In the *region III,* the rate of generation of bubbles at numerous nucleation sites result in the Formation of continuous columns of vapour and high heat fluxes.
- Due to large heat fluxes obtainable with small temperature differences, the nucleate boiling regime is most desirable.

$$\Delta T = (T_W - T_S)\ {}^0\mathrm{C}$$

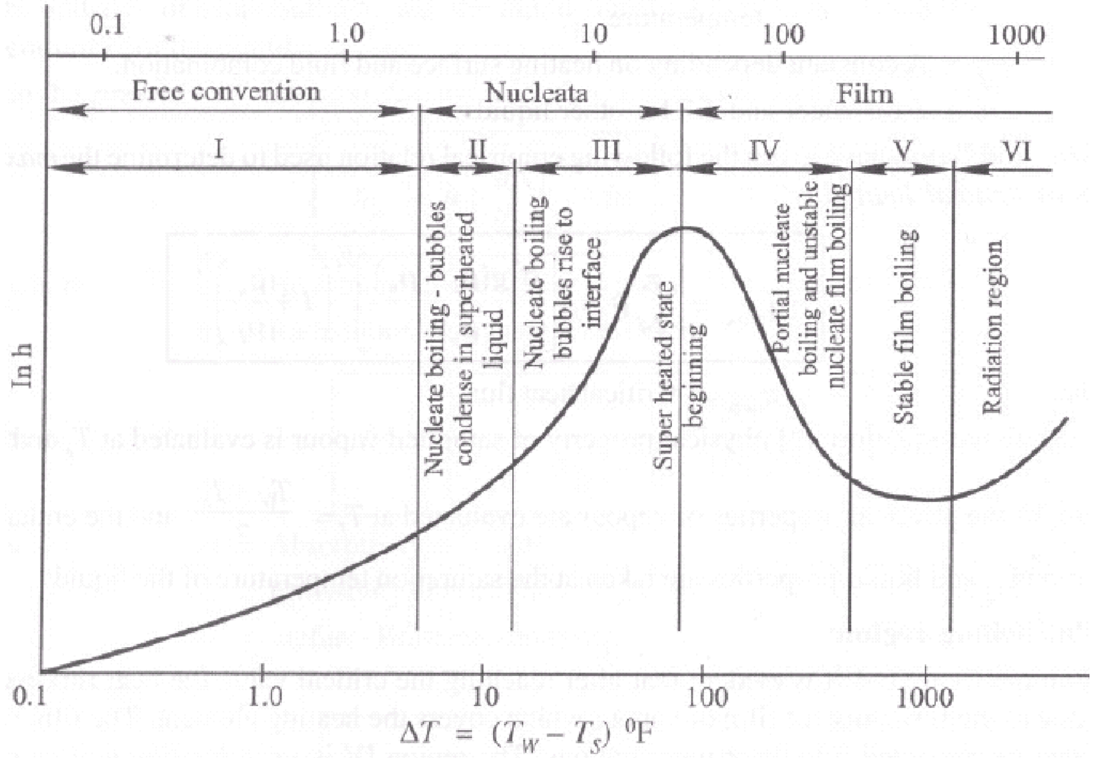


Fig 4: Pool boiling regimes

In the nucleate boiling regime heat flux increases rapidly until a peak value. This location is known as *burnout point* or *departure from nucleate boiling* (DNB), or the *critical heat flux* (CHF). Beyond this point a large temperature difference *is* needed to realize *the resulting* heat *flux.* This high temperature difference may burn or melt the heating element. The following empirical relation is used to correlate the heat flux in the entire nucleate as Boiling regime.

$$\boxed{q = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma^*}\right]^{0.5}\left[\frac{c_{pl}\Delta T}{h_{fg}\,Pr_l^n}\times\frac{1}{c_{sf}}\right]^3}$$

where, q = Boiling heat flux

μ_l = Viscosity of saturated liquid

h_{fg} = Latent heat of vaporization

g = Gravitational acceleration = 9.81 m/s^2

ρ_l, ρ_v = Density of liquid and saturated vapour

σ^* = Surface tension of liquid-vapour interface

c_{pl} = Specific heat of saturated liquid

$Pr_l = c_{pl}\,\mu_l / k_l$ = Prandtl number of saturated liquid

$\Delta T = T_W - T_S$ = Temperature difference between the wall and the saturation temperature.

c_{sf} = A constant depending on heating surface and fluid combination.

n = 1 for water and 1.7 for other liquids

Zuber and *Tribus* have given the following empirical relation used to determine the *maximum or peak or critical heat flux.*

$$q_{\max} = \frac{\pi}{24}\rho_v h_{fg}\left[\frac{\sigma^* g(\rho_l - \rho_v)}{\rho_v^{\,2}}\right]^{1/4}\left(1 + \frac{\rho_v}{\rho_l}\right)^{1/2}$$

3. Film boiling regime

- From the figure 4 it is evident that after reaching the critical value the heat flux reduces. This is due to the formation of mm of vapour which covers the heating element. The film boiling regime can be separated into three more regions.
- The region IV is *unstable film boiling* region, where the unstable vapour film collapses and reforms due to convective currents & surface tension.
- As the average wetted area of the heater surface decreases the heat flux decreases due to increased surface temperature. The region V is *stable film boiling* region in which heat flux drops to a minimum due to continuous formation of vapour film on the heater surface.
- In the region VI the high surface temperature of the heater gives way to thermal radiation effect and hence the heat flux begins to increase.

The average heat transfer coefficient ho for stable film boiling on the outside of a horizontal tube or cylinder in the absence of radiation is given by,

$$\boxed{h_o = 0.62\left[\frac{k_v^3\rho_v(\rho_l - \rho_v)g\,h_{fg}}{\mu_v D_0 \Delta T}\left(1 + \frac{0.4\,c_{pv}\Delta T}{h_{fg}}\right)\right]^{1/4}}$$

k_v = Thermal conductivity of saturated vapour

D_0 = Outside diameter of tube

c_{pv} = Specific heat of saturated vapour.

$\Delta T = T_W - T_S$ = Temperature difference between wall and saturation temperature

In the *presence of radiation,* the average heat transfer coefficient is given by,

$$\boxed{h_m = h_o\left(\frac{h_o}{h_m}\right)^{1/3} + h_r}$$

Where h_o = Heat transfer coefficient without the radiation effects.

h_r =Radiation heat transfer coefficient.

$$\boxed{h_r = \frac{1}{\left[\left(1/\varepsilon\right)+\left(1/\alpha\right)-1\right]} \times \frac{\sigma\left(T_W^4 - T_S^4\right)}{\left(T_W - T_S\right)}}$$

Where

a =Absorptivity of liquid

s =Emissivity of hot tube

o =Stefan - Boltzman constant

$$\boxed{h_m = h_o + \frac{3}{4}h_r}$$

HEAT EXCHANGERS

INTRODUCTION

- *The devices that are used to facilitate heat transfer between two or more fluids at different temperatures are known as heat exchangers.*
- Different types and sizes of heat exchangers are used in steam power plants, chemical processing units, building heating and air conditioning, house hold refrigerators, car radiators, radiators for space vehicles etc.
- This chapter deals with classification of heat exchangers, the overall heat transfer coefficient, LMTD, NTU method and Effectiveness of heat exchangers.

CLASSIFICATION OF HEAT EXCHANGERS

Heat exchangers are broadly classified based on the following considerations.

1. Classification based on Transfer Process

Based on heat transfer process heat exchangers are classified as *direct contact* and *indirect contact*

a) Direct contact

In direct contact heat exchangers, heat transfer takes place between two immiscible fluids like a gas and a liquid coming into direct contact.

e.g.: Cooling towers, jet condensers for water vapour, and other vapors utilizing water spray.

b) Indirect contact

In indirect - contact type of heat exchangers the hot and cold fluids are separated by an impervious surface. There is no mixing of the two fluids and these heat exchangers are also known as *surface heat exchangers.*

e.g: Automobile radiators.

2. Classification based on Compactness

The ratio of the heat transfer surface area on one side of the heat exchanger to the volume is used as a measure of compactness. The heat exchanger having a surface area density on anyone side greater than about 700 m2/m3 is known as a *compact heat exchanger.*

e.g.: Automobile radiators (1100 m^2/m^3*),Gas turbine engines (6600* m^2/m^3*),*

Human lungs (20,000 m^2/m^3*)*

3. Classification based on type of construction

Based on the type of construction heat exchangers are classified as follows.

a) Tubular heat exchangers

- Tubular heat exchangers are available in many sizes, flow arrangements and types. They can withstand a wide range of operating pressures and temperatures.
- A commonly used design is *shell-and-tube* heat exchanger which consists ofround tubes mounted on cylindrical shells with their axes parallel to that of the shell.
- The combination of fluids may be liquid-to-liquid, liquid-to -gas or gas-to-gas.

b) Plate heat exchangers

- In these types thin plates are used to affect heat transfer. The plates may be either smooth or corrugated.
- These heat exchangers are suitable only for moderate temperature or pressure as the plate geometry restricts the use of high pressure and temperature differentials.
- The compactness factor for plate exchangers ranges from *120* to *230* m^2/m^3.

c) Plate fin heat exchangers

- These heat exchangers use louvered or corrugated fins separated by flat plates. Fins can be arranged on each side of the plate to get *cross-flow, counter-flow* or *parallel-flow* arrangements.
- These heat exchangers are used for gas-to-gas applications at low pressures (10 atm.) and temperatures not exceeding 800°C.
- They also find use in cryogenic applications. The compactness factor for these heat exchangers is upto 6000 m^2/m^3.

d) Tube-fin heat exchangers

- Such heat exchanges are used when a high operating pressure or an extended surface is needed on one side. The tubes may be either round or flat.
- Tube-fin heat exchangers are used in gas- 252 Heat and Mass Transfer *turbine, nuclear, fuel cell, automobile, airplane, heat pump, refrigeration, Cryogenics* etc.
- The operating pressure is about 30 atm. and the operating temperature ranges from low cryogenic temperatures to about 870 Dc.
- The maximum compactness ratio is about 330 m^2/m^3

e) Regenerative heat exchangers

- Regenerative heat exchangers may be either *static* type or *dynamic* type.
- The *static* type has no moving parts and consists of a porous mass like balls, pebbles, powders etc. through which hot and cold fluids pass alternatively.

  e.g.: air preheaters used in coke manufacturing and glass melting plants.
- In *dynamic type* regenerators, the matrix is arranged in the form of a drum which rotates about an axis in such a manner that a given portion of the matrix passes periodically through the hot stream and then through the cold stream.
- The heat absorbed by the matrix from the hot stream is transferred to the cold stream during its run.

4. Classification based on flow Arrangement

Based on flow arrangement heat exchangers are classified into the following principal types.

a) Parallel-flow

In this heat exchanger, the hot and the cold fluids enter at the same end of the heat exchanger and flow through in the same direction and leave together at the other end as shown in Fig 5(a).

b) Counter flow

In this heat exchanger hot and cold fluids enter in the opposite ends of the heat exchanger and flow in opposite directions as shown in Fig 5(b).

c) Cross flow

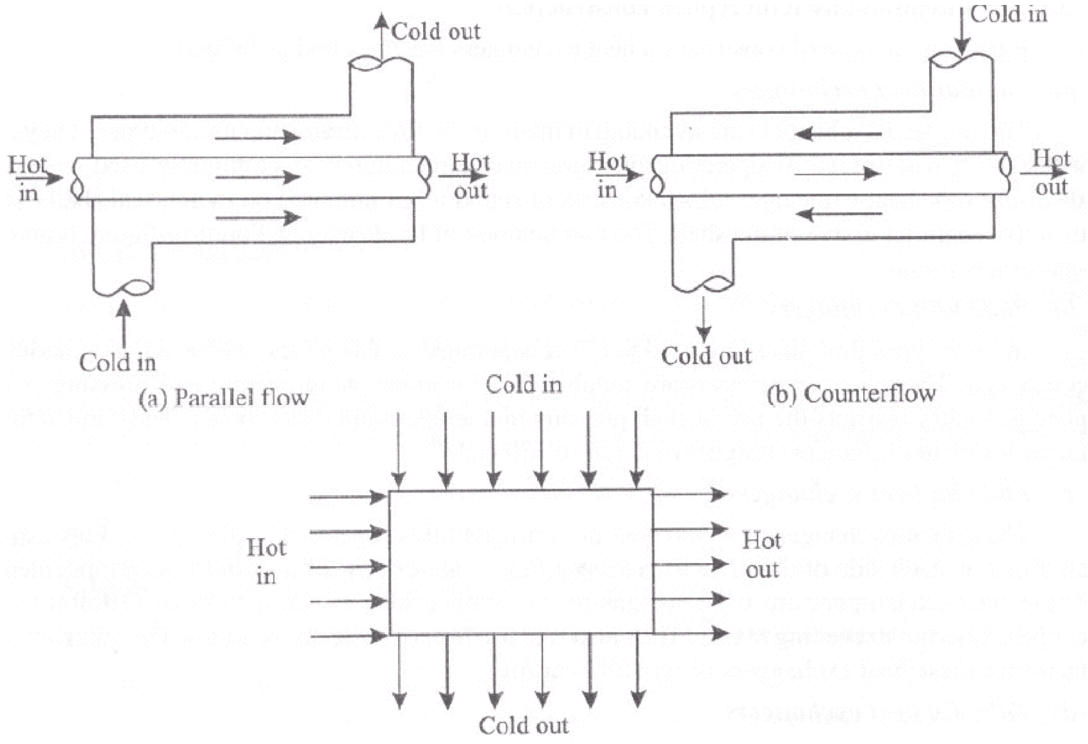
- In this heat exchanger, the two fluids flow at right angles to each other as shown in Fig 5 (c).
- In this arrangement the flow may be mixed or unmixed.

In general, in a cross flow exchanger, three idealized flow arrangements are possible

1. The fluids are unmixed
2. One fluid is mixed, and the other is unmixed
3. Both fluids are mixed.

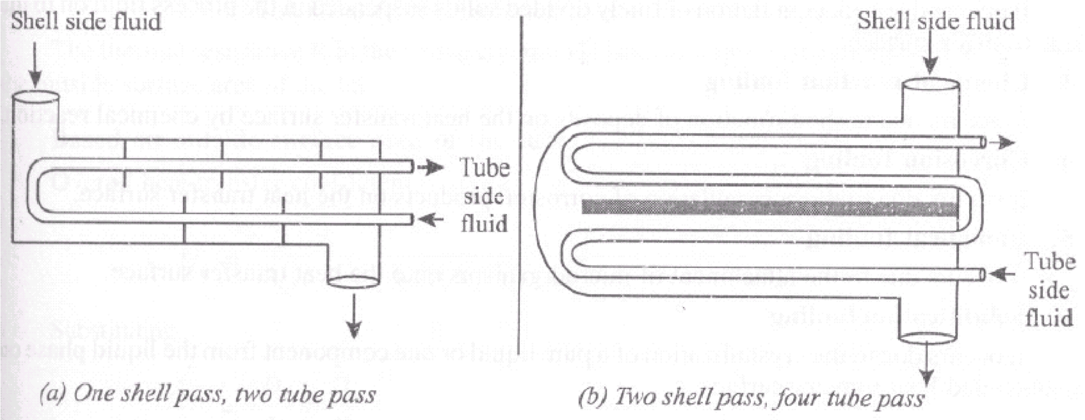
d) Multipass flow

- Since multi passing increases the overall effectiveness over individual effectiveness they are frequently used in heat exchanger design.
- Different multipass flow arrangements are *"One shell pass, two tube pass"* known as *"one - two"* heat exchanger, *"two shell pass, two tube pass",* etc. as shown in Fig 6.



(a) Parallel flow (b) Counterflow

Fig. 5: Classification by flow arrangement



(a) One shell pass, two tube pass

(b) Two shell pass, four tube pass

Fig. 6: Multi pass flow arrangement

5. Classification based on heat transfer mechanism

Heat exchangers are classified based on the following modes of heat transfer.

1. Single phase forced or free convection.
2. Phase change due to boiling and condensation.
3. Radiation or combined convection and radiation.

FOULING FACTOR

- In heat exchanger applications, the heat transfer surface is fouled with the accumulation of deposits.
- Due to this accumulation thermal resistance in the path of heat flow increases reducing heat transfer rate.
- The factor which is introduced to include the effect of fouling is known as *fouling factor, F.* It is expressed in $m^2. C / W$.

Ill effects

1. Due to fouling, the size of the heat exchanger considerably increases resulting in higher capital cost.
2. Due to fouling thermal efficiency of the heat exchanger reduces which results in energy loss.
3. Fouling necessitates periodic cleaning of heat exchangers which increases the maintenance cost.
4. For periodic cleaning the heat exchangers are shut down which means loss of production during this period.

Types of Fouling

1. Scaling or precipitation fouling

It occurs mainly due to crystallization from solution of dissolved substance on to the heat transfer surface.

2. Particulate fouling

It occurs due to accumulation of finely divided solids suspended in the process fluid on to the heat transfer surface.

3. Chemical reaction fouling

It occurs due to the formation of deposits on the heat transfer surface by chemical reaction.

4. Corrosion fouling

It occurs due to the accumulation of corrosion products on the heat transfer surface.

5. Biological fouling

It occurs due to the attachment of microorganisms onto the heat transfer surface.

6. Solidification fouling

It occurs due to the crystallization of a pure liquid or one component from the liquid phase on a sub cooled heat transfer surface.

MECHANISM OF FOULING

- Mechanism of fouling is very much complicated and its prediction is also very difficult.
- When a new heat exchanger is put into service its efficiency decreases progressively due to the build up of fouling resistance.
- The rate at which fouling occurs is mainly dependent of fluid velocity and temperature.
- Higher velocity decreases both the rate of deposit and the amount, whereas higher temperature increases both the rate of deposit and the amount.
- The fouling factors in heat transfer calculations are prepared by the Tubular Equipment Manufacturers Association (TEMA) and are available in the heat transfer tables.

OVER ALL HEAT TRANSFER COEFFICIENT

- For the analysis of heat exchangers it is necessary to combine the various thermal resistances in the path of heat flow from the hot to the cold fluid.
- These combined resistances are expressed in terms of overall heat transfer coefficient, U.

The total *thermal resistance* R to the heat flow across a tube, between the inside and the Outside flow is given by,

R = Thermal resistance of (Inside flow + Tube material + Outside flow)

$$R = \frac{1}{A_i h_i} + \frac{t}{kA_m} + \frac{1}{A_0 h_0}$$

Where A_o, A_i = Surface areas of tube outside and inside surfaces respectively, m^2

$$A_m = \frac{A_0 - A_i}{ln(A_0/A_i)} = \text{Logarithmic mean area, m}^2$$

h_i, h_o = Inside and outside heat transfer coefficients respectively.

k = Thermal conductivity of tube material W/m^{o}.C

t = Thickness of tube material, m

The thermal resistance R in the above equation can be expressed either based on inside or the outside surface area of the tube.

Based on outside surface area of the tube

Overall heat transfer coefficient

$$U_o = \frac{1}{A_0 R} = \frac{1}{(A_0/A_i)(1/h_i) + (A_0/A_m)(t/k) + (1/h_0)}$$

$$\frac{A_0}{A_m} = \frac{D_0}{2t} ln \frac{D_0}{D_i}$$

$$2t = D_o - D_i$$

$$U_o = \frac{1}{A_0 R} = \frac{1}{(D_0/D_i)(1/h_i) + (1/2k) D_0 \, ln(D_0/D_i) + (1/h_0)}$$

Based on inside surface area of the tube

Overall heat transfer coefficient

$$U_i = \frac{1}{A_i R} = \frac{1}{(1/h_i) + (A_i/A_m)(t/k) + (A_i/A_0)(1/h_0)}$$

$$U_i = \frac{1}{(1/h_i) + (1/2k) D_i \, ln(D_0/D_i) + (D_i/D_0)(1/h_0)}$$

Where D_i and D_o are the inside and outside diameters of the tubes, respectively. When the thermal conductivity of the tube is high but its thickness is small, equation [5] reduces to

$$U_i = \frac{1}{\left(\frac{1}{h_0}\right)+\left(\frac{1}{h_i}\right)}$$

If F_i and F_o are the *fouling factors* on the inside and outside surfaces of the tube, then the thermal resistance R in the heat flow path is given by,

$$R = \frac{1}{A_i h_i} + \frac{F_i}{A_i} + \frac{t}{kA_m} + \frac{F_0}{A_0} + \frac{1}{A_0 h_0}$$

Since in heat exchanger applications, the overall heat transfer coefficient is expressed based on the outer tube surface, equation is *expressed as*

$$U_o = \frac{1}{(D_0/D_i)(1/h_i) + (D_0/D_i)F_i + (D_0/2k)\ln(D_0/D_i) + F_0 + (1/h_0)}$$

$$U_o = \frac{1}{(D_r/h_i) + D_r F_i + (D_0/2k)\ln D_r + F_0 + (1/h_0)}$$

LMTD METHOD FOR PARALLEL AND COUNTER FLOW HEAT EXCHANGERS

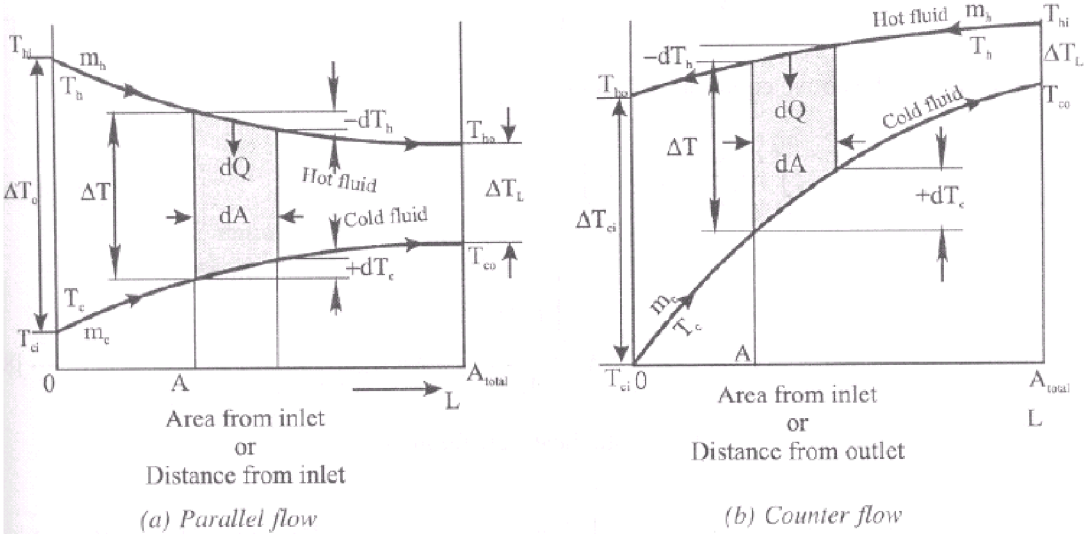
Consider a single flow arrangement of heat exchangers as shown in Figure 5.3

Let A = Heat transfer area measured at inlet, m^2.

m_c = Mass flow rate of cold fluid, kg/h

m_h = Mass flow rate of hot fluid

U = Local overall heat transfer coefficient between two fluids, W/m^2°C.



(a) Parallel flow (b) Counter flow

Fig 7: LMTD method for analysis of heat exchangers

The rate of heat transfer dQ from the hot fluid to the cold fluid through an elemental strip of area dA about location A is given by,

$$dQ = UdA\Delta T$$

The rate of heat transfer dQ is equal to the amount of heat lost by the hot fluid or the amount of heat gained by the cold fluid. Hence,

$$dQ = -m_h c_{ph} d T_h \text{ (hot fluid)}$$
$$= m_c c_{pc} d T_c \text{ (cold fluid)}$$

$$Q = U_m A_t \frac{\Delta T_o - \Delta T_L}{ln\left(\frac{\Delta T_o}{\Delta T_L}\right)}$$

$$LMTD = \frac{\Delta T_0 - \Delta T_L}{ln\left(\frac{\Delta T_0}{\Delta T_L}\right)}$$

s- NTU METHOD FOR PARALLEL AND COUNTER FLOW HEAT EXCHANGERS

- For the analysis of the heat exchangers two problems that are mainly encountered are *rating* and *sizing* of heat exchangers.
- The *rating* problem deals with the determination of the heat transfer rate, the fluid outlet temperatures, and the pressure drops either for the existing or already sized heat exchanger.
- The *sizing* problem deals with the determination of matrix dimensions to meet the specified heat transfer and pressure drop requirements.
- If the inlet and outlet temperatures of the hot fluid and the cold fluid and overall heat transfer coefficient are known then LMTD method is used to solve both rating and sizing problems.
- However, if heat transfer coefficient is not known (with known inlet temperatures of cold and hot fluids) determination of LMTD is very difficult due to tedious iterations equations.
- This difficulty is overcome by using *E-NTU method* or *effectiveness* method.

"Heat exchanger effectiveness s is defined as the ratio of actual heat transfer rate to maximum possible heat transfer rate".

$$\varepsilon = \frac{Q}{Q_{max}}$$

- The maximum possible value Q_{max} is obtained by counter flow arrangement if the temperature change of the fluid having minimum $m.c_p = T_{hi} - T_{ci}$
- *Minimum value of* $m.c_p$ *is because the heat lost by hot fluid must be equal to the heat gained by the cold fluid.*

- If maximum value of $m.c_p$ is considered, then the other fluid should undergo a temperature change greater than the maximum available temperature difference.

i.e., OT for other fluid > $T_{hi} - T_{ci}$. Which is not possible

$$Q_{max} = (mc_p)_{min} \; (T_{hi} - T_{ci})$$
$$Q = \varepsilon (mc_p)_{min} \; (T_{hi} - T_{ci})$$

Parallel Flow Heat Exchanger

Consider a parallel flow heat exchanger as shown in fig 8 (a)

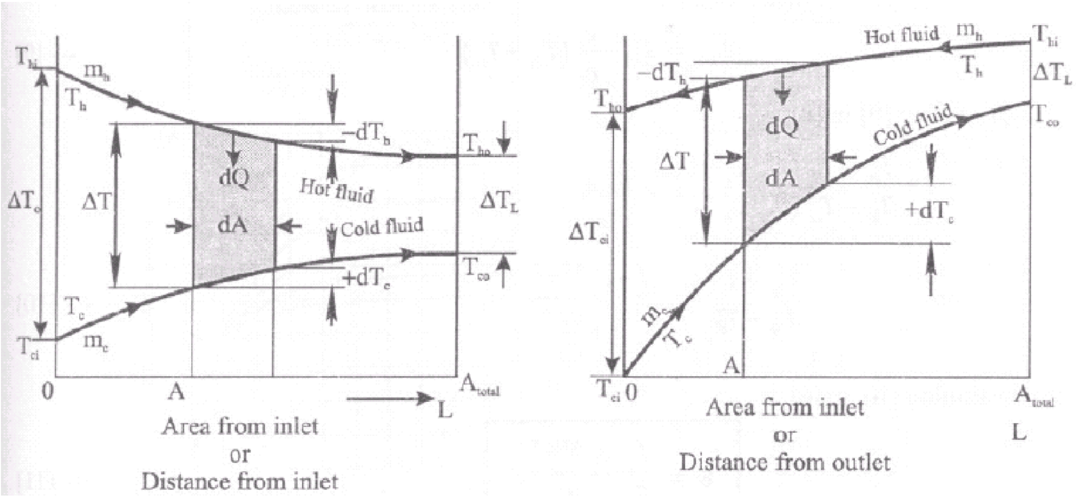


Fig 8: LMTD method for analysis of heat exchangers

$$NTU = \frac{AU_m}{c_{min}} \equiv N$$

$$\varepsilon = \frac{1 - exp\left[-N\left(\frac{c_{min}}{c_c} + \frac{c_{min}}{c_h}\right)\right]}{\left(\frac{c_{min}}{c_c}\right) + \left(\frac{c_{min}}{c_h}\right)}$$

$$\boxed{\varepsilon = \frac{1 - exp\left[-N(1 + c)\right]}{1 + c}}$$

Physical Significance of NTU

We know that

$$NTU = N = \frac{AU_m}{c_{min}}$$

MASS TRANSFER

INTRODUCTION

- In a system consisting of one or more components whose concentrations vary from point to point, there is a natural tendency for the transport of different species from the region of high to those of low concentration.
- This *process of transfer of mass as a result of the species concentration difference in a system / mixture is called* mass transfer. So long as there is concentration difference mass transfer will occur.

Some *examples of* mass transfer are:

A. Examples of Industrial importance

1. Refrigeration by the evaporation of liquid ammonia in the atmosphere of H_2 is Electrolux refrigerator.
2. Humidification of air in cooling tower.
3. Evaporation of petrol in the carburetor of an I.C. engine.
4. Neutron diffusion within nuclear reactors.
5. Estimation of depth to which carbon will penetrate in a mild steel specimen during the act of carburizing.

B. Examples of day-to-day life

1. Dissolution of sugar added to a cup of coffee.
2. The separation of the components of a mixture by or absorption.
3. The transfer of water vapour into dry air, drying and evaporation.
4. Diffusion of smoke through tall chimneys into the environment.

MODES OF MASS TRANSFER

The mechanism of mass transfer depends greatly on the dynamics of the system in which it occurs. Like those of heat transfer, there are different modes of mass transfer, which are:

1. Mass transfer by diffusion;
2. Mass transfer by convection;
3. Mass transfer by change of phase.

1. Mass transfer by diffusion (molecular or eddy diffusion)

- *The transport of water on a microscopic level as a result of diffusion from a region of high concentration to a region of low concentration in a system/mixture of liquids or gases is* called molecular diffusion.
- It occurs when a substance diffuses through a layer of stagnant fluid and may be due to concentration, temperature or pressure gradients. In a gaseous mixture, molecular

2. Mass transfer by convection

- Mass transfer by convection involves *transfer between a moving fluid and a surface,* or *between two relatively immiscible moving fluids.*
- The convective mass transfer depends on the transport properties and on the dynamic (laminar or turbulent) characteristics of the flowing fluid.

Example: The evaporation of ether.

3. Mass transfer by change of phase

- Mass transfer occurs whenever a change from one phase to another takes place.
- The mass transfer in such a case occurs due to simultaneous action of convection and diffusion.

Some examples are:

1. Hot gases escaping from the chimney rise by convection and then diffuse into the air above the chimney.
2. Mixing of water vapour with air during evaporation of water from the lake surface (partly by convection and partly by diffusion).
3. Boiling of water in open air - there is first transfer of mass from liquid to vapour state and then vapour mass from the liquid interface is transferred to the open air by convection as well as by diffusion.

CONCENTRATIONS, VELOCITIES AND FLUXES

Concentrations

Mass concentration (or mass density):

The mass concentration or mass density ρ_A of species A in a multi-component mixture is defined as the *mass of A per unit volume of the mixture.* It is expressed in kg/m^3 units.

Molar concentration (or molar density):

The molar concentration C_A of species A is defined *as the number of moles of species A per unit volume of the mixture.* It is expressed in\kg *mole/m*^{3} units.

The mass concentration and molar, concentration are related by the expression,

$$C_A = \frac{\rho_A}{M_A}$$

Where, M_A =molecular weight of component *A.*

Mass fraction:

The mass fraction, m_A is defined as the ratio of *mass concentration of species A to the total mass density,* ρ *of the mixture.*

$$m_A^* = \frac{\rho_A}{\rho}$$

Mole fraction:

The mole fraction, X_A in terms of total mole concentration of the mixture, C is defined as

$$x_A = \frac{C_A}{C}$$

In a binary mixture of *A* and *B,* by definition, the following summation rules hold good.

$$\rho_A + \rho_B = \rho$$
$$C_A + C_B = C$$
$$m_A^* + m_B^* = 1$$
$$x_A + x_B = 1$$

$$C_A = \frac{n_A}{V} = \frac{p_A}{GT}$$

$$x_A = \frac{C_A}{C} = \frac{p_A/GT}{p/GT} = \frac{p_A}{p}$$

$$C = C_A + C_B = \frac{p_A}{GT} + \frac{p_B}{GT} = \frac{p}{GT}$$

Where G =universal gas constant =*MR* = 8314 J/kg mole *K.*

Velocities:

The bulk velocity of a mixture, in which different components may have different mobility's, is computed either on mass-average or molar-average basis.

In a fluid mixture of two components *A* and *B,* if u_A and u_B is the mean velocities (of the components, respectively), then: The *mass-average velocity ("mass)* is defined by

$$u_{mass} = \frac{\rho_A u_A + \rho_B u_B}{\rho_A + \rho_B}$$
$$= \frac{\rho_A u_A + \rho_B u_B}{\rho}$$

$$u_{mass} = m_A^* u_A + m_B^* u_B$$

The molar-average velocity ("molar) is defined by

$$u_{molar} = \frac{u_A\, C_A + u_B\, C_B}{C_A + C_B}$$

$$= \frac{u_A\, C_A + u_B\, C_B}{C}$$

$$u_{molar} = x_A u_A + x_B u_B$$

Fluxes

Flux of mass transfer is caused by the existence of different velocities and concentrations.

For species *A* of the multi-component mixture:

Absolute flux = $\rho_A\, u_A$

$$\rho_A\, u_A = \frac{m_A}{A} + \rho_A\, u_{mass}$$

$$\frac{m_A}{A} = \rho_A\, u_A - \rho_A\, u_{mass} = \rho_A\, (u_A - u_{mass})$$

$$\text{Diffusion flux} = m_A^*\, (u_A - u_{molar})$$

- In order to understand the mass diffusion (a transport process originating from molecular activity), consider a chamber in which two different gas species *A* and *B* at the same temperature and pressure are initially separated by a partition.
- The left compartment has a high concentration *(i.e.,* more molecules per unit volume) of gas *A* (open circles) whereas the right compartment is rich in gas *B* (dark circles).
- When the partition wall is removed a driving potential comes into existence which tends to equalize the concentration difference.
- Mass transfer by diffusion will be in the direction of decreasing concentration and subsequently there will be a net transport of species *A* to the right and of species *B* to the left.
- After a sufficiently long period, equilibrium conditions prevail *i.e.,* uniform concentrations of species *A* and *B* are achieved and then the mass diffusion ceases.

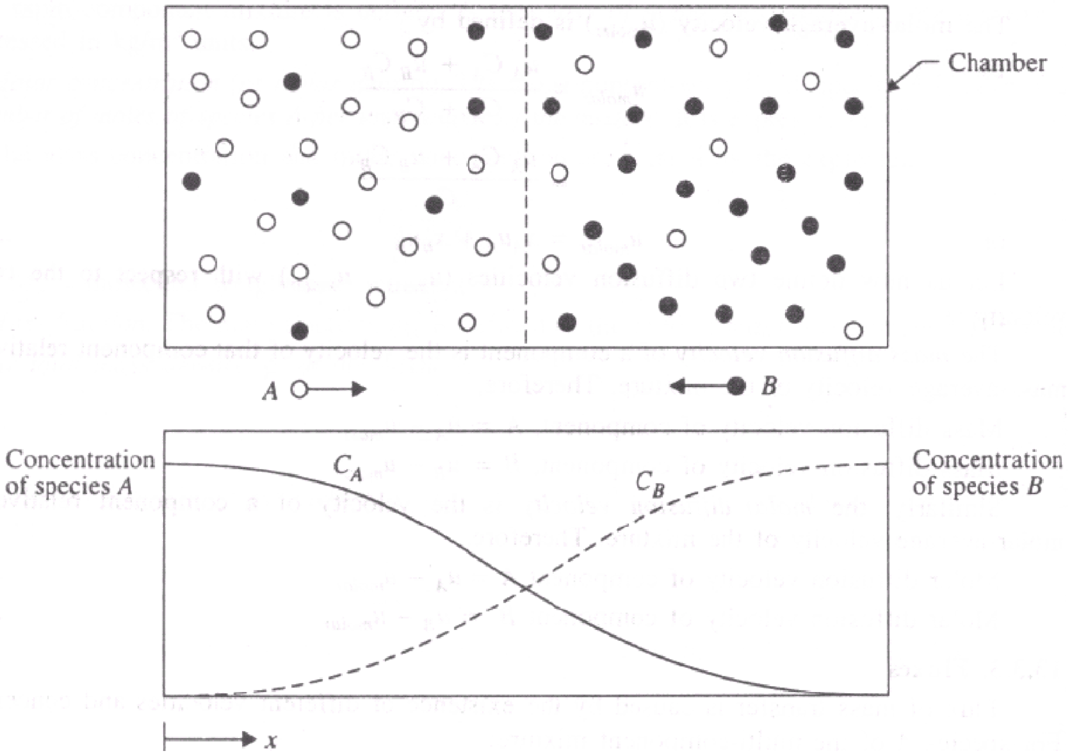


Fig. 1. Mass transfer by diffusion in a binary gas mixture.

A = area through which mass is flowing, m^2;

$N_A = \frac{m_A}{A}$ = mass flux of species A *i.e.,* amount of species A that is transferred per unit time and per unit area perpendicular to the direction of transfer, kg/s-m^2 or kg mole/s-m^2;

D_{AB} = diffusion coefficient or mass diffusivity for binary mixture of species A and B, m^2/s;

C_A = concentration or molecules per unit volume of species A, kg/m^3;

$\frac{dC_A}{dx}$ = concentration of gradient for species A; this acts as driving potential, kg/m^3.

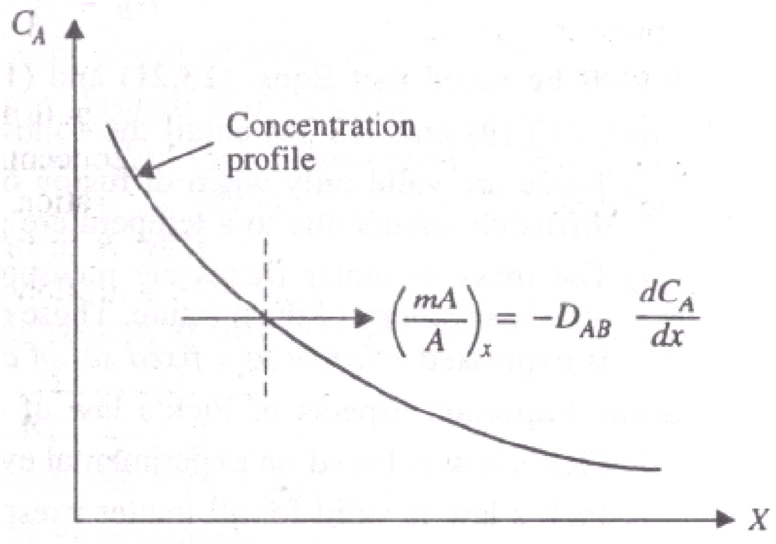


Fig. 2. Concentration profile.

In above Equation - ve sign indicates that diffusion takes place in the direction opposite to that of increasing concentration. The diffusion rate for species *B* is given by

$$N_B = \frac{m_B}{A} = -D_{BA}\frac{dC_B}{dx}$$

It may be noted that diffusion coefficient *D (i.e.,* D_{AB} or D_{BA} in this case) is dependent upon the temperature, pressure and nature of the components of the system.

(*i*) Fourier law of heat conduction: $\frac{Q}{A} = -k\frac{dt}{dx}$; and

(*ii*) Newton's law of viscosity : $\tau = \mu\frac{du}{dy}$

- By comparison of the above equations, we find that the Fourier equation describes the transport of heat energy due to temperature gradient, the shear equation describes.
- The transport of momentum due to velocity gradient while the Fick's law describes the mass transport due to concentration gradient.

$$D = -\frac{N}{\left(\frac{dC}{dx}\right)} = \frac{(M/L^2T)}{[(M/L^3)\times(1/L)]} = \frac{L^2}{T} = \mathrm{m^2/s}$$

Thus units of mass diffusion coefficient are identical to those of thermal diffusivity (α) and kinematic viscosity (ν). Thus, *diffusion coefficient is a transport property of the fluid.*

Further, by using perfect gas equation the Fick's law may be expressed in terms of partial pressures of species as follows:

$$\rho_A = C_A = \frac{p_A M_A}{G.T}$$

Substituting the value of P_A is Equation we get

$$N_A = \frac{m_A}{A} = -D_{AB}\frac{d}{dx}\left[\frac{p_A M_A}{G.T}\right]$$

$$N_A = \frac{m_A}{A} = -D_{AB}\frac{M_A}{G.T}\cdot\frac{dp_A}{dx}$$

Similarly, for species *B:*

$$N_B = \frac{m_B}{A} = -D_{AB}\frac{M_B}{G.T.}\cdot\frac{dp_B}{dx}$$

- These are valid only when diffusion occurs due to concentration gradient and fail when diffusion occurs due to a temperature gradient, pressure gradient or an external force.
- The mass or molar fluxes are measured *relative to co-ordinates which move with some average velocity* of the mixture. These equations become invalid if the flux (mass or molar) is expressed *relative to a fixed set of coordinates.*

Some important aspects of Pick's law of diffusion:

1. Pick's law is based on experimental evidence and cannot be derived from first principles.
2. Pick's law is valid for all matter irrespective of its state *(e.g.,* solid, liquid or gas).
3. The mass diffusion, besides *consultation gradient,* may occur due to a temperature gradient, a pressure gradient or an external force; however, while applying Pick's law it is assumed that these additional effects are either absent or negligibly small.
4. The movement of a diffusion substance is *in .the direction of decreasing concentration.* In a diffusion process, the concentration difference is similar to temperature difference in a heat transfer process.
5. Diffusion coefficient *(D),* in general, is dependent upon temperature, pressure and nature of the system component; however, for ideal gases and dilute liquids it can be assumed to remain practically constant for a given range of temperature and pressure.

Mass diffusion coefficient

By using kinetic theory of gases it is possible to predict the mass diffusion coefficient D_{AB} for the binary mixture of two gases, *A* and *B;* it is of the form given below:

$$D_{AB} \sim \frac{T^{3/2}}{p}$$

$$D_{AB} = 0.0043 \frac{(T^{3/2})}{p\,(V_A{}^{1/3} + V_B{}^{1/3})^2}\left[\frac{1}{M_A} + \frac{1}{M_B}\right]^{1/2}$$

P =total pressure in atmosphere $=P_A + P_B$

T =absolute temperature of the binary gas mixture, *K,*

$$\frac{D_1}{D_2} = \left(\frac{T_1}{T_2}\right)^{3/2}\left(\frac{p_2}{p_1}\right)$$

D_{AB}-for liquids is wholly available from experiments.

- *Liquid mass diffusivities are considerably smaller than those for the gases.* This is due to high molecular density in me liquid phase.
- However, increase in D_{AB} with increase in temperature has been observed. *Diffusion in solids is even slower than in liquids,* only very little information in the experimental form is available.

An effective diffusivity in case of steady state diffusion through a non-diffusing, in multi component mixture of constant composition is given by:

$$D = \frac{1}{\left(\frac{x_B}{D_{AB}}\right) + \left(\frac{x_C}{D_{AC}}\right) + \left(\frac{x_D}{D_{AD}}\right)}$$

where, x_B, x_C, x_D = mole fraction composition of the mixture on a free basis.
D_{AB}, D_{AC}, D_{AD} = diffusivities of species *A* through *B, C, D,* etc.

For gas pairs of non-polar, non-reacting molecules, the diffusion coefficient is given by:

$$D_{AB} = 0.001858 \frac{T^{3/2}}{p\,(\sigma_{AB})^2\,\Omega}\left[\frac{1}{M_A} + \frac{1}{M_B}\right]^{1/2}$$

D_{AB} =mass diffusivity of gas species *B* diffusing through another gas species *A,* cm^2/s;

T =absolute temperature, K;

P =total pressure in atmospheres $=P_A + P_B$;

σ_{AB} =collision diameter in A (Angstroms);

Ω =collision integral, a dimensionless function of the temperature and the intermolecular potential field for one molecule of *A* and one molecule of *B;*

M_A, M_B =molecular weights of gas species *A* and *B* respectively.

In case of a binary system, composed of non-polar molecule pairs, we have:

$$\sigma_{AB} = \frac{\sigma_A + \sigma_B}{2}$$

$$\varepsilon_{AB} \doteq \sqrt{\varepsilon_A \varepsilon_B}$$

The diffusion coefficient for dilute liquids is calculated from the following empirical relation:

$$D_{AB} = \frac{T}{F \mu_B}$$

D_{AB} =diffusivity of solute *A* through a solvent *B,* m^2/s;

T =absolute temperature, K;

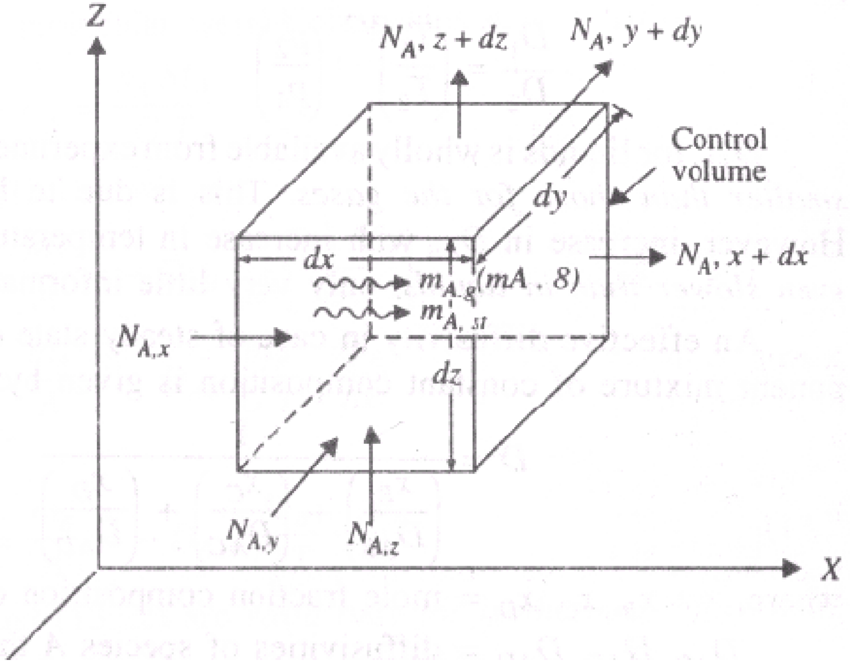


Fig. 3. Control volume for species conservation equation.

- Consider a homogeneous medium consisting of binary mixture of species *A* and *B.* Let the medium be stationary *(i.e.,* the mass average or molar average velocity of the mixture is zero) and mass transfer may occur only by diffusion.
- Now, consider a differential control volume *dx dy dz* as shown in Fig. 3. The *y* mass balance of species *A* diffusing through the control volume in the stationary medium *B* is given by:

Along X-direction

$$\text{Mass influx at the left face} = N_{A, x} \, . \, dy \, dz \qquad \left(\text{where } N_A = \frac{m_A}{A}\right)$$

$$\begin{aligned}\text{Mass efflux at the right face} &= N_{A,\, x + dx} \, . \, dy \, dz \\ &= N_{A,x} \, . \, dy \, dz + \frac{\partial}{\partial x} [N_{A,x} \, dy \, dz] \, dx \\ &= \left[N_{A,x} + \frac{\partial}{\partial x}(N_{A,x}) \, dx\right] dy \, dz\end{aligned}$$

Accumulations of mass of species *A* in the control volume due to its mass diffusion in the X-direction is given by the difference between the mass influx and mass efflux.

:. Mass of species accumulated/stored, due to diffusion, within the control volume

$$\begin{aligned}&= N_{A,x} \, dy \, dz - \left[N_{A,x} + \frac{\partial}{\partial x}(N_{A,x}) \, dx\right] dy \, dz \\ &= -\frac{\partial}{\partial x}(N_{A,x}) \, dx \, dy \, dz\end{aligned}$$

Similarly the mass accumulation of species *A* along Y and Z directions in given by:

$$\textit{Along Y-direction} = -\frac{\partial}{\partial y}(N_{A,y}) \, dx \, dy \, dz$$

As a result of volumetric chemical reactions occurring throughout the medium, there may be a generation of species *A* within the control volume, which may be expressed as

$$m_{A,g} = N_{A,g} \, . \, dx \, dy \, dz$$

$N_{A,g}$ =rate of increase of the mass of species *A* due to chemical reactions per unit volume of the mixture, kg/s.m^3.

- The total mass of species *A* accumulated in the control volume due to mass diffusion along the coordinate axes and the mass generated within the control volume serves to increase the mass concentration of species *A.*
- This increase is reflected by the time rate of change in mass concentration of species *A* in the control volume and is

$$= \frac{\partial C_A}{\partial \tau} \, . \, dx \, dy \, dz$$

Now, from mass-balance considerations, we have

$$-\left[\frac{\partial}{\partial x}(N_{A,x}) + \frac{\partial}{\partial y}(N_{A,y}) + \frac{\partial}{\partial z}(N_{A,z})\right] dx \, dy \, dz + N_{A,g} \, . \, dx \, dy \, dz = \frac{\partial C_A}{\partial \tau} \, . \, dx \, dy \, dz$$

Dividing both sides by *dx dy dz.* we get

$$-\left[\frac{\partial}{\partial x}(N_{A,x}) + \frac{\partial}{\partial y}(N_{A,y}) + \frac{\partial}{\partial z}(N_{A,z})\right] + N_{A,g} = \frac{\partial C_A}{\partial \tau}$$

For a stationary medium, using Fick's law, the above Eqn. reduces to

$$\frac{\partial}{\partial x}\left(D_{AB}\frac{\partial C_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{AB}\frac{\partial C_A}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_{AB}\frac{\partial C_A}{\partial z}\right) + N_{A,g} = \frac{\partial C_A}{\partial \tau}$$

If D_{AB} and C are constant the above equation becomes

$$\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} + \frac{N_{A,g}}{D_{AB}} = \frac{1}{D_{AB}}\frac{\partial C_A}{\partial \tau}$$

The above equation is analogous to the heat conduction equation. A few typical boundary conditions are:

1. *Specified boundary concentration:* $C_A = C_{A0}$ at $x = 0$
2. *Impermeable surface at boundary:* $\frac{\partial C_A}{\partial x} = 0$ at $x = 0$
3. *Specified mass flux at a surface :* $N_A = \frac{m_A}{A} = -D_{AB}\frac{\partial C_A}{\partial x}$ at $x = 0$
4. *Specified mass transfer coefficient* $N_A = h_m(C_{As} - C_{A\infty})$

$$\frac{d^2 C_A}{dx^2} = 0$$

Upon integration, we have

$$\frac{dC_A}{dx} = C_1 \text{ and}$$

$$C_A = C_1 x + C_2$$

Using the boundary conditions, we have

$$C_A = C_{A1} \text{ at } x = 0, \quad C_A = C_{A2} \text{ at } x = L$$

$$C_2 = C_{A1} \text{ and } \quad C_1 = \left[\frac{C_{A2} - C_{A1}}{L}\right]$$

Substituting these values in equation we get

$$C_A = (C_{A2} - C_{A1})\frac{x}{L} + C_{A1}$$

The mass transfer rate is given by

$$\frac{m_A}{A} = -D\frac{dC_A}{dx}$$

$$= -D\frac{d}{dx}\left[(C_{A2} - C_{A1})\frac{x}{L} + C_{A1}\right]$$

$$= -D\left(\frac{C_{A2} - C_{A1}}{L}\right)$$

$$\frac{m_A}{A} = \frac{D}{L}(C_{A1} - C_{A2})$$

$$\frac{m_A}{A} = \frac{C_{A1} - C_{A2}}{(L/D)}$$

(L/D) is known as *diffusional resistance.*

The above expression can be used for solving the problems on composite membranes.

The diffusion rate in the radial direction of a *cylindrical system* of inner and outer radii of $r_1$and r_2 respectively and length *L* is

$$m_A = \frac{D\,(C_{A1} - C_{A2})}{\Delta x} A_m$$

$$\Delta x = (r_2 - r_1) \text{ and } A_m = \frac{2\pi L\,(r_2 - r_1)}{\ln (r_2/r_1)}$$

$$\Delta x = (r_2 - r_1) \text{ and } A_m = 4\pi r_1 r_2$$

STEADY STATE EQUIMOLAR COUNTER DIFFUSION

Equimolar counter diffusion between species *A* and *B of* a binary gas mixture is defined as an *isothermal diffusion process in which each molecule of component A is replaced by each*

$$N_A = \frac{m_A}{A} = -D_{AB}\frac{A}{G.T}\cdot\frac{dp_A}{dx}$$

$$N_B = \frac{m_B}{A} = -D_{BA}\frac{A}{G.T}\cdot\frac{dp_B}{dx}$$

P_A, P_B = partial pressures, and

N_A, N_B = molar diffusion rates of the gases/species *A* and *B* respectively.

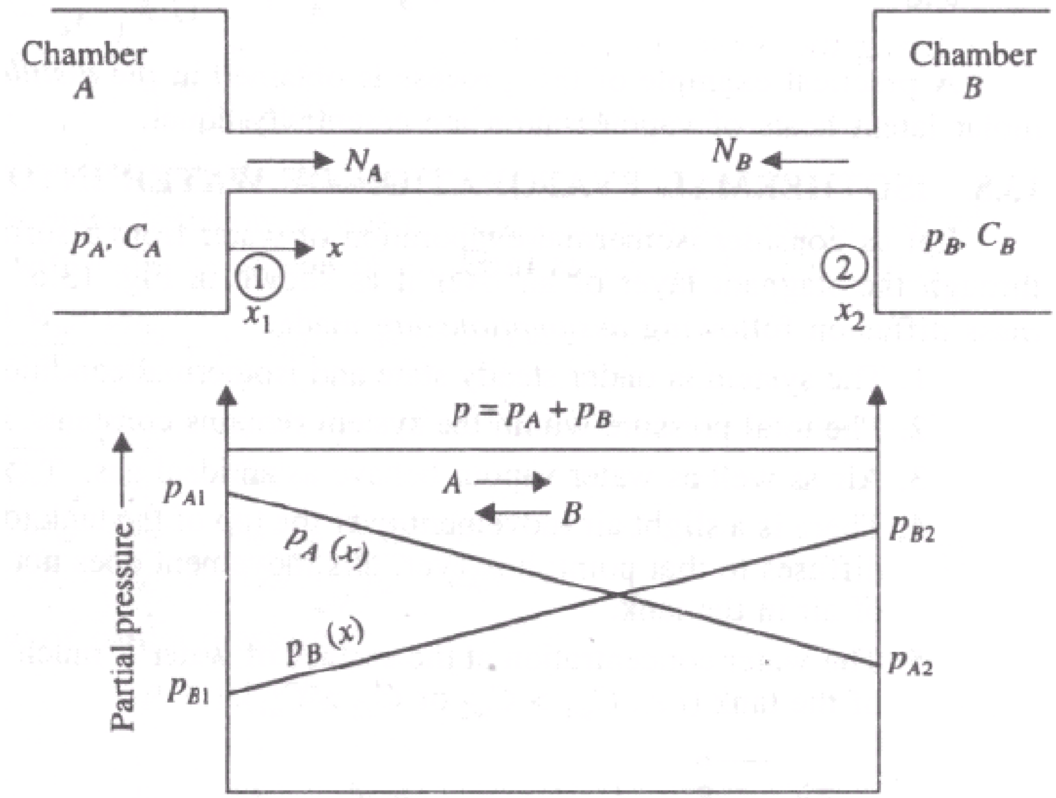


Fig. 5. Equimolar counter diffusion.

It is evident from Fig. 5 that species *A* and *B* are Chamber diffusing in the direction of *A* their decreasing concentration gradient *(i.e.,* opposite direction).

As per Dalton's law of partial pressures, the total pressure *(P)* is equal to the sum of the partial pressures of the constituents (P_A, P_B) *i.e.,* $P = P_A + P_B$ Differentiating with respect to *x,* we obtain

$$\frac{dp}{dx} = \frac{dp_A}{dx} + \frac{dp_B}{dx}$$

Since the total pressure of the system remains constant under steady conditions, therefore,

$$\frac{dp}{dx} = \frac{dp_A}{dx} + \frac{dp_B}{dx} = 0$$

$$\frac{dp_A}{dx} = -\frac{dp_B}{dx}$$

Further, under steady state conditions, the total molar flux, relative to stationary coordinates must be zero. Thus,

$$N_A + N_B = 0$$

$$N_A = -N_B$$

$$-D_{AB}\frac{A}{G.T}\cdot\frac{dp_A}{dx} = +D_{BA}\frac{A}{G.T}\cdot\frac{dp_B}{dx}$$

$$N_A = \frac{m_A}{A} = D\frac{A}{G.T}\left[\frac{p_{A1}-p_{A2}}{x_2-x_1}\right]$$

$$N_B = \frac{m_B}{A} = \mathrm{D}\frac{A}{G.T}\left[\frac{p_{B1}-p_{B2}}{x_2-x_1}\right]$$

A practical example of this process is obtained in the *distillation* of two constituents whose molar latent heats of vaporization are essentially equal.

ISOTHERMAL EVAPORATION OF WATER INTO Am FROM A SURFACE

Let us consider isothermal evaporation of water from a surface and its subsequent diffusion through the stagnant layer of air over it as shown in Fig.6. For the analysis of this type of mass diffusion following *assumptions* are made:

1. The system is under steady state and isothermal conditions.
2. The total pressure within the system remains constant.
3. Air as well as water vapour behaves as an ideal gas.
4. There is a slight air movement over the top of the tank to remove the water vapour which diffuses to that point; however, this movement does not disturb the concentration profile of air in the tank.
5. The water concentration at the surface of water is much more compared to that at the top of the tank

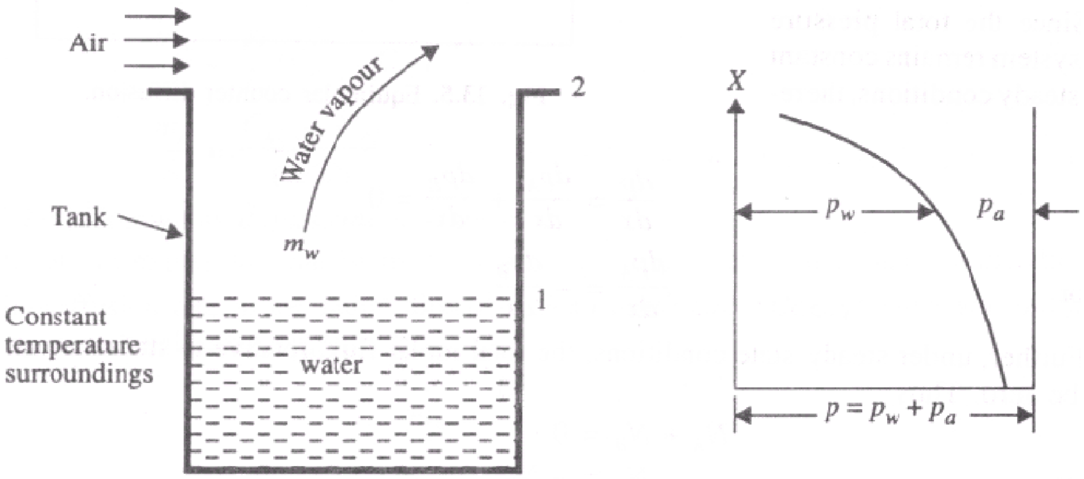


Fig.6. Diffusion of water vapour through air.

Under steady state conditions the upward movement of water must be balanced by a downward diffusion of air so that concentration at any distance from the water surface remains constant.

Mass diffusion of air in the downward direction is given, by

$$(m_a)_{down} = -D\,\frac{A\,M_a}{GT}\cdot\frac{dp_a}{dx}$$

$$D\,\frac{A\,M_a}{GT}\cdot\frac{dp_A}{dx} = p_a\,\frac{M_a}{GT}\,A\,u$$

$$u = \frac{D}{p_a}\cdot\frac{dp_a}{dx}$$

$$(m_w)_{total} = \frac{DAM_w}{G.T.}\cdot\frac{p}{(x_2 - x_1)}\cdot\frac{p_{a2} - p_{a1}}{LMPa}$$

$$= \frac{DAM_w}{G.T.}\cdot\frac{p}{x_2 - x_1}\cdot\frac{p_{w1} - p_{w2}}{LMPa}$$

MASS TRANSFER COEFFICIENT

Mass transfer coefficient h_{mc} similar to convective heat transfer coefficient, h may also be defined of species *A* as follows:

$$m_A = \frac{DA\,(C_{A1} - C_{A2})}{(x_2 - x_1)} = h_{mc}.A\;(C_{A1} - C_{A2})$$

h_{mc}= diffusion mass transfer coefficient of species *A* based on concentration difference,

A =area of cross-section. m^2

C_{A1},C_{A2}= fluid concentrations at the two faces.

The mass transfer coefficient can also be expressed in terms of partial pressure differences for species *A.*

$$m_A = DA\,\frac{M_A}{G.T}\,\frac{(p_{A1} - p_{A2})}{(x_2 - x_1)}$$

$$= \frac{DA}{(x_2 - x_1)}\cdot\frac{M_A}{G.T}(p_{A1} - p_{A2})$$

$$= h_{mc}\,\frac{A.M_A}{G.T}\,(p_{A1} - p_{A2}) = h_{mp}\,A\,(p_{A1} - p_{A2})$$

$$h_{mp} = h_{mc.}\cdot\frac{M_A}{G.T} = \frac{h_{mc}}{RT}$$

Above equation gives relationship between h_{mp} and h_{me}

For diffusion of water vapour through a layer of stagnant air, we have

$$m_w = \frac{DA}{G.T}\,\frac{M_w p}{(x_2 - x_1)} \ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$$

$$= h_{mp}\,A\,(p_{w1} - p_{w2})$$

Therefore, the mass transfer coefficient based on *pressure differences,* h_{mp} works out to be

$$h_{mp} = \frac{D\,p}{(x_2 - x_1)\,(p_{w1} - p_{w2})} \cdot \frac{M_w}{G.T}\,\ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$$

The corresponding expression for mass transfer coefficient based upon *concentration difference* is given by

$$h_{mc} = \frac{D\,p}{(x_2 - x_1)\,(p_{w1} - p_{w2})} \cdot \ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$$

CONVECTIVE MASS TRANSFER

- Whereas molecular diffusion mass transfer is analogous to conduction heat transfer (during molecular diffusion, the bulk velocities are, insignificant and only diffusion velocities are considered), the convective mass transfer is analogous to convective heat transfer (this is particularly true for low concentrations of mass in the fluid and low mass transfer rates).
- Mass transfer by convection takes place in cases *where the bulk velocity is appreciable or when both the species, in a binary mixture, are moving with significant velocities.*
- Mass convection, like heat convection, may occur under *free* or *forced conditions*. The buoyancy force causing circulation in free convection mass transfer results from the differences in density of the vapour-air mixtures of varying compositions.
- The evaporation of alcohol is an example of *free convention* mass transfer, whereas the evaporation of water from an ocean when *air blows over it* is a case of *forced convection* mass transfer.
- The fluid flow may be *laminar* or *turbulent.* If the fluid flow is laminar then all of the transport between the surface and moving fluid will be by molecular means.
- On the other hand, if the fluid flow is turbulent, there will be physical movement of the material across streamlines, transported by eddies present in the turbulent flow. As in the case of heat transfer, higher mass transfer rates are associated with turbulent conditions.

- Therefore, in any convective situation, the distribution between laminar or turbulent flow will be an important consideration.
- Mass transfer by convection involves the transporation of material between a boundary surface and a moving fluid or between two immiscible moving fluids. It is expressed as

$$\frac{m_A}{A} = h_m \, (\Delta C_A)$$

CORRELATIONS FOR CONVECTIVE MASS TRANSFER

The equations for conservation of momentum and energy for the boundary layer development on a flat plate are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial t}{\partial x} + v \frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2}$$

Similarly, the concentration equation may be written as

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}$$

C= concentration of the component which is diffusing through the boundary layer

D =diffusion coefficient.

In the correlations the following parameters are used:

1. Prandtl number (Pr):

Prandtl number, *Pr* = ν/α. It forms the connecting link between velocity and temperature profiles; these profiles become identical when *Pr* =1.

2. Schmidt number (Sc):

Schmidt number, *(Sc)* = ν/D. It forms the connecting link between velocity and concentration profiles; these profiles show the identical behavior when Sc =1.

3. Lewis number (Le)

Lewis number, *Le* =α / D. It forms a connecting link between the temperature and concentration profiles; these profiles become identical when *Le* =1.