Unit -1

Classifications of Machine Design

1. ***Adaptive design***

- Adaptation of existing designs.
- Needs no special knowledge or skill and can be attempted by designers of ordinary technical training.
- The designer only makes minor alternation or modification in the existing designs of the product.

2. ***Development design***

- Needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture.
- In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. ***New design***

- Needs lot of research, technical ability and creative thinking.
- Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows :

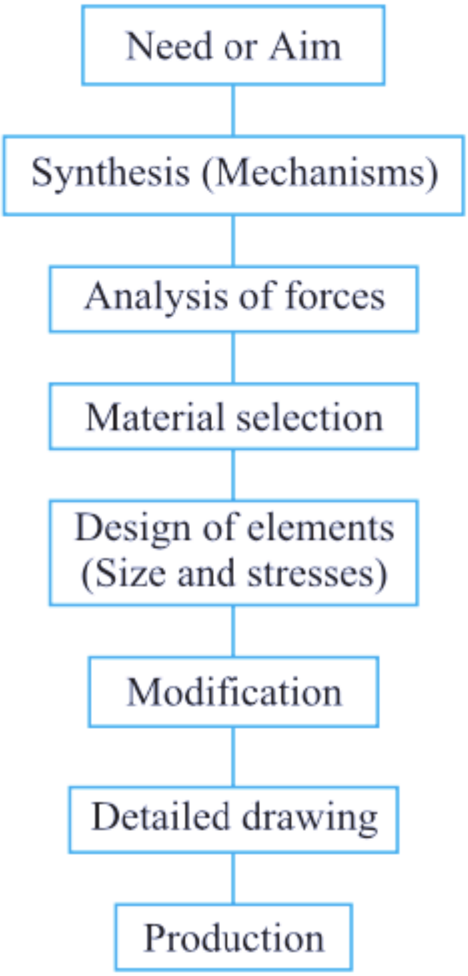
- Rational design.
- Empirical design
- Industrial design
- Optimum design
- System design
- Element design
- Computer aided design

General Consideration in Machine Design:

1. Type of load and stresses caused by the load
2. Motion of the parts or kinematics of the machine
3. Selection of materials.
4. Form and size of the parts
5. Frictional resistance and lubrication.
6. Convenient and economical features
7. Use of standard parts
8. Safety of operation
9. Workshop facilities
10. Number of machines to be manufactured
11. Cost of construction
12. Assembling

General Procedure in Machine Design

1. Recognition of need
2. Synthesis(Mechanisms)
3. Analysis of forces
4. Material selection
5. Design of elements (Size and Stresses)
6. Modification
7. Detailed drawing
8. Production



BASIC REQUIREMENTS OF MACHINE ELEMENTS

- A machine consists of machine elements. Each part of a machine, which has motion with respect to some other part, is called a machine element.

- Machine elements can be classified into two groups—general-purpose and special-purpose machine elements.
- General- purpose machine elements include shafts, couplings, clutches, bearings, springs, gears and machine frames Special-purpose machine elements include pistons, valves or spindles

1. The broad objective of designing a machine element is to ensure that it preserves its operating capacity during the stipulated service life with minimum manufacturing and operating costs

2. A machine element should satisfy the following basic requirements:

   i. **Strength**: A machine part should not fail under the effect of the forces that act on it. It should have sufficient strength to avoid failure either due to fracture or due to general yielding.

   ii. **Rigidity**: A machine component should be rigid, that is, it should not deflect or bend too much due to forces or moments that act on it.

   iii. **Wear Resistance**: Wear is the main reason for putting the machine part out of order. It reduces useful life of the component. Wear also leads to the loss of accuracy of machine tools.

   iv. **Minimum Dimensions and Weight**: A machine part should be sufficiently strong, rigid and wear- resistant and at the same time, with minimum possible dimensions and weight. This will result in minimum material cost.

   v. **Manufacturability**: Manufacturability is the ease of fabrication and assembly. The shape and material of the machine part should be selected in such a way that it can be produced with minimum labour cost.

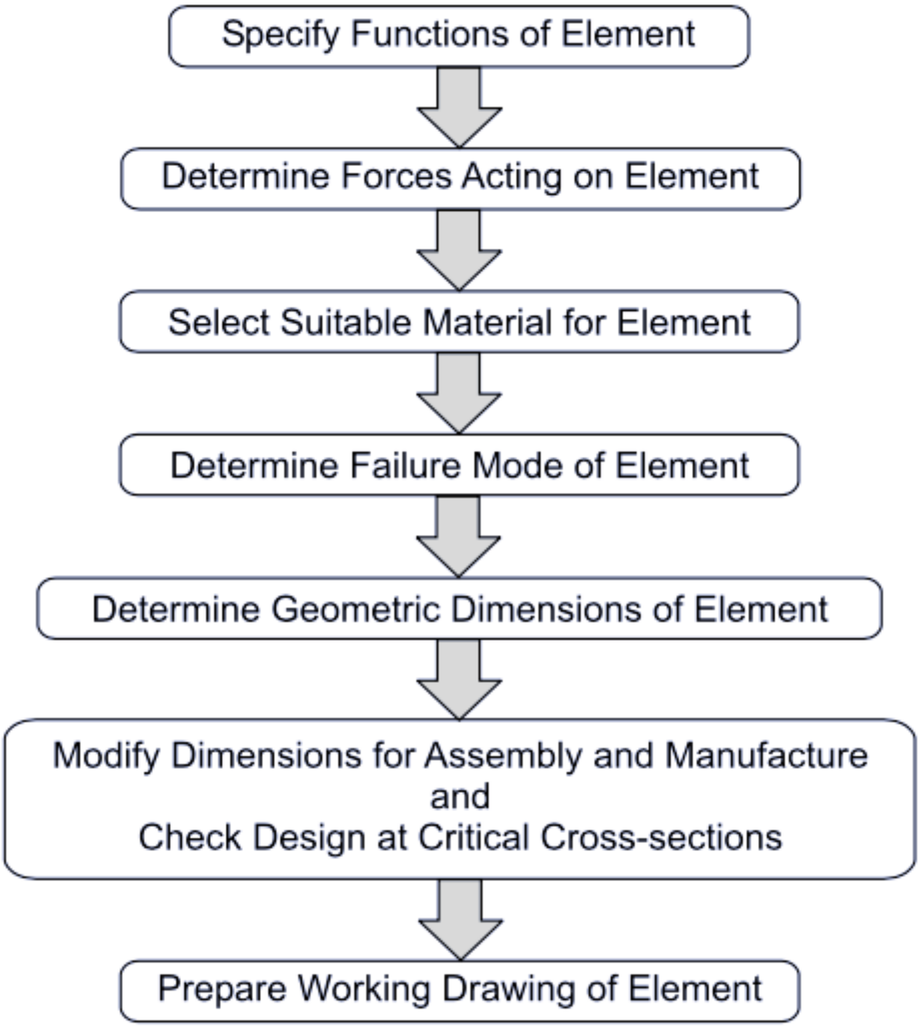
   vi. **Safety**: The shape and dimensions of the machine parts should ensure safety to the operator of the machine. The designer should assume the worst possible conditions and apply 'fail-safe' or 'redundancy' principles in such cases.

   vii. **Conformance to Standards**: A machine part should conform to the national or international standard covering its profile, dimensions, grade and material.

   viii. **Reliability:** Reliability is the probability that a machine part will perform its intended functions under desired operating conditions over a specified period of time. A machine part should be reliable, that is, it should perform its function satisfactorily over its lifetime.

   ix. **Maintainability**: A machine part should be maintainable. Maintainability is the ease with which a machine part can be serviced or repaired.

x. **Minimum Life-cycle Cost**: Life-cycle cost of the machine part is the total cost to be paid by the purchaser for purchasing the part and operating and maintaining it over its life span.



Selection of Materials:

Classification of Engineering Materials

1. **Metals and their alloys, such as iron, steel, copper, aluminium, etc.**

2. **Non-metals, such as glass, rubber, plastic, etc.**

The metals may be further classified as :
(*a*) Ferrous metals, and (*b*) Non-ferrous metals

Ferrous Metals: iron as their main constituent, such as cast iron, wrought iron and steel.

Non Ferrous Metals : metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc

Selection of Materials for Engineering Purposes

1. **Availability of the materials,**

2. **Suitability of the materials for the working conditions in service, and**

3. **The cost of the materials.**

Physical Properties of Metals

1. luster,
2. colour,
3. size and shape,
4. density,
5. electric and thermal conductivity, and
6. melting point

Metal	*Density* (kg/m^3)	*Melting point* (°C)	*Thermal conductivity* (W/m°C)	*Coefficient of linear expansion at* 20°C (μm/m/°C)
Aluminium	2700	660	220	23.0
Brass	8450	950	130	16.7
Bronze	8730	1040	67	17.3
Cast iron	7250	1300	54.5	9.0
Copper	8900	1083	393.5	16.7
Lead	11 400	327	33.5	29.1
Monel metal	8600	1350	25.2	14.0
Nickel	8900	1453	63.2	12.8
Silver	10 500	960	420	18.9
Steel	7850	1510	50.2	11.1
Tin	7400	232	67	21.4
Tungsten	19 300	3410	201	4.5
Zinc	7200	419	113	33.0
Cobalt	8850	1490	69.2	12.4
Molybdenum	10 200	2650	13	4.8
Vanadium	6000	1750	—	7.75

Mechanical Properties of Metals

Properties associated with the ability of the material to resist mechanical forces and load

1. ***Strength.***

- It is the ability of a material to resist the externally applied forces without breaking or yielding.
- The internal resistance offered by a part to an externally applied force is called stress.

2. ***Stiffness***

- It is the ability of a material to resist deformation under stress.
- The modulus of elasticity is the measure of stiffness.

3. ***Elasticity***

- It is the property of a material to regain its original shape after deformation when the external forces are removed.
- This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. ***Plasticity***

- It is property of a material which retains the deformation produced under load permanently.
- This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. ***Ductility***

- It is the property of a material enabling it to be drawn into wire with the application of a tensile force.
- A ductile material must be both strong and plastic.
- The ductility is usually measured by the terms, percentage elongation and percentage reduction in area.
- The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. ***Brittleness***

- It is the property of a material opposite to ductility.
- Property of breaking of a material with little permanent distortion.
- Brittle materials when subjected to tensile loads, snap off without giving any sensible elongation.
- Cast iron is a brittle material.

7. ***Malleability***

- A special case of ductility which permits materials to be rolled or hammered into thin sheets.
- A malleable material should be plastic but it is not essential to be so strong.

- The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. *Toughness*
   - Property of a material to resist fracture due to high impact loads like hammer blows.
   - This property is desirable in parts subjected to shock and impact loads.

9. *Machinability*
   - Property of a material which refers to a relative case with which a material can be cut.
     e.g : brass can be easily machined than steel.

10. *Resilience*
    - It is the property of a material to absorb energy and to resist shock and impact loads.
    - It is measured by the amount of energy absorbed per unit volume within elastic limit.
    - This property is essential for spring materials.

11. *Creep*
    - When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called ***creep.***
    - This property is considered in designing internal combustion engines, boilers and turbines.

12. *Fatigue*
    - When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue***.
    - This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. *Hardness*
    - It is a very important property of the metals
    - A combination of many different properties such as resistance to wear, scratching, deformation and machinability etc.
    - Also means the ability of a metal to cut another metal.

BIS SYSTEM OF DESIGNATION OF STEELS

Standard to designate your steels

designated by a group of letters or numbers indicating any one of the following three properties:

(i) tensile strength;
(ii) carbon content; and
(iii) composition of alloying elements.

— Carbon content
— alloy material

Steels, which are standardised on the basis of their tensile strength without detailed chemical composition,

- Fe 360 indicates a steel with a minimum tensile strength of 360 $\frac{N}{mm^2}$.
- Similarly, FeE 250 indicates a steel with a minimum yield strength of 250 $\frac{N}{mm^2}$

The designation of **plain carbon steel** consists of the following three quantities:

1. a figure indicating 100 times the average percentage of carbon;
2. a letter C; and
3. a figure indicating 10 times the average percentage of manganese.

- 55C4 indicates a plain carbon steel with 0.55% carbon and 0.4% manganese.

  x100 x10

- A steel with 0.35–0.45% carbon and 0.7–0.9% manganese is designated as 40C8.

The designation of unalloyed free cutting steels consists of the following quantities:

Sulphur in them

1. a figure indicating 100 times the average percentage of carbon;
2. a letter C;
3. a figure indicating 10 times the average percentage of manganese;
4. a symbol 'S', 'Se', 'Te' or 'Pb' depending upon the element that is present and which makes the steel free cutting; and
5. a figure indicating 100 times the average percentage of the above element that makes the steel free cutting.

- 25C12S14 indicates a free cutting steel with 0.25% carbon, 1.2% manganese and 0.14% sulphur.
- Similarly, a free cutting steel with an average of 0.20% carbon, 1.2% manganese and 0.15% lead is designated as 20C12Pb15.

Alloy Steel:

'alloy' steel is used for low and medium alloy steels containing total alloying elements not exceeding 10%.

The designation of alloy steels consists of the following quantities:

- a figure indicating 100 times the average percentage of carbon; and
- chemical symbols for alloying elements each followed by the figure for its average percentage content multiplied by a factor.
  - The multiplying factor depends upon the alloying element. The values of this factor are as follows:

C% x100)
alloy

Elements	Multiplying factor
Cr, Co, Ni, Mn, Si and W	4
Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr and Mo	10
P, S, N	100

Alloys added
Cobalt, Chromium, Nickel, Silicon, Mn etc;

4% Chromium x4

- 25Cr4Mo2 is an alloy steel having average 0.25% of carbon, 1% chromium and 0.2% molybdenum.

Cr x10
Cr16

- Similarly, 40Ni8Cr8V2 is an alloy steel containing average 0.4% of carbon, 2% nickel, 2% chromium and 0.2% vanadium.

Selection of Preferred sizes:

- Designer has to specify the size of the product.
- The '*size*' of the product is a general term, which includes different parameters like power transmitting capacity, load carrying capacity, speed, dimensions of the component such as height, length and width, and volume or weight of the product.
- System is based on the use of geometric progression to develop a set of numbers.
- There are five basic series, denoted as R5, R10, R20, R40 and R80 series, which increase in steps of 58%, 26%, 12%, 6%, and 3%, respectively.
- Each series has its own *series factor*.

R5 Series	$\sqrt[5]{10} = 1.58$
R10 Series	$\sqrt[10]{10} = 1.26$
R20 Series	$\sqrt[20]{10} = 1.12$
R40 Series	$\sqrt[40]{10} = 1.06$
R80 Series	$\sqrt[80]{10} = 1.03$

1.58

$1.26 \quad \sqrt[10]{10}$

$\sqrt[20]{10}$

0.1mm x 1.58 = next size,

0.158 x 1.58 = 3rd size

0.1 - 1mm

0.1, 0.158, 0.25, ---

1mm

1 x 1.26 = 1.26 ≈ 1.25mm

R20 = 1 x 1.12 = 1.12 mm

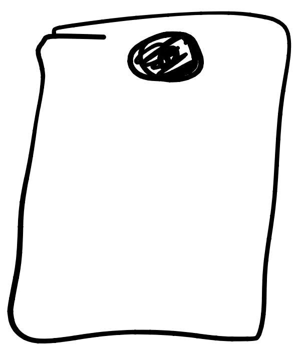
1.2mm

R5	*R10*	*R20*	*R40*
1.00	1.00	1.00	1.00
			1.06
		1.12	1.12
			1.18
	1.25	1.25	1.25
			1.32
		1.40	1.40
			1.50

R5	*R10*	*R20*	*R40*
1.60	1.60	1.60	1.60
			1.70
		1.80	1.80
			1.90
	2.00	2.00	2.00
			2.12
		2.24	2.24
			2.36
2.50	2.50	2.50	2.50
			2.65
		2.80	2.80
			3.00
	3.15	3.15	3.15
			3.35
		3.55	3.55
			3.75
4.00	4.00	4.00	4.00
			4.25
		4.50	4.50
			4.75
	5.00	5.00	5.00
			5.30
		5.60	5.60
			6.00

Other factors considered:

1. **AESTHETICS** — Looks (or) good looks
2. **ERGONOMICS** — Ease of use



Simple Stresses in Machine Elements

Load

- It is defined as any external force acting upon a machine part.

1. Dead or steady load. A load is said to be a dead or steady load, when it does not change in magnitude or direction.
2. Live or variable load. A load is said to be a live or variable load, when it changes continually.
3. Suddenly applied or shock loads. A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.
4. Impact load. A load is said to be an impact load, when it is applied with some initial velocity.

Stress

Stress: $\frac{\text{mass}}{\text{unit Area}}$, pressure (or) tension

Stress: $\frac{\text{Force}}{\text{Unit Area}}$ = Pressure

$$\boxed{\sigma = \frac{F}{A}}$$ → N/m²

1 MPa = 10^6 Pa

1 KPa = 10^3 Pa

1 Pa = 1 N/m²

10 Pa

Strain: $\frac{\text{Change in length}}{\text{original length}} = \frac{\delta l}{l}$

$$\boxed{e = \frac{\delta l}{l}}$$

no units

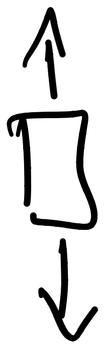
Youngs Modulus (or) Modulus of Elasticity:

$E = \frac{\text{Stress}}{\text{Strain}}$ $\left(\frac{N/m^2}{-}\right)$ N/m²

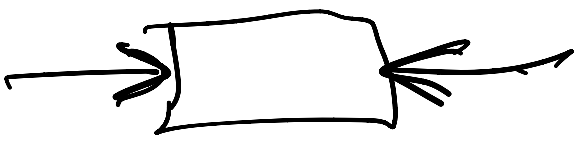
Hooke's Law:

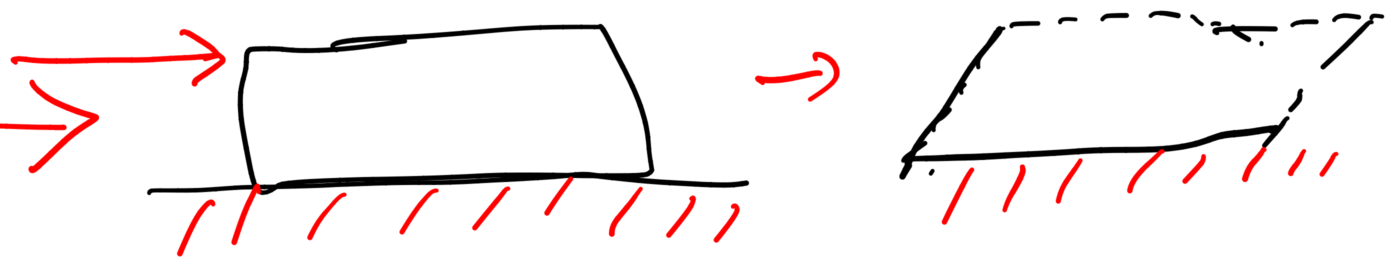
$\sigma \propto e$

$\boxed{\sigma = E \times e}$ Gpa

Stress: 1) Tensile —  → Elongates.

2) Compressive —

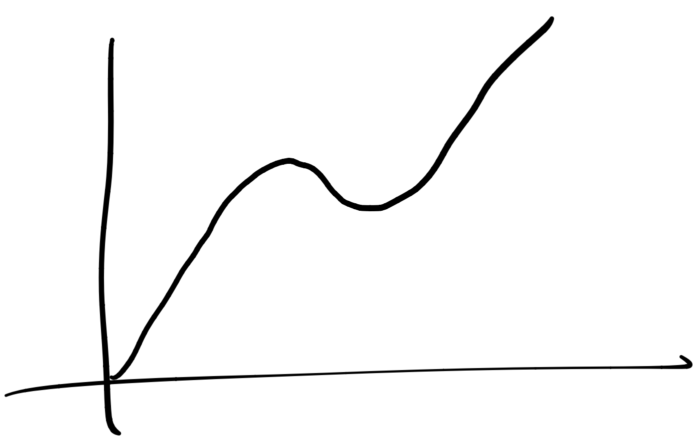
3) Shear stress  Contracted



$$E = \frac{\sigma}{e} = \frac{\frac{F}{A}}{\frac{\delta l}{l}} \Rightarrow E = \frac{Fl}{A(\delta l)}$$

$$\boxed{\delta l = \frac{Fl}{AE}} = \frac{PL}{AE}$$

Elongation

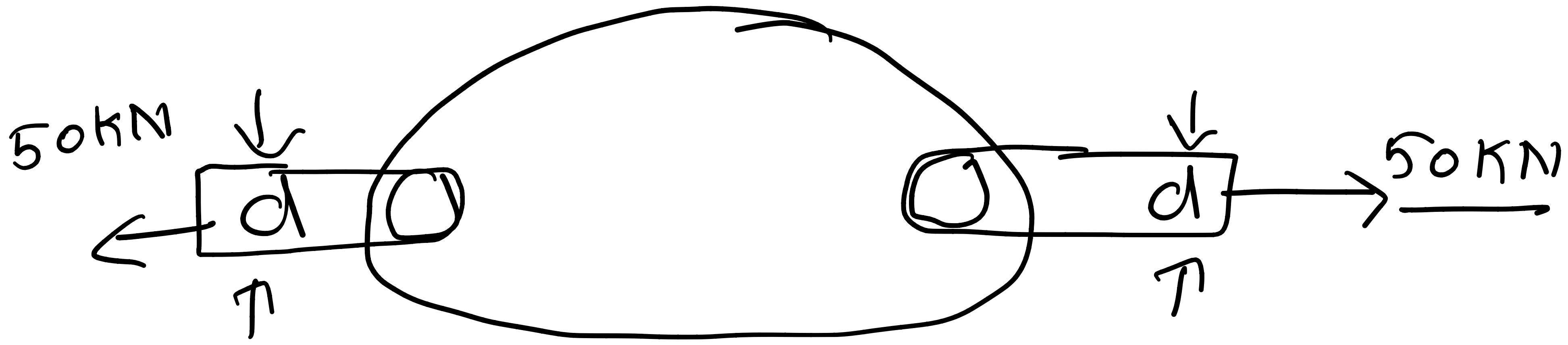


Find the diameter of the link stock, if the permissible tensile stress in the link material is not to exceed 75 MPa.

Given data :

$$\sigma_t = 75\ \text{MPa} = 75 \times 10^6\ \text{Pa}$$

$$F = 50\ \text{KN} = 50 \times 10^3\ \text{N}$$



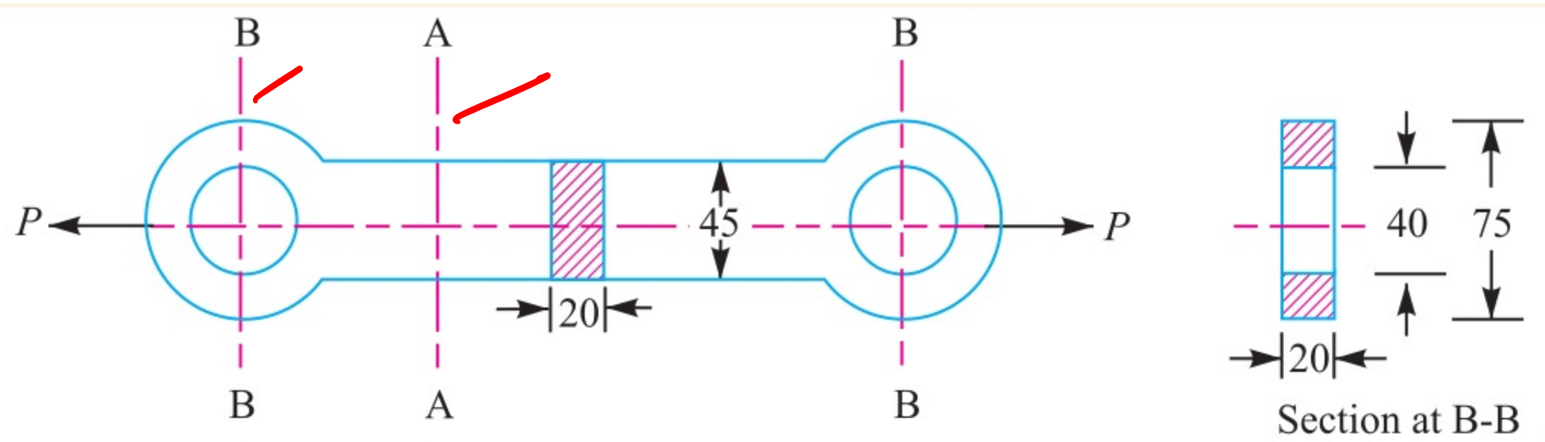
Area $A = \frac{\pi}{4} d^2$

$$\sigma_t = \frac{F}{A} \Rightarrow 75 \times 10^6 = \frac{50 \times 10^3}{\frac{\pi}{4} d^2}$$

$$d = \underline{0.029}\ \text{m}$$

$$\boxed{d = 29\ \text{mm}}$$

A cast iron link, as shown in Fig. is required to transmit a steady tensile load of 45 kN. Find the tensile stress induced in the link material at sections A-A and B-B.



mm

Given data:

$$F = 45\,kN = 45 \times 10^3\ N$$

Stress @ Section A-A:

$$Area = 45 \times 20 = 900\ mm^2$$

$$\sigma = \frac{F}{A} = \frac{45 \times 10^3}{900} = 50\ N/mm^2 = 50 \times 10^6\ Pa$$

$$= 50\ mPa$$

$$1\ Pa = 1\ N/m^2$$

$$1\ N/mm^2 = \_\_\_\_\ N/m^2$$

$$1\ m = 10^3\ mm$$

$$1\ mm = \frac{1}{1000}\ m$$

$$1\ N/(10^{-3} m)^2 = \frac{N}{(10^{-6}) m^2}$$

$$1\ mm = 10^{-3}\ m$$

$$1\ N/mm^2 = 10^6\ N/m^2$$

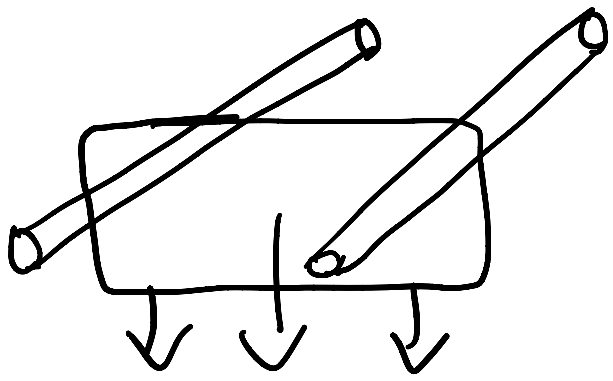
Stress @ Section B-B

$$\text{Area} = 20(75-40)$$

$$= 700\ mm^2$$

$$\sigma_{B-B} = \frac{45\times10^3}{700} = 64.28\ N/mm^2 = 64.28\ mpa$$

A hydraulic press exerts a total load of 3.5 MN. This load is carried by two steel rods, supporting the upper head of the press. If the safe stress is 85 MPa and $E = 210\ kN/mm^2$, find : 1. diameter of the rods, and 2. extension in each rod in a length of 2.5 m.



$P = 3.5\,MN$

$\sigma = 85\ mpa$

$E = 210\ kN/mm^2$

$d = ?$ $\qquad \sigma = \frac{P}{A}$ $\qquad Area = \frac{\pi}{4} d^2$

Load carried by each rod

$\frac{3.5}{2}$ = Load carried by each rod = 1.75 MN

$P = 1.75 \times 10^6\ N$

$\sigma_{safe} = 85\ mPa = 85 \times 10^6\ Pa = 85 \times 10^6\ N/m^2$

$\sigma = \frac{P}{A} \Rightarrow A = \frac{P}{\sigma} = \frac{1.75 \times 10^6}{85 \times 10^6}$

$Area = 0.02\ m^2 = \frac{\pi}{4} d^2$

$d = 0.159\ m \approx 160\ mm$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\delta l / l} = \frac{P/A}{\delta l / l}$$

$$\text{Strain} = \frac{\delta l}{l}$$

$P = 1.75 \text{ MN}$

$$210 \times 10^9 = \frac{85 \times 10^6 \times 2.5}{\delta l}$$

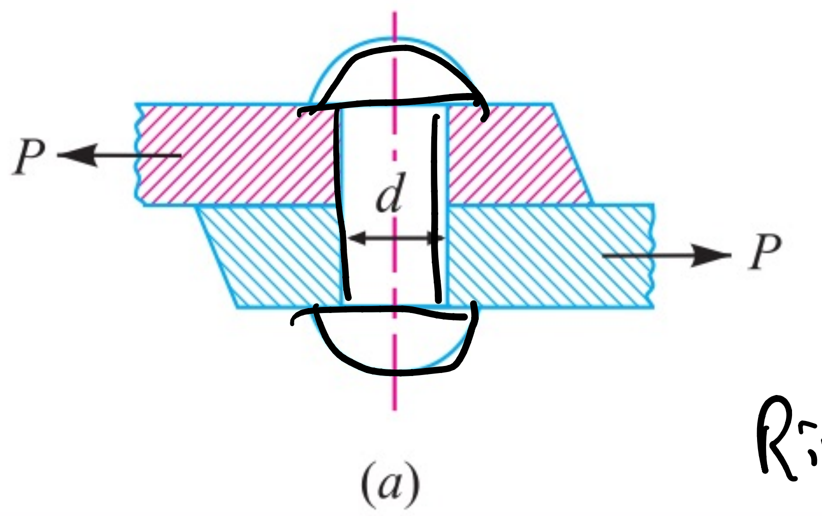
$l = 2.5 \text{ m}$

$A = 0.02 \text{ m}^2$

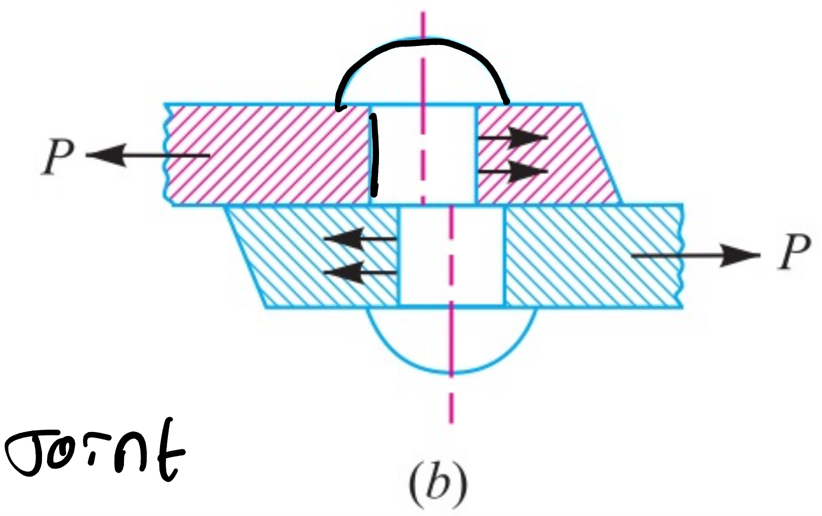
$$\boxed{\delta l = 1.011 \text{ mm}}$$

$E = 210 \times 10^3 \times 10^6 \text{ N/m}^2$

Shear Stress & Shear Strain:



Riveted Joint



Shearing.

Single shear

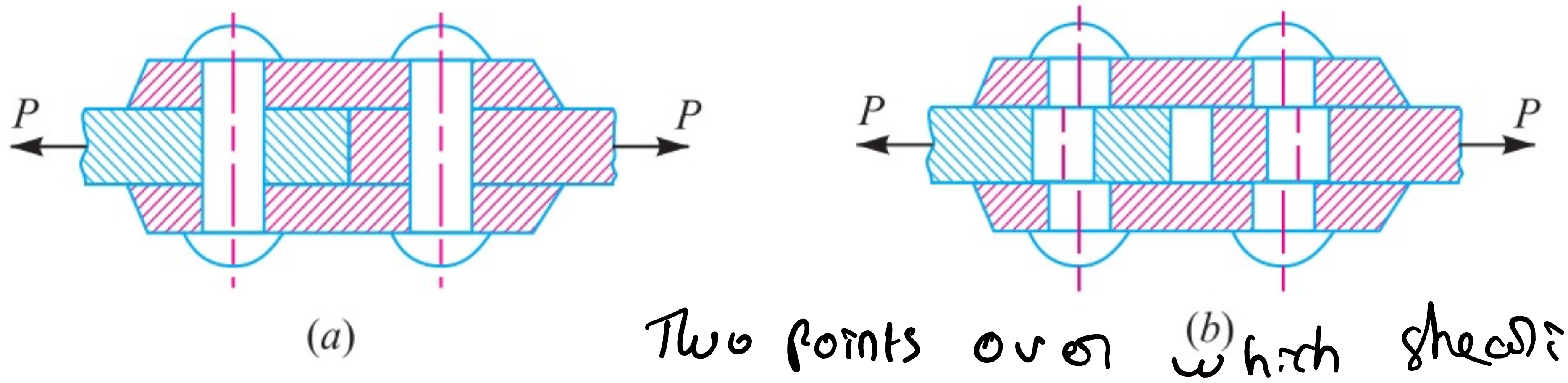
$$\text{Shear stress } (\tau) = \frac{\text{Tangential force}}{\text{Resisting Area}}$$

$$\text{Area} = \frac{\pi}{4} d^2$$

$$\tau = \frac{P}{A}$$

Area that is parallel to the force

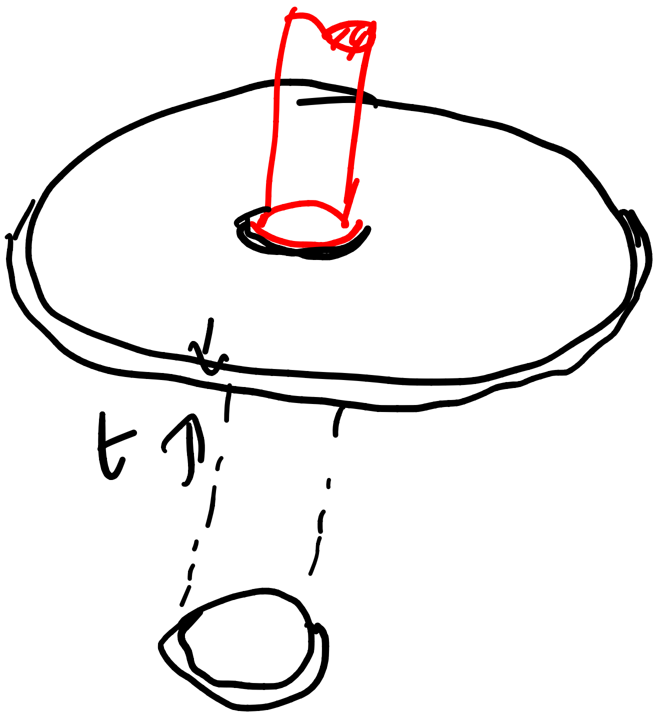
$$\boxed{\tau = \frac{P}{\frac{\pi}{4} d^2} = \frac{4P}{\pi d^2}}$$

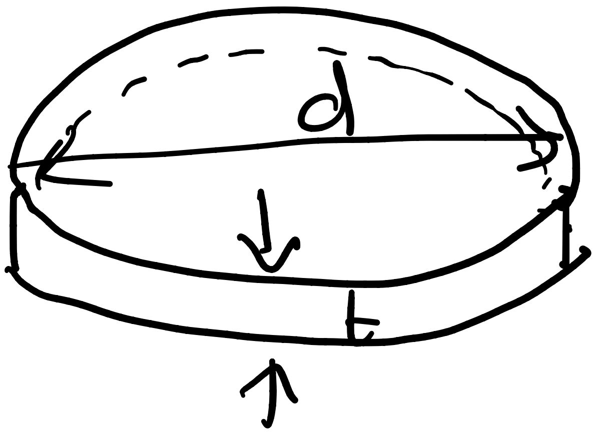


Two points over which shearing takes place.

So this is called Double shear. $\longrightarrow$ less than in Single shear

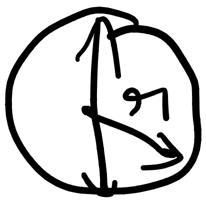
$\tau =$





t, Circumference of the circle

t, πd



$$= 2\pi r$$

$$= \pi(2r)$$

$$= \pi d$$

Area $= \pi d \times t$

$$\boxed{P = (\pi d \times t) \times \tau_u}$$

ultimate shear stress

Calculate the force required to punch a circular blank of 60 mm diameter in a plate of 5 mm thick. The ultimate shear stress of the plate is 350 N/mm^2.

$d = 60\,mm$
$= 60 \times 10^{-3}\,m$

$t = 5\,mm$
$= 5 \times 10^{-3}\,m$

$\tau_u = 350\,N/mm^2$
$= 350 \times 10^{6}\,N/m^2$

Force = ?

$P = \tau_u \times Area = \tau_u \times \pi d t$

$P = 350 \times 10^{6} \times \pi \times 60 \times 10^{-3} \times 5 \times 10^{-3}$

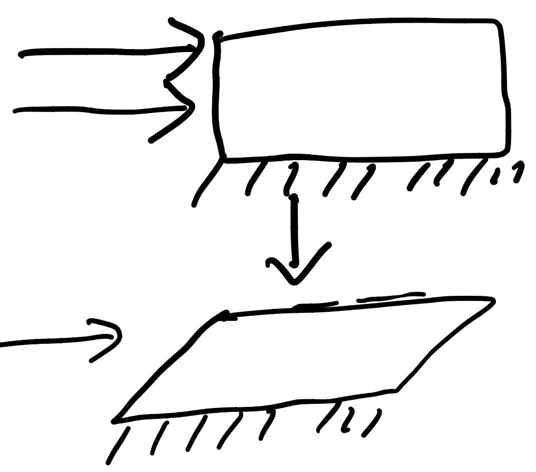
$P = 329867.22\,N$

$\boxed{P = 0.329\,MN}$

Rigidity modulus:

youngs modulus → Ratio of Stress & strain
↓
Tensile (or) Comp

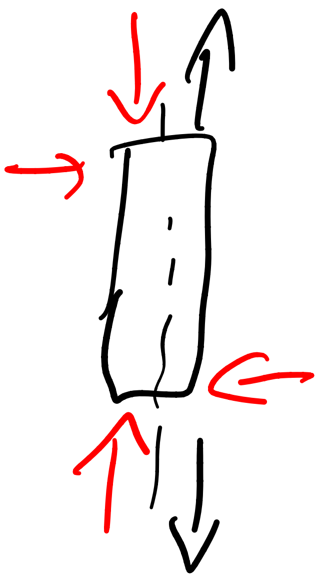
Rigidity modulus $= \dfrac{\text{Shear Stress}}{\text{Shear Strain}}$



$C \text{ (or) } G = \dfrac{\tau}{\varepsilon_t}$

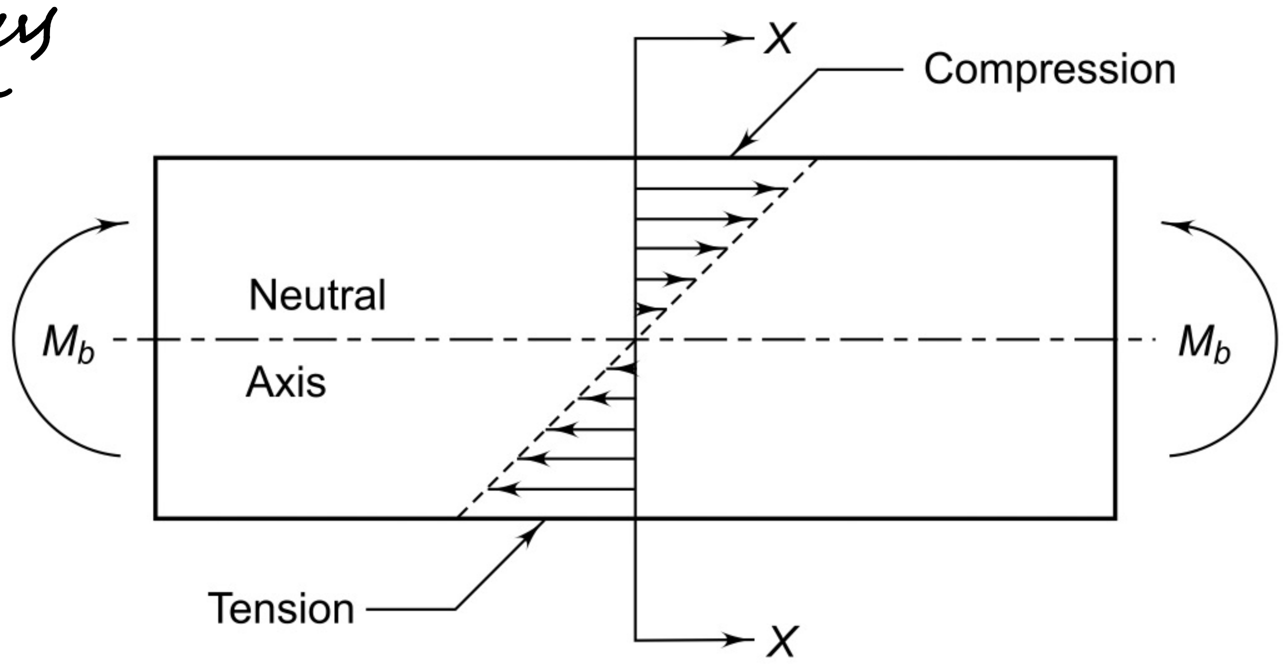
$\boxed{C = \dfrac{\tau}{\varepsilon_t}}$ → Strength in lateral dirⁿ.

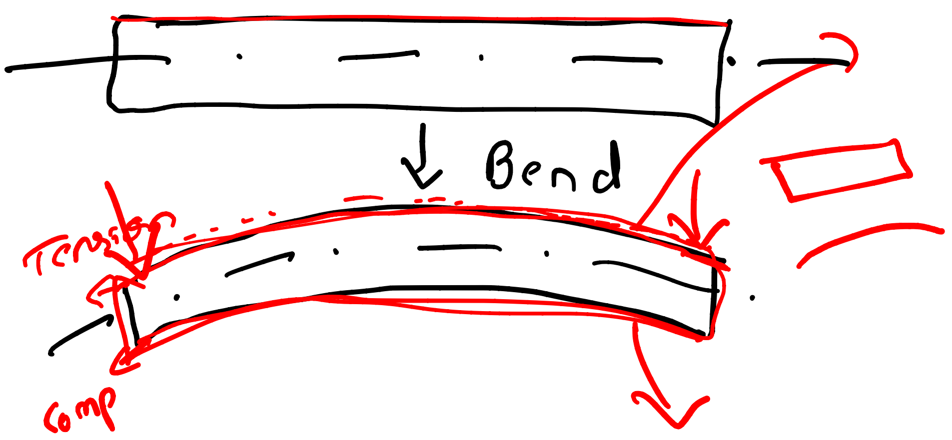
Stresses due to bending:



Bending

Bending Stress





Combination of tensile, comp & shear stress.

Neutral axis → No Stress developed

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

σ → Bending Stress

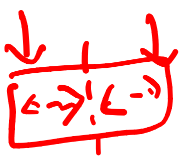
R → Radius of curvature

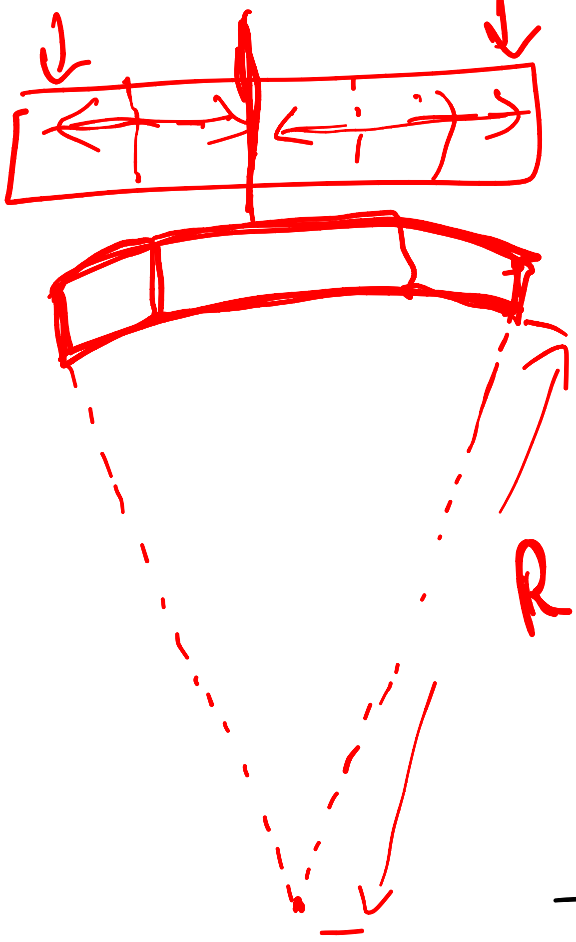
M → Bending moment

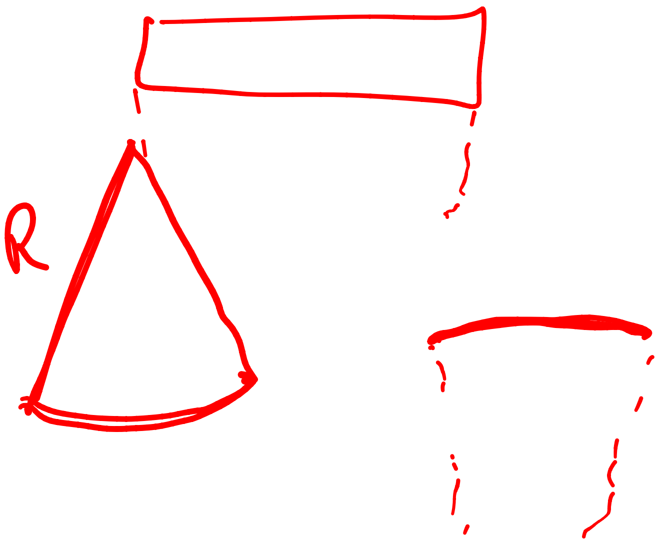
$\boxed{\text{Nm}}$ → units

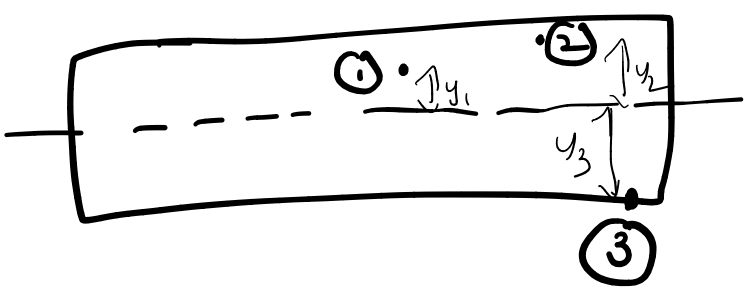
I → moment of Inertia

y → Distance from neutral axis









$y_3 > y_2 > y_1$

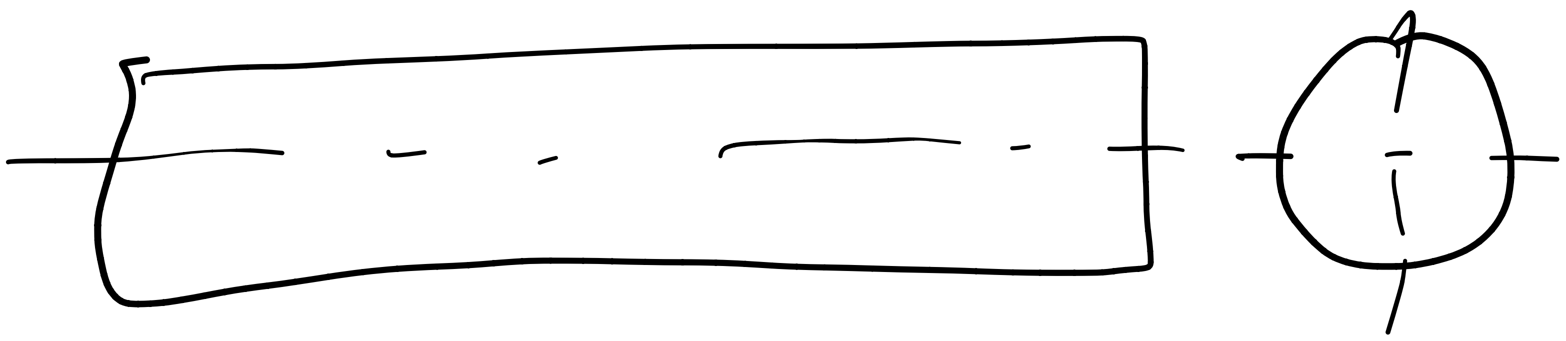
$$\boxed{\sigma = \frac{My}{I}}$$ N/m² (or) pa

$y \uparrow \rightarrow \sigma \uparrow$

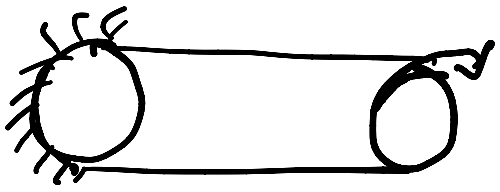
$$\sigma = \frac{M}{(I/y)}$$

$I/y = Z$ → section modulus

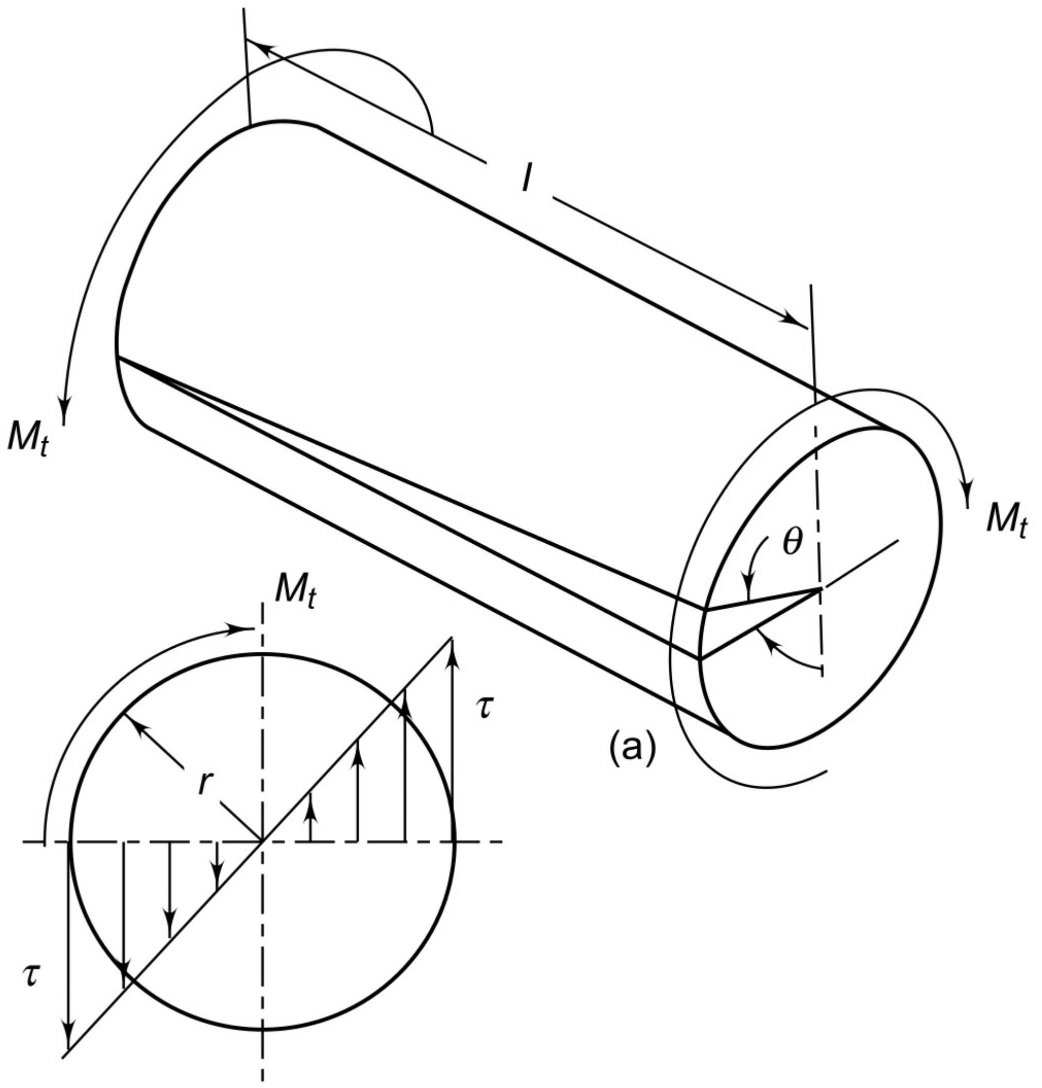
$$\boxed{\sigma = \frac{M}{Z}}$$



Stresses due to torsion:



Stress induced due to twisting action (or) moment is called Torsional stresses.



$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{C\,\theta}{l}}$$

J → polar moment of Inertia

Applied torsion (or) Torque

Torsional stress

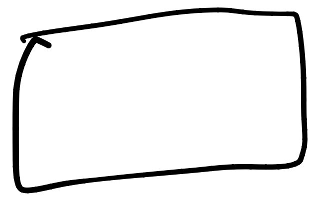
Rigidity modulus

$$\boxed{\tau = \frac{T}{J} \times R = \frac{C \cdot \theta \cdot R}{l}}$$

Factor of Safety:

Brittle → Breaks (or) fractures

Ductile → yields



Max allowable load

↙

Max allowable Stress

Max Stress ≠ working Stress

$$\boxed{F.O.S = \frac{\text{Max allowable Stress}}{\text{working Stress}}}$$

Selection of Factor of Safety

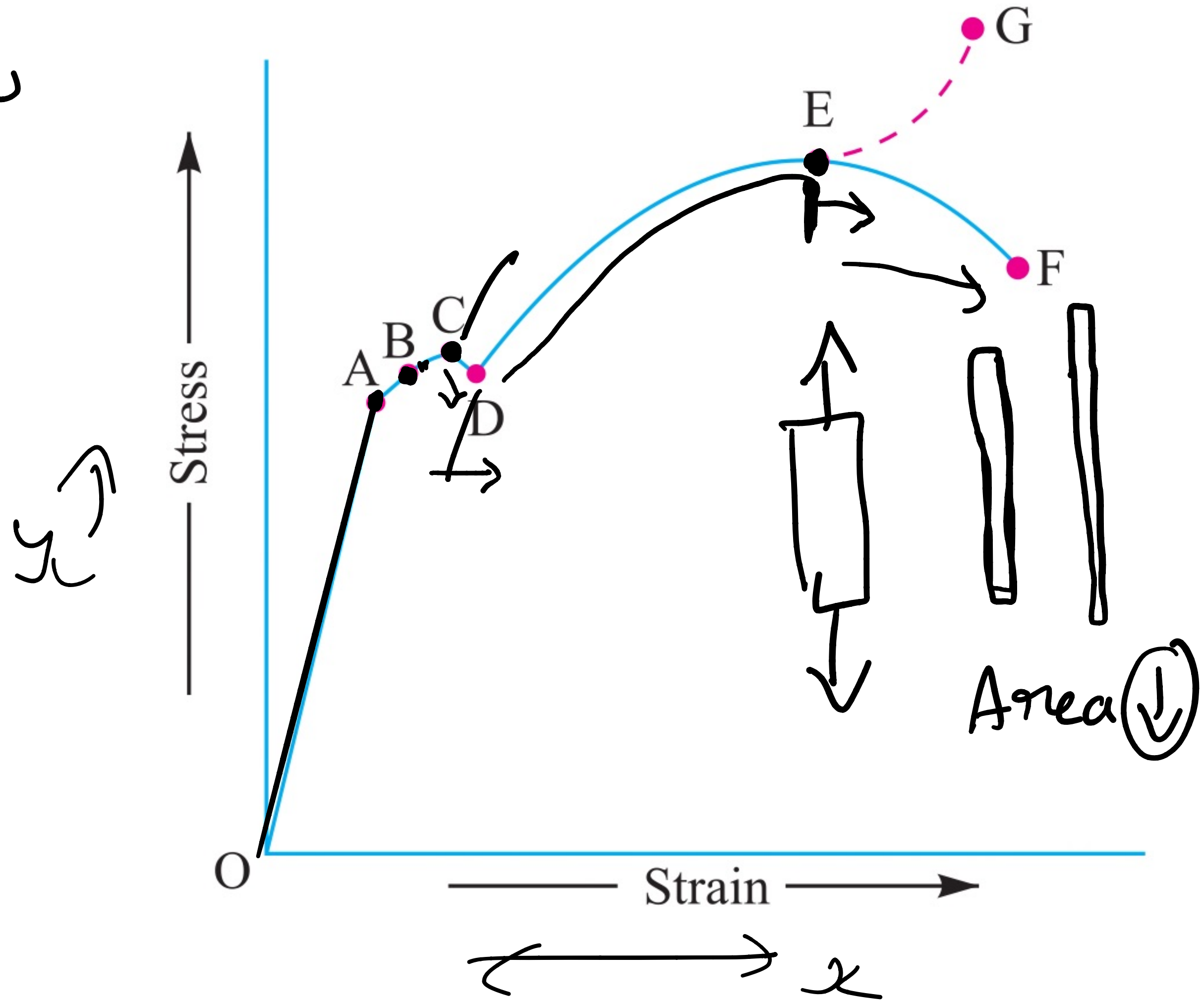
Depends upon a number of considerations, such as the material, mode of manufacture, type of stress, general service conditions and shape of the parts.

1. The reliability of the properties of the material and change of these properties during service ;
2. The reliability of test results and accuracy of application of these results to actual machine parts ;
3. The reliability of applied load ;
4. The certainty as to exact mode of failure ;
5. The extent of simplifying assumptions ;
6. The extent of localised stresses ;
7. The extent of initial stresses set up during manufacture ;
8. The extent of loss of life if failure occurs ; and
9. The extent of loss of property if failure occurs.

Stress - Strain Relation:

1) Proportional limit:

$\boxed{\sigma \propto \varepsilon}$ → hooke's law



2) Elastic Limit:

when you remove load, component reverts to its original shape.

3) yield point:

On removal of Load, the component wont return to its original shape.

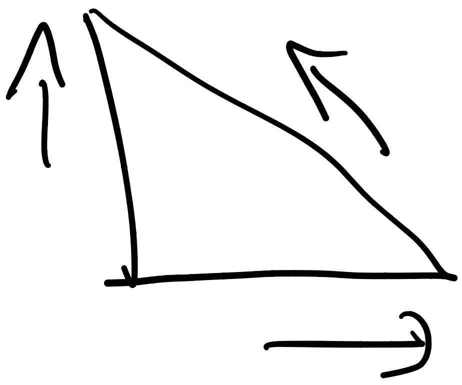
$$\sigma = \frac{Load}{Area}$$

4) ultimate Stress:

Max strength

5) Breaking Stress:

Principal Stresses:



Shear stress = 0

$\downarrow$

Principal planes

Principal stresses

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

$\sigma_1 \rightarrow \sigma_{max}$

$\sigma_2 \rightarrow \sigma_{min}$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

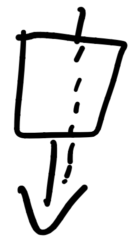
Example *A hollow shaft of 40 mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously, it is subjected to an axial thrust of 10 kN and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses.*

Ans:

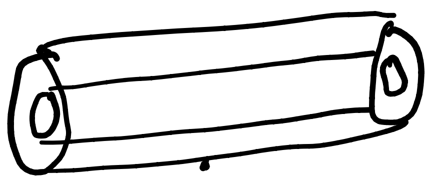
$d_o = 40\,mm = 40\times10^{-3}\,m$ $\quad T = 120\ N\text{-}m$

$d_i = 25\,mm = 25\times10^{-3}\,m$ $\quad P = 10\ KN$

$M = 80\ N\text{-}m$



$\sigma_{c(max)}$ & τ_{max}



Area $a = \frac{\pi}{4}(d_o^2 - d_i^2)$

$= \frac{\pi}{4}\left((40\times10^{-3})^2 - (25\times10^{-3})^2\right)$

$= 7.65\times10^{-4}\ m^2$

Direct stress due to axial load

$\sigma_o = \frac{P}{A} = \frac{10\times10^3}{7.65\times10^{-4}} = 13.07\times10^6\ N/m^2$

$= 13.07$ mpa

$=$

$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$

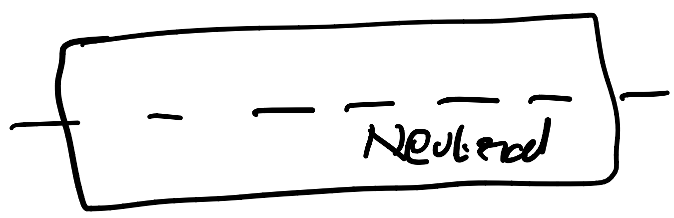
$I/y = Z$

$\sigma_b = M/(I/y)$

$\sigma_b = \frac{M}{Z}$ — (1)

$\sigma_b = \frac{M}{Z} = \frac{80}{5.32\times10^{-6}}$

$I = \frac{\pi}{64}(d_o^4 - d_i^4)$ $\quad mm^4$ 3 mm



$y = \frac{d_o}{2}$

$I/y = Z = \frac{\pi}{32}\frac{(d_o^4 - d_i^4)}{d_o} = 5.32\times10^{-6}\ m^3$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\cdot\theta}{l}$$

$\sigma_b = 15.02$ mpa

$$J = \frac{\pi}{32} d^4$$

Resultant Compressive Stress

$$J_H = \frac{\pi}{32}(d_o^4 - d_i^4)$$

$$\sigma_c = \sigma_b + \sigma_o = 15.02 + 13.07$$

$$\sigma_c = 28.09 \text{ Mpa}$$

Twisting moment → Causes Shear Stress

$$T_{solid} = \frac{\pi}{16} \tau d^3$$

$$R = d/2$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$J = \frac{\pi}{32} d^4$$

Polar moment of Inertia

$$\tau = \frac{T}{J} \times R = \frac{T \times d/2}{\frac{\pi}{\cancel{32}_{16}}\left[d_o^4 - d_i^4\right]}$$

$$\tau = \frac{16T}{\frac{\pi\left[d_o^4 - d_i^4\right]}{d_o}}$$

$$\tau = \frac{16 \times 120 \times 40 \times 10^{-3}}{\pi\left[(40\times10^{-3})^4 - (25\times10^{-3})^4\right]}$$

$$\tau = 11.02 \text{ mpa}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau)^2}$$

$$\sigma_y = 0$$

$$\sigma_{c(max)} = \frac{\sigma_c + 0}{2} + \sqrt{\left(\frac{\sigma_c - 0}{2}\right)^2 + \tau^2}$$

$$= 14.045 + \sqrt{(14.045)^2 + (11.27)^2}$$

$$\boxed{\sigma_{c_{max}} = 32.05 \text{ mpa}}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + t^2}$$

$\sigma_x = \sigma_c$

$\sigma_y = 0$

$\tau = t$

$$\boxed{\tau_{max} \simeq 18 \text{ mpa}}$$

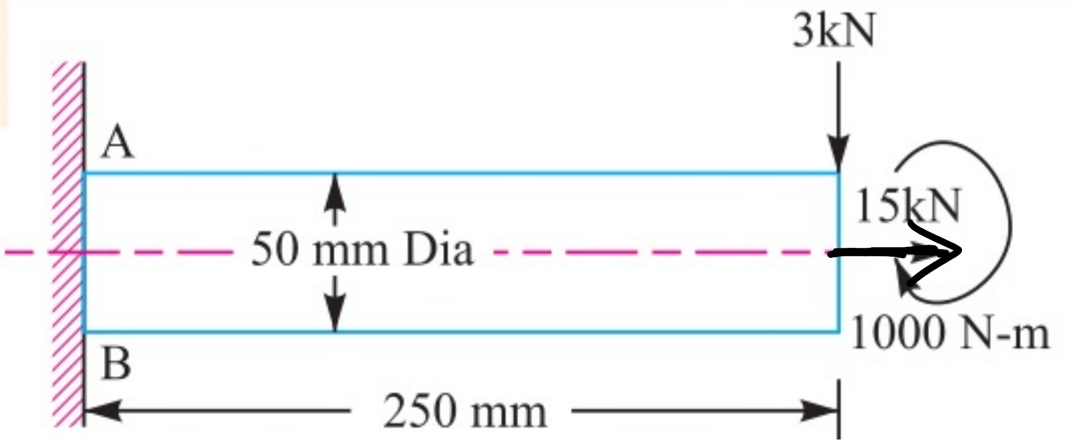
yield strength

$\sigma_y = 30$ mpa

$\frac{\sigma_y}{2} = 15$ mpa

Example *A shaft, as shown in Fig. 5.17, is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN.*

Calculate the stresses at A and B.



Solution. Given : $W = 3$ kN $= 3000$ N ; $T = 1000$ N-m $= 1 \times 10^6$ N-mm ; $P = 15$ kN $= 15 \times 10^3$ N ; $d = 50$ mm; $x = 250$ mm

Ans:

$$A = \frac{\pi}{4}(50 \times 10^{-3})^2 = 1.963 \times 10^{-3} \ m^2$$

$$\sigma_0 = \frac{P}{A} = \frac{15 \times 10^3}{1.963 \times 10^{-3}} = 7.64 \ mpa$$

$$Z = \frac{\frac{\pi d^4}{64}}{d/2}$$

$$M = W \times x = 3000 \times 0.25 = 750 \ N\text{-}m$$

$$\sigma_b = \frac{M}{Z}$$ Z → Section modulus

$$Z = \frac{\pi}{32} d^3 = 1.22 \times 10^{-5} \ m^3$$

$$\sigma_b = 61.5 \ mpa$$

Tensile @ point A

Compressive @ point B

Stresses @ point A:

$$\sigma_A = \sigma_0 + \sigma_b = 7.64 + 61.5 = 69.11 mpa$$

Stresses @ point B:

$$\sigma_B = \sigma_0 - \sigma_b = 7.64 - 61.5 = -53.9 \ mpa = 53.9 \ mpa \ (comp)$$

Shear Stress @ A&B:

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 1000}{\pi \times (50 \times 10^{-3})^3} = 40.74 \text{ mpa}$$

@ point A:

max principal stress:

$$\sigma_1 = \frac{\sigma_A}{2} + \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau^2}$$

$\sigma_x = 69.11 \text{ mpa} = \sigma_A$

$\sigma_y = 0$

$\tau = 40.74 \text{ mpa}$

$\sigma_{A(max)} = 88 \text{ mpa}$

min principal stress:

$$\sigma_2 = \frac{\sigma_A}{2} - \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau^2}$$

$\sigma_{A(min)} = -18.87 \text{ mpa (tensile)} = 18.87 \text{ mpa (comp)}$

$\tau_{A\,max} = 53.42 \text{ mpa}$

@ point B:

$\sigma_y = 0$

$\sigma_x = \sigma_B = 53.9 \text{ mpa}$

$\sigma_{B(max)} = 75.8 \text{ mpa (comp)}$

$\sigma_{B(min)} = 22.03 \text{ mpa (tensile)}$

$\tau_{B(max)} = 48.96 \text{ mpa}$

Max stress:

$$\sigma_{1B} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau^2}$$

$$= \frac{53.46}{2} + \sqrt{\left(\frac{53.46}{2}\right)^2 + (40.74)^2}$$

$$= 75.46 \text{ MPA (comp)}$$

Min stress

$$(\sigma_{2B})_{min} = \frac{\sigma_B}{2} - \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau^2}$$

$$= -22 \text{ MPA}$$

$$= 22 \text{ MPA (Tensile)}$$

$$(\tau_{max})_B = \sqrt{\left(\frac{53.46}{2}\right)^2 + (40.74)^2}$$

$$= 48.73 \text{ MPA}.$$

Theories of failure:

i) Max principal Stress Theory (or) Rankine theory

Failure (or) yielding in a member will happen when max normal stress reaches limiting strength of the material in simple tension test.

$\sigma_1 = \sigma_{max}$ then your member fails.

$\sigma_x, \sigma_y, \tau_{xy}$

$\sigma_1 = \dfrac{\sigma_{yt}}{FS} \rightarrow$ Ductile — yield stress

$= \dfrac{\sigma_u}{FS} \rightarrow$ Brittle — ultimate stress

$\sigma_1 = 180\,mpa$ $\xleftarrow{\text{safe}}$ $200\,mpa$

$\sigma_1 = 210\,mpa$ $\xleftarrow{\text{fail}}$

Shear stress is not considered

This theory is useful for Brittle materials as they are strong in shear.

principal

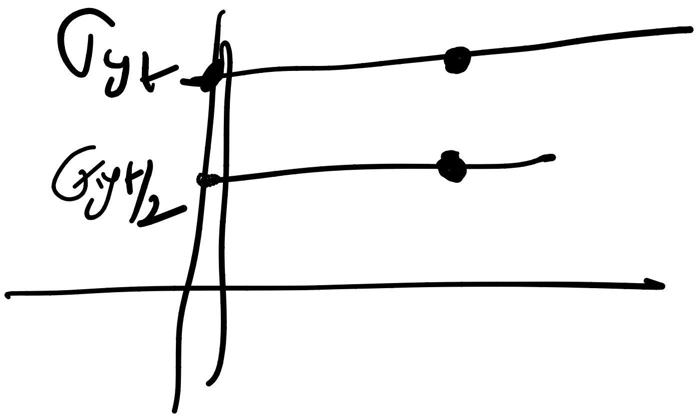
2) max shear stress theory:

max shear stress = yield point shear stress

$$\boxed{\tau_{max} = \frac{\tau_{yt}}{FS}}$$

$$\boxed{\tau_{yt} = \frac{\sigma_{yt}}{2}}$$

$$\boxed{\tau_{max} = \frac{\sigma_{yt}}{2(FS)}}$$



used mostly for ductile materials.

Max Principal Strain Theory (St. Venant's Theory)

will yield (or) fail, when max principal strain reaches the limiting value of strain (yield point strain) from a simple tensile test for a biaxial stress system.

$$\text{Strain} = \frac{\text{Stress}}{E}$$

poisson's ratio

$\sigma_x \quad \sigma_y$

$\sigma_1 \quad \sigma_2$

$$\varepsilon_{max} = \frac{\sigma_1 - \mu\sigma_2}{E}$$

$$\parallel$$

$$\varepsilon = \frac{\sigma_{yt}}{E \times (F.S)}$$

$\mu \rightarrow$ Poisson's ratio

σ_1, σ_2

max & min principal stresses

$$\frac{\sigma_1}{\cancel{E}} - \frac{\mu\sigma_2}{\cancel{E}} = \frac{\sigma_{yt}}{\cancel{E} \times (F.S)}$$

$$\boxed{\sigma_1 - \mu\sigma_2 = \frac{\sigma_{yt}}{F.S}}$$

$\sigma_{yt} \rightarrow$ yield point stress

04/09/2020

Max Strain Energy theory (or) Haigh's Theory:

Failure (or) yielding will happen when Max strain Energy will be equal to limiting (or) yielding strain Energy value of a Simple Tensile test.

Strain energy per unit volume in biaxial stress system

$$U_0 = \frac{1}{2E}\left[(\sigma_1)^2 + (\sigma_2)^2 - 2\mu\sigma_1\sigma_2\right]$$

μ → Poisson's ratio

Limiting strain energy

$$U = \frac{1}{2E}\left[\frac{\sigma_{yt}}{FS}\right]^2$$

Stress is v.high

Strain

fail

$$\varepsilon = \frac{\sigma}{E}$$

$$S.E = \frac{1}{2}\sigma \times \varepsilon = \frac{\sigma^2}{2E}$$

$$U_0 = U$$

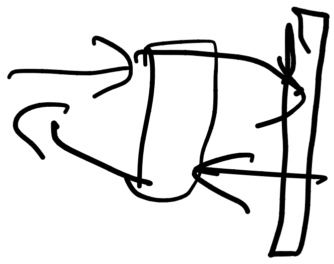
$$\boxed{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = \left[\frac{\sigma_{yt}}{FS}\right]^2}$$

Max Distortion Energy theory (or) Hencky & von-mises theory

↓

Shear Strain Energy

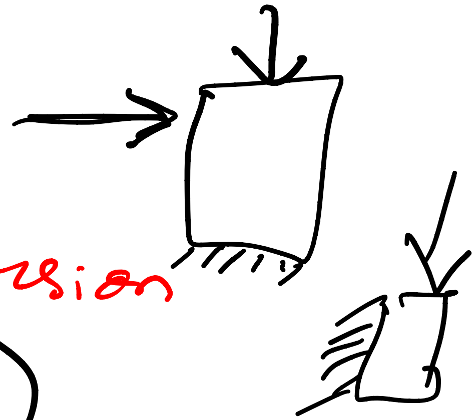
$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \left[\frac{\sigma_{yt}}{F.S}\right]^2$$



Max distortion energy per unit volume for a bi-axial stress system. = Limiting Distortion energy in a simple Tensile test

Then your component will fail

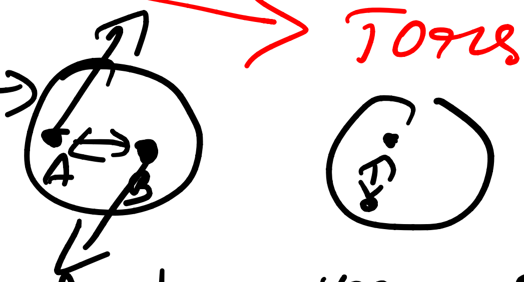
Concept of Stiffness: → Rigidity



Tension

Bending

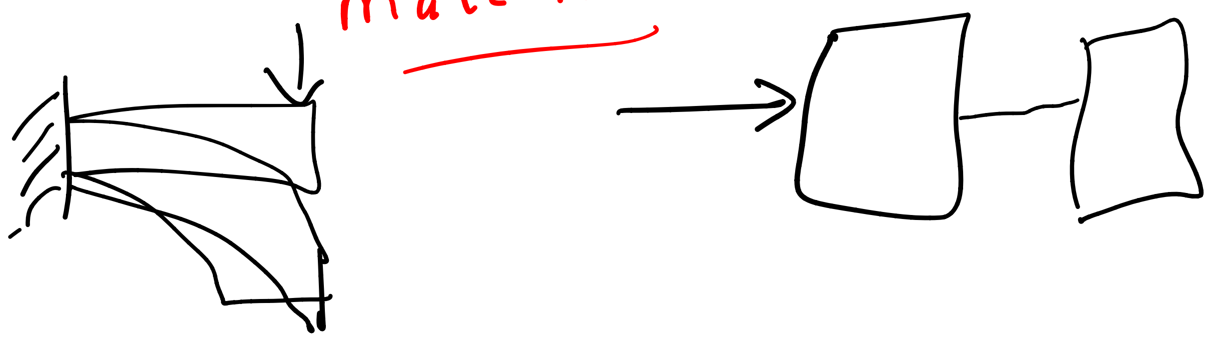
Torsion



- Stiffness is resistance offered by the object against displacement (or) deflection.

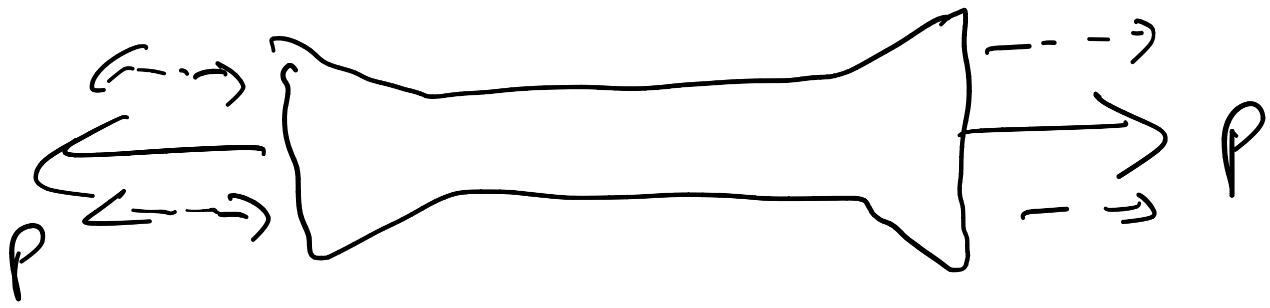
says how rigid is the material

- $S = \frac{\text{Force}}{\text{Unit displacement (or) deflection}}$



1) Axial Stiffness

Resistance to the axial loads



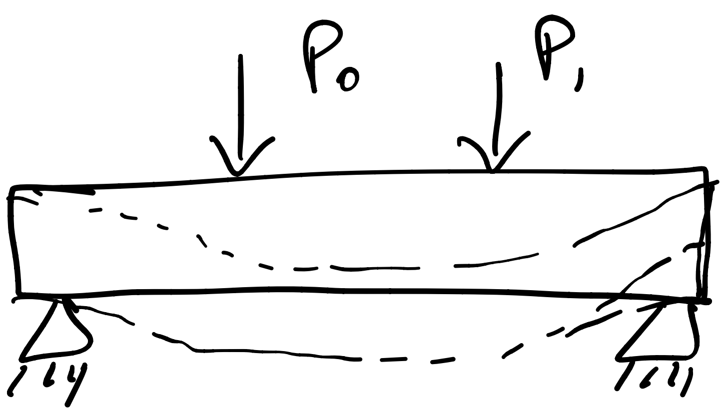
$\frac{EA}{L}$

E → youngs modulus

A → Area

L → length of the component

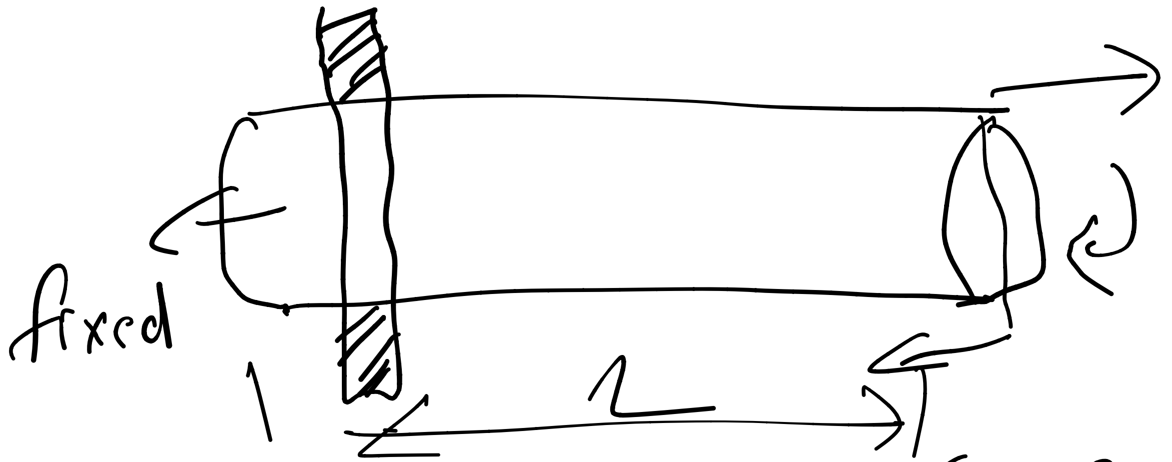
2) Bending Stiffness



$\frac{EI}{L}$ I - moment of Inertia

→ Flexural rigidity

3) Torsional Stiffness



$\frac{GJ}{L}$ (or) $\frac{CJ}{L}$

C → Rigidity modulus

J → polar M.I

dia (or) length → Stiffer component.

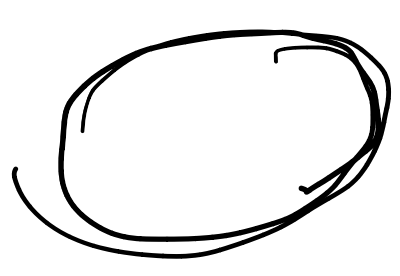
Impact Stress:

One component in motion → collides with other component → motion (or) @ rest

→ Impact load

Impact stress

Hammers, Springs, Ropes, Punching machines.



A → Area of the bar

P → Impact force

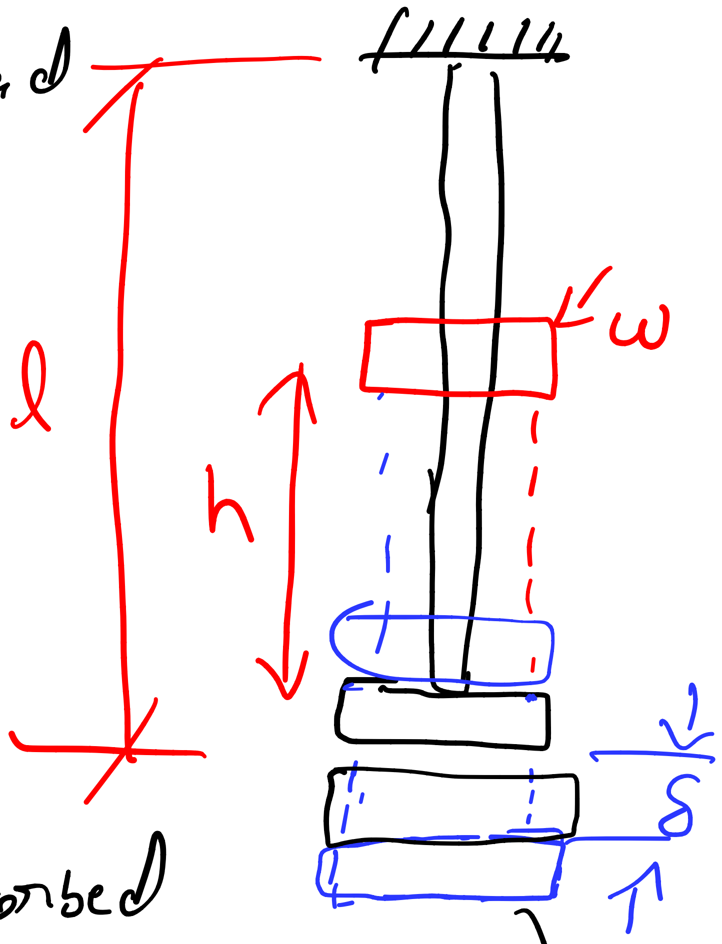
δ → deflection

E → Youngs modulus

σ_I → Impact stress

Avg load $= \frac{0+P}{2}$

$= \frac{P}{2}$



Energy released by falling object = Energy absorbed by the system —(bar + object)

$\Rightarrow \quad w(h+\delta) = \frac{(0+P)}{2} \times \delta$

$\Rightarrow \quad wh + w\delta = \frac{P}{2}\delta$

$\Rightarrow \quad w\left(h + \frac{\sigma_i l}{E}\right) = \frac{\sigma_i \times A}{2} \times \frac{\sigma_i l}{E}$

$$\frac{\frac{wl}{E} + \sqrt{\frac{w^2l^2}{E^2} + \frac{4 AL}{2E}wh}}{\cancel{2}\times\frac{Al}{2E}}$$

$\boxed{P = \sigma_i \times A}$

δ → strain × l

$= \varepsilon \times l$

$\boxed{\delta = \frac{\sigma_i l}{E}}$

$$(\sigma_i)^2\left[\frac{Al}{2E}\right] - (\sigma_i)\left[\frac{wl}{E}\right] - wh = 0$$

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

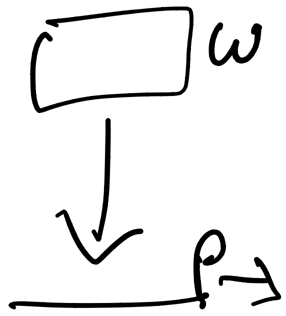
σ_i

$$\boxed{\sigma_I = \frac{w}{A}\left[1 + \sqrt{1 + \frac{2hAE}{wl}}\right]}$$ → Impact stress

Impact load $P = \sigma_i \times A$

$$P = W\left[1+\sqrt{1+\frac{2hAE}{Wl}}\right]$$

$$\frac{P}{W} = 1+\sqrt{1+\frac{2hAE}{Wl}}$$



Shock factor

How much the load has magnified due to impact

The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to

1. Maximum principal stress theory;
2. Maximum shear stress theory;
3. Maximum strain energy theory
4.. Maximum distortion energy theory.

Solution. Given : $P_{t1} = 10$ kN ; $P_s = 5$ kN ; $\sigma_{t(el)} = 100$ MPa $= 100$ N/mm^2 ; $1/m = 0.3$

Let d = Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\, d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\, d^2} = \frac{6.365}{d^2} \text{ kN/mm}^2$$

1. *According to maximum principal stress theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4+4}\right] = \frac{15.365}{d^2} \text{ kN/mm}^2 = \frac{15\,365}{d^2} \text{ N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

∴ $d^2 = 15\,365/100 = 153.65$ or $d = 12.4$ mm **Ans.**

2. *According to maximum shear stress theory*

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{9}{d^2}\ \text{kN/mm}^2 = \frac{9000}{d^2}\ \text{N/mm}^2$$

According to maximum shear stress theory,

$\tau_{max} = \tau_{yt}$

$\boxed{\tau_{yt} = \frac{\sigma_{yt}}{2}}$

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

$$\therefore \quad d^2 = 9000 / 50 = 180 \quad \text{or} \quad d = 13.42 \text{ mm } \textbf{Ans.}$$

3. *According to maximum principal strain theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = -\frac{2.63}{d^2}\ \text{KN/mm}^2$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$\frac{1}{m} = 0.3$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-2.635}{d^2}\ \text{kN/mm}^2$$

$\sigma_{t(el)} = 100$

$$= \frac{-2635}{d^2}\ \text{N/mm}^2$$

We know that according to maximum principal strain theory,

$\frac{1}{m} = \mu$

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$$d^2 = 16\,156 / 100 = 161.56 \quad \text{or} \quad d = 12.7 \text{ mm } \textbf{Ans.}$$

4. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-\,2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$$\therefore \qquad d^4 = 26\,724 \quad \text{or} \quad d = 12.78 \text{ mm}$$ **Ans.**

5. *According to maximum distortion energy theory*

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-\,2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\,391}{d^4} = 1$$

$$\therefore \qquad d^4 = 32\,391 \quad \text{or} \quad d = 13.4 \text{ mm}$$ **Ans.**

05/09/2020

Interchangeability

- The term interchangeability is normally employed for the mass production of identical items within the prescribed limits of sizes
- If the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines and mechanisms. Therefore, certain variations are recognised and allowed in the sizes of the mating parts to give the required fitting
- In order to control the size of finished part, with due allowance for error, for interchangeable parts is called ***limit system.***
- When an assembly is made of two parts, the part which enters into the other, is known as ***enveloped surface*** (or **shaft** for cylindrical part) and the other in which one enters is called ***enveloping surface*** (or **hole** for cylindrical part).

Important Terms used in Limit System

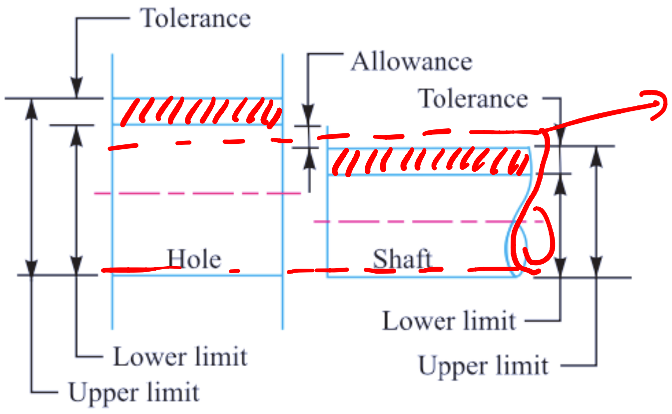
50mm ±1mm 50mm

50.01m 50.05mm 49.5mm

1. *Nominal size.* It is the size of a part specified in the drawing as a matter of convenience.

2. *Basic size.* It is the size of a part to which all limits of variation (*i.e.* tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.

505mm



50.03mm
Actual size

3. *Actual size.* It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit, otherwise it will interfere with the interchangeability of the mating parts.

4. *Limits of sizes.* There are two extreme permissible sizes for a dimension of the part as shown in Fig. 3.1. The largest permissible size for a dimension of the part is called ***upper*** or ***high*** or ***maximum limit***, whereas the smallest size of the part is known as ***lower*** or ***minimum limit.***

50 ±1mm

5. *Allowance.* It is the difference between the basic dimensions of the mating parts. The allowance may be ***positive*** or ***negative***. When the shaft size is less than the hole size, then the allowance is ***positive*** and when the shaft size is greater than the hole size, then the allowance is ***negative.***

51mm 49mm

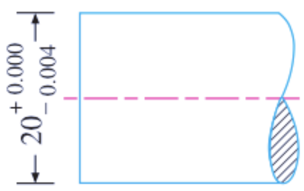
6. *Tolerance.* It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be ***unilateral*** or ***bilateral.***

(51-49) = 2mm
Tolerance

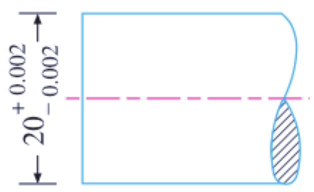
$50^{+0}_{-0.5}$

50+0

50-0.5= 49.5



(*a*) Unilateral tolerance.



(*b*) Bilateral tolerance.

The unilateral system is mostly used in industries as it permits changing the tolerance value while still retaining the same allowance or type of fit.

7. *Tolerance zone.* It is the zone between the maximum and minimum limit size

8. *Zeroline.*Itisastraightlinecorrespondingtothebasicsize.Thedeviationsaremeasured from this line. The positive and negative deviations are shown above and below the zero line respectively.

9. ***Upper deviation***. It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol ES (Ecart Superior) and of a shaft, it is represented by es.

10. *Lower deviation*. It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol EI (Ecart Inferior) and of a shaft, it is represented by ei.

11. *Actual deviation*. It is the algebraic difference between an actual size and the corresponding basic size.

12. *Mean deviation*. It is the arithmetical mean between the upper and lower deviations.

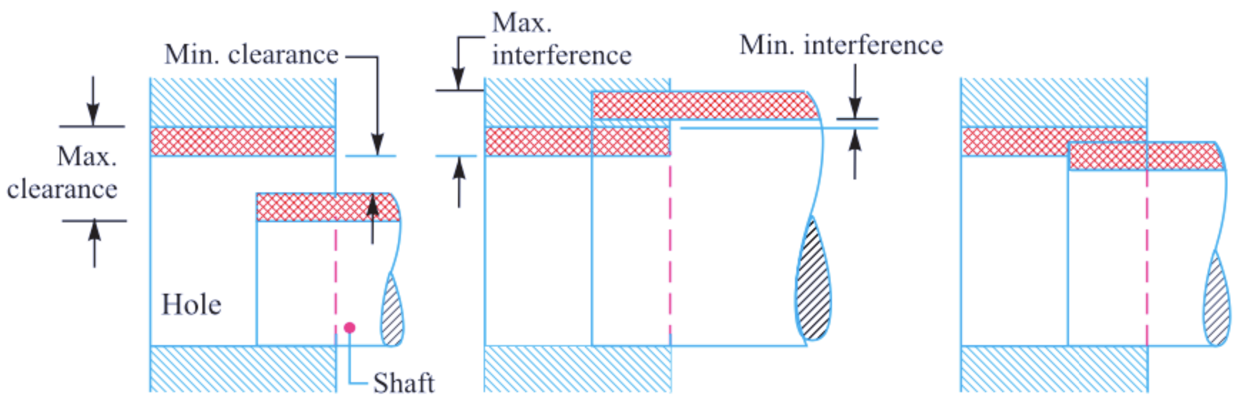
13. *Fundamental deviation*. It is one of the two deviations which is conventionally chosen to define the position of the tolerance zone in relation to zero line

The degree of tightness or looseness between the two mating parts is known as a *fit* of the parts.

The nature of fit is characterised by the presence and size of clearance and interference.

The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly as shown in Fig. 3.5 (*a*). In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be ***positive.*** → Shaft < Hole

The ***interference*** is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly as shown in Fig. 3.5 (*b*). In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be ***negative.*** → Shaft > Hole



(*a*) Clearance fit. (*b*) Interference fit. (*c*) Transition fit.

Indian Standard System of Limits and Fits

According to Indian standard [IS : 919 (Part I)-1993], the system of limits and fits comprises 18 grades of fundamental tolerances *i.e.* grades of accuracy of manufacture and 25 types of fundamental deviations indicated by letter symbols for both holes and shafts (capital letter *A* to *ZC* for holes and small letters *a* to *zc* for shafts) in diameter steps ranging from 1 to 500 mm.

The 18 tolerance grades are designated as IT 01, IT 0 and IT 1 to IT 16. These are called ***standard tolerances.*** The standard tolerances for grades IT 5 to IT 7 are determined in terms of standard tolerance unit (i) in microns, where

i (microns) $= 0.45\ \sqrt[3]{D} + 0.001\ D$, where D is the size or geometric mean diameter in mm.

The following table shows the relative magnitude for grades between IT 5 and IT 16.

Table 3.2. Relative magnitude of tolerance grades.

Tolerance grade	IT 5	IT 6	IT 7	IT 8	IT 9	IT 10	IT 11	IT 12	IT 13	IT 14	IT 15	IT 16
Magnitude	7 i	10 i	16 i	25 i	40 i	64 i	100 i	160 i	250 i	400 i	640 i	1000 i

The values of standard tolerances corresponding to grades IT 01, IT 0 and IT 1 are as given below:

For IT 01, i (microns) $= 0.3 + 0.008\,D$,

For IT 0, i (microns) $= 0.5 + 0.012\,D$, and

For IT 1, i (microns) $= 0.8 + 0.020\,D$,

where D is the size or geometric mean diameter in mm.

50 ± 1mm

51 ± 1mm

A fit is designated by its basic size followed by symbols representing the limits of each of its two components, the hole being quoted first. For example, 100 $H6/g5$ means basic size is 100 mm and the tolerance grade for the hole is 6 and for the shaft is 5.

Calculation of Fundamental Deviation for Shafts

For holes, the upper deviation is denoted by ES and the lower deviation by EI. Similarly for shafts, the upper deviation is represented by es and the lower deviation by ei.

The other deviation may be calculated by using the absolute value of the standard tolerance (IT) from the following relation:

$ei = es - IT$ or $es = ei + IT$

$es = ei + IT$

Table 3.7. Formulae for fundamental shaft deviations.

Upper deviation (es)		*Lower deviation (ei)*	
Shaft designation	*In microns (for D in mm)*	*Shaft designation*	*In microns (for D in mm)*
a	$= -(265 + 1.3\,D)$	$J\,5$ to $j\,8$	No formula
	for $D \le 120$	$k\,4$ to $k\,7$	$= +0.6\,\sqrt[3]{D}$
	$= -3.5\,D$		
	for $D > 120$	k for grades	$= 0$
		≤ 3 and ≤ 8	
b	$= -(140 + 0.85\,D)$	m	$= +(IT\,7 - IT\,6)$
	for $D \le 160$		
	$= -1.8\,D$	n	$= +5\,(D)^{0.34}$
	for $D > 160$	p	$= +IT\,7 + 0$ to 5
c	$= -52\,(D)^{0.2}$	r	= Geometric mean of values of ei
	for $D \le 40$		for shaft p and s
	$= -(95 + 0.8\,D)$	s	$= +(IT\,8 + 1$ to $4)$ for $D \le 50$
	for $D > 40$		$= +(IT\,7 + 0.4\,D)$ for $D > 50$
d	$= -16\,(D)^{0.44}$	t	$= +(IT\,7 + 0.63\,D)$
e	$= -11\,(D)^{0.41}$	u	$= +(IT\,7 + D)$
f	$= -5.5\,(D)^{0.41}$	v	$= +(IT\,7 + 1.25\,D)$
		x	$= +(IT\,7 + 1.6\,D)$
g	$= -2.5\,(D)^{0.34}$	y	$= +(IT\,7 + 2\,D)$
		z	$= +(IT\,7 + 2.5\,D)$
h	$= 0$	za	$= +(IT\,8 + 3.15\,D)$
		zb	$= +(IT\,9 + 4\,D)$
		zc	$= +(IT\,10 + 5\,D)$

Table 3.8. Rules for fundamental deviation for holes.

<table>
<tr><td colspan="3">All deviation except those below</td><td>General rule
Hole limits are identical with the shaft limits of the same symbol (letter and grade) but disposed on the other side of the zero line.
EI = Upper deviation es of the shaft of the same letter symbol but of opposite sign.</td></tr>
<tr><td rowspan="2">For sizes above 3 mm</td><td>N</td><td>9 and coarser grades</td><td>$ES = 0$</td></tr>
<tr><td>J, K, M and N

P to ZC</td><td>Upto grade 8 inclusive

upto grade 7 inclusive</td><td>Special rule
ES = Lower deviation ei of the shaft of the same letter symbol but one grade finer and of opposite sign increased by the difference between the tolerances of the two grades in question.</td></tr>
</table>

The fundamental deviation for Indian standard holes for diameter steps from 1 to 200 mm may be taken directly from the following table.

Table 3.9. Indian standard 'H' Hole
Limits for H5 to H13 over the range 1 to 200 mm as per IS : 919 (Part II) -1993.

Diameter steps in mm		*Deviations in micron (1 micron = 0.001 mm)*									
		H5	H6	H7	H8	H9	H10	H11	H12	H13	H5 – H13
Over	To	High +	High +	High +	High +	High +	High +	High +	High +	High +	Low
1 3 6	3 6 10	5 5 6	7 8 9	9 12 15	14 18 22	25 30 36	40 48 58	60 75 90	90 120 150	140 180 220	0 0 0
10 14	14 18	8	11	18	27	43	70	110	180	270	0
18 24	24 30	9	13	21	33	52	84	130	210	330	0
30 40	40 50	11	16	25	39	62	100	160	250	460	0
50 65	65 80	13	19	30	46	74	120	190	300	390	0
80 100	100 120	15	22	35	54	87	140	220	350	540	0
120 140	140 160	18	25	40	63	100	160	250	400	630	0
160 180	180 200	20	29	46	72	115	185	290	460	720	0

Example . *The dimensions of the mating parts, according to basic hole system, are given as follows :*

Hole : 25.00 mm *Shaft : 24.97 mm*

25.02 mm *24.95 mm*

Find the hole tolerance, shaft tolerance and allowance.

Ans: Lower limit of hole = 25.00 mm

upper " " " = 25.02 mm

Lower " " shaft = 24.95 mm

upper " " " = 24.97 mm

Hole tolerance = (upper limit − lower limit) of hole

= 25.02 − 25.00

= 0.02 mm

shaft tolerance = (upper limit − lower limit)

= 24.97 − 24.95 = 0.02 mm

Allowance = lower limit of Hole − upper limit of shaft

= 25.00 − 24.97

= 0.03 mm

Example . *Calculate the tolerances, fundamental deviations and limits of sizes for the shaft designated as 40 H8 / f7.*

Ans: 40 H8/f7

40 → Basic size

Tolerance grade for hole → IT8

" " " shaft → IT7

$$i = 0.45 \sqrt[3]{D} + 0.001 D \ \mu$$

D = ?

40 lies b/n 30 & 50

$$D = \sqrt{30 \times 50}$$

$$= 38.73 \text{ mm}$$

$$i = 0.45 \sqrt[3]{38.73} + 0.001 \times 38.73$$

$$i = 1.56 \text{ microns} = 1.56 \times 10^{-3} \text{ mm}$$

$$\boxed{1 \text{ micron} = 0.001 \text{ mm}}$$

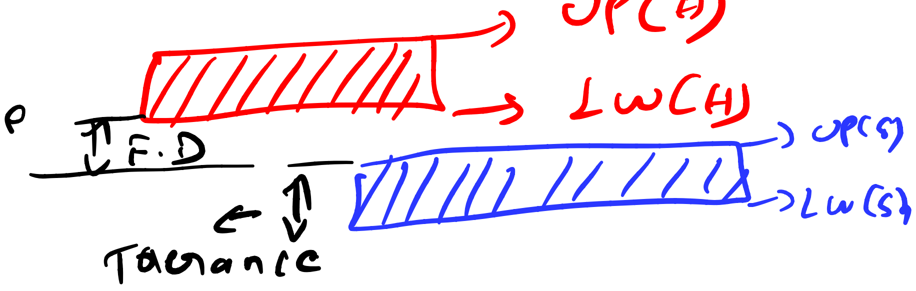
$$\boxed{1 \text{ micron} = 10^{-6} \text{ m}}$$

For IT8 = 25 i

$$= 25 \times 1.56 \times 10^{-3}$$

$$= 0.039 \text{ mm} \rightarrow \text{Hole}$$

For shaft, the Standard tolerance



$$= 16 i$$

$$= 16 \times 1.56 \times 10^{-3}$$

$$= \underline{0.025} \text{ mm}$$

Fundamental deviation (Lower deviation) for hole H

$$EI = 0$$

Fundamental deviation (upper deviation) for shaft f

$$= -5.5 \times (D)^{0.41}$$

$= -5.5\,(38.73)^{0.41}$

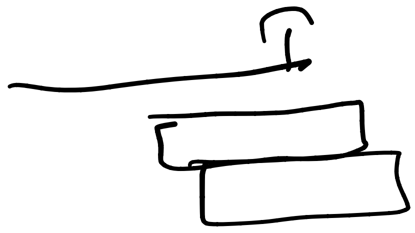
$es = -25$ microns $= -0.025\,mm$

Fundamental (lower deviation) for shaft f

$\underline{ei = es - IT} \Rightarrow ei = -0.025 - 0.025$

$es = ei + IT$

$\underline{ei = -0.05\,mm}$



Basic size = 40mm

upper limit of hole = Lower limit of hole + Tolerance

$= 40 + 0.039$

$= 40.039\,mm$

upper limit of shaft = Lower limit for hole − Fundamental deviation

Allowance = Lower limit of hole − upper limit of shaft

$= 40 - 0.025$

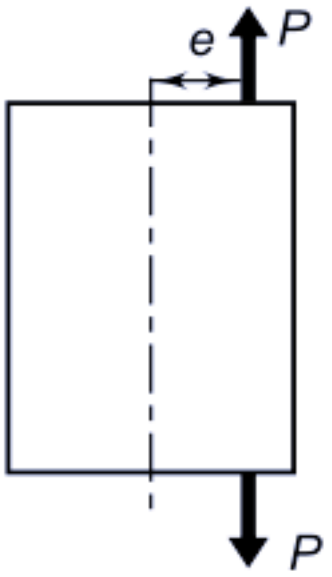
$= 39.975\,mm$

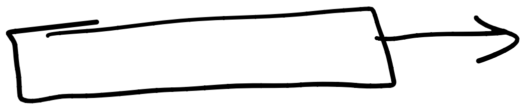
Lower limit for shaft = upper limit − Tolerance

$= 39.975 - 0.025$

$= 39.95\,mm$ //

Eccentric Axial Loading:





e → eccentricity

1) Tensile = P

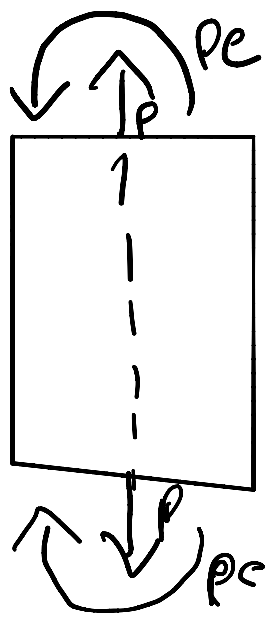
2) Moment = P × e

$$\sigma_t = \frac{P}{A}$$

$$\sigma_b = \frac{Pey}{I}$$

$$\sigma_b = \frac{My}{I}$$

$$\sigma = \sigma_t \pm \sigma_b$$



$$\boxed{\sigma = \frac{P}{A} \pm \frac{Pey}{I}}$$

Fracture Mechanics:

Fracture strength of a brittle material, like glass, is inversely proportional to the square root of the crack length.

- The concept of fracture mechanics begins with the assumption that all components contain microscopic cracks.
- In case of ductile materials, there is stress concentration in the vicinity of a crack. When the localized stress near the crack reaches the yield point, there is plastic deformation, resulting in redistribution of stresses.
- Therefore, the effect of crack is not serious in case of components made of ductile materials.

The effect of crack is much more serious in case of components made of brittle materials due to their inability of plastic deformation

Fracture mechanics is normally concerned with materials that are in the brittle state.

- They include high-strength, low-alloy steels, high-strength aluminium alloys, titanium alloys and some polymers.
- They also include 'normally ductile' materials, which under certain conditions of thermal and corrosive environment, behave like brittle materials.
- For example, low carbon steels at temperatures below 0°C behave like brittle materials.

Fracture mechanics is the science of predicting the influence of cracks and crack like defects on the brittle fracture of components.

i. When a component containing a small microscopic crack is subjected to an external force, there is an almost instantaneous propagation of the crack leading to sudden and total failure

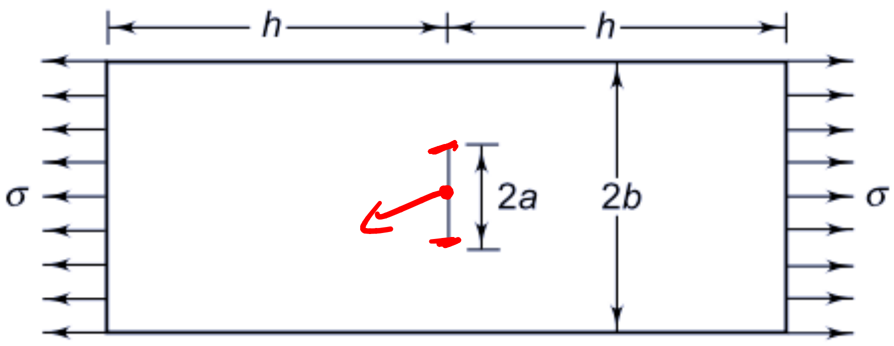
ii. Less force is required to propagate a crack than to initiate it

iii. Fracture failure occurs at a stress level which is well below the yield point of the material.

300mpa

100mpa

fail



The stress intensity factor K_0 specifies the stress intensity at the tip of the crack.

$$K_0 = \sigma\sqrt{\pi a}$$

where,

K_0 = stress intensity factor (in units of N/mm^2 $\sqrt{\text{m}}$)

σ = nominal tensile stress at the edge $\left(\dfrac{P}{2bt}\right)$ (N/mm^2)

t = plate thickness (mm)

a = half crack length (m)

P → Load applied

t → thickness of plat

The fracture toughness is the critical value of stress intensity at which crack extension occurs. The fracture toughness is denoted by K_I

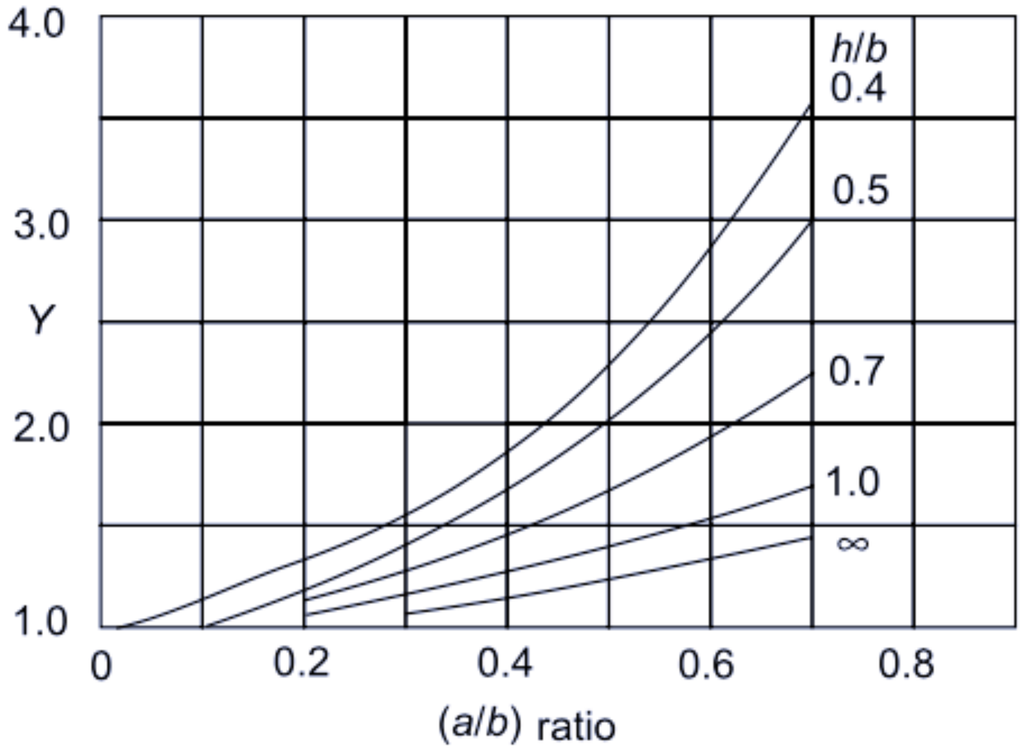
$$K_I = Y K_0 = Y\sigma\sqrt{\pi a}$$

where,

Y = dimensionless correction factor that accounts for the geometry of the part containing the crack

K_I = fracture toughness (in units of N/mm^2 m)

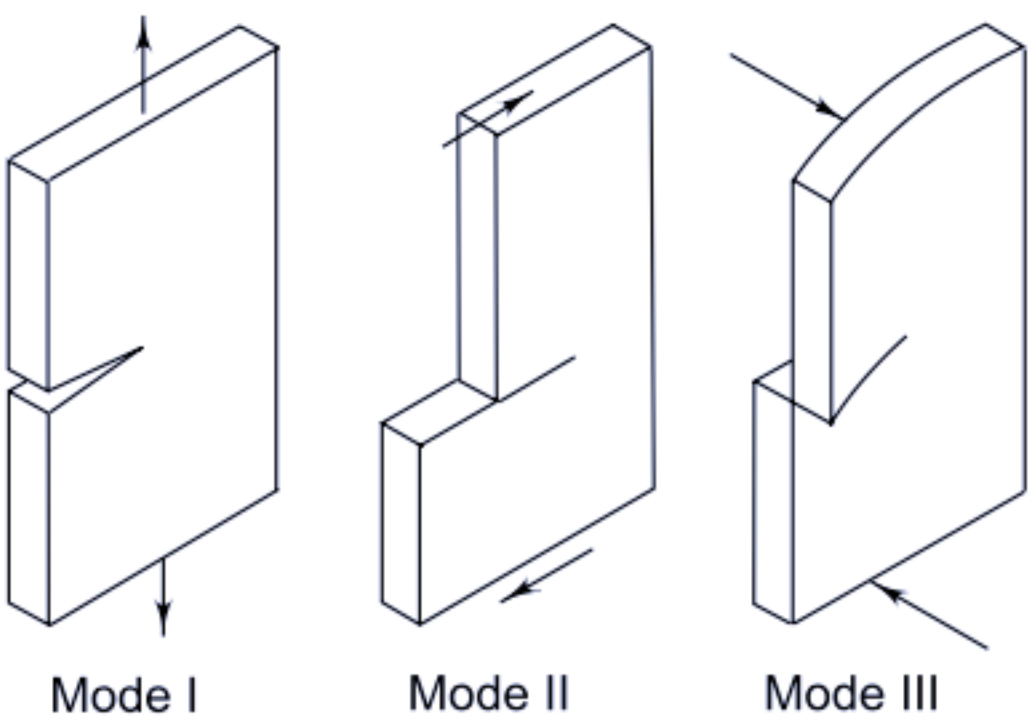
Variation of Y :



- The stress intensity factor K_0 represents the stress level at the tip of the crack in the machine part.
- On the other hand, fracture toughness K_I is the highest stress intensity that the part can withstand without fracture at the crack.

There are three basic modes of crack propagation

- *Mode-I* is called the *opening* or *tensile mode*. It is the most commonly observed mode of crack propagation. In this case, the crack faces separate symmetrically with respect to the crack plane.
- *Mode-II* is called *sliding* or *in- plane shearing mode*
- *Mode-III* is called *tearing mode. Both Mode II & III failures are shear modes.*



Curved Beams:

A curved beam is defined as a beam in which the neutral axis in unloaded condition is curved instead of straight

Assumptions made are:

1. Plane sections perpendicular to the axis of the beam remain plane after bending.
2. The moduli of elasticity in tension and compression are equal.
3. The material is homogeneous and obeys Hooke's law.

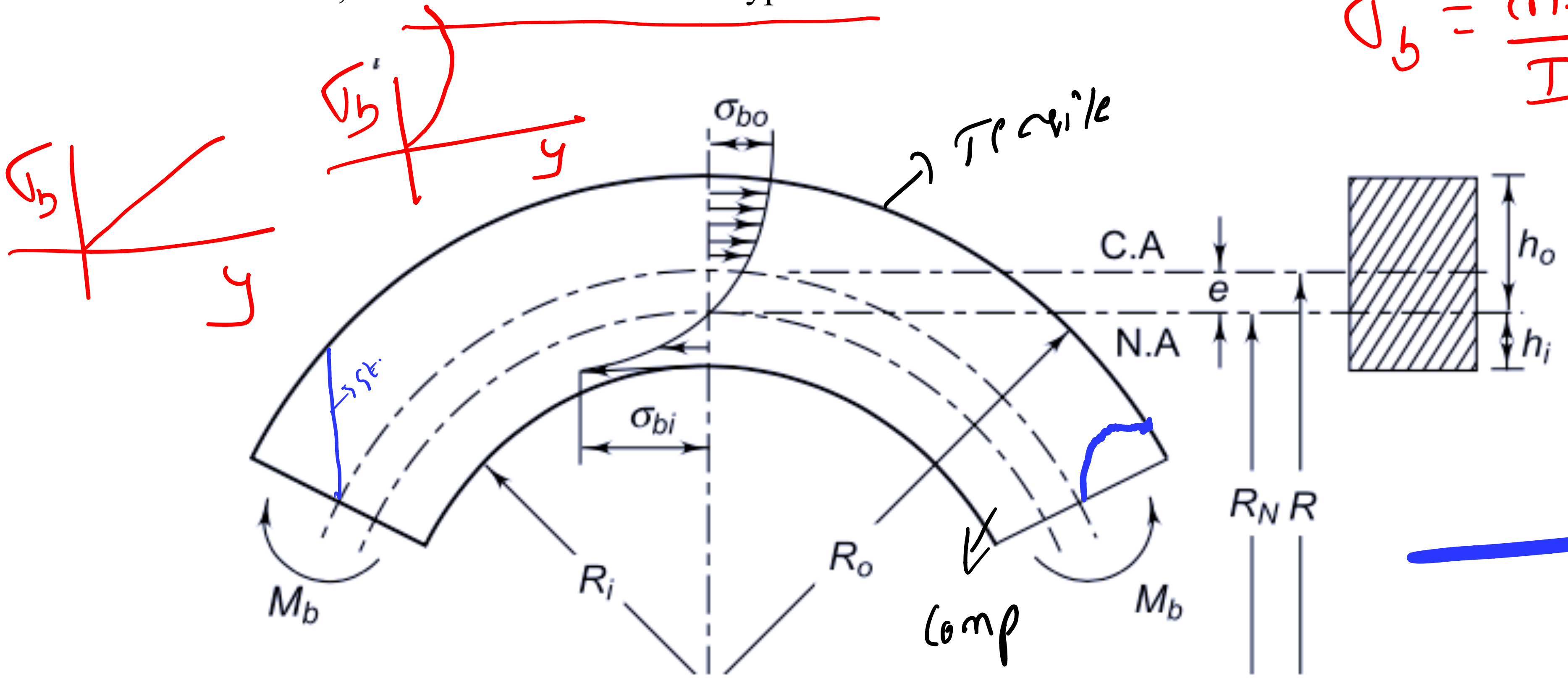
The differences between a straight beam and curved beam:

(i) The neutral and centroidal axes of the straight beam are coincident. However, in a curved beam the neutral axis is shifted towards the centre of curvature.

(ii) The bending stresses in a straight beam vary linearly with the distance from the neutral axis. However in curved beams, the stress distribution is hyperbolic.

$\sigma_b = \frac{M.y}{I}$ → dist from axis

$\sigma_b \propto y$



Neutral axis → axis on which no stress acts

R_o = radius of outer fibre (mm)
R_i = radius of inner fibre (mm)
R = radius of centroidal axis (mm)
R_N = radius of neutral axis (mm)
h_i = distance of inner fibre from neutral axis (mm)
h_o = distance of outer fibre from neutral axis (mm)
M_b = bending moment with respect to centroidal axis (N-mm)
A = area of the cross-section (mm^2)

Eccentricity $e = R - R_N$

Bending stress $\sigma_b = \frac{M_b y}{Ae(R_N - y)}$

The equation indicates the hyperbolic distribution of bending stress with respect to *y*. The maximum stress occurs either at the inner fibre or at the outer fibre.

The bending stress at the inner fibre is

$$\sigma_{bi} = \frac{M_b h_i}{AeR_i}$$

Similarly, the bending stress at the outer fibre is given by,

$$\sigma_{bo} = \frac{M_b h_o}{AeR_o}$$

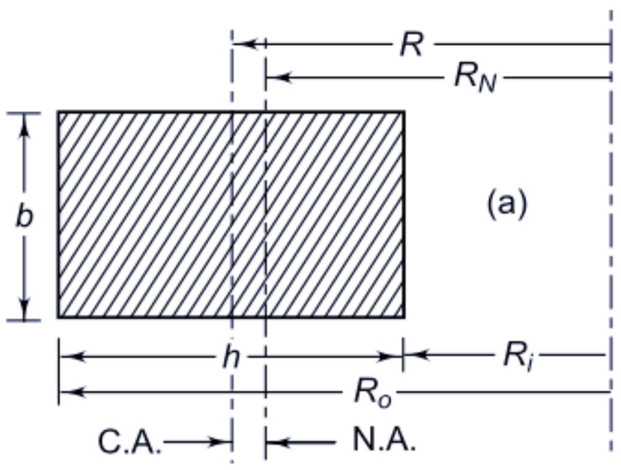
- In symmetrical cross-sections, such as circular or rectangular, the maximum bending stress always occurs at the inner fibre.
- In unsymmetrical cross- sections, it is necessary to calculate the stresses at the inner as well as outer fibres to determine the maximum stress.

In most of the engineering problems, the **magnitude of e is very small** and it should be **calculated precisely** to avoid a large percentage error in the final results.

For rectangular section

$$R_N = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)}$$

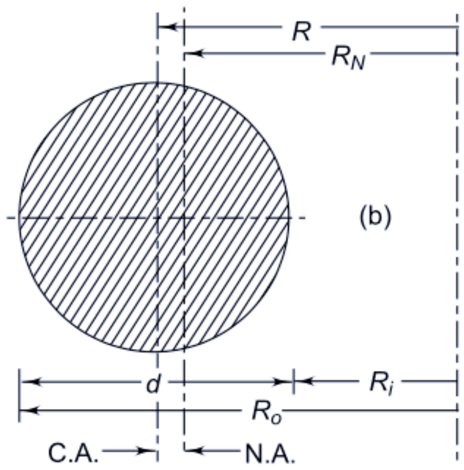
$$R = R_i + \frac{h}{2}$$



For circular section

$$R_N = \frac{\left(\sqrt{R_o} + \sqrt{R_i}\right)^2}{4}$$

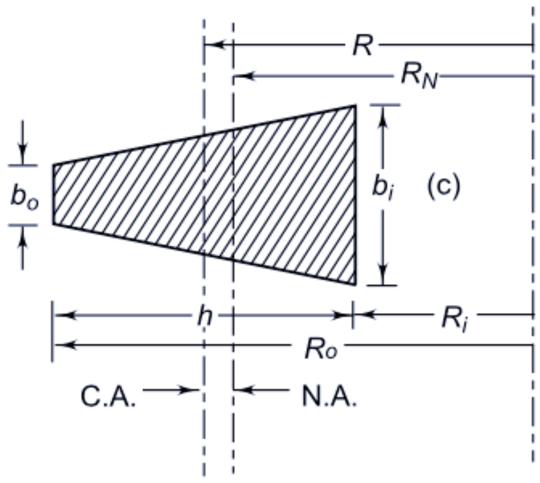
and $$R = R_i + \frac{d}{2}$$

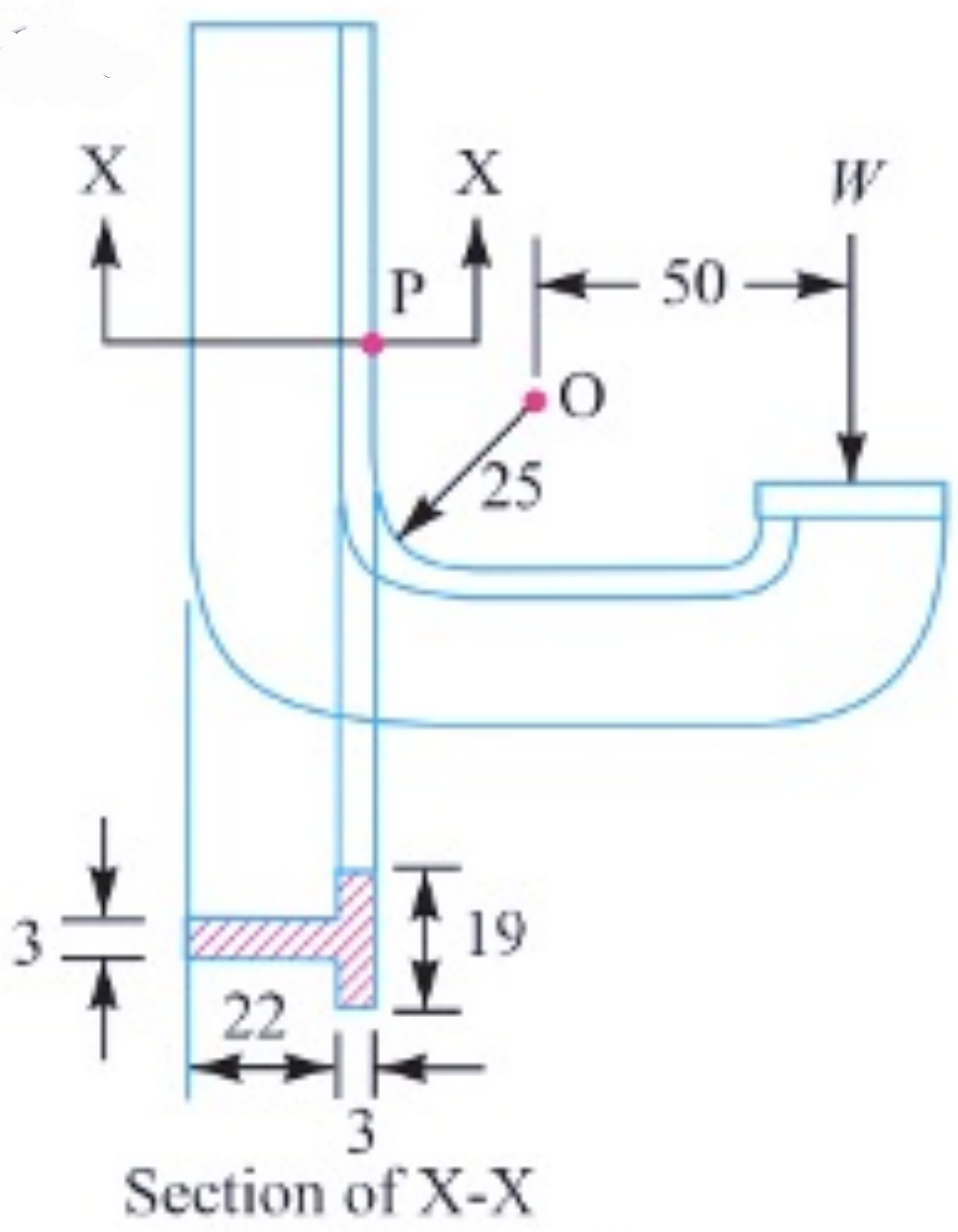


For Trapezoidal Section

$$R_N = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)} \quad (4.62)$$

and $$R = R_i + \frac{h\,(b_i + 2b_o)}{3(b_i + b_o)} \quad (4.63)$$





All dimensions in mm.

$$\delta l = \frac{Pl}{AE}$$

Example 4.5. *The piston rod of a steam engine is 50 mm in diameter and 600 mm long. The diameter of the piston is 400 mm and the maximum steam pressure is 0.9 N/mm². Find the compression of the piston rod if the Young's modulus for the material of the piston rod is 210 kN/mm².*

Solution. Given : $d = 50$ mm ; $l = 600$ mm ; $D = 400$ mm ; $p = 0.9$ N/mm² ; $E = 210$ kN/mm² $= 210 \times 10^3$ N/mm²

Let δl = Compression of the piston rod.

We know that cross-sectional area of piston,

$$= \frac{\pi}{4} \times D^2 = \frac{\pi}{4} (400)^2 = 125\,680 \text{ mm}^2$$

∴ Maximum load acting on the piston due to steam,

$$P = \text{Cross-sectional area of piston} \times \text{Steam pressure}$$

$$= 125\,680 \times 0.9 = 113\,110 \text{ N}$$

We also know that cross-sectional area of piston rod,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} (50)^2$$

$$= 1964 \text{ mm}^2$$

and Young's modulus (E),

$$210 \times 10^3 = \frac{P \times l}{A \times \delta l}$$

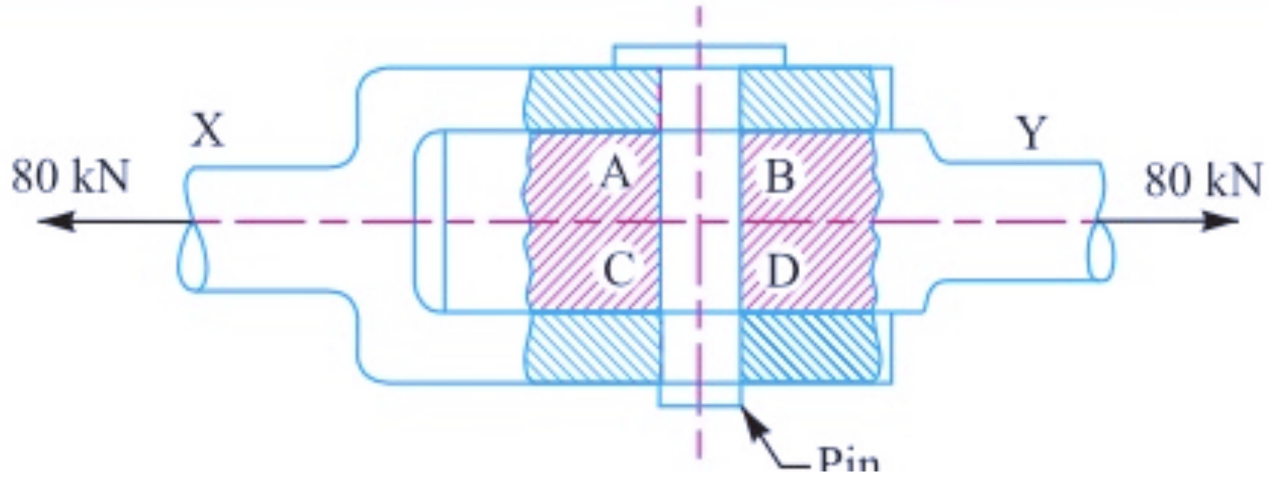
$$= \frac{113\,110 \times 600}{1964 \times \delta l} = \frac{34\,555}{\delta l}$$

$$\therefore \quad \delta l = 34\,555 / (210 \times 10^3)$$

$$= 0.165 \text{ mm} \quad \textbf{Ans.}$$

Example 4.7. *A pull of 80 kN is transmitted from a bar X to the bar Y through a pin as shown in Fig. 4.8.*

If the maximum permissible tensile stress in the bars is 100 N/mm² and the permissible shear stress in the pin is 80 N/mm², find the diameter of bars and of the pin.



Solution. Given : $P = 80\text{ kN} = 80 \times 10^3\text{ N}$; $\sigma_t = 100\text{ N/mm}^2$; $\tau = 80\text{ N/mm}^2$

Diameter of the bars

Let D_b = Diameter of the bars in mm.

$\therefore$ Area, $A_b = \frac{\pi}{4}(D_b)^2 = 0.7854\,(D_b)^2$

We know that permissible tensile stress in the bar (σ_t),

$$100 = \quad = \frac{80 \times 10^3}{0.7854\,(D_b)^2} = \frac{101\,846}{(D_b)^2}$$

$\therefore \quad (D_b)^2 = 101\,846 / 100 = 1018.46$

or $\quad D_b = 32\text{ mm}$ **Ans.**

Diameter of the pin

Let D_p = Diameter of the pin in mm.

Since the tensile load P tends to shear off the pin at two sections *i.e.* at AB and CD, therefore the pin is in double shear.

$\therefore$ Resisting area,

$$A_p = 2 \times \frac{\pi}{4}(D_p)^2 = 1.571\,(D_p)^2$$

We know that permissible shear stress in the pin (τ),

$$80 = \frac{P}{A_p} = \frac{80 \times 10^3}{1.571\,(D_p)^2} = \frac{50.9 \times 10^3}{(D_p)^2}$$

$\therefore \quad (D_p)^2 = 50.9 \times 10^3/80 = 636.5$ or $D_p = 25.2\text{ mm}$ **Ans.**

Example 4.8. *Two plates 16 mm thick are joined by a double riveted lap joint as shown in Fig. 4.11. The rivets are 25 mm in diameter.*

Find the crushing stress induced between the plates and the rivet, if the maximum tensile load on the joint is 48 kN.

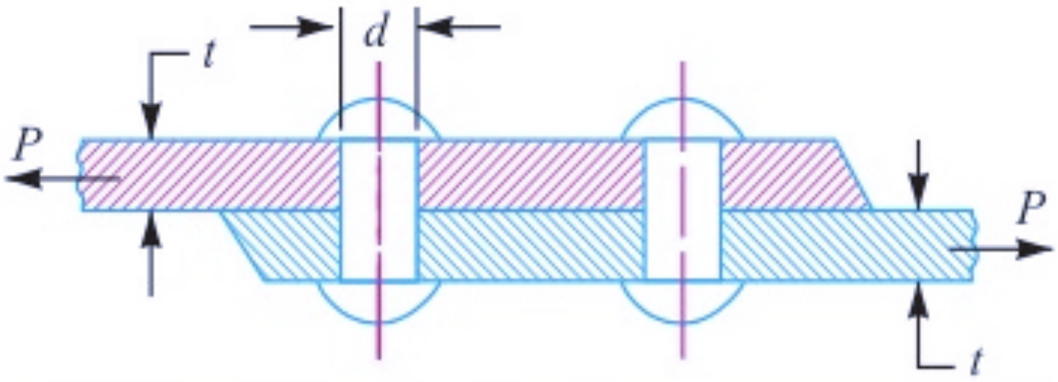


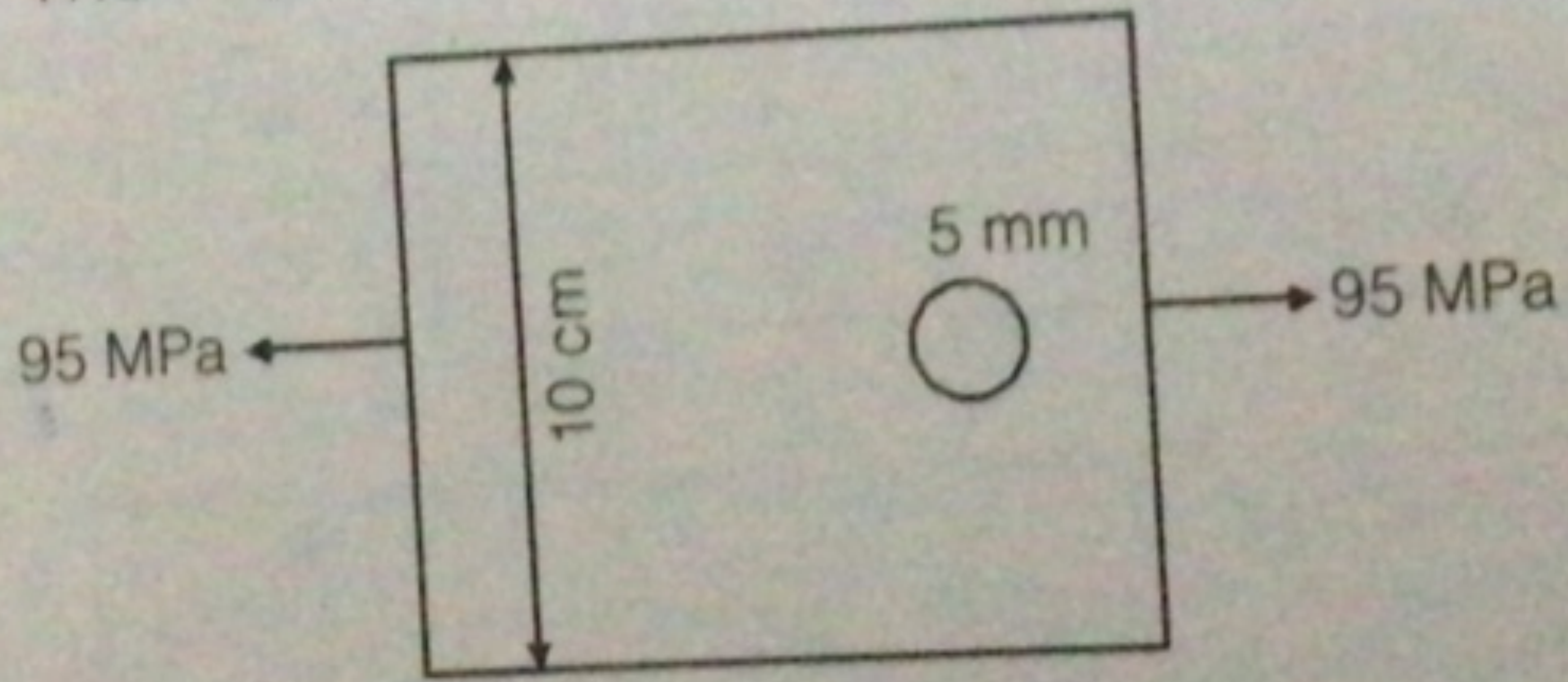
Fig. 4.11

Solution. Given : $t = 16$ mm ; $d = 25$ mm ; $P = 48$ kN $= 48 \times 10^3$ N

Since the joint is double riveted, therefore, strength of two rivets in bearing (or crushing) is taken. We know that crushing stress induced between the plates and the rivets,

$$\sigma_c = \frac{P}{d.t.n} = \frac{48 \times 10^3}{25 \times 16 \times 2} = 60 \text{ N/mm}^2 \textbf{ Ans.}$$

1.3 A large uniform plate containing a rivet hole subjected to uniform uniaxial tension of 95 MPa. The maximum stress in the plate is



(a) 100 MPa (b) 285 MPa

(c) 190 MPa (d) Indeterminate

[1992 : 1 Mark]

$$\text{Max. stress} = \frac{\text{Avg. Force}}{\text{Min. area}} = \frac{95 \times 100 \times \cancel{t}}{(100-5) \times \cancel{t}}$$

$$= \frac{95 \times 10 \times t}{(10-0.5)t} = 100 \text{ MPa}$$

Example An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm^2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take $E = 200$ kN/mm^2.

<u>Impact stress</u>

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$

$\delta l = 2$ mm

$E = 200$ KN/mm^2

$l = 3$m $= 3000$ mm

$$\text{Strain} = \frac{\delta l}{l}$$

$$\text{Young's Modulus, } E = \frac{\text{Stress}}{\text{Strain}}$$

$$E = \frac{\sigma}{\varepsilon} \qquad \sigma = E \times \varepsilon$$

$$= \frac{E \times \delta l}{l} = \frac{200 \times \cancel{10^3} \times 2}{3 \times \cancel{10^3}}$$

1. $\boxed{\sigma = 133.3 \text{ N/mm}^2}$

$A = 600$ mm^2

$h = 10$ mm

$$133.3 = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times \cancel{10^3}}{W \times 3\cancel{000}}}\right]$$

$$\frac{400}{3} \times 600 = W\left[1 + \sqrt{1 + \frac{8 \times 10^5}{W}}\right]$$

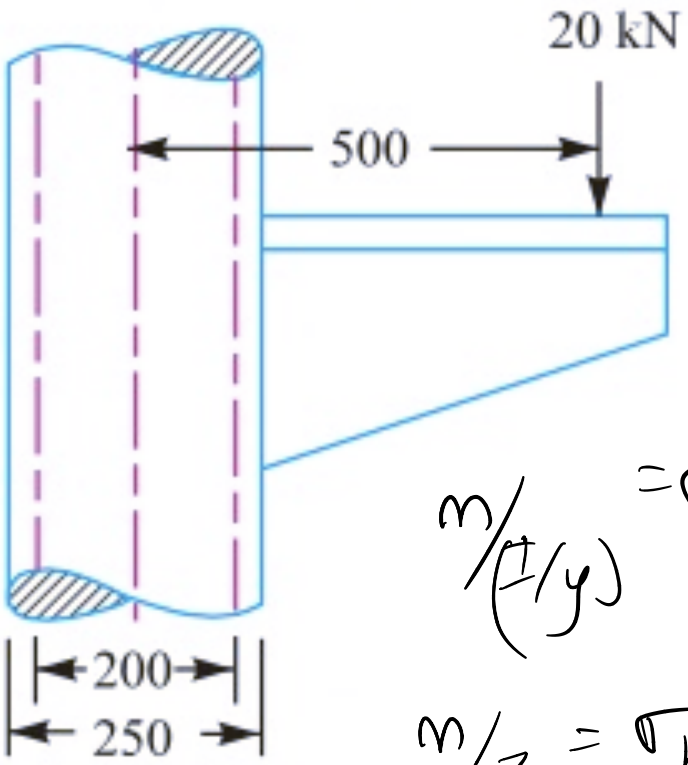
$$\left(\frac{80,000}{W} - 1\right)^2 = \left(\sqrt{1 + \frac{800,000}{W}}\right)^2$$

$$\left(\frac{80{,}000}{\omega}\right)^2 + \cancel{1} - \frac{2\times 80{,}000}{\omega}\times 1 = \cancel{1} + \frac{800{,}000}{\omega}$$

$$\frac{6.4\times 10^{4}}{\omega^{\cancel{2}}} = \frac{160{,}000 + 800{,}000}{\cancel{\omega}}$$

$$\boxed{\omega = 6666.7\cancel{1}}$$

Example *A hollow circular column of external diameter 250 mm and internal diameter 200 mm, carries a projecting bracket on which a load of 20 kN rests, as shown in Fig. The centre of the load from the centre of the column is 500 mm. Find the stresses at the sides of the column.*



9.91

Solution. Given : $D = 250$ mm ; $d = 200$ mm ; $P = 20$ kN $= 20 \times 10^3$ N ; $e = 500$ mm

We know that cross-sectional area of column,

$$A = \frac{\pi}{4}(D^2 - d^2)$$
$$= \frac{\pi}{4}[(250)^2 - (200)^2]$$
$$= 17\,674 \text{ mm}^2$$

$\therefore$ Direct compressive stress,

$$\sigma_o = \frac{P}{A} = \frac{20 \times 10^3}{17\,674} = 1.13 \text{ N/mm}^2$$
$$= 1.13 \text{ MPa}$$

Section modulus for the column,

$$Z = \frac{\frac{\pi}{64}\left[D^4 - d^4\right]}{D/2} = \frac{\frac{\pi}{64}\left[(250)^4 - (200)^4\right]}{250/2}$$
$$= 905.8 \times 10^3 \text{ mm}^3$$

Bending moment,

$$M = P.e$$
$$= 20 \times 10^3 \times 500$$
$$= 10 \times 10^6 \text{ N-mm}$$

$\therefore$ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{10 \times 10^6}{905.8 \times 10^3}$$
$$= 11.04 \text{ N/mm}^2$$
$$= 11.04 \text{ MPa}$$

Since σ_o is less than σ_b, therefore right hand side of the column will be subjected to compressive stress and the left hand side of the column will be subjected to tensile stress.

$\therefore$ Maximum compressive stress,

$$\sigma_c = \sigma_b + \sigma_o =$$
$$= 12.17 \text{ MPa} \quad \textbf{Ans.}$$

and maximum tensile stress,

$$\sigma_t = \sigma_b - \sigma_o = 11.04 - 1.13 = 9.91 \text{ MPa}$$

$M/(I/y) = \sigma_b$

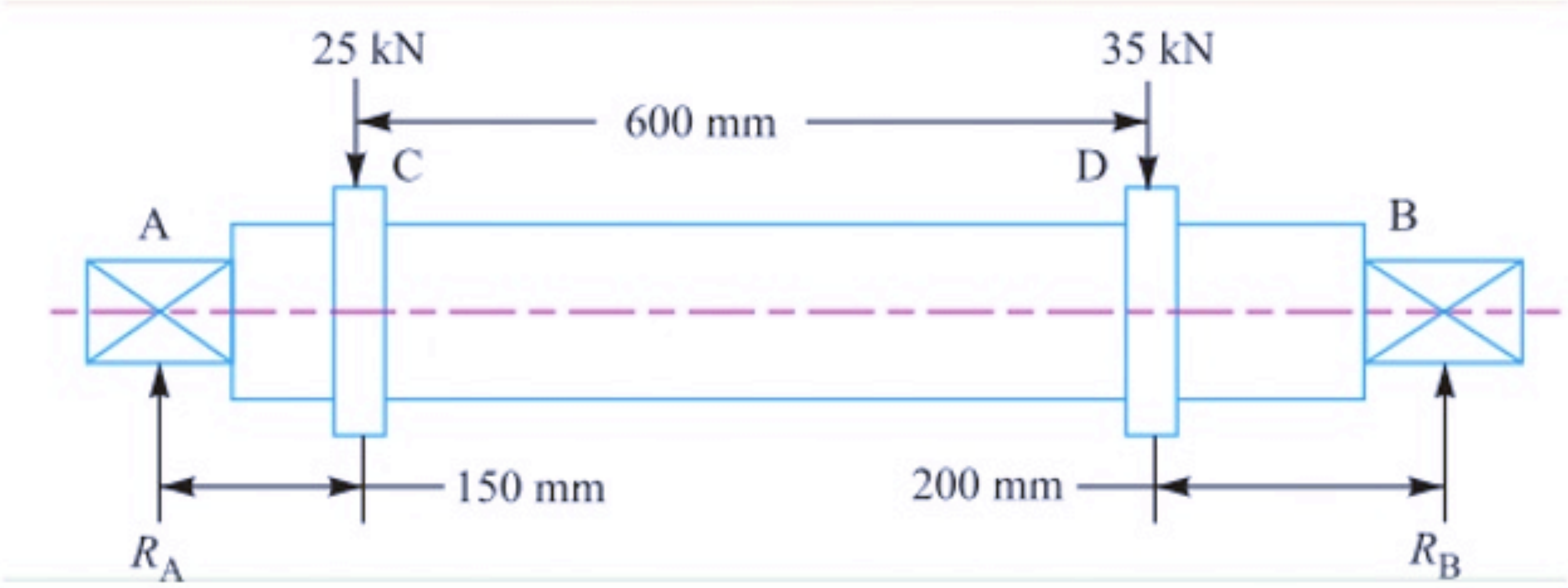
$M/Z = \sigma_b$

$1 \text{ Pa} = 1 \text{ N/m}^2$

$= 1 \text{ N}/(1000 \text{ mm})^2$

$1 \text{ Pa} = 10^{-6} \text{ N/mm}^2$

A pump lever rocking shaft is shown in Fig. below. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.



R_A & R_B → Reactions @ ends A & B

$$R_A + R_B = 25 + 35 = 60 \quad \text{— (1)}$$

B.m @ A

$$(25 \times 150) + (35 \times 750) - 950 \times R_B = 0$$

$$R_B = 31.57 \text{ KN}$$

$$R_A = 60 - R_B \Rightarrow R_A = 28.43 \text{ KN}$$

B.m @ C:

$$m_c = R_A \times 150 = 4.264 \times 10^3 \text{ N-mm}$$

B.m @ D:

$$m_D = R_B \times 200 = 6.3 \times 10^3 \text{ N-mm}$$

$$\frac{M}{I} = \frac{\sigma_b}{y} \Rightarrow M = \sigma_b \times Z$$

$$Z = M/\sigma_b = \frac{6300\ \cancel{N}\text{-mm}}{100\ \cancel{N}/\text{mm}^2}$$

$$Z = 63 \quad \text{mm}^3$$

$$Z = I/y = \frac{\frac{\pi}{64} d^4}{d/2} = \pi/32\, d^3$$

$$\frac{\pi}{32} d^3 = 63$$

$$\boxed{d = 86.3\,\text{mm}}$$

Example 4.19 *A crane hook having an approximate trapezoidal cross-section is shown in Fig. 4.66. It is made of plain carbon steel 45C8 (S_{yt} = 380 N/mm²) and the factor of safety is 3.5. Determine the load carrying capacity of the hook.*

Ans:

$S_{yt} = 380\ N/mm^2$

fos = 3.5

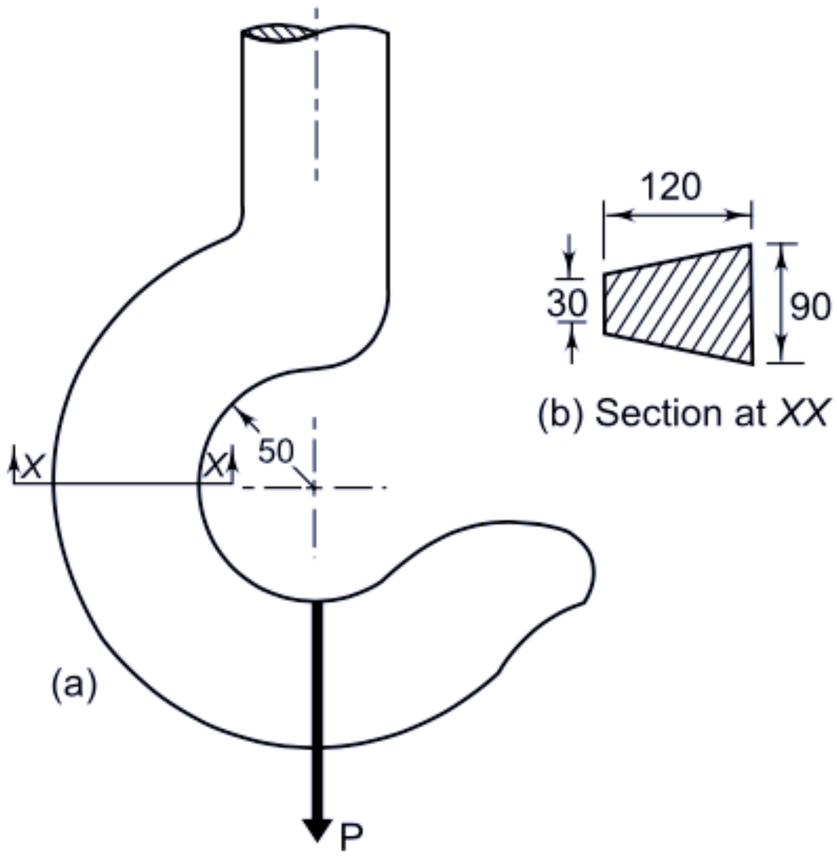
$b_i = 90\,mm$

$h = 120\,mm$

$b_o = 30\,mm$

$R_i = 50\,mm$

$R_o = 50+120 = 170\,mm$



(i) Max permissible tensile Stress :

$$\sigma_{max} = \frac{S_{yt}}{Fos} = \frac{380}{3.5} = 108.57\ N/mm^2$$

$$R_N = \frac{\left(\frac{b_i+b_o}{2}\right)h}{\left(\frac{b_iR_o-b_oR_i}{h}\right)\log_e\left[\frac{R_o}{R_i}\right]-(b_i-b_o)}$$

$$= \frac{\left(\frac{90+30}{2}\right)120}{\left(\frac{90\times170-30\times50}{120}\right)\log_e\left[\frac{170}{50}\right]-(90-30)} = 89.181\ mm$$

$$R = R_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)}$$

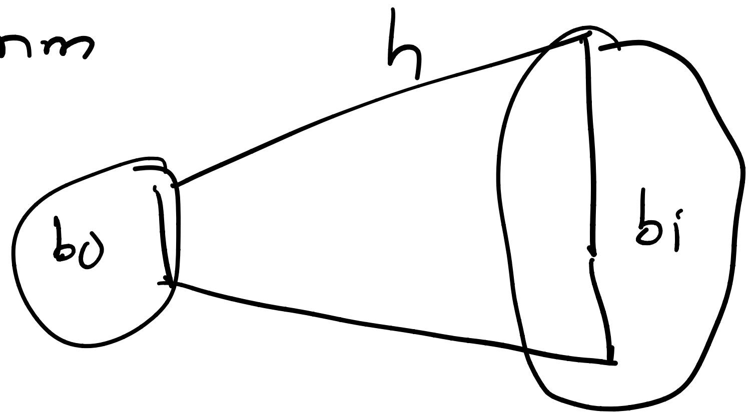
$$= 50 + \frac{120(90 + 2 \times 30)}{3(90 + 30)} = 100 mm$$

$$e = R - R_N = 100 - 89.181 = 10.818 mm$$

$$h_i = R_N - R_i = 89.181 - 50 = 39.181 mm$$

$$A = \frac{1}{2}\left[h[b_i + b_o]\right] =$$

$$= \frac{1}{2}\left[120 \times 120\right] = 7200 mm^2$$



$$M_b = P \times R = P \times 100 = 100P$$

Bending stress on the inner fibre

$$\sigma_i = \frac{M_b h_i}{A e R_i} = \frac{(100P)(39.181)}{(7200)(10.818)(50)}$$

$$\sigma_i = \left(\frac{7.2435}{7200}\right) P \quad N/mm^2$$

direct tensile stress

$$\sigma_t = \frac{P}{A} = \frac{P}{7200} \quad N/mm^2$$

$$\boxed{P = 94.627 KN}$$

$$\sigma_i + \sigma_t \leq \sigma_{max} \qquad \sigma_i + \sigma_t = \sigma_{max}$$

$$\frac{7.2435P + P}{7200} = 108.57$$

$$\frac{6400 \times 10^6}{w^2} \cancel{+1} - \frac{160,000}{w} = \cancel{17}\ \frac{800,000}{w}$$

$$\frac{6400 \times 10^6}{w^{\cancel{2}}} = \frac{800,000 + 160,000}{\cancel{w}}$$

$$w = \frac{6400 \times 10^2}{96}$$

$$\boxed{w = 6666.66\ N}$$

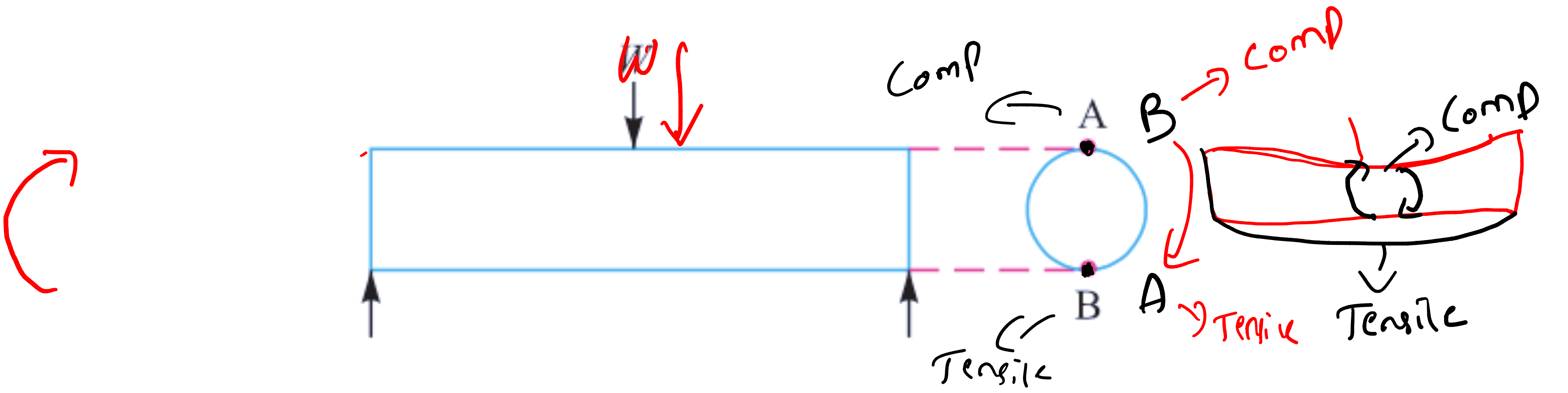
Unit -2

Design for Variable load

Completely Reversed or Cyclic Stresses

80% of failures of mechanical components are due to 'fatigue failure' resulting from fluctuating stresses

Consider a rotating beam of circular cross-section and carrying a load *W*

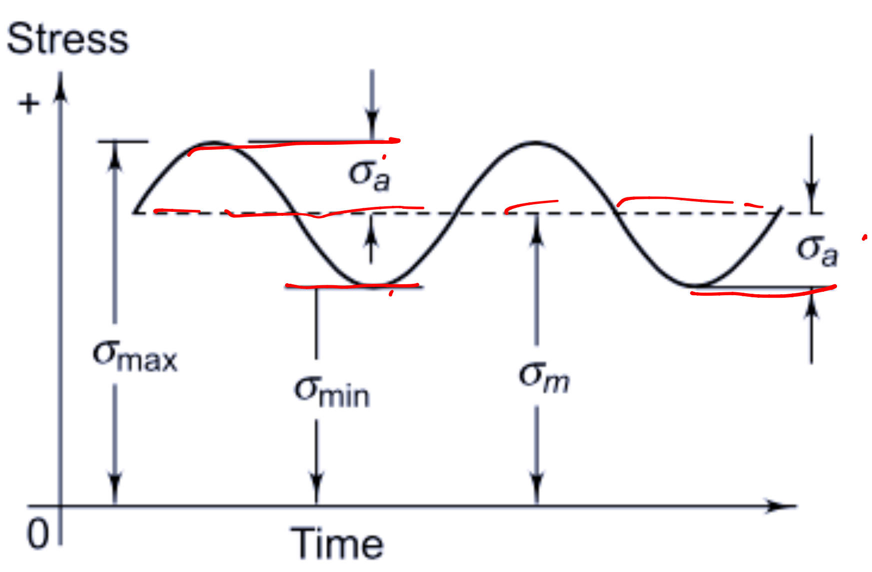


Upper fibres of the beam (*i.e.* at point *A*) are under compressive stress and the lower fibres (*i.e.* at point *B*) are under tensile stress

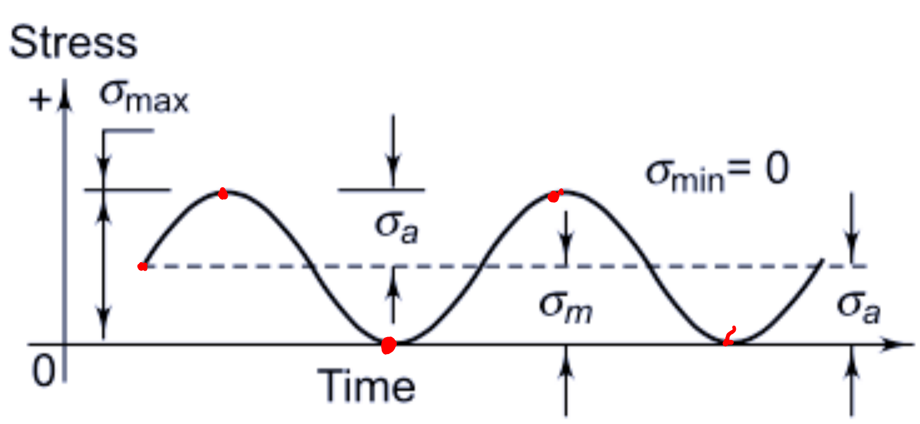
After half a revolution, points get reversed and now B will be under compressive stress and A is under Tensile stress.
The speed of variation depends on the speed of the beam.

The stresses which vary from one value of compressive to the same value of tensile or *vice versa*, are known as ***completely reversed*** or ***cyclic stresses.***

1. The stresses which vary from a minimum value to a maximum value of the same nature, (i.e. tensile or compressive) are called **fluctuating stresses**.
2. The stresses which vary from zero to a certain maximum value are called **repeated stresses.**
3. The stresses which vary from a minimum value to a maximum value of the opposite nature (i.e. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called **alternating stresses.**



(a) Fluctuating stresses

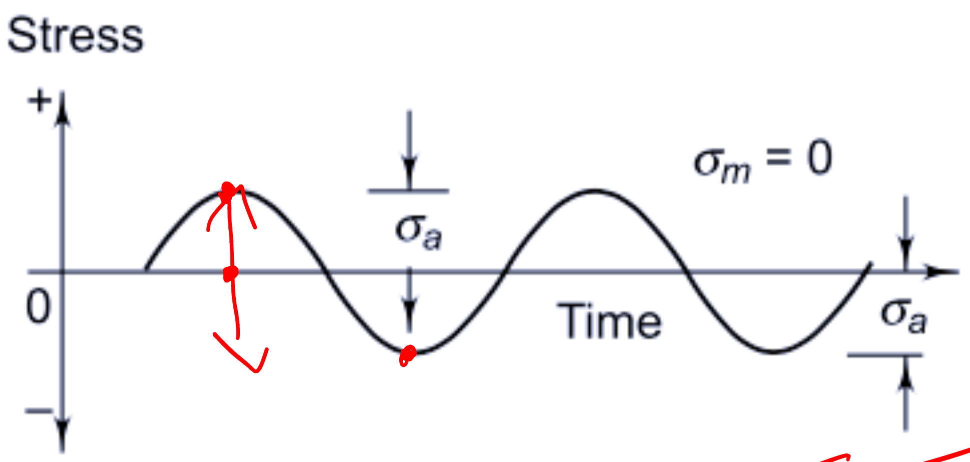


(b) Repeated stresses

$\sigma_{min} = 0$

$\sigma_m = \frac{1}{2}\sigma_{max}$ & $\sigma_a = \frac{1}{2}\sigma_{max}$

$\sigma_m = \sigma_a$



(c) Reversed stresses

$\sigma_{max} = -\sigma_{min}$

$\sigma_m = \frac{1}{2}(\sigma_{max} - \sigma_{max})$

$= 0$

$\sigma_m = 0$

Fatigue or Endurance Limit

It has been observed that materials fail under fluctuating stresses at a stress magnitude which is lower than the ultimate tensile strength of the material.

Sometimes, the magnitude is even lower than the yield strength.

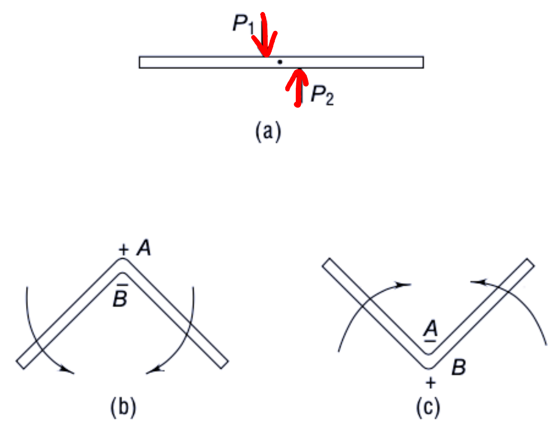
Further, the magnitude of the stress causing fatigue failure decreases as the number of stress cycles increase.

This phenomenon of decreased resistance of the materials to fluctuating stresses is the main characteristic of fatigue failure

Fatigue failure is defined as time delayed fracture under cyclic loading

The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size

The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

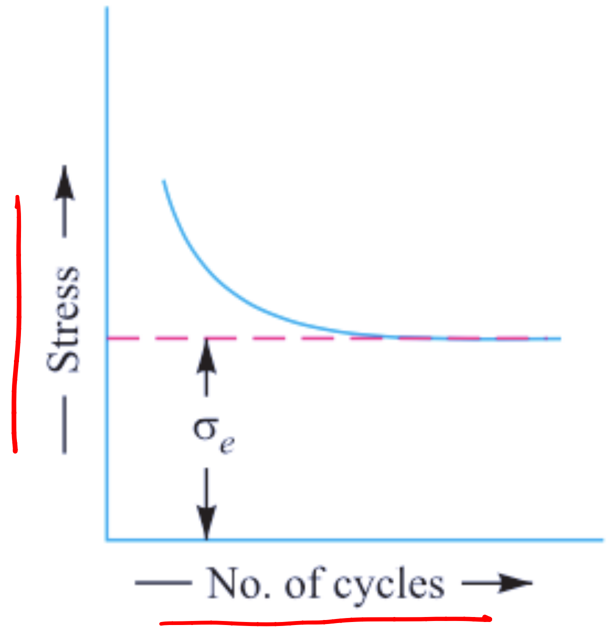


A machine part is being turned on a Lathe.

As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive.

The specimen is subjected to a completely reversed stress cycle.

A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve



(c) Endurance or fatigue limit.

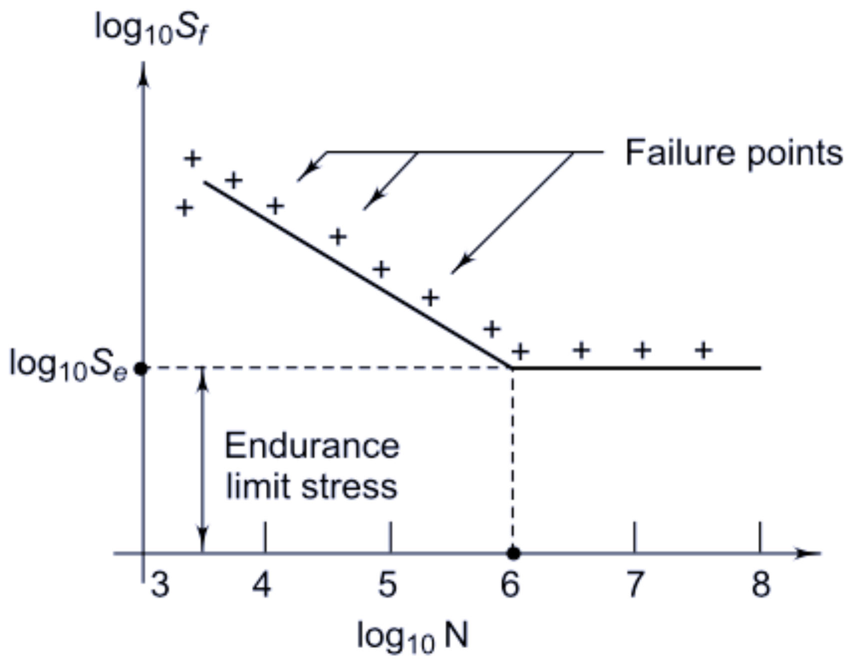


Fig. 5.20 *S-N Curve for Steels*

If the stress is kept below a certain value as shown by dotted line, the material will not fail whatever may be the number of cycles.

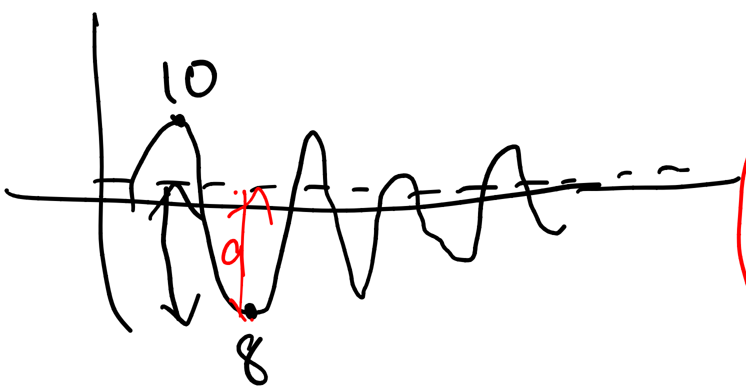
This stress, as represented by dotted line, is known as **endurance** or **fatigue limit**

The fatigue or endurance limit of a material is defined as the maximum amplitude of completely reversed stress that the standard specimen can sustain for an unlimited number of cycles without fatigue failure.

We have seen the fluctuation of stress from a min to max which is of same magnitude but opposite in nature.
In actual practice, many machine members undergo different range of stress than the completely reversed stress.

But each type of stress has a max and min value,

$\sigma_m \to$ mean stress



$$\sigma_m = \frac{1}{2}(\sigma_{max.} + \sigma_{min})$$

$$\sigma_a = \frac{1}{2}(\sigma_{max.} - \sigma_{min.})$$

amplitude

It can be observed that repeated stress and reversed stress are special cases of fluctuating stress

Effect of Loading on Endurance Limit—Load Factor

- The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load.
- There are many machine members which are subjected to loads other than reversed bending loads.
- Thus the endurance limit will also be different for different types of loading

K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

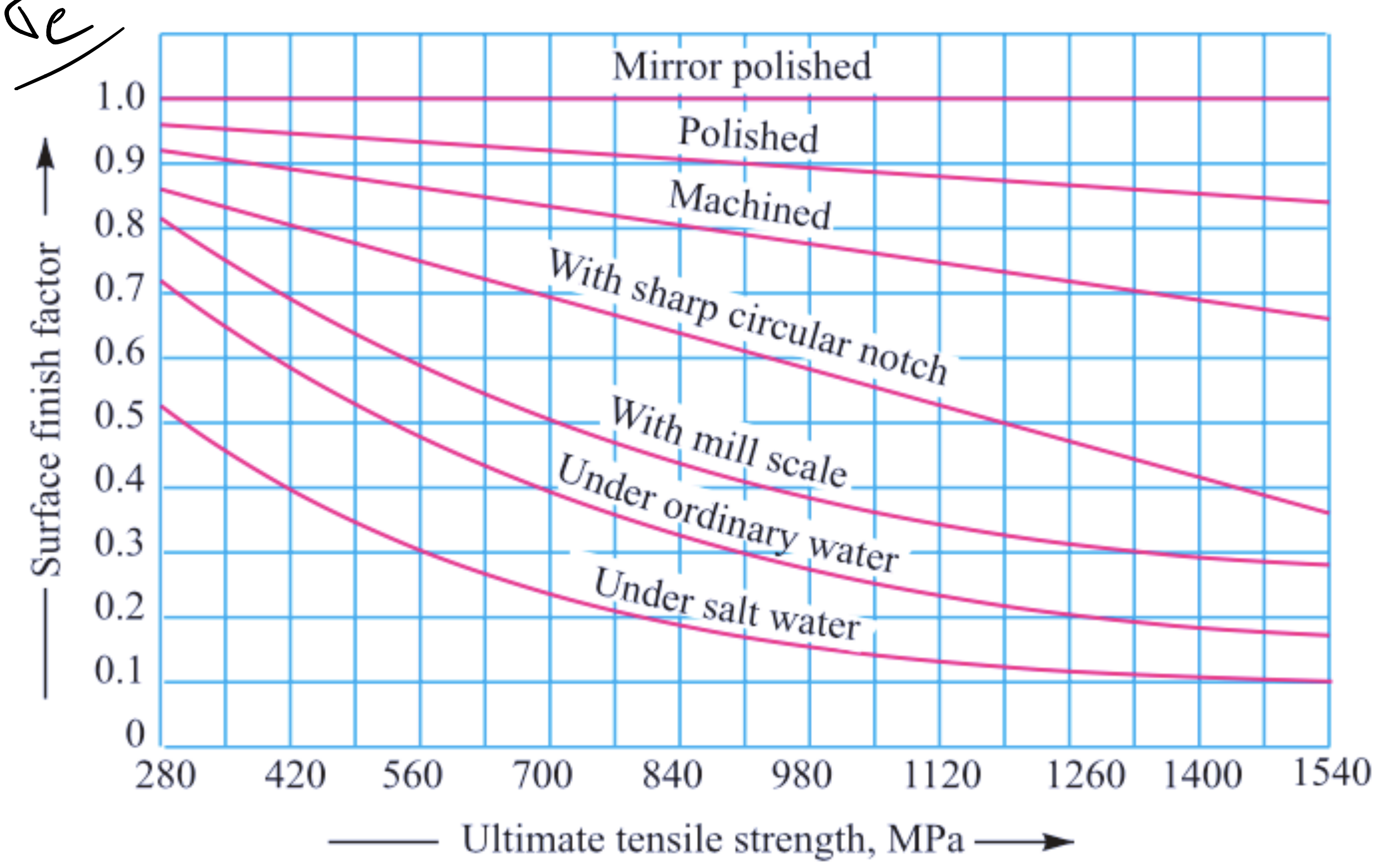
∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e . K_b = \sigma_e$ ...(∵ $K_b = 1$)

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e . K_a$ $K_a \cdot \sigma_e$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e . K_s$ → $K_s \cdot \sigma_e$

Effect of Surface Finish on Endurance Limit—Surface Finish Factor

$\sigma_{ef} = K_{sur} \times \sigma_e$



0.9×350
0.9×350
$K_{sur} = 0.9$
for 350 MPa & machined

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur} \quad \text{...(∵ } K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad \text{...(For reversed torsional or shear load)}$$

300 mpa

$K_{sur} = 0.95$

$K_{sz} = 0.85$

Effect of Size on Endurance Limit—Size Factor

⇒ If the size of the standard specimen is increased, then the endurance limit of the material will decrease.

$K_a = 0.75$

⇒ This is due to the fact that a longer specimen will have more defects than a smaller one.

$\sigma_{e_2} = 0.75 \times 0.85 \times 0.95 \times 300$

$\sigma_{e_2} = 230$

Let K_{sz} = Size factor.

∴ Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$

$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$

$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur.}K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

$(\sigma_{e_2})_{final} = \frac{230}{2} = 115$

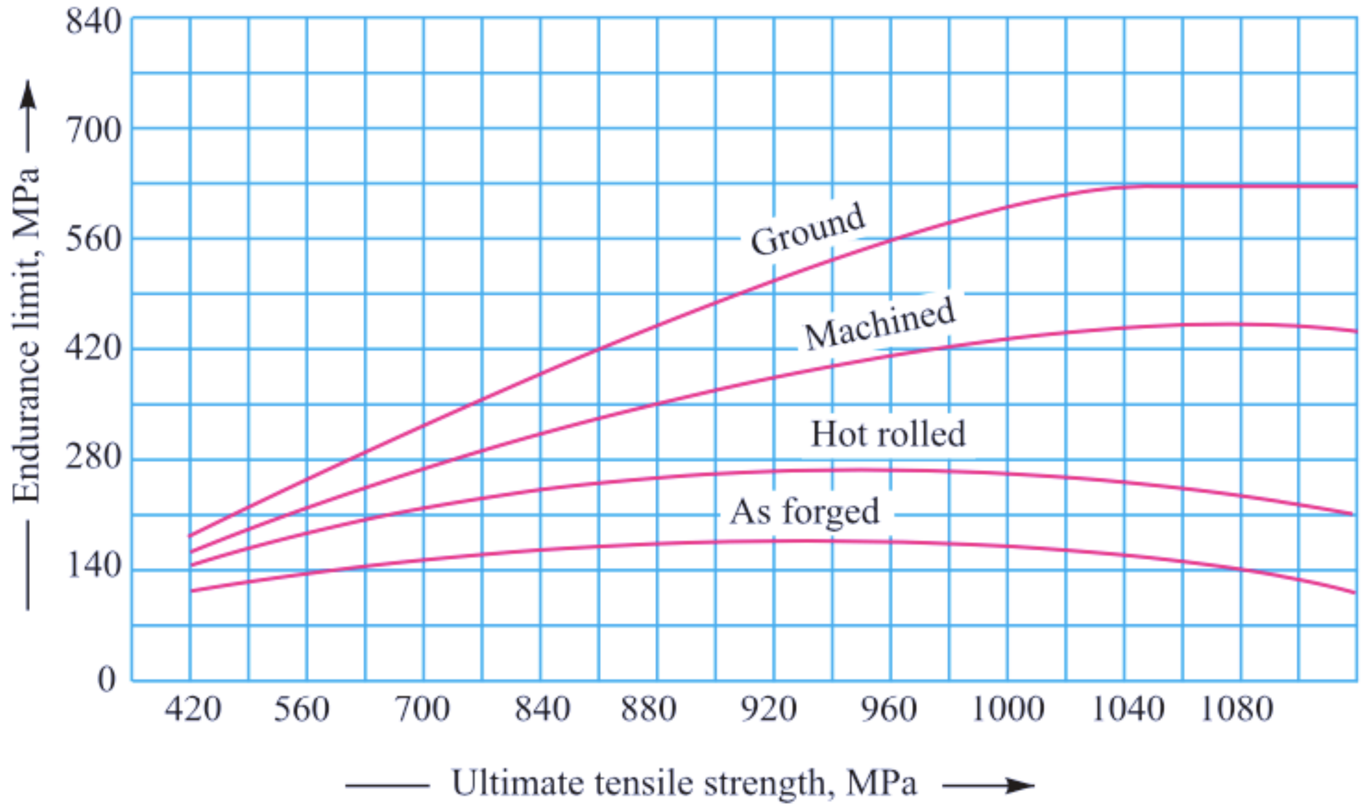
2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

There are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc.

Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u)



For steel, $\sigma_e = 0.5\ \sigma_u$;

For cast steel, $\sigma_e = 0.4\ \sigma_u$;

For cast iron, $\sigma_e = 0.35\ \sigma_u$;

For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure.

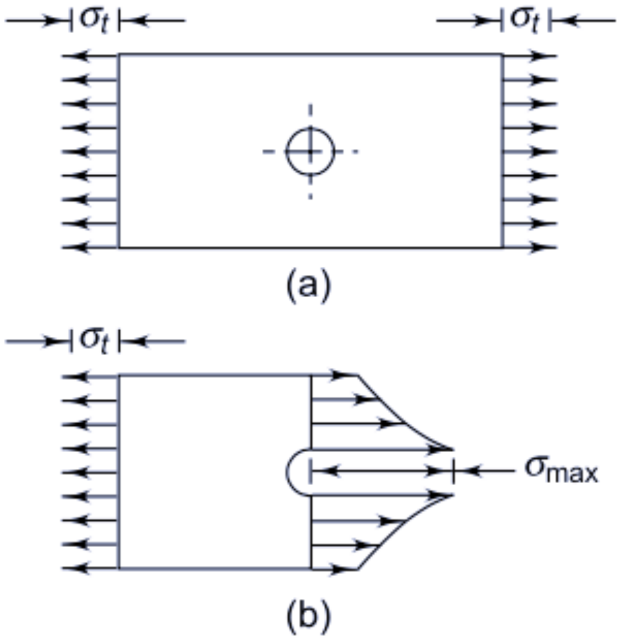
$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

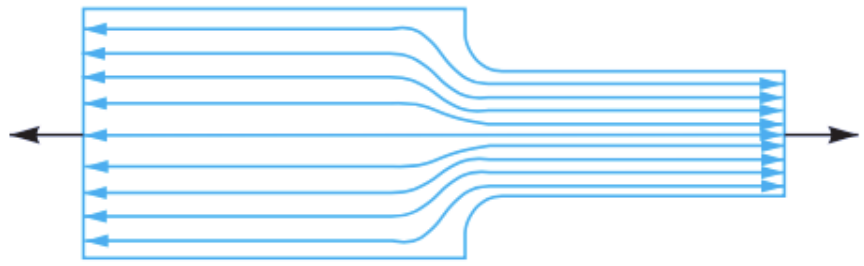
σ_y - yield strength

For steel, $\sigma_e = 0.8$ to $0.9\ \sigma_y$

Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.***





$$K_t = \frac{\text{Highest value of actual stress near discontinuity}}{\text{Nominal stress obtained by elementary equations for minimum croos-section}}$$

$$K_t = \frac{\sigma_{max.}}{\sigma_0} = \frac{\tau_{max.}}{\tau_0}$$

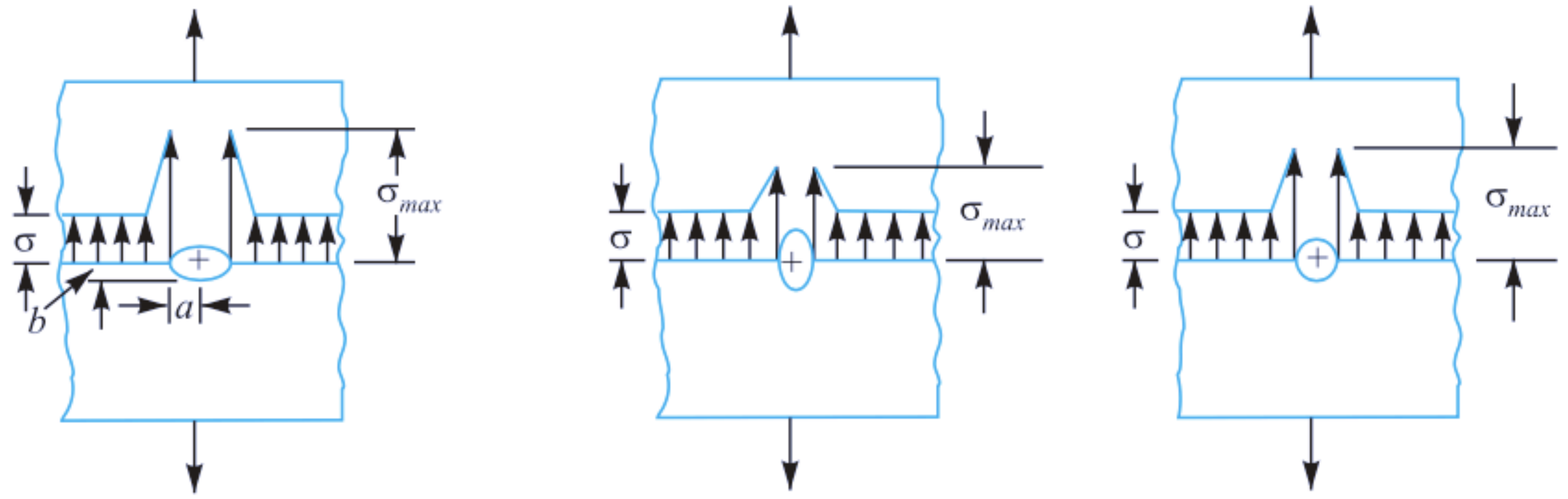
The subscript t denotes the '*theoretical*' stress concentration factor. The magnitude of stress concentration factor depends upon the geometry of the component.

- ***Variation in Properties of Materials***
- ***Load Application***
- ***Abrupt Changes in Section***
- ***Discontinuities in the Component***
- ***Machining Scratches***

In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration

In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

Stress Concentration due to Holes and Notches



$a/b = 2$, $\sigma_{max} = 5\sigma$ $\quad$ $a/b = 1/2$, $\sigma_{max} = 2\sigma$ $\quad$ $a/b = 1$, $\sigma_{max} = 3\sigma$

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

2a → hori. axis

2b → vert axis

theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{b}\right)$$

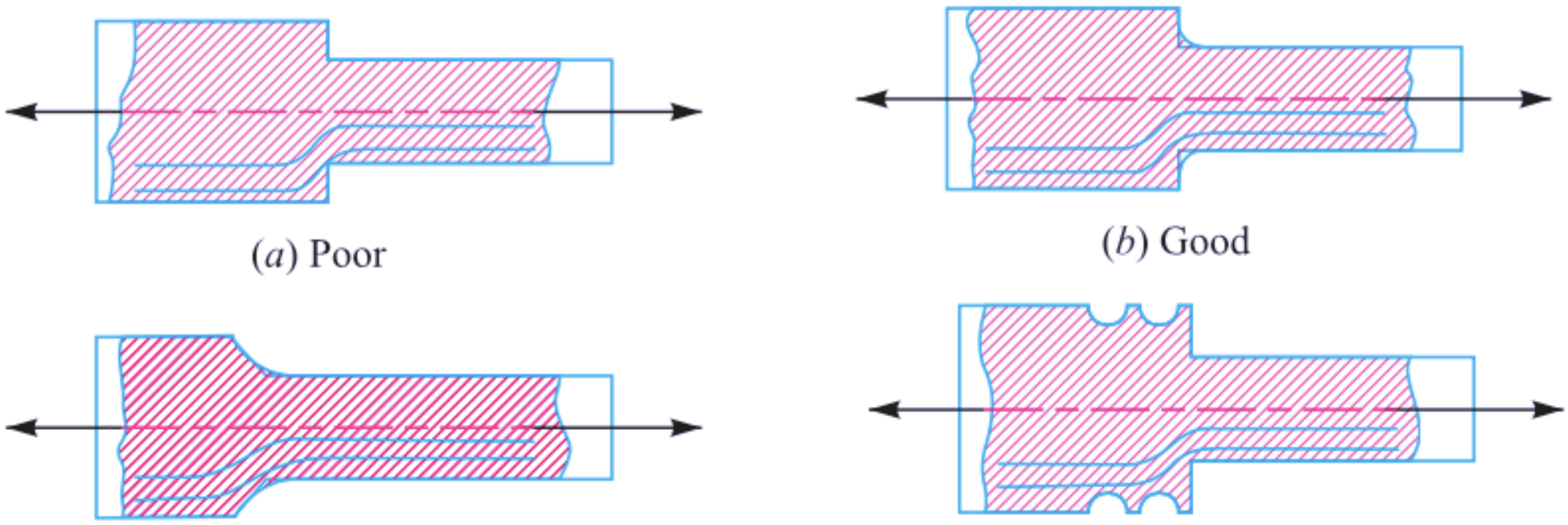
When *a*/*b* is large, the ellipse approaches a crack transverse to the load and the value of K_t becomes very large.

When *a*/*b* is small, the ellipse approaches a longitudinal slit and the increase in stress is small

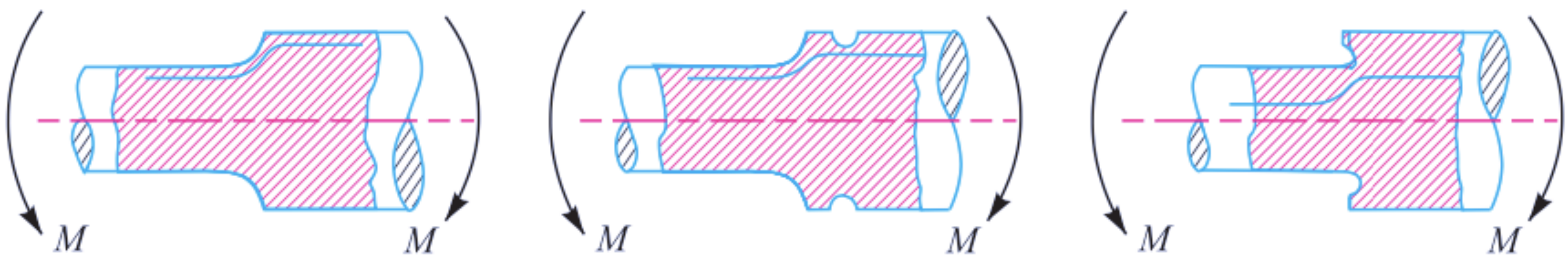
Methods of Reducing Stress Concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results

Stress concentration cannot be totally eliminated but it may be reduced to some extent



(*a*) Poor (*b*) Good (*c*) Preferred (*d*) Preferred



(*a*) Poor (*b*) Good (*c*) Preferred

Factors to be Considered while Designing Machine Parts to Avoid Fatigue Failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. **The variation in the size of the component should be as gradual as possible.**
2. **The holes, notches and other stress raisers should be avoided.**
3. **The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.**
4. **The parts should be protected from corrosive atmosphere.**
5. **A smooth finish of outer surface of the component increases the fatigue life.**
6. **The material with high fatigue strength should be selected.**
7. **The residual compressive stresses over the parts surface increases its fatigue strength.**

Table 6.1. Theoretical stress concentration factor (K_t) for a plate with hole (of diameter d) in tension.

$\frac{d}{b}$	0.05	0.1	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
K_t	2.83	2.69	2.59	2.50	2.43	2.37	2.32	2.26	2.22	2.17	2.13

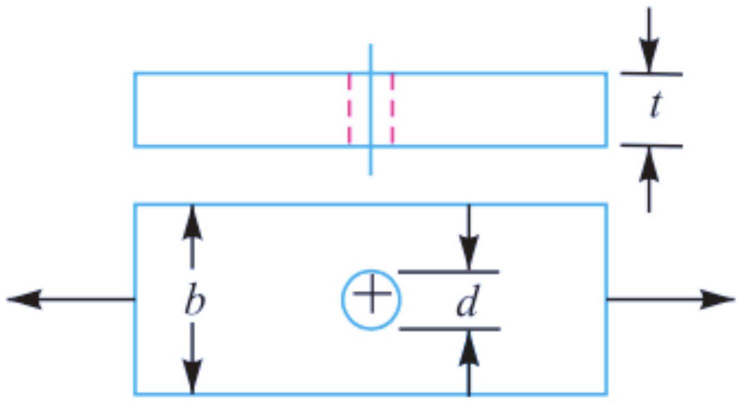


Fig. for Table 6.1

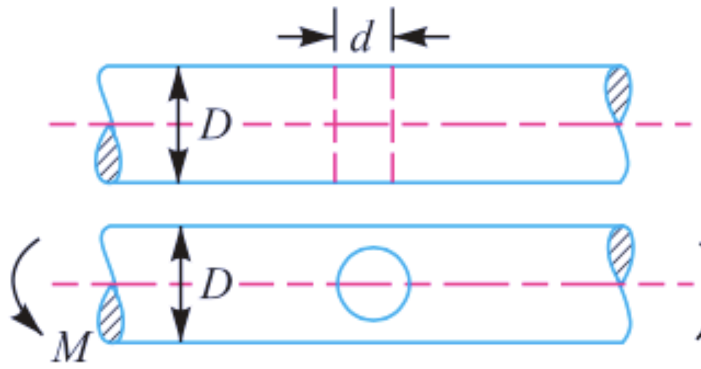


Fig. for Table 6.2

Table 6.2. Theoretical stress concentration factor (K_t) for a shaft with transverse hole (of diameter d) in bending.

$\frac{d}{D}$	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
K_t	2.70	2.52	2.33	2.26	2.20	2.11	2.03	1.96	1.92	1.90

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor.

The actual reduction in the endurance limit of a material due to stress concentration is less than the amount indicated by the theoretical stress concentration factor

Determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$\boxed{K_f = \frac{\text{Endurance Limit without stress conc}}{\text{Endurance limit with Stress Conc}}} \rightarrow \text{General}$$

Notch Sensitivity

σ_{e_1} σ_e

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed

$\sigma_{e_2} > \sigma_{e_1}$

The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material

$\sigma_e = 300$
$\sigma_{e_1} = 225$
$(\sigma_{e_2})_B = 245$

$$K_f = \frac{Endurance\ Limit\ of\ notch\ free\ specimen}{Endurance\ limit\ of\ notched\ specimen}$$

This factor Kf is applicable to actual materials and depends upon the grain size of the material. It is observed that there is a greater reduction in the endurance limit of fine-grained materials as compared to coarse-grained materials, due to stress concentration.

Notch sensitivity is defined as the susceptibility of a material to succumb to the damaging effects of stress raising notches in fatigue loading.

Defined as the degree to which the theoretical effect of stress concentration is actually reached

$$\text{Notch Sensitivity } q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

σ_o = nominal stress as obtained by elementary equations

actual stress = $Kf\,\sigma_o$

theoretical stress = $Kt\,\sigma_o$

increase of actual stress over nominal stress = $(Kf\,\sigma_o - \sigma_o)$

increase of theoretical stress over nominal stress = $(Kt\,\sigma_o - \sigma_o)$

Therefore,

$$q = \frac{(K_f\sigma_o - \sigma_o)}{(K_t\sigma_o - \sigma_o)} = \frac{\sigma_o(K_f - 1)}{\sigma_o(K_t - 1)}$$

$$\boxed{q = \frac{(K_f - 1)}{(K_t - 1)}}$$

$q(K_t - 1) = K_f - 1$

$K_f = 1 + q(K_t - 1)$

$$\boxed{K_f = q(K_t - 1) + 1}$$

fatigue stress conc factor

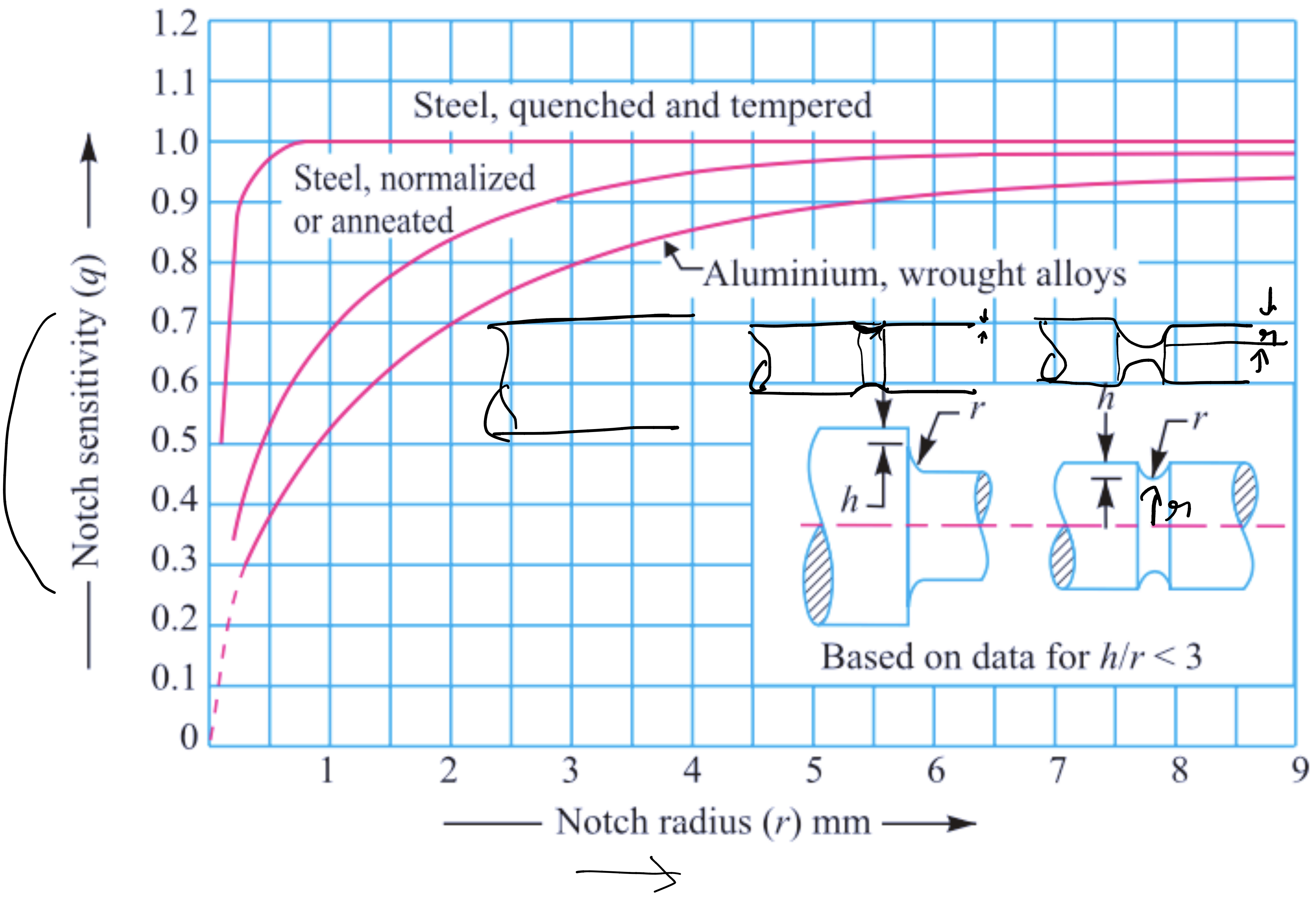
The following conclusions are drawn with the help of equation

1. When the material has no sensitivity to notches,

$$q = 0 \text{ and } K_f = 1$$

2. When the material is fully sensitive to notches,

$$q = 1 \text{ and } K_f = K_t$$



Combined Steady and Variable Stress

We have two stress components

1. Static
2. Variable

When a component is subjected to fluctuating stresses as shown in Fig. 5.15 (a), there is mean stress (σ_m) as well as stress amplitude (σ_a).

When mean stress is compressive (negative) the failure by fatigue is rare.

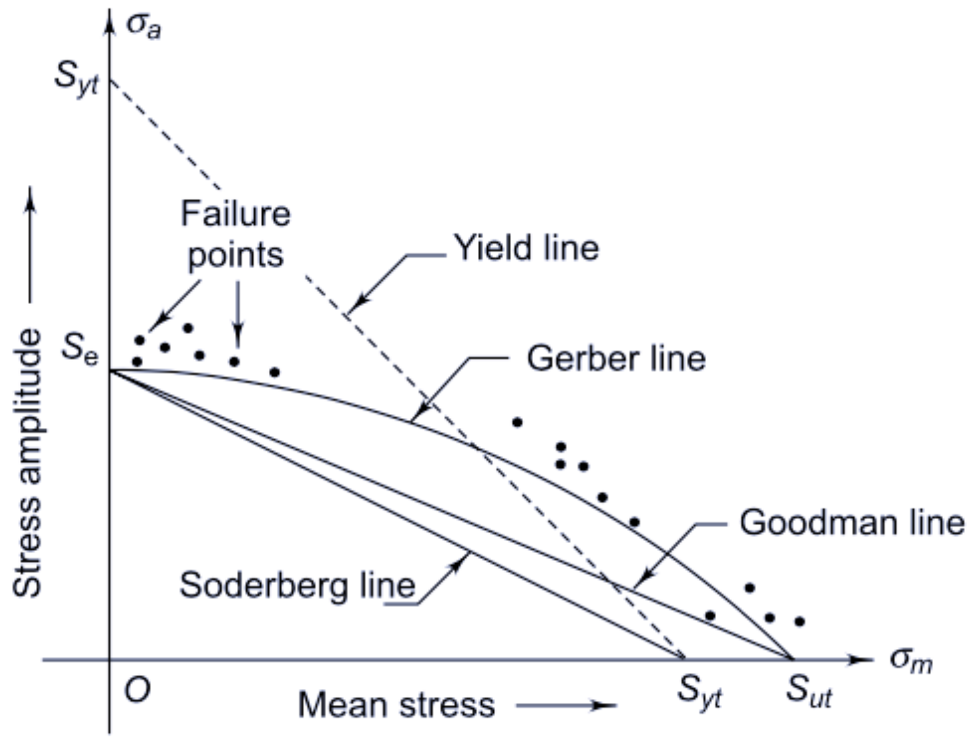
$\sigma_m = 0$

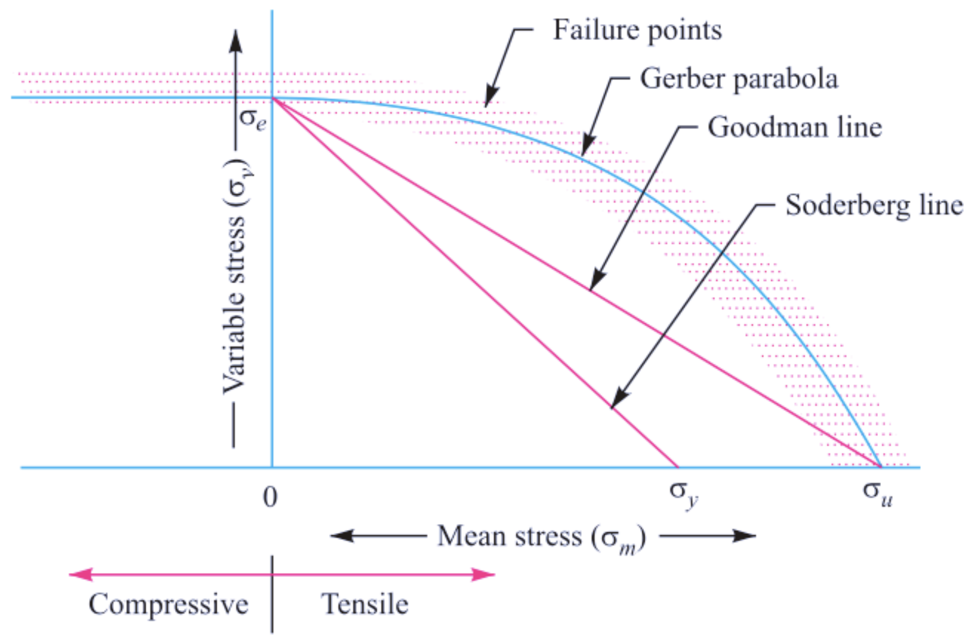
Completely reversed

The point σ_e represents the fatigue strength corresponding to the case of complete reversal (σ_m = 0) and the point σ_u represents the static ultimate strength corresponding to σ_a= 0.

$\sigma_a = 0 \rightarrow$ Completely Static loading

$\sigma_m = 0 \rightarrow$,, reversed loading $\rightarrow$ Cyclic loading





- Gerber method,
- Goodman method, and
- Soderberg method.

Gerber Method for Combination of Stresses

A parabolic curve drawn between the endurance limit (σ_e) and ultimate tensile strength (σ_u) was proposed by Gerber

Mainly used for Ductile materials

$$\sigma_a \text{ (or) } \sigma_v = \sigma_e \left[\frac{1}{Fos} - \left(\frac{\sigma_m}{\sigma_u}\right)^2 Fos \right]$$

$$\boxed{\left[\left(\frac{\sigma_m}{\sigma_u}\right)^2 \times Fos\right] + \frac{\sigma_a}{\sigma_e} = \frac{1}{Fos}}$$

$\sigma_m \rightarrow$ mean stress (Tensile (or) Comp)

$\sigma_a \rightarrow$ stress amp

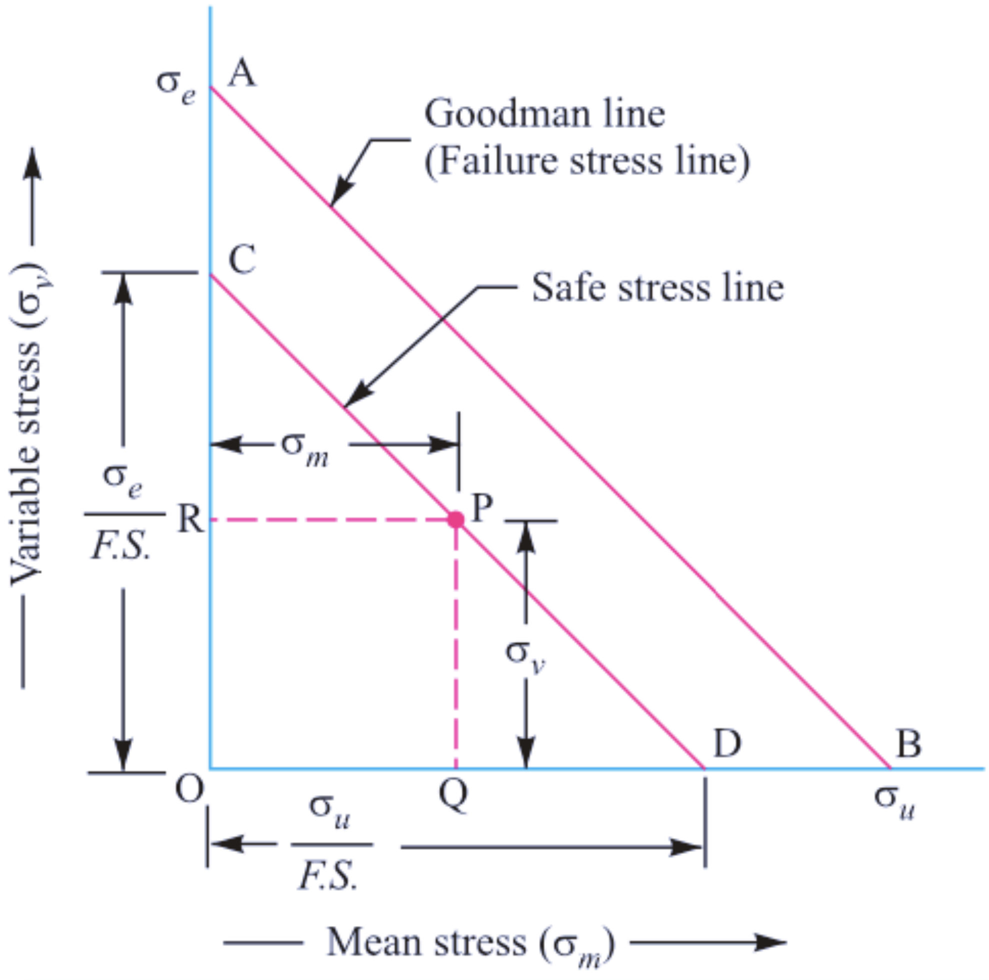
$\sigma_u \rightarrow$ ultimate Strength

$\sigma_e \rightarrow$ Endurance limit

$$\boxed{\frac{1}{Fos} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 Fos + \frac{k_f \sigma_a}{\sigma_e}}$$

Goodman Method for Combination of Stresses

More Safe than Gerber method



$$\frac{PQ}{OC} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$\frac{PQ}{OC} = 1 - \frac{OQ}{OD}$$

$$\frac{\sigma_v}{\left(\frac{\sigma_e}{FoS}\right)} = 1 - \frac{\sigma_m}{\left(\frac{\sigma_u}{FoS}\right)}$$

$$\frac{\sigma_v \times (FoS)}{\sigma_e} + \frac{\sigma_m \times (FoS)}{\sigma_u} = 1$$

$$\Rightarrow \frac{\sigma_m}{\sigma_u} + \frac{\sigma_a \,(\text{or})\, \sigma_v}{\sigma_e} = \frac{1}{FoS}$$

$$\Rightarrow \boxed{\frac{\sigma_m}{\sigma_u} + \frac{K_f\,\sigma_a}{\sigma_e} = \frac{1}{FoS}}$$

A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

$$\frac{1}{Fos} = \frac{K_t \times \sigma_m}{\sigma_u} + \frac{K_f \times \sigma_a}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

K_t

Similarly, for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For ductile materials)}$$

and

$$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For brittle materials)}$$

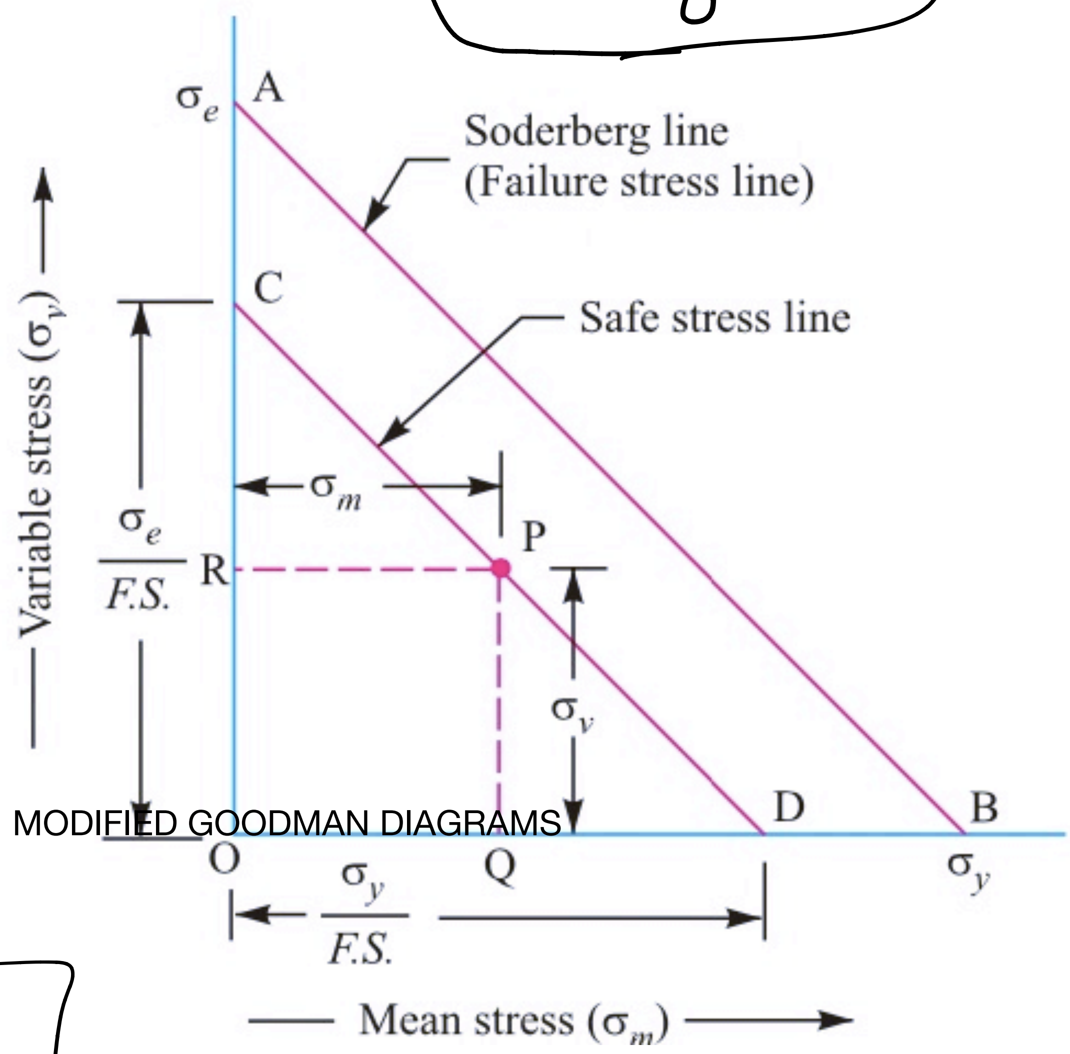
Soderberg Method for combination of stresses

Considering yielding

For most $\sigma_y < \sigma_u$

$$\frac{PQ}{CO} = \frac{QD}{OD} = 1 - \frac{OQ}{OD}$$

$$\frac{\sigma_v}{\left(\frac{\sigma_e}{Fos}\right)} = 1 - \frac{\sigma_m}{\left(\frac{\sigma_y}{Fos}\right)}$$



$$\boxed{\frac{1}{Fos} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_a \text{ (or) } \sigma_v}{\sigma_e}}$$

$$\boxed{\frac{1}{Fos} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_a \times K_f}{\sigma_e}}$$

Ductile materials

$$\boxed{\tau_y = 0.55\,\sigma_y}$$

$$\boxed{\frac{1}{Fos} = \frac{\tau_m}{\tau_y} + \frac{K_{fs} \times \tau_a}{\tau_e}}$$

MODIFIED GOODMAN DIAGRAMS

For the purpose of design, the problems are classified into two groups:

(i) components subjected to fluctuating axial or bending stresses; and
(ii) components subjected to fluctuating torsional shear stresses.

The region OABC is called modified Goodman diagram

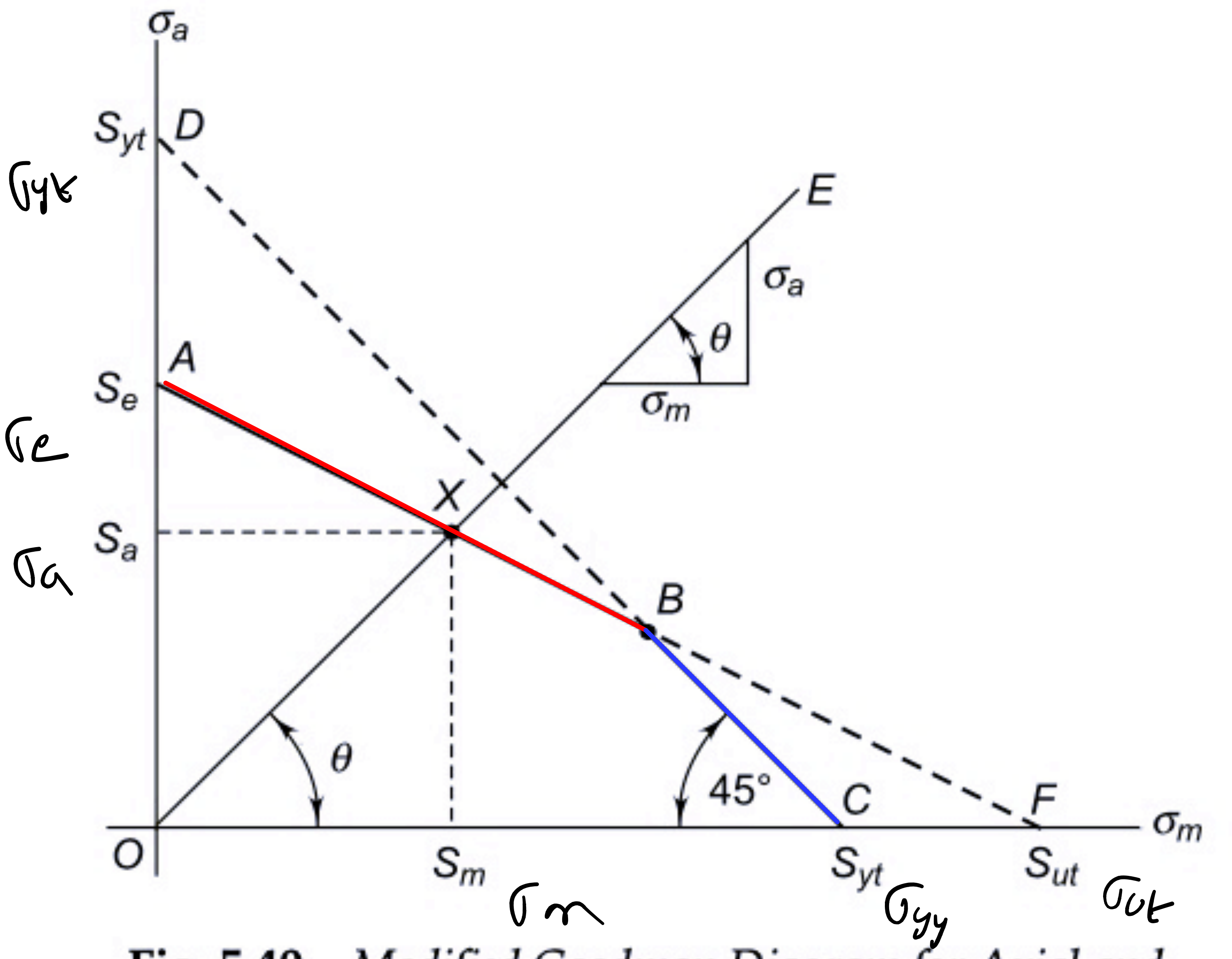


Fig. 5.40 *Modified Goodman Diagram for Axial and Bending Stresses*

If the mean component of stress (σ_m) is very large and the alternating component (σ_a) very small, their combination will define a point in the region BCF that would be safely within the Goodman line but would yield on the first cycle.

This will result in failure, irrespective of safety in fatigue failure.

The point of intersection of lines AB and OE is X.

The point X indicates the dividing line between the safe region and the region of failure.

$$\tan\theta = \sigma_a/\sigma_m = \frac{P_A/A}{P_m/A} = P_A/P_m = \frac{(M_b)_a}{(M_b)_m}$$

$$\sigma_a = \frac{S_a}{(fs)} \quad \text{and} \quad \sigma_m = \frac{S_m}{(fs)}$$

$S_{se} \rightarrow$ Endurance limit in shear

Till this point there is no effect of σ_m on endurance limit (σ_e)

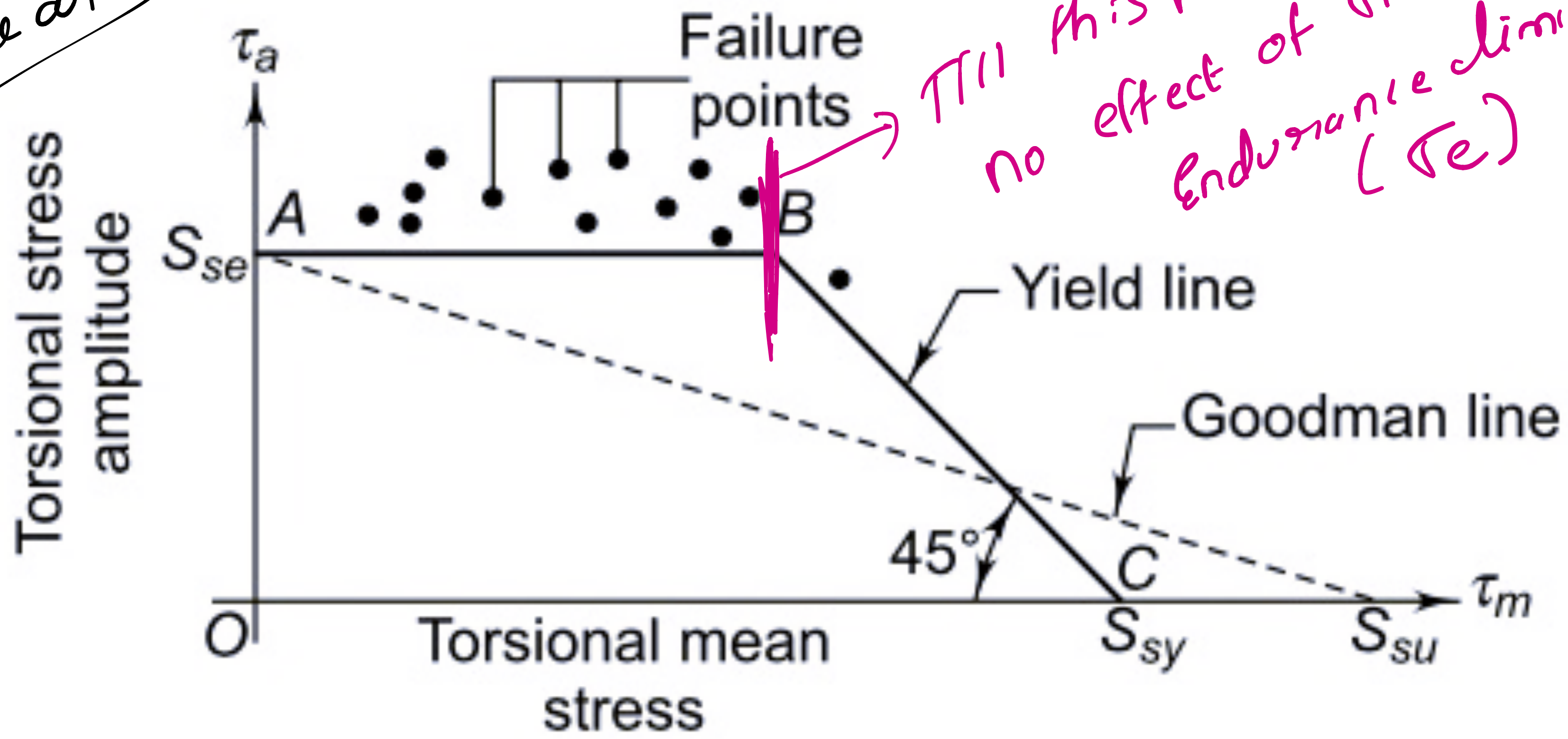


Fig. 5.41 *Modified Goodman Diagram for Torsional Shear Stresses*

Up to a certain point, the torsional mean stress has no effect on the torsional endurance limit.

$$\tau_a = S_{se}$$

and a static failure is indicated if,

$$\tau_{max} = \tau_a + \tau_m = S_{sy}$$

The permissible stresses are as follows:

$$\tau_a = \frac{S_{se}}{(fs)}$$

and

$$\tau_{max} = \frac{S_{sy}}{(fs)}$$

Example *A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m² and – 150 MN/m². Determine the value of minimum ultimate strength according to 1. Gerber relation; 2. Modified Goodman relation; and 3. Soderberg relation.*

Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2.

$\sigma_{max} = 300\ MN/m^2$

$\sigma_{min} = -150\ MN/m^2$

$(\sigma_u)_{min} = ?$

$\sigma_y = 0.55\,\sigma_u$

$\sigma_e = 0.5\,\sigma_u$

$FOS = 2.$

$$\sigma_m = \frac{1}{2}\left[\sigma_{max} + \sigma_{min}\right] = \frac{1}{2}\left[300 + (-150)\right] = 75\ MN/m^2$$

$$\sigma_a = \frac{1}{2}\left[\sigma_{max} - \sigma_{min}\right] = 225\ MN/m^2$$

1. Gerber Relation:

$$\sigma_u^2 - 900\sigma_u - 22500 = 0$$

$$\left[\left(\frac{\sigma_m}{\sigma_u}\right)^2 FOS\right] + \frac{\sigma_a}{\sigma_e} = \frac{1}{FOS}$$

924, −24

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u}$$

$$\Rightarrow \frac{1}{2} = \frac{11{,}250}{\sigma_u^2} + \frac{450}{\sigma_u} \Rightarrow \frac{11{,}250 + 450\sigma_u}{\sigma_u^2} = \frac{1}{2}$$

$$\Rightarrow \sigma_u^2 - 900\sigma_u - 22{,}500 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{900 \pm \sqrt{(900)^2 + 4 \times 22{,}500}}{2}$$

$$\sigma_u = \frac{900 \pm 948.68}{2}$$

$$\boxed{\sigma_u = 924\ MN/m^2}$$

2. Modified Goodman Relation

$$\frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} = \frac{1}{Fos}$$

$$\frac{75}{\sigma_u} + \frac{225}{0.5\sigma_u} = \frac{1}{2} \Rightarrow \frac{75+450}{\sigma_u} = \frac{1}{2}$$

$$\boxed{\sigma_u = 1050\, MN/m^2}$$

3. Soderberg:

$$\frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{Fos}$$

$$\frac{75}{0.55\sigma_u} + \frac{450}{\sigma_u} = \frac{1}{2} \Rightarrow \frac{1}{\sigma_u}\left[450 + 136.36\right] = \frac{1}{2}$$

$$\sigma_u = 2\left[586.36\right]$$

$$\boxed{\sigma_u = 1172.7\, MN/m^2}$$

Example A bar of circular cross-section is subjected to alternating tensile forces varying from a minimum of 200 kN to a maximum of 500 kN. It is to be manufactured of a material with an ultimate tensile strength of 900 MPa and an endurance limit of 700 MPa. Determine the diameter of bar using safety factors of 3.5 related to ultimate tensile strength and 4 related to endurance limit and a stress concentration factor of 1.65 for fatigue load. Use Goodman straight line as basis for design.

$F_{max} = 500\,kN$; $F_{min} = 200\,kN$; $\sigma_u = 900\,mpa$; $\sigma_e = 700\,mpa$

$(Fos)_u = 3.5$; $(Fos)_e = 4$; $K_f = 1.65$; $d = ?$ "m"

$$Area = \frac{\pi}{4} d^2 = 0.7854\, d^2\ m^2$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

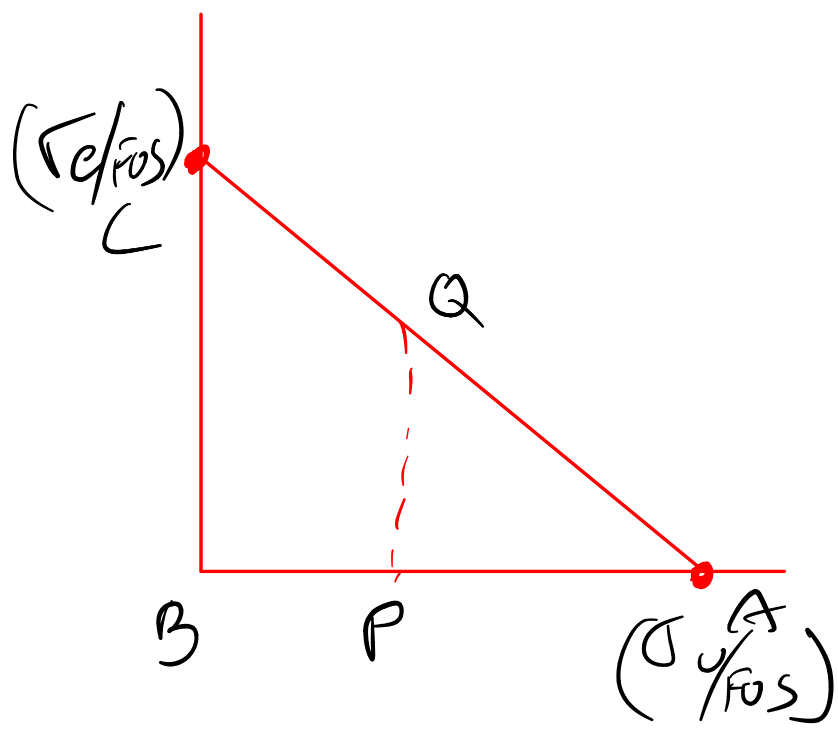
$F_m = 350\,kN$

$$\sigma_m = \frac{F_m}{A} = \frac{350 \times 10^3}{0.7854\, d^2} \Rightarrow \sigma_m = \frac{445.63}{d^2}\ kN/m^2$$

$$F_a = \frac{1}{2}\left[500 - 200\right] = 150\,kN$$

$$\sigma_a = \frac{F_a}{A} = \frac{150}{0.7854\, d^2} = \frac{191}{d^2}\ kN/m^2$$

$$\frac{PQ}{CD} = 1 - \frac{OD}{OQ}$$



$$\frac{\sigma_v}{\frac{\sigma_e}{(Fos)_e}} = 1 - \frac{\sigma_m \times K_f}{\frac{\sigma_u}{(Fos)_u}} \Rightarrow$$

$$\frac{\frac{191 \times 10^3}{d^2}}{\frac{700 \times 10^6}{4}} = 1 - \frac{\frac{446 \times 10^3}{d^2} \times 1.65}{\frac{900 \times 10^6}{3.5}}$$

$$\frac{4 \times 191 \times 10^3}{d^2 \times 700 \times 10^6} = 1 - \frac{3.5 \times 735.9 \times 10^3}{d^2 \times 900 \times 10^6}$$

$$\frac{1.091 \times 10^{-3}}{d^2} = 1 - \frac{2.859 \times 10^{-3}}{d^2}$$

$$\frac{(1.091 + 2.859) \times 10^{-3}}{d^2} = 1 \Rightarrow d^2 = 3.95 \times 10^{-3}$$

$$d = 0.0628 \text{ m}$$

$$\boxed{d = 62.8 \text{ mm}}$$

$$\boxed{d = 65 \text{ mm}}$$

Example *Find the maximum stress induced in the following cases taking stress concentration into account:*

1. A rectangular plate 60 mm × 10 mm with a hole 12 diameter as shown in Fig. 6.13 (a) and subjected to a tensile load of 12 kN.

2. A stepped shaft as shown in Fig. 6.13 (b) and carrying a tensile load of 12 kN.

Case (i)

$b = 60\,mm$

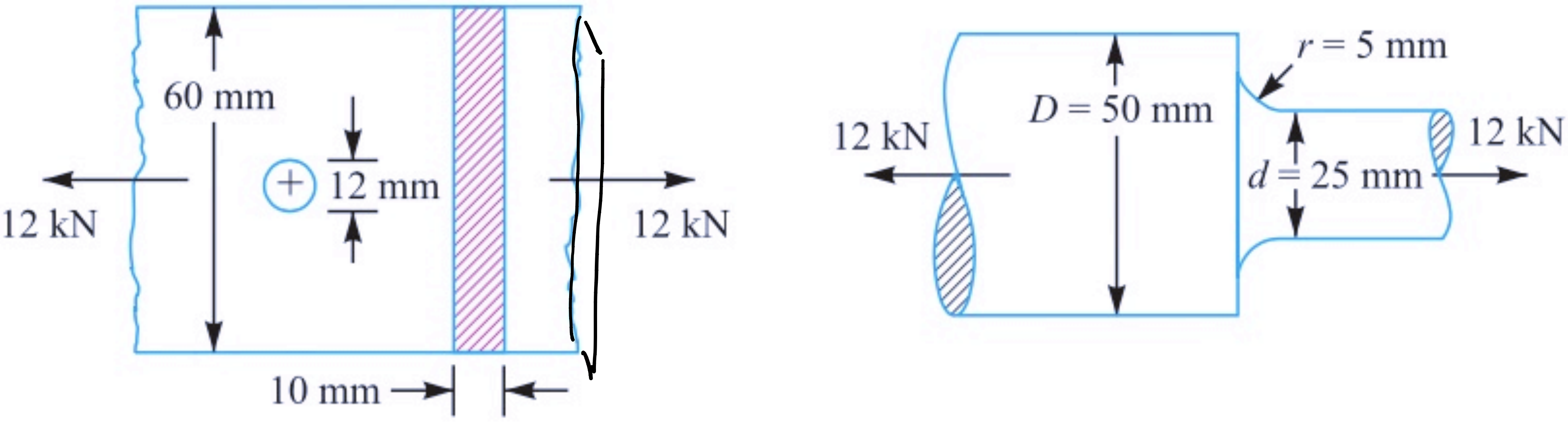
$t = 10\,mm$

$d = 12\,mm$

$P = 12\,kN = 12 \times 10^3\,N$

$A = (b - d)\,t = (60 - 12)\,10$

$A = 480\,mm^2$



(a) (b)

Table 6.3. Theoretical stress concentration factor (K_t) for stepped shaft with a shoulder fillet (of radius r) in tension.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.08	0.10	0.12	0.16	0.18	0.20	0.22	0.24	0.28	0.30
1.01	1.27	1.24	1.21	1.17	1.16	1.15	1.15	1.14	1.13	1.13
1.02	1.38	1.34	1.30	1.26	1.24	1.23	1.22	1.21	1.19	1.19
1.05	1.53	1.46	1.42	1.36	1.34	1.32	1.30	1.28	1.26	1.25
1.10	1.65	1.56	1.50	1.43	1.39	1.37	1.34	1.33	1.30	1.28
1.15	1.73	1.63	1.56	1.46	1.43	1.40	1.37	1.35	1.32	1.31
1.20	1.82	1.68	1.62	1.51	1.47	1.44	1.41	1.38	1.35	1.34
1.50	2.03	1.84	1.80	1.66	1.60	1.56	1.53	1.50	1.46	1.44
2.00	2.14	1.94	1.89	1.74	1.68	1.64	1.59	1.56	1.50	1.47

Case (i) Nominal Stress :

$$\sigma_0 = \frac{P}{A} = \frac{12 \times 10^3}{480} = 25 \ N/mm^2 = 25 \ MPa$$

$$\frac{d}{b} = \frac{12}{60} = \frac{1}{5} = 0.2 \Rightarrow K_t = 2.5 \ [\text{From the table}]$$

$$K_t = \frac{\sigma_{max}}{\sigma_0} \Rightarrow \sigma_{max} = K_t \times \sigma_0$$

$$= 2.5 \times 25$$

$$\boxed{\sigma_{max} = 62.5 \ mpa}$$

Case (ii) $D = 50mm$ $d = 25mm$ $r = 5mm$ $P = 12 \times 10^3 N$

$$\text{Area} = \frac{\pi}{4}(25)^2 = 491 \ mm^2$$

$$\sigma_0 = \frac{P}{A} = \frac{12 \times 10^3}{491} = 24.43 \ N/mm^2 = 24.43 \ mpa$$

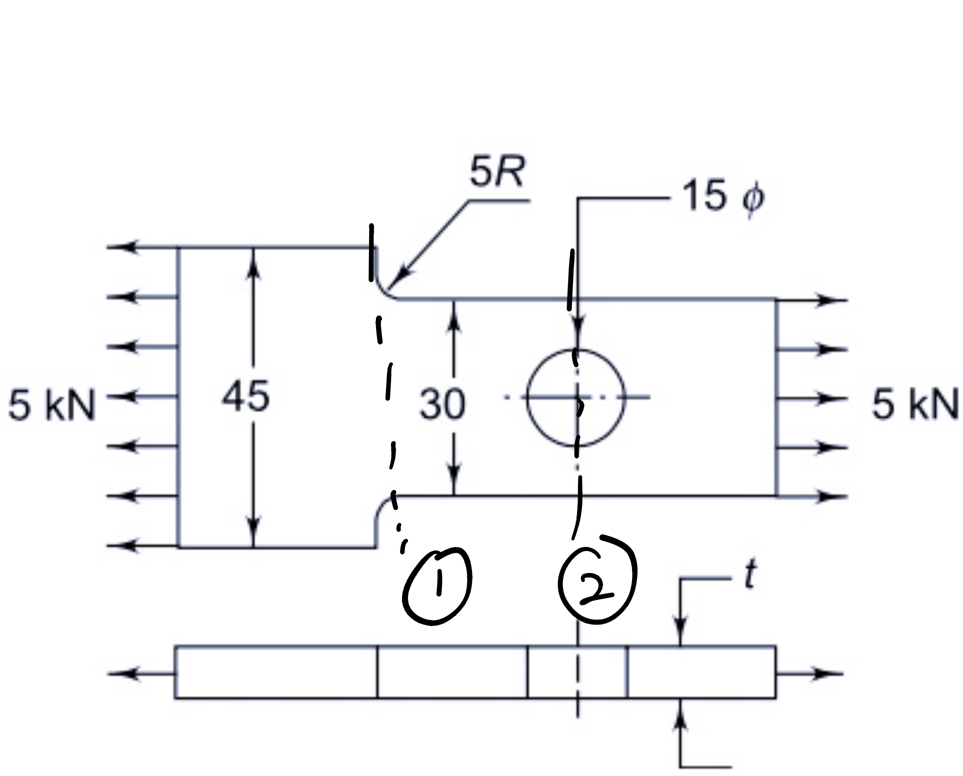
$$D/d = \frac{50}{25} = 2 \qquad r/d = 5/25 = 0.2$$

$$K_t = 1.64 \ [\text{From table}]$$

$$\sigma_{max} = 1.64 \times 24.43$$

$$\boxed{\sigma_{max} = 40.1 \ mpa}$$

Example *A flat plate subjected to a tensile force of 5 kN is shown in Fig. The plate material is grey cast iron FG 200 and the factor of safety is 2.5. Determine the thickness of the plate.*



Take ultimate strength = 200 MPa

Permissible stress (or) working $= \dfrac{\sigma_u}{(FOS)} = \dfrac{200}{2.5}$

$(\sigma_p) = 80\ N/mm^2 = 80\ MPa$

@ Fillet Section:

$$\sigma_0 = \frac{P}{dt} = \left(\frac{5\times10^3}{30t}\right) = \frac{166.67}{t}$$

$$\frac{D}{d} = \frac{45}{30} = 1.5$$

$$\frac{r}{d} = \frac{5}{30} = \frac{1}{6} = 0.167$$

From table $K_t = 1.66$

$$(\sigma_{max})_1 = 1.66 \times \frac{5000}{30t} = \frac{276.66}{t} \text{ N/mm}^2 \text{ @ } 1$$

@ Hole:

$$\sigma_0 = \frac{P}{(b-d)t} = \frac{5\times10^3}{(30-15)t} = \frac{5000}{15t}$$

$$\frac{d}{b} = \frac{15}{30} = 0.5$$

$$K_t = \frac{\sigma_{max}}{\sigma_0}$$

$$K_t = 2.17$$

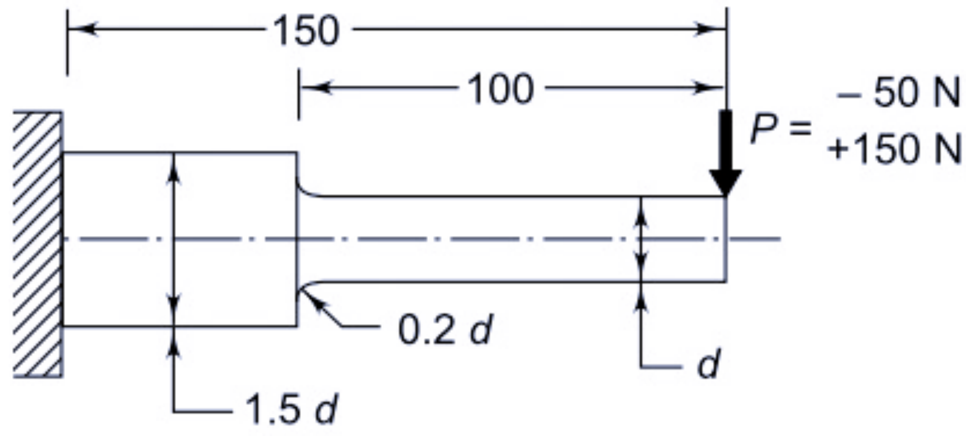
$$(\sigma_{max})_2 = 2.17 \times \frac{5000}{15t} = \frac{723}{t} \text{ N/mm}^2 \text{ @ } 2$$

This is our design criterion

$$(\sigma_{max})_H = \sigma_p \Rightarrow \frac{723}{t} = 80 \Rightarrow \boxed{t = 9.03\text{mm}}$$

$$\left((\sigma_{max})_2 = 210.72\ \text{N/mm}^2\right)$$

Example *A cantilever beam made of cold drawn steel 40C8 (S_{ut} = 600 N/mm^2 and S_{yt} = 380 N/mm^2) is shown in Fig. The force P acting at the free end varies from –50 N to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter 'd' of the beam at the fillet cross-section.*



Reliability Factor

- The laboratory values of endurance limit are usually mean values.

- There is considerable dispersion of the data when a number of tests are conducted even using the same material and same conditions.

The reliability factor Kc depends upon the reliability that is used in the design of the component.

The greater the likelihood that a part will survive, the more is the reliability and lower is the reliability factor.

The reliability factor is one for 50% reliability. This means that 50% of the components will survive in the given set of conditions.

To ensure that more than 50% of the parts will survive, the stress amplitude on the component should be lower than the tabulated value of the endurance limit.

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Modifying Factor to Account for Stress Concentration

The endurance limit is reduced due to stress concentration. The stress concentration factor used for cyclic loading is less than the theoretical stress concentration factor due to the notch sensitivity of the material

$$K_d = \frac{1}{K_f}$$

$K_f < K_t$

$P = -50\ N$

$+150\ N$

$\sigma_{ut} = 600\ N/mm^2$

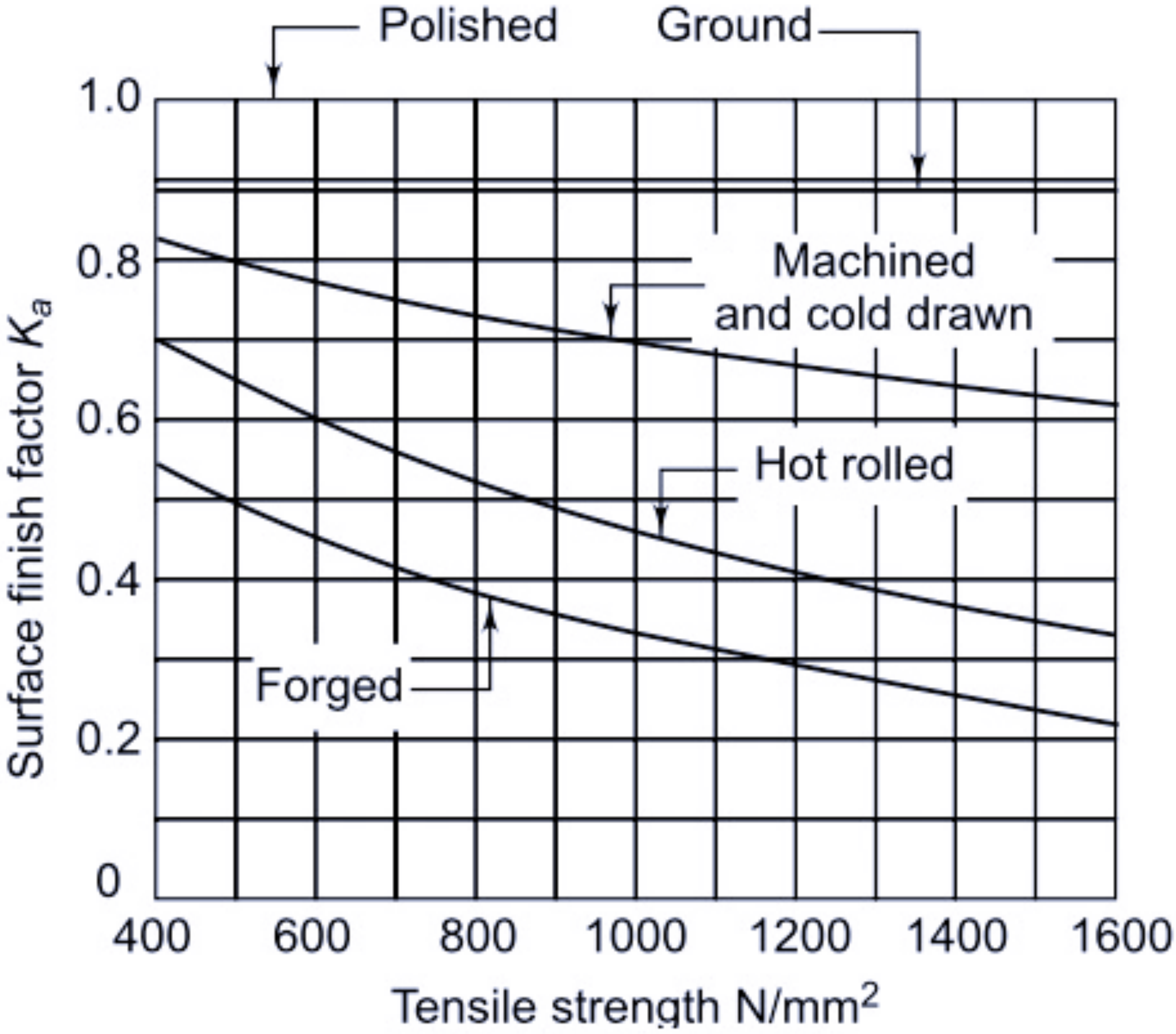
$\sigma_{yt} = 380\ N/mm^2$

$R = 90\%$

$K_c = 0.897$

$FOS = 2$

$q = 0.9$



$\sigma_e' = 0.5\sigma_u$

$= 0.5 \times 600$

$= 300\ N/mm^2$

K_{sur} (or) $K_a = 0.77$

Assume d is in b/n 7.65 mm & 50 mm

$7.65 < d < 50$

Now K_b (or) $K_{sz} = 0.85$

$$q = \frac{K_f - 1}{K_t - 1} \Rightarrow K_f = 1 + q(K_t - 1)$$

$$K_d = \frac{1}{K_f}$$

$$\frac{r}{d} = 0.2 \qquad \frac{D}{d} = 1.5$$

$K_t = 1.44$

$K_f = 1 + 0.9(1.44 - 1)$

$K_f = 1.396$

$$K_d = \frac{1}{1.396} = 0.716$$

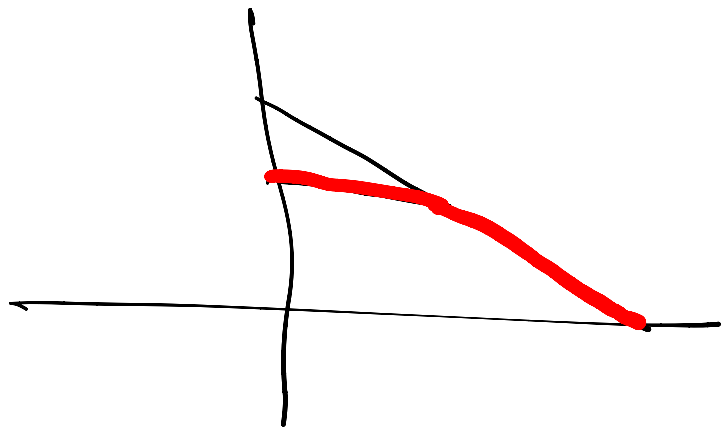
Actual Endurance limit :

$$\sigma_e = K_a K_b K_c K_d \sigma_e'$$

$$= (0.77)(0.85)(0.897)(0.716)\, 300$$

$$\sigma_e = 126.10 \ N/mm^2$$

Modified Goodmann condition :



$$(M_b)_{max} = 150 \times 100 = 15{,}000 \ Nmm$$

$$(M_b)_{min} = -50 \times 100 = -5000 \ Nmm$$

$$(M_b)_m = \frac{1}{2}\left[(M_b)_{max} + (M_b)_{min}\right]$$

$$= 5000 \ N\text{-}mm$$

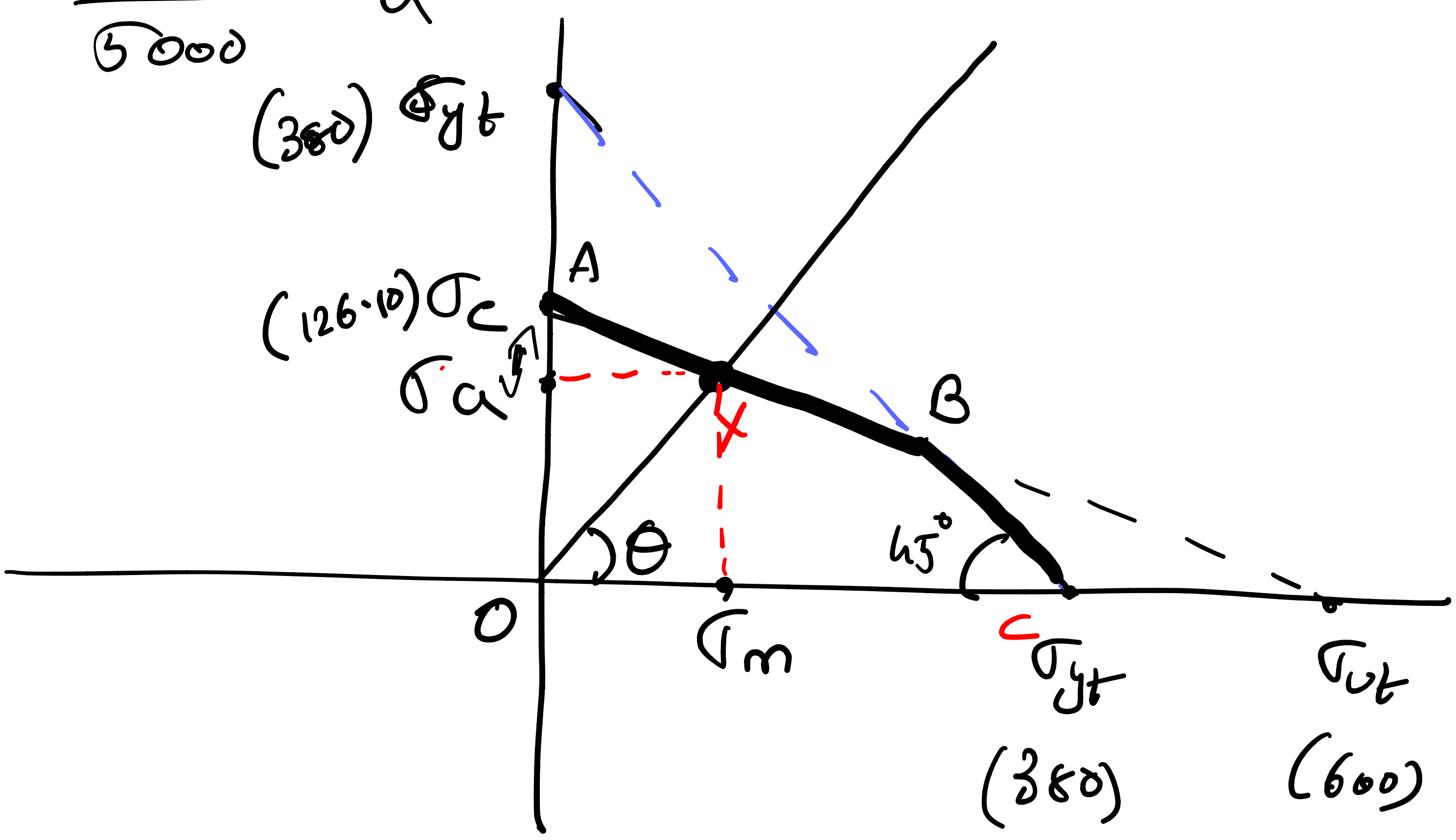
$$(M_b)_a = \frac{1}{2}\left[(M_b)_{max} - (M_b)_{min}\right]$$

$$= \frac{1}{2}\left[15{,}000 - (-5000)\right]$$

$$= 10{,}000 \ N\text{-}mm$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} = \frac{10{,}000}{5000} = 2$$

$$\theta = 63.43^\circ$$



$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_U} = \frac{1}{FOS} \quad \text{—} \quad (1)$$

Eqn of line ox

$$\frac{\sigma_a}{\sigma_m} = \tan\theta = 2 \quad \text{—} \quad (2)$$

$$\sigma_a = 2\sigma_m$$

$$\frac{2\sigma_m}{126.1} + \frac{\sigma_m}{600} = \frac{1}{2}$$

$$\sigma_m \left[\frac{1}{63.05} + \frac{1}{600} \right] = \frac{1}{2}$$

$$\sigma_m = 28.52 \ N/mm^2$$

$$\sigma_a = 57.04 \ N/mm^2$$

without considering FOS

$$\sigma_m' = 57.04 \ N/mm^2$$

$$\sigma_a' = 114.08 \ N/mm^2$$

$$\sigma = \frac{My}{I} \quad \frac{d}{2} \quad \frac{\pi}{64} d^4$$

$$\sigma_a = \frac{32 M_a}{\pi d^3}$$

$$\frac{32 \times 10{,}000}{\pi \times d^3} = 57.04$$

$$d^3 = \frac{32 \times 10{,}000}{\pi \times 57.04}$$

$$\boxed{d = 12.13 \ mm}$$

Example 5.13 *A transmission shaft of cold drawn steel 27Mn2 (S_{ut} = 500 N/mm² and S_{yt} = 300 N/mm²) is subjected to a fluctuating torque which varies from –100 N-m to + 400 N-m. The factor of safety is 2 and the expected reliability is 90%. Neglecting the effect of stress concentration, determine the diameter of the shaft.*

Assume the distortion energy theory of failure.

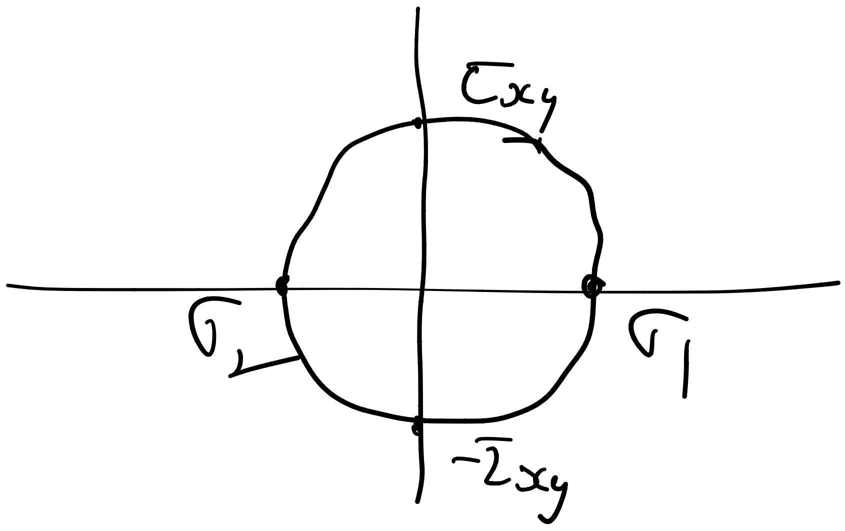
Pure Shear

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left[\frac{\sigma_{yt}}{FOS}\right]^2 \rightarrow \text{Distortion Energy theory}$$

$$\sigma_{yt} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

Assume, where only pure shear is applied

$$\sigma_1 = -\sigma_2 = \tau_{xy}.$$



$$\sigma_{yt} = \sqrt{\sigma_1^2 + \sigma_1^2 + \sigma_1^2}$$

$$\sigma_{yt} = \sqrt{3\sigma_1^2}$$

$$\boxed{\sigma_{yt} = \sqrt{3}\,\tau_{xy}}$$

Replace τ_{xy} by σ_{sy} σ_{yt}

$$\sigma_{yt} = \sqrt{3}\,\sigma_{sy} \Rightarrow \sigma_{sy} = \frac{1}{\sqrt{3}}\sigma_{yt}$$

$$\boxed{\sigma_{sy} = 0.577\,\sigma_{yt}}$$

Goodman

0.577×600

For steel

$$\sigma_e' = 0.5\,\sigma_{ut} \Rightarrow \sigma_{et}' = 0.5 \times 500$$

$$(\sigma_{et}' = 250 \text{ N/mm}^2)$$

From the figure for $\sigma_u = 500\ N/mm^2$ & cold drawn steel

$K_a = 0.79$

Assuming $7.65 < d < 50$

$K_b = 0.85$

For 90% reliability $\rightarrow$ $K_c = 0.897$

$\sigma_{et} = (K_a) K_b K_c \sigma_e'$

$\boxed{\sigma_{et} = 150.58\ N/mm^2}$

$\sigma_{es} = 0.577\ \sigma_{et} = 0.577 \times 150.58 = 86.88\ N/mm^2$

$\sigma_{sy} = 0.577\ \sigma_{yt} = 173.1\ N/mm^2$

$\rightarrow$ Construct modified Goodmann diagram :

$(M_t)_m = \frac{1}{2}\left[(M_t)_{max} + (M_t)_{min}\right]$

$(M_t)_{max} = 400\ N\text{-}m$

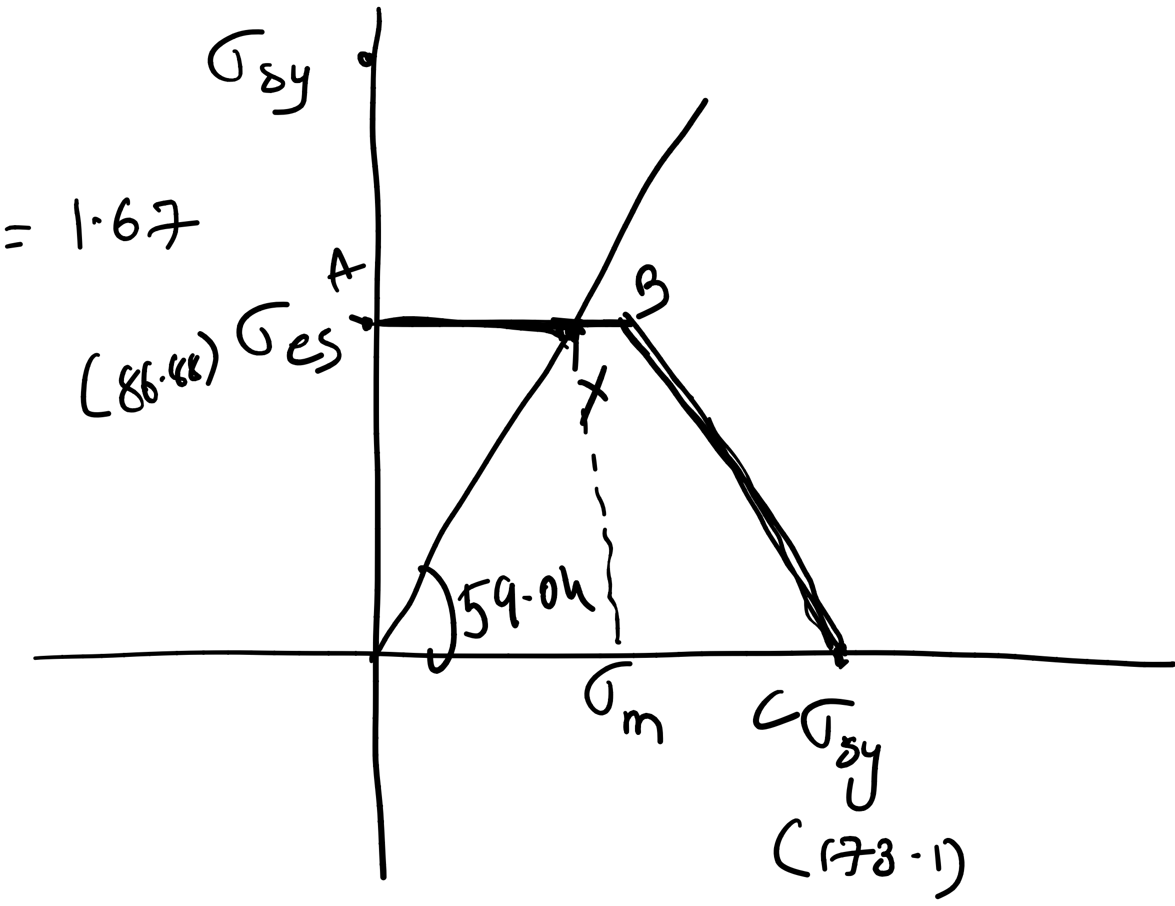
$(M_t)_{min} = -100\ N\text{-}m$

$(M_t)_m = 150\ N\text{-}m$

$(M_t)_a = 250\ N\text{-}m$

$\tan\theta = \frac{(M_t)_a}{(M_t)_m} = \frac{250}{150} = 1.67$

$\boxed{\theta = 59.04^\circ}$



$\sigma_{sa} = 86.88 \ N/mm^2$

$$\tau_a = \frac{\sigma_{sa}}{(FOS)} = \frac{16 (M_t)_a}{\pi d^3}$$

$$\frac{86.88}{2} = \frac{16 \times (250 \times 10^3)}{\pi d^3}$$

$$d^3 = \frac{16 \times 250 \times 1000 \times 2}{\pi \times 86.88}$$

$$d^3 = 29.31 \times 10^3$$

$$\boxed{d = 30.83 mm}$$

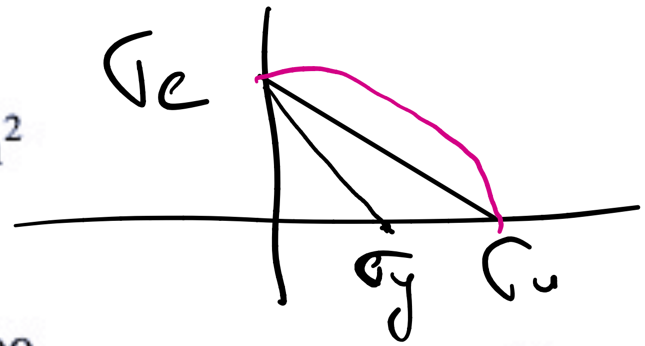
$$\tau = \frac{16T}{\pi d^3}$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C \cdot \theta}{l}$$

2. A bar of circular cross-section is subjected to alternating tensile forces varying from a minimum of 200 kN to a maximum of 500 kN. It is to be manufactured of a material with an ultimate tensile strength of 900 MPa and an endurance limit of 700 MPa. Determine the diameter of bar using safety factors of 3.5 related to ultimate tensile strength and safety factor of 4 related to endurance limit and a stress concentration factor of 1.65 for fatigue load. Use Goodman straight line as basis for design.

Solution. Given : $W_{min} = 200$ kN ; $W_{max} = 500$ kN ; $\sigma_u = 900$ MPa $= 900$ N/mm^2 ; $\sigma_e = 700$ MPa $= 700$ N/mm^2 ; $(F.S.)_u = 3.5$; $(F.S.)_e = 4$; $K_f = 1.65$

Let d = Diameter of bar in mm.



$\therefore$ Area, $A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$ mm^2

We know that mean or average force,

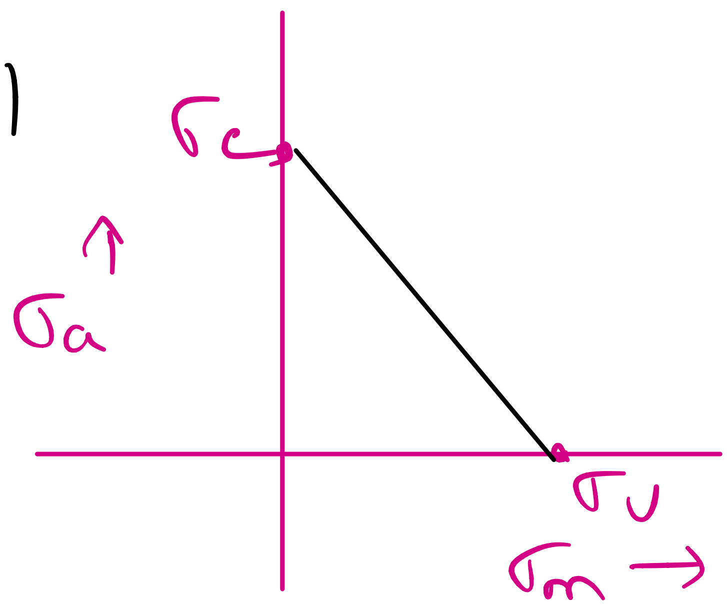
$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{500 + 200}{2} = 350 \text{ kN} = 350 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{350 \times 10^3}{0.7854\, d^2} = \frac{446 \times 10^3}{d^2} \text{ N/mm}^2$$

$$\text{Variable force, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{500 - 200}{2} = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{150 \times 10^3}{0.7854\, d^2} = \frac{191 \times 10^3}{d^2} \text{ N/mm}^2$$

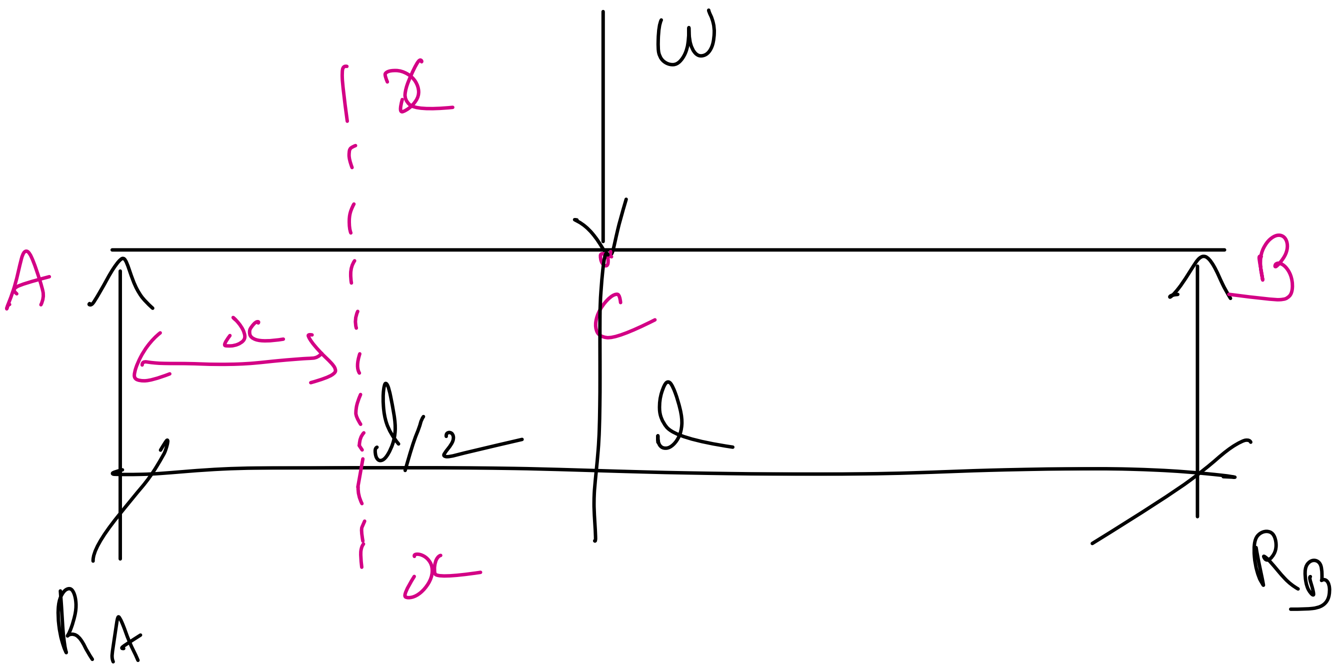
$$\frac{\sigma_a}{\left(\sigma_e / Fos\right)_e} + \frac{\sigma_m \times K_f}{\left(\sigma_u / Fos\right)_u} = 1$$



$$\frac{\frac{446 \times 10^3}{d^2} \times 1.65}{\left(\frac{900}{3.5}\right)} + \frac{\frac{191 \times 10^3}{d^2}}{\left(700/4\right)} = 1$$

$$\frac{1}{d^2}\left[\frac{446 \times 3.5 \times 10^3}{900} \times 1.65 + \frac{191 \times 4 \times 10^3}{700}\right] = 1$$

$d^2 = 3953.26$ $\quad d = 63$mm



$R_A + R_B = W$ — (1)

B.M @ point A = 0

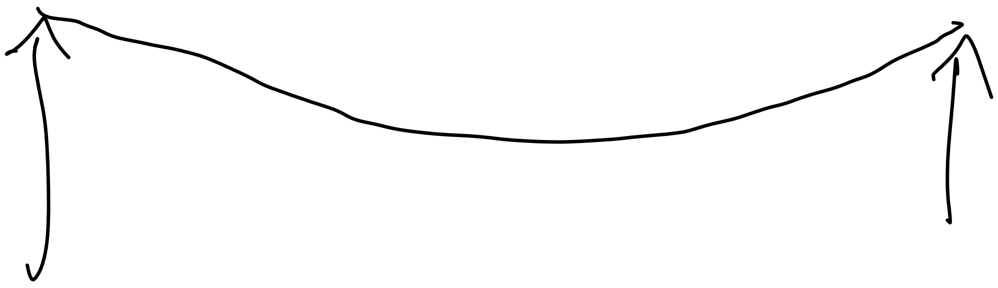
$W \times l/2 - l \times R_B = 0 \Rightarrow R_B = W/2$

$R_A = W/2$

moment @ section x-x $= R_A \times x = \frac{W}{2} \times x$

@ point C $\quad x = l/2 \Rightarrow M = \frac{Wl}{4}$

$M_{max} = \frac{W_{max} \times l}{4}$ & $M_{min} = \frac{W_{min} \times l}{4}$



$\sigma_{max} = \frac{M_{max}}{Z}$

B.M equation $\Rightarrow \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$\sigma_{min} = \frac{M_{min}}{Z}$

$\sigma = \frac{My}{I} = \frac{M}{Z}$

Example 6.8. *A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by : ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.*

$l = 500 mm$, $W_{min} = 20 kN$, $W_{max} = 50 kN$

$FOS = 1.5$, $\sigma_S = 0.85$, $\sigma_{Surf} = 0.9$

$\sigma_U = 650 MPa$, $\sigma_y = 500$ ", $\sigma_e = 350$ "

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

$$M_{min} = \frac{W_{min} \times l}{4} = 2500 \times 10^3 \text{ N-mm}$$

$$M_m = \frac{M_{max} + M_{min}}{2} = 4375 \times 10^3 \text{ N-mm}$$

$$M_v = \frac{M_{max} - M_{min}}{2} = 1875 \times 10^3 \text{ N-mm}$$

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.098 d^3}$$

$$\sigma_{v(or)} = \sigma_a = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.098 d^3}$$

$$Z = I/y = \frac{\frac{\pi}{64} d^4}{d/2} = \frac{\pi}{32} d^3 = 0.098 d^3$$

$$\frac{\sigma_a}{\sigma_e \times K_{Surf} \times K_{SZ}} + \frac{\sigma_m}{\sigma_U} = \frac{1}{FoS}$$

$$\frac{1875\times10^3}{0.098d^3\times350\times0.9\times0.85}+\frac{4375\times10^3}{0.098d^3\times650}=\frac{1}{1.5}$$

$$\frac{10^3}{d^3}\left[\frac{1875}{26.23}+\frac{4375}{63.7}\right]=\frac{1}{1.5}$$

$$d^3=1.5\times1000\left[71.48+68.68\right]$$

$$d^3=1500\times140.16=210241.90$$

$$d=\underline{59.46}\ mm\simeq\underline{60\ mm}$$

Example 6.9. *A 50 mm diameter shaft is made from carbon steel having ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates between 2000 N-m to – 800 N-m. Using Soderberg method, calculate the factor of safety. Assume suitable values for any other data needed.*

Assume the yield stress (σ_y) for carbon steel in reversed bending as 510 N/mm², surface finish factor (K_{sur}) as 0.87, size factor (K_{sz}) as 0.85 and fatigue stress concentration factor (K_{fs}) as 1.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C \cdot \theta}{l} \qquad \tau = \frac{TR}{J}$$

$$\tau = \frac{T \times D/2}{\frac{\pi}{32} D^4}$$

$$\boxed{\tau = \frac{16T}{\pi D^3}}$$

$$T_{mean} = \frac{T_{max} + T_{min}}{2} = 600 \times 10^3 \text{ N-mm}$$

$D = 50$ mm

$$T_v = \frac{T_{max} - T_{min}}{2} = 1400 \times 10^3 \text{ N-mm}$$

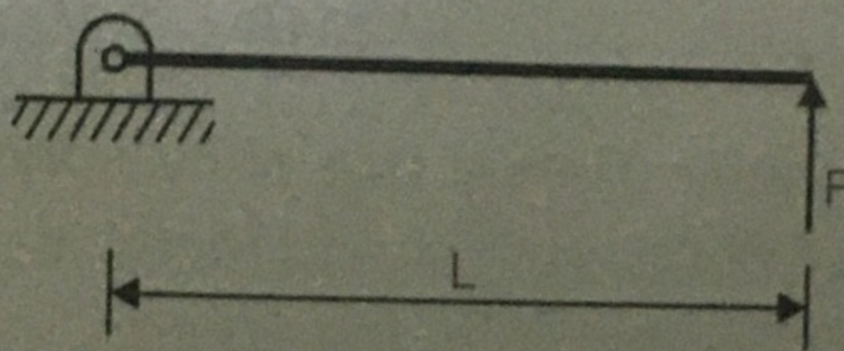
$$\tau_{mean} = \frac{16 \times 600 \times 10^3}{\pi \times (50)^3} = 24.44 \text{ N/mm}^2$$

$$\tau_v = 57.04 \text{ N/mm}^2$$

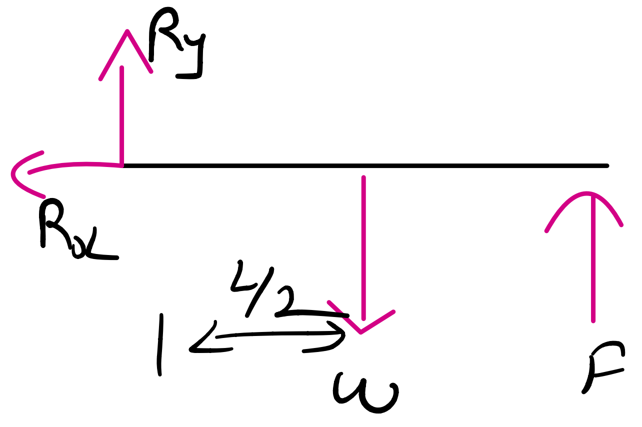
$$\frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{Fos}$$

$$\frac{\tau_m}{\tau_y} + \frac{\tau_v}{\tau_e} = \frac{1}{fos}$$

1.12 A pin jointed uniform rigid rod of weight W and length L is supported horizontally by an external force F as shown in the figure below. The force F is suddenly removed. At the instant of force removal, the magnitude of vertical reaction developed at the support is



(a) zero (b) $W/4$
(c) $W/2$ (d) W



$\Sigma F_x = 0 \Rightarrow R_x = 0$

$\Sigma F_y = 0 \Rightarrow R_y = W - F$ — (1)

When force is removed, the rod swings about the hinge. Hence body possesses angular acceleration.

$$T = I\alpha$$

$I = \dfrac{mL^2}{3}$ [For a thin rod about axis @ one end of the rod]

$= \left(\dfrac{W}{g}\right)\dfrac{L^2}{3}$

$T = W \times L/2$

$$\Rightarrow \frac{\cancel{W}\cancel{L}}{2} = \left(\frac{\cancel{W}}{g}\right)\frac{L^{\cancel{2}}}{3} \times \alpha$$

$$\Rightarrow \frac{L\alpha}{3g} = \frac{1}{2} \Rightarrow \boxed{\alpha = \frac{3g}{2L}} \text{ — (2)}$$

Now $F = m a_{c.m}$

$$W - R_y = \frac{W}{g} a_{c.m}$$ [∵ $F = W - R_y$ from (1)]

$$a_{c.m} = \left(\frac{L}{2}\right)\alpha$$

$$W - R_y = \frac{W}{\cancel{g}}\left(\frac{\cancel{L}}{2}\right)\left(\frac{3\cancel{g}}{2\cancel{L}}\right)$$ [from eq (2)]

$$W - R_y = \frac{3W}{4} \Rightarrow \boxed{R_y = W/4}$$

Unit - 5

Design of Shafts, Keys & Couplings

Shafts

A shaft is a rotating machine element which is used to transmit power from one place to another

In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

Shafts are used to transmit torque and bending moment

Shaft may bend(or) twist

Usually mountings are done by means of Keys, splines etc;

1. An ***axle***, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave. An axle may rotate with the wheel or simply support a rotating wheel.
2. A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g.* drill press spindles) or to a work piece (*e.g.* lathe spindles).

Other types or names are:

1. Jackshaft - auxiliary or intermediate shaft between two shafts
2. Countershaft - used in multi-stage gearboxes.
3. Line Shaft - consists of a number of shafts, which are connected in axial direction by means of couplings

Material Used for Shafts

- The material used for shafts should have the following properties :
- High strength.
- Good machinability.
- Low notch sensitivity factor.
- Good heat treatment properties.
- High wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4, 50 C 12, 30C8 or 40C8.

30C8 or 40C8 steels are commonly called *machinery steels*.

Indian standard designation	Ultimate tensile strength, MPa	Yield strength, MPa
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

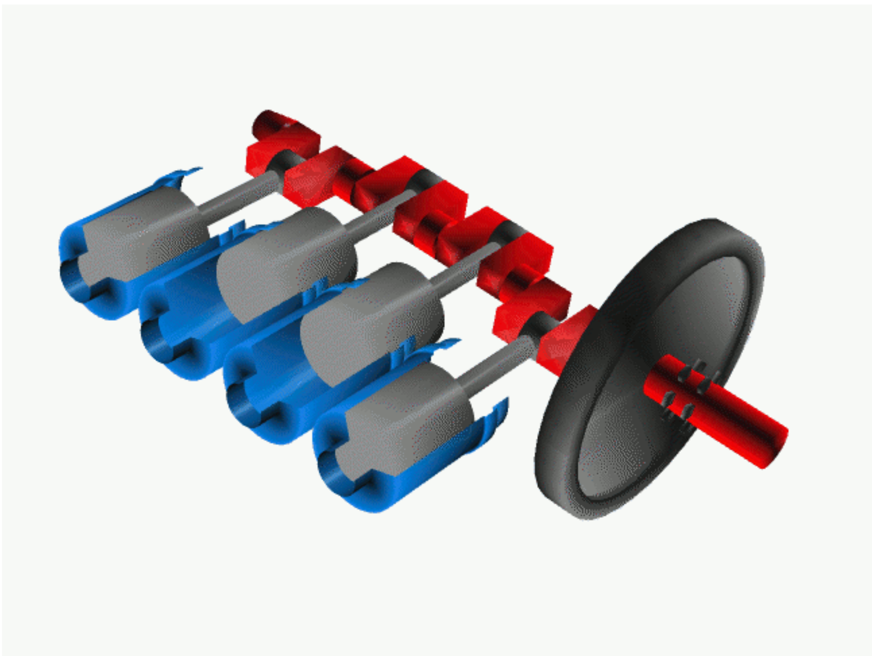
When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used

Manufacturing of Shafts

- Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding.
- The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.
- The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut.
- Shafts of larger diameter are usually forged and turned to size in a lathe.

Types of Shafts

1. ***Transmission shafts.*** These shafts transmit power between the source and the machines absorbing power.
   a. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts.
   b. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.
2. ***Machine shafts.*** These shafts form an integral part of the machine itself.
   a. The crank shaft is an example of machine shaft.



Standard Sizes of Transmission Shafts

25 mm to 60 mm with 5 mm steps;

60 mm to 110 mm with 10 mm steps ;

110 mm to 140 mm with 15 mm steps ; and

140 mm to 500 mm with 20 mm steps.

The standard length of the shafts are 5 m, 6 m and 7 m.

Stresses in Shafts

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses(tensile or compressive)due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

Torsion equation

$$\boxed{\frac{T}{J} = \frac{\tau}{R}} = \frac{C\theta}{l} \rightarrow \text{Torsion equation}$$

$T \rightarrow$ Torque

$R \rightarrow$ dist from neutral axis

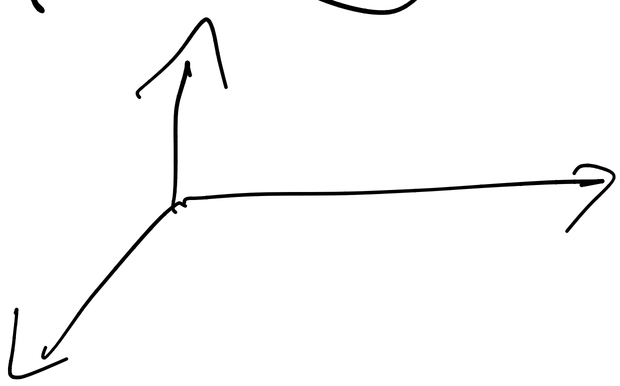
$R = d/2$

$$\frac{T}{\frac{\pi}{\cancel{32}_{16}} d^{\cancel{4}3}} = \frac{\tau}{\cancel{d}/2}$$

$J =$ polar moment of inertia I

$$\boxed{T = \frac{\pi}{16} \tau d^3}$$

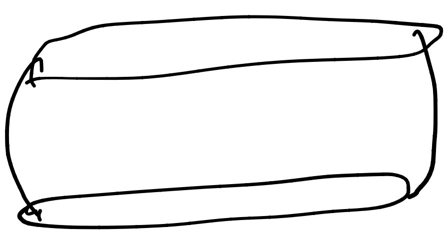
$$\boxed{J = \frac{\pi}{32} d^4}$$



$$\boxed{\tau = \frac{16T}{\pi d^3}}$$

$$\boxed{I = \frac{\pi}{64} d^4}$$

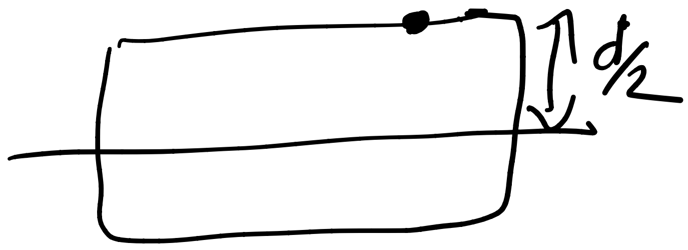
$$J = \frac{\pi}{32}\left[d_o^4 - d_i^4\right]$$



d_o - outer dia

d_i - inner dia

$$\frac{T}{\frac{\pi}{\cancel{32}\,16}\left(d_o^4 - d_i^4\right)} = \frac{\tau}{d_o/\cancel{2}}$$



$$T = \frac{\pi}{16}\,\frac{\left[d_o^4 - d_i^4\right]}{d_o} \times \tau$$

$$= \frac{\pi}{16}\,\frac{d_o^{\cancel{4}\,3}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]}{\cancel{d_o}} \times \tau$$

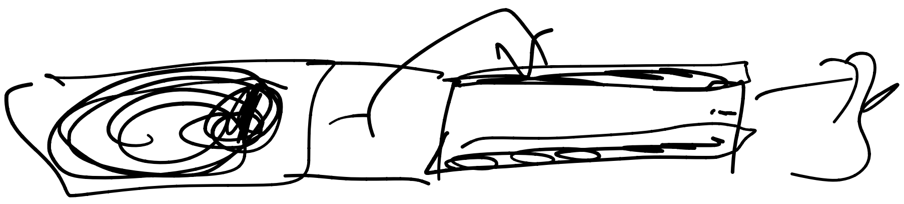
Let take

$$d_i/d_o = K$$

$$T = \frac{\pi}{16}\,d_o^3\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] \times \tau$$

$$\boxed{T = \frac{\pi}{16}\,\tau\, d_o^3\left[1 - K^4\right]}$$ → For a hollow shaft

The hollow shafts are usually used in marine work. These shafts are stronger per kg of material



Shafts Subjected to Bending Moment only

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

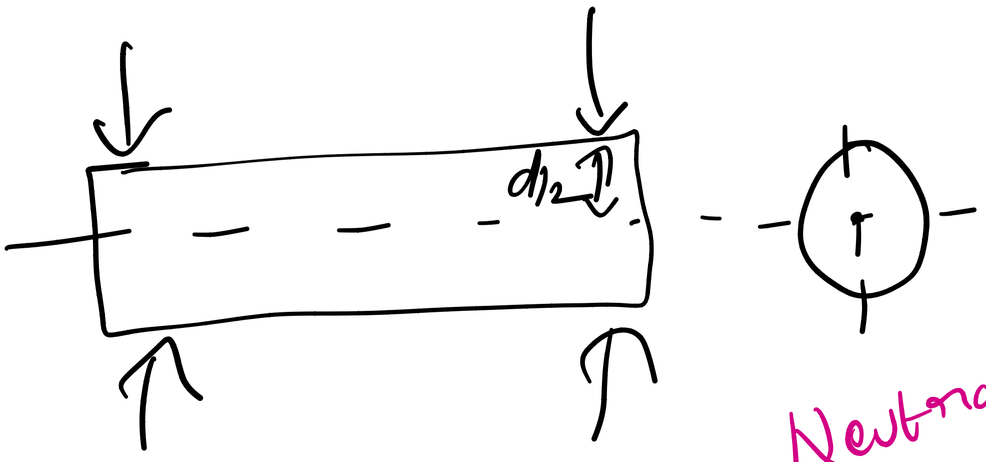
M - moment

I - MoI

σ_b - Bending Stress

y - dist from neutral axis

$= d/2$



Neutral axis → Has no effect from Bending moment

$$I = \frac{\pi}{64} d^4$$

$$\frac{M}{\frac{\pi}{\cancel{64}\,32} d^{\cancel{4}\,3}} = \frac{\sigma_b}{\cancel{d/2}} \Rightarrow$$

$$\boxed{M = \frac{\pi}{32} \sigma_b d^3}$$

Solid shaft

$$\boxed{M = \frac{\pi}{32} \sigma_b (1-k^4) d_o^3}$$

Hollow shaft

Shafts Subjected to Combined Twisting Moment and Bending Moment

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_1)^2 + (2\tau)^2}$$

$$\sigma_b = \frac{32M}{\pi d^3} \qquad \tau = \frac{16T}{\pi d^3}$$

Substituting the values of τ and σ_b

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2+T^2}\right]$$

or

$$\boxed{\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}}$$

$$\tau_{max} = \frac{16}{\pi d^3}\left[\sqrt{M^2+T^2}\right]$$

Equivalent Twisting moment

$$\boxed{T_e = \sqrt{M^2+T^2} = \frac{\pi}{16}\tau d^3}$$

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\sigma_{b(max)} = \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)\right]$$

$$\frac{\pi}{32}\sigma_{b(max)} d^3 = \frac{1}{2}\left[M + \sqrt{M^2+T^2}\right]$$

Equivalent Bending moment

$$\frac{1}{2}\left[M + \sqrt{M^2+T^2}\right] = \frac{\pi}{32}(\sigma_b)(d_o)^3(1-k^4)$$

Hollow shaft

$$k = \frac{d_i}{d_o}$$

Shafts Subjected to Fluctuating Loads

In order to design shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M)

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

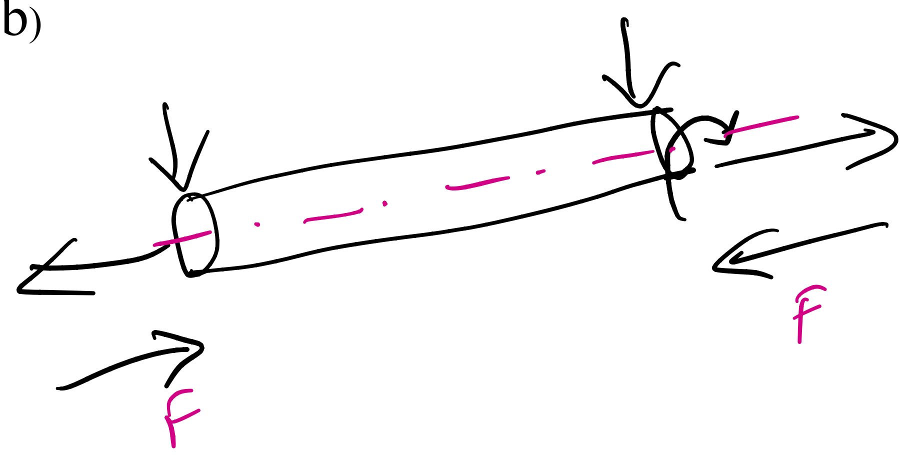
K_t = Combined shock and fatigue factor for torsion.

Table 14.2. Recommended values for K_m and K_t.

Nature of load	K_m	K_t
1. Stationary shafts		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σb)



$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32\,M}{\pi\, d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4\,F}{\pi\, d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4\,F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \qquad \text{... } (\because k = d_i/d_o)$$

∴ Resultant stress (tensile or compressive) for solid shaft,

$$\boxed{\sigma_1 = \frac{32\,M}{\pi\, d^3} + \frac{4\,F}{\pi\, d^2} = \frac{32}{\pi\, d^3}\left(M + \frac{F \times d}{8}\right)} \qquad \text{...}(i)$$

$$= \frac{32\,M_1}{\pi\, d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32\,M}{\pi\,(d_o)^3\,(1 - k^4)} + \frac{4\,F}{\pi\,(d_o)^2\,(1 - k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1 - k^4)}\left[M + \frac{F\,d_o\,(1 + k^2)}{8}\right] = \frac{32\,M_1}{\pi (d_o)^3\,(1 - k^4)}$$

$$\text{...}\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1 + k^2)}{8}\right]$$

Design of Shafts on the basis of Rigidity

1. Torsional rigidity. The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected.

The permissible amount of twist should not exceed 0.25° per metre length of such shafts.

$$\frac{T}{J} = \cancel{\frac{\tau}{R}} = \frac{C \cdot \theta}{L}$$

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G \,.\, \theta}{L} \quad \text{or} \quad \theta = \frac{T \,.\, L}{J \,.\, G}$$

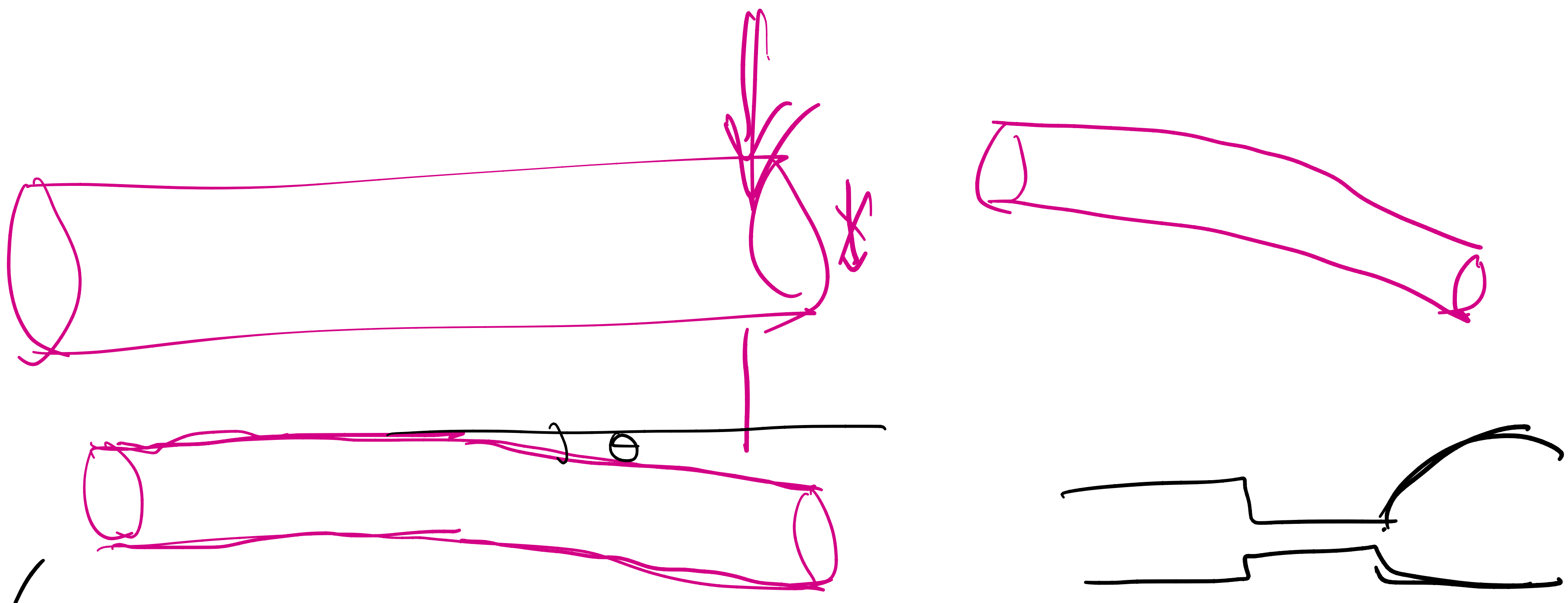
where
θ = Torsional deflection or angle of twist in radians,
T = Twisting moment or torque on the shaft,
J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \qquad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \qquad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and
L = Length of the shaft.

2. Lateral rigidity. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment



$$\delta = \frac{PL}{AE} \rightarrow \text{Constant Cross Section}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \rightarrow \text{Variable Cross-Section}$$

$EI \rightarrow$ Flexural Rigidity

Example 14.5. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

$$FOS = 6 = \frac{ult}{wor}$$

$$\text{tensile working stress} = \frac{700}{6} = 116.6 \text{ MPa}$$

$$\text{allowable shear stress} = \frac{500}{6} = 83.33 \text{ MPa}$$

Keys & Keyways:

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.

It is always inserted parallel to the axis of the shaft.

Keys are used as temporary fastenings and are subjected to consider- able crushing and shearing stresses.

A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys,
2. Saddle keys,
3. Tangent keys,
4. Round keys, and
5. Splines.

Sunk Keys:

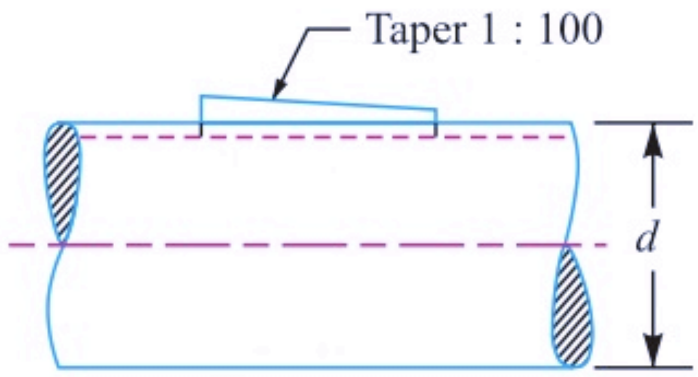
The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

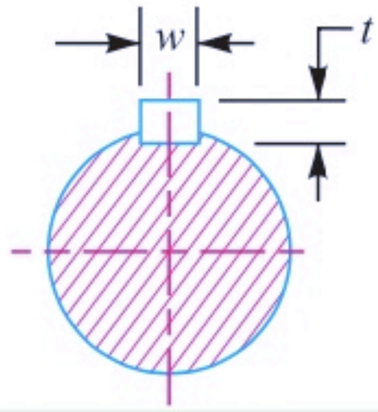
1. ***Rectangular sunk key.*** A rectangular sunk key is shown in Fig. 13.1. The usual proportions of this key are :

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.



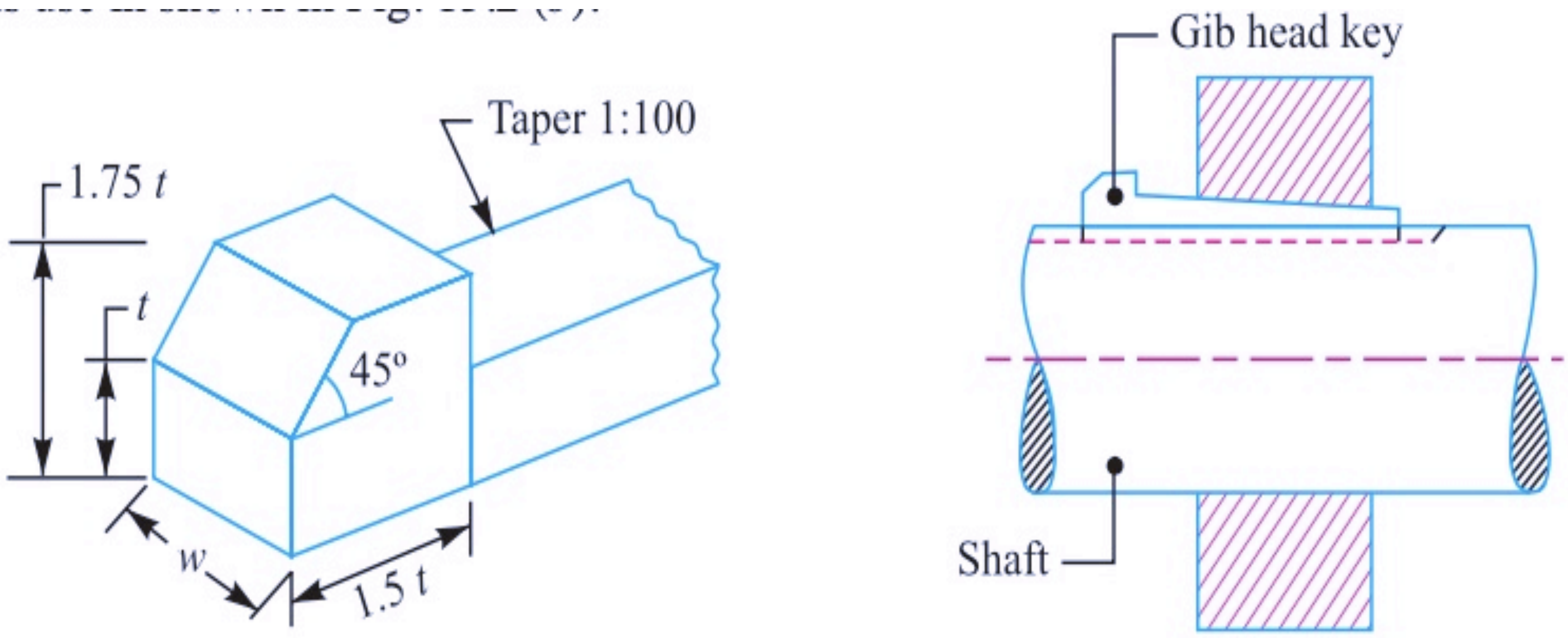


2. ***Square sunk key.*** The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

3. ***Parallel sunk key.*** The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. ***Gib-head key.*** It is a rectangular sunk key with a head at one end known as ***gib head.*** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. 13.2 (*a*) and its use in shown in Fig. 13.2 (*b*).



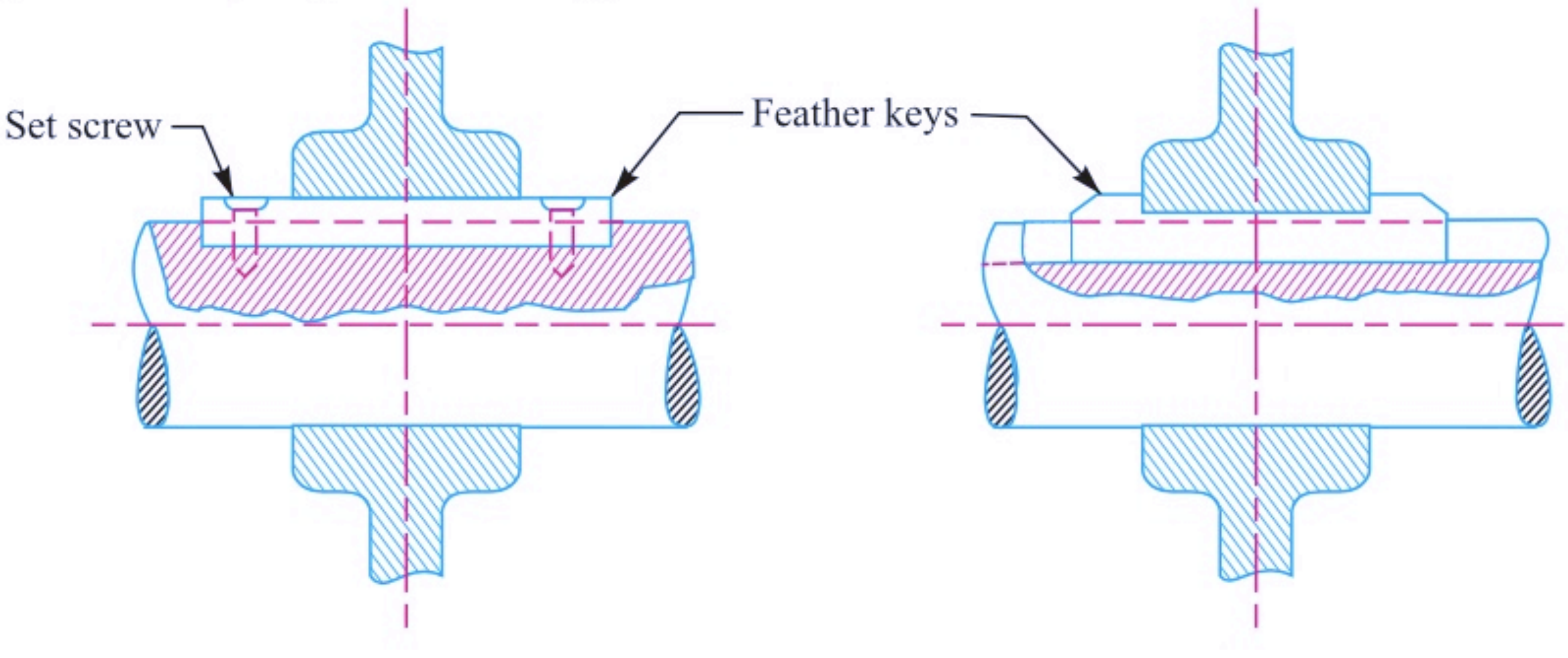
(*a*) (*b*)

The usual proportions of the gib head key are :

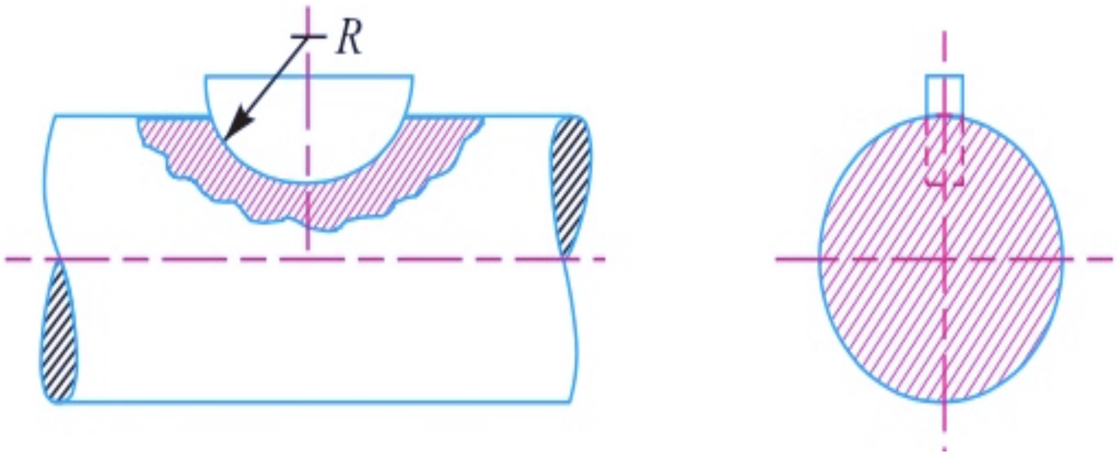
Width, $w = d / 4$;

and thickness at large end, $t = 2w / 3 = d / 6$

5. ***Feather key.*** A key attached to one member of a pair and which permits relative axial movement is known as ***feather key.*** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



6. ***Woodruff key.*** The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. 13.4. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.



1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft *prevents any tendency to turn over in its keyway.

3. The depth of the keyway weakens the shaft

Saddle keys

The saddle keys are of the following two types :

1. Flat saddle key, and **2.** Hollow saddle key.

A ***flat saddle key*** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. . It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

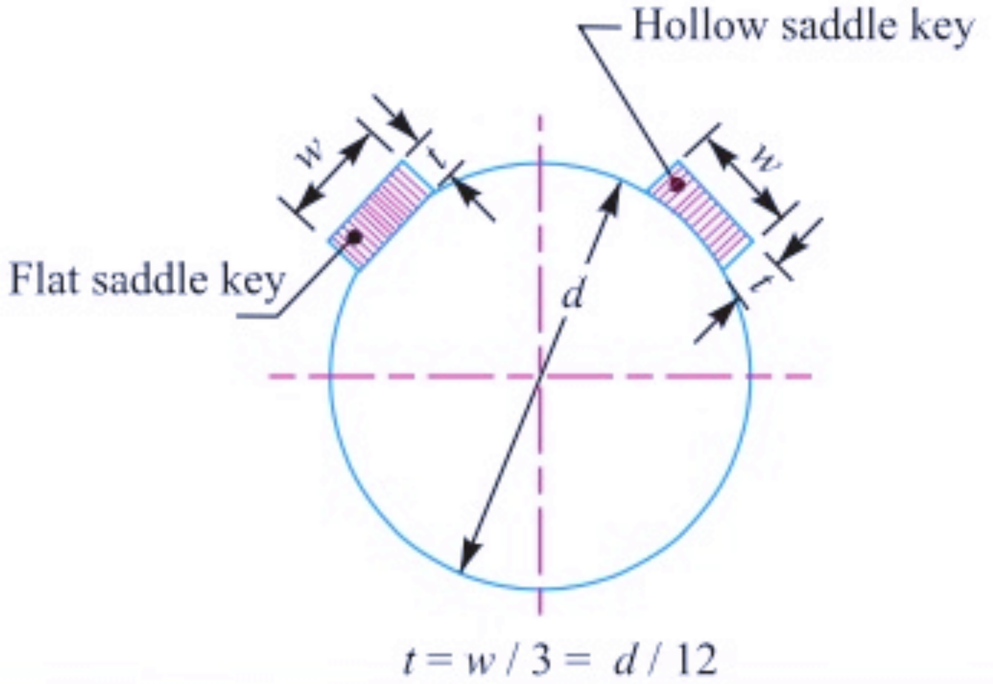


Fig. . Saddle key.

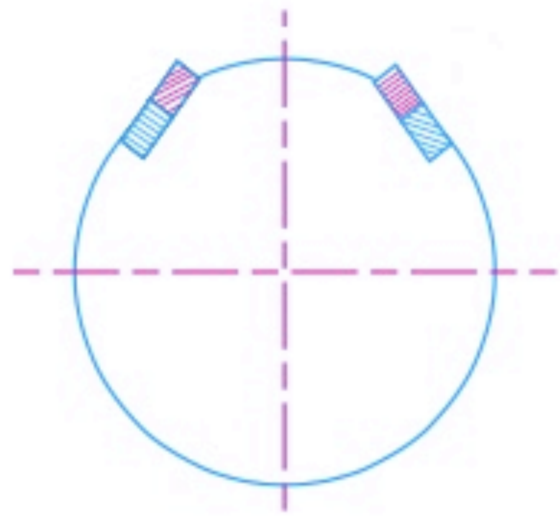


Fig. . Tangent key.

A ***hollow saddle key*** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. . Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

Round Keys

The round keys, as shown in Fig. (*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

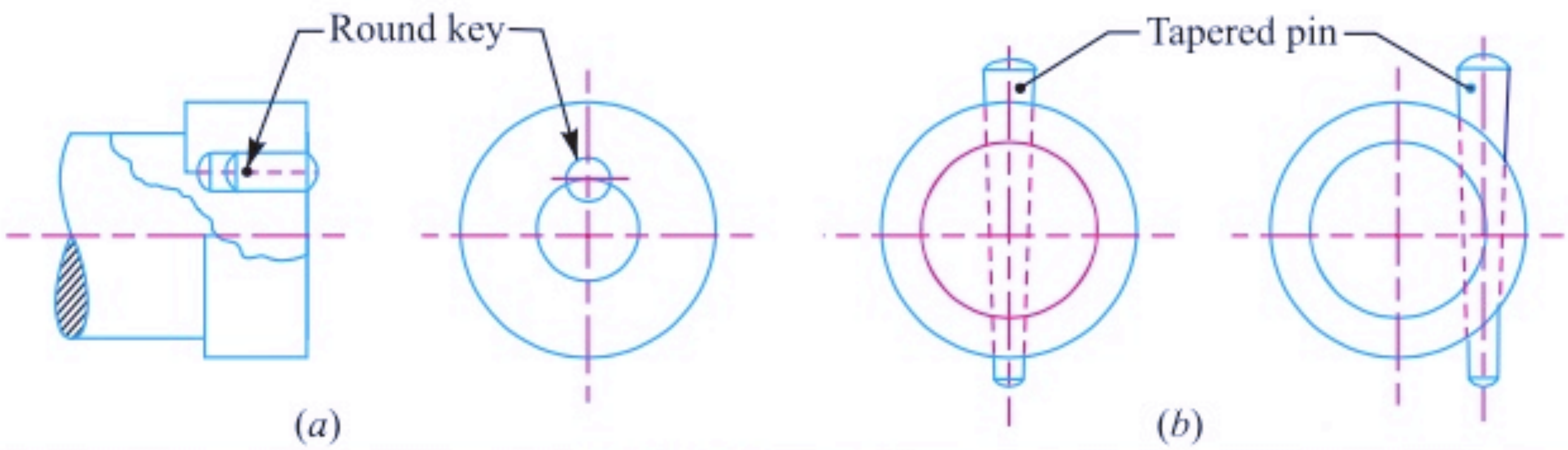
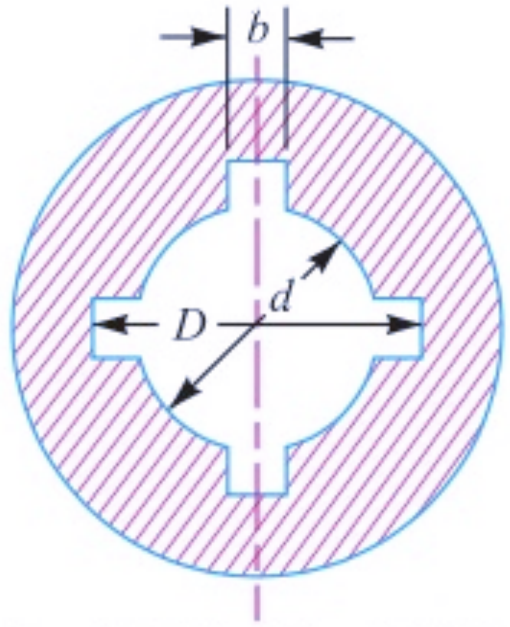


Fig. . Round keys.

Sometimes the tapered pin, as shown in Fig. (*b*), is held in place by the friction between the pin and the reamed tapered holes.

Splines



Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig. . These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

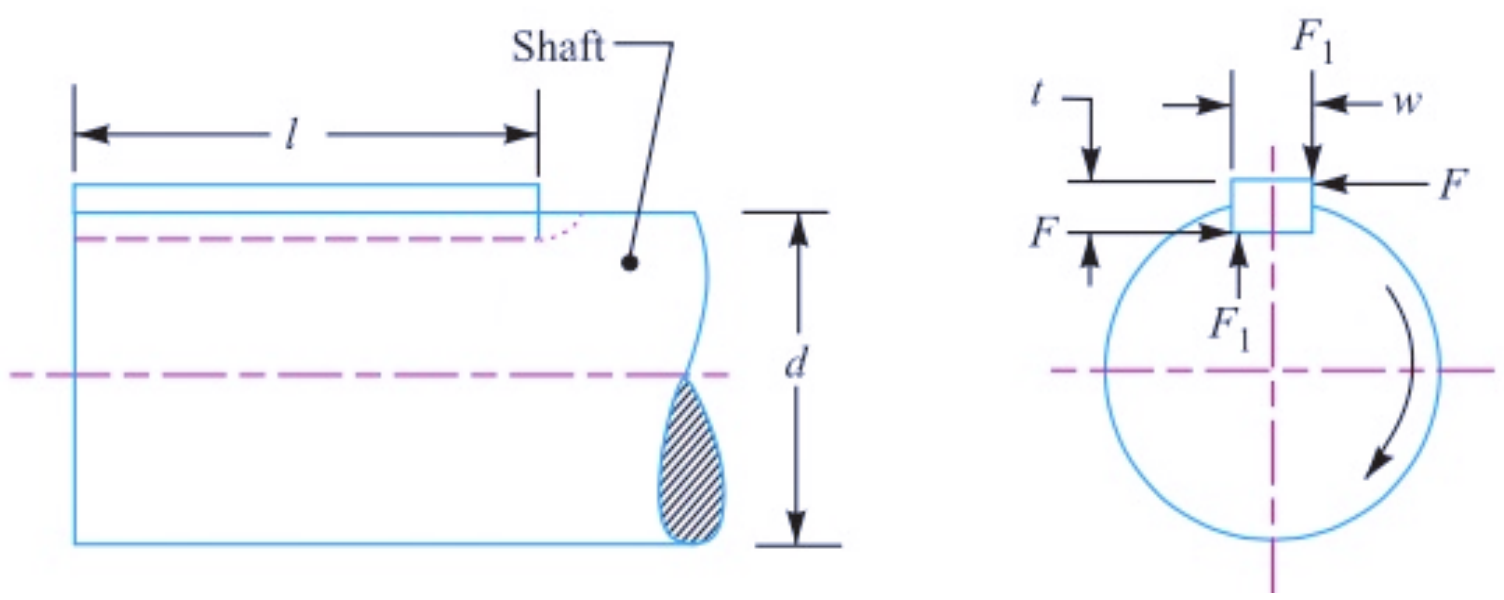
The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.

Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.



The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. .

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

T – Torque transmitted

F → tangential force @ circumference of shaft

d → dia of shaft

w → width of key

t → thickness of key

Shearing force = Shear stress × Resisting Area

$$F = \tau \times l \times w$$

Torque transmitted

$$T = F \times S = F \times d/2 = \tau \times l \times w \times d/2$$

Crushing force = Resisting area × Crushing Stress

$$F = w \times t/2 \times \sigma_c$$

$$T = \sigma_c \times \frac{w \times t \times d}{4}$$

Key will be equally strong in shearing & Crushing when

$$\sigma_c = \frac{1}{2}\sigma_u$$

$$l \times w \times \frac{\cancel{d}}{\cancel{2}} \times \tau = \frac{\cancel{l} \times t \times \cancel{d}}{\cancel{2}\,2} \times \sigma_c$$

$$\boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}}$$

$$\boxed{\sigma_c = 2\tau}$$

$$\boxed{\frac{w}{t} = 1}$$

Square

<u>length</u>: Shearing Strength of Key = Torsional shear Strength of shaft

$$\frac{l\,w\,d}{\cancel{2}}\,\tau = \frac{\pi}{\cancel{16}\,8}\,\tau_1\,d^{\cancel{3}\,2}$$

Torque $\boxed{T = \frac{\pi}{16}\,\tau_1\,d^3}$ $\Rightarrow$ $l = \frac{\pi\,\tau_1\,d^2}{8\,\tau\,w}$

let $w = \frac{d}{4} \rightarrow$ shaft

$\downarrow$

Key

$$l = \frac{\pi}{8} \times \left(\frac{\tau_1}{\tau}\right)\left(d^2/d/4\right)$$

$$l = \frac{\pi d}{2} \times \left(\tau_1/\tau\right)$$

$\tau_1 \rightarrow$ shear stress of shaft

$\tau \rightarrow$ shear stress of key

when <u>key & shaft are of same material.</u>

$\boxed{\tau_1 = \tau}$ then $\boxed{l = 1.57\,d}$

Effect of Keyways

- The keyway cut into the shaft reduces the load carrying capacity of the shaft.
- This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced.
- Weakening effect of Keyway cut is given by the equation:

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = $\dfrac{\text{Thickness of key } (t)}{2}$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the *angle of twist* is increased in the ratio K_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Shaft Coupling

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following :

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. It should have no projecting parts.

A coupling is termed as a device used to make permanent or semi-permanent connection where as a clutch permits rapid connection or disconnection at the will of the operator.

Requirements of a Good Shaft Coupling

A good shaft coupling should have the following requirements :

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows :

1. Rigid coupling: It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view :

a) Sleeve or muff coupling.
b) Clamp or split-muff or compression coupling, and
c) Flange coupling.

2. Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view :

a) Bushed pin type coupling,
b) Universal coupling, and
c) Oldham coupling.

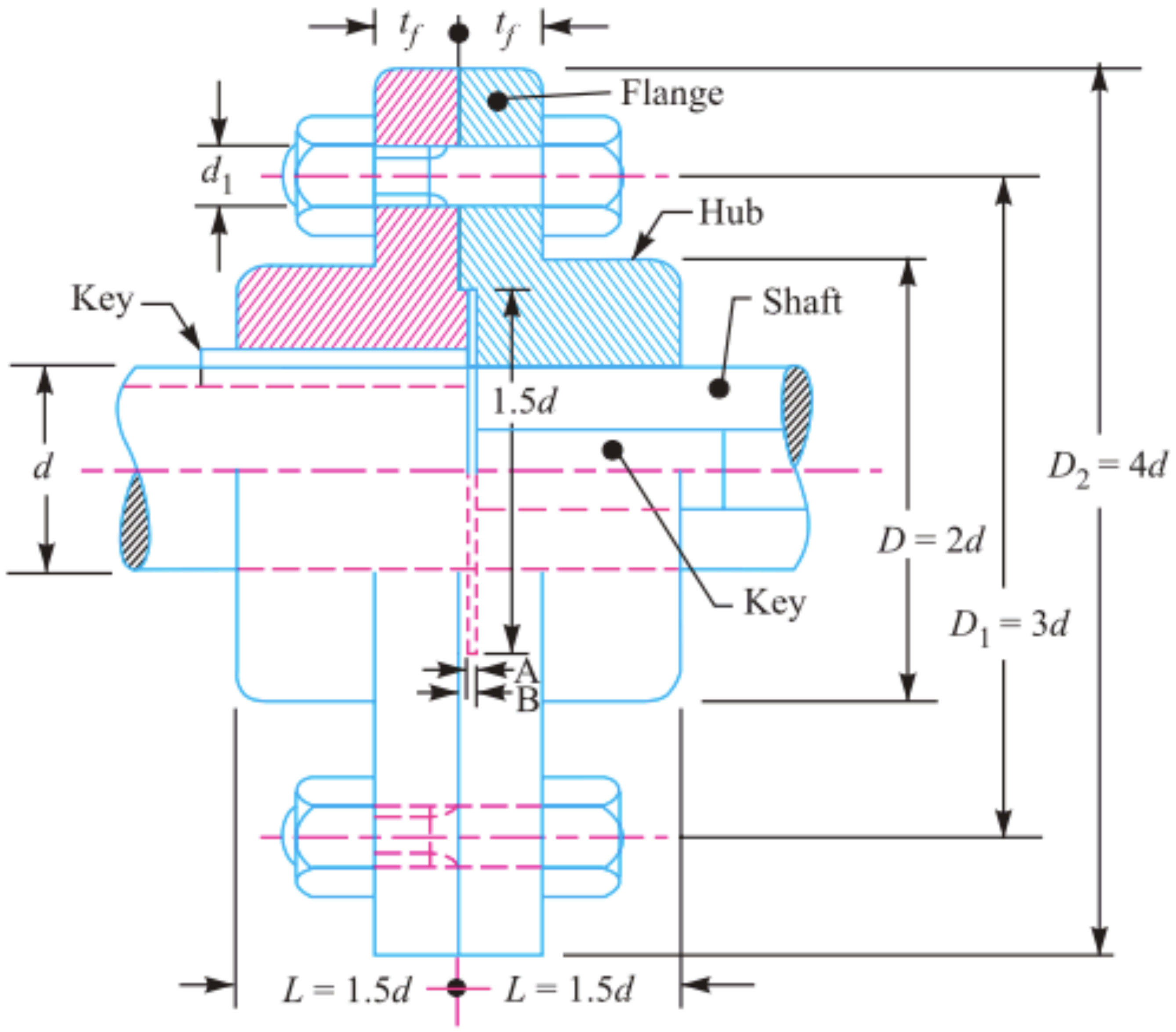
Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess.

This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adopted to heavy loads and hence it is used on large shafting. The flange couplings are of the following three types :

Unprotected type flange coupling. In an unprotected type flange coupling, as shown in Fig, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.

d–dia of shaft



If d is the diameter of the shaft or inner diameter of the hub, then

Outside diameter of hub,

$$D = 2d$$

Length of hub, $L = 1.5\,d$

Pitch circle diameter of bolts,

$$D_1 = 3d$$

Outside diameter of flange,

$$D_2 = D_1 + (D_1 - D) = 2D_1 - D = 4\,d$$

Thickness of flange, $t_f = 0.5\,d$

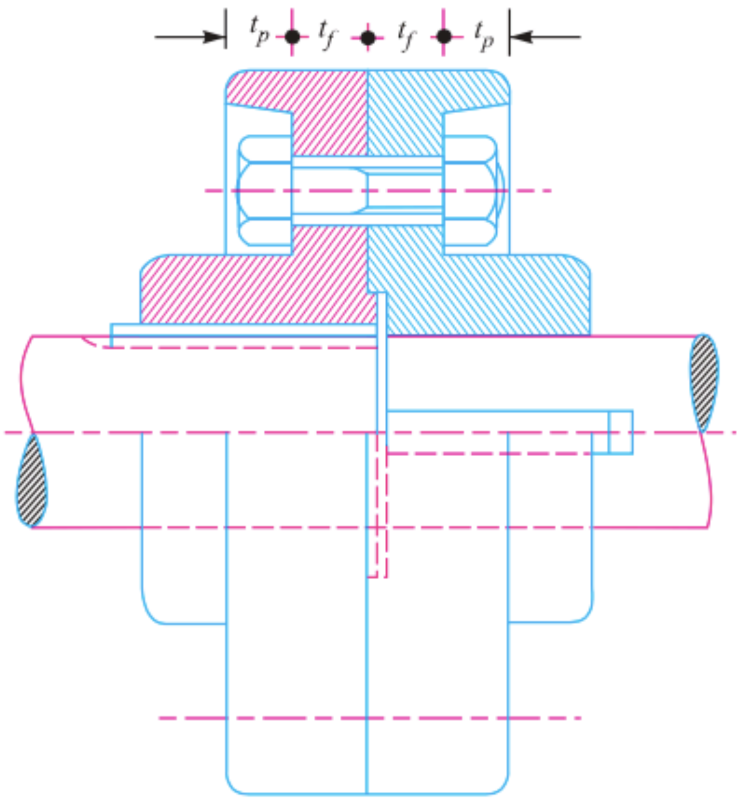
Number of bolts $= 3$, for d upto 40 mm

$= 4$, for d upto 100 mm

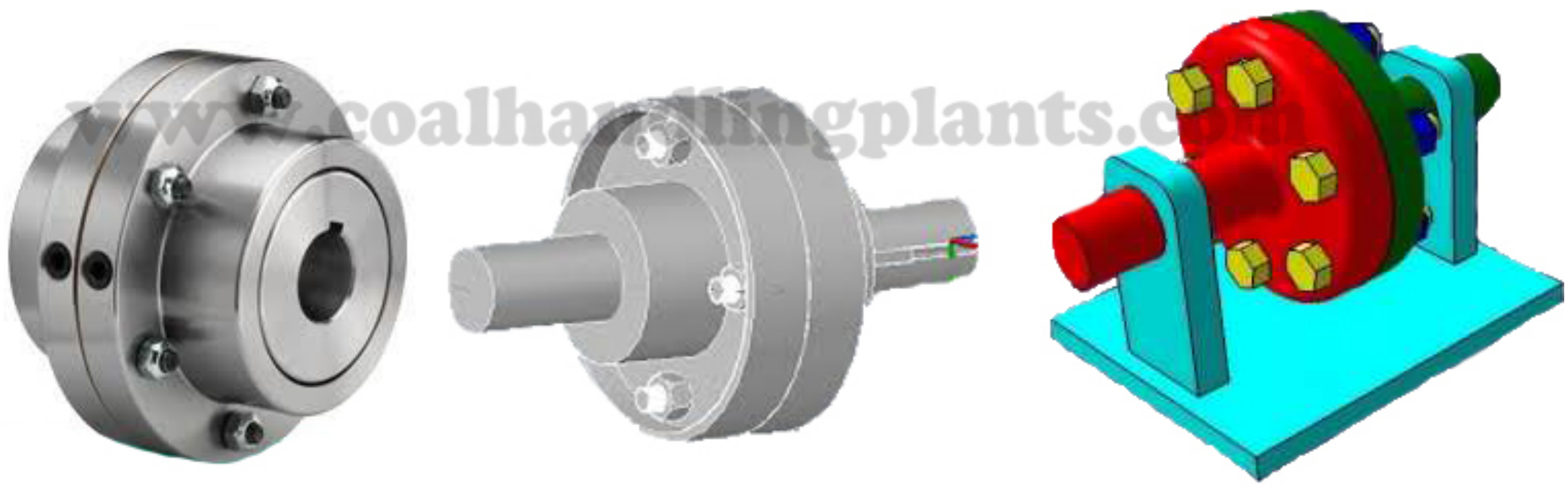
$= 6$, for d upto 180 mm

Protected type flange coupling. In a protected type flange coupling, as shown.

The protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.



The thickness of the protective circumferential flange (tp) is taken as 0.25 d. The other proportions of the coupling are same as for unprotected type flange coupling.



UNPROTECTED TYPE FLANGE COUPLING

PROTECTED TYPE FLANGE COUPLING

MARINE TYPE FLANGE COUPLING

Marine type flange coupling. In a marine type flange coupling, the flanges are forged integral with the shafts. The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft

Table Number of bolts for marine type flange coupling.
(According to IS : 3653 – 1966 (Reaffirmed 1990))

Shaft diameter (mm)	35 to 55	56 to 150	151 to 230	231 to 390	Above 390
No. of bolts	4	6	8	10	12

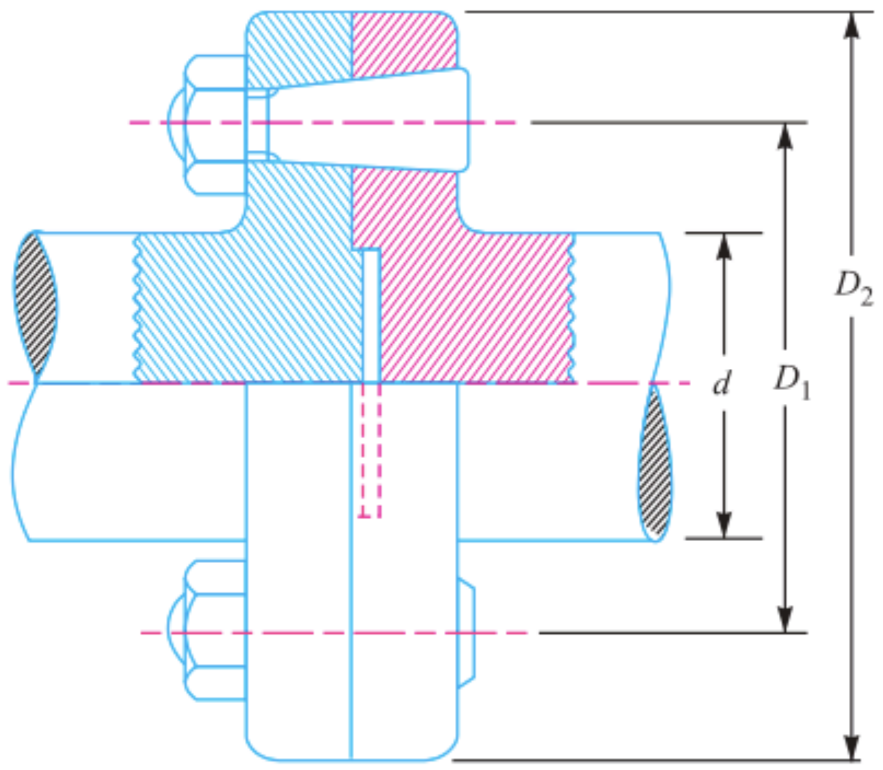
The other proportions for the marine type flange coupling are taken as follows :

Thickness of flange $= d / 3$

Taper of bolt $= 1$ in 20 to 1 in 40

Pitch circle diameter of bolts, $D_1 = 1.6\, d$

Outside diameter of flange, $D_2 = 2.2\, d$



13.17 Design of Flange Coupling

Consider a flange coupling as shown in Fig. 13.12 and Fig. 13.13.

Let d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

d_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively

τ_c = Allowable shear stress for the flange material *i.e.* cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

The flange coupling is designed as discussed below :

1. *Design for hub*

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$\therefore \qquad T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2. Design for key

The key is designed with usual proportions and then checked for shearing and crushing stresses.

The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. Design for flange

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the torque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. *Design for bolts*

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d. We know that

$$\text{Load on each bolt} = \frac{\pi}{4}(d_1)^2\ \tau_b$$

∴ Total load on all the bolts

$$= \frac{\pi}{4}(d_1)^2\ \tau_b \times n$$

and torque transmitted, $$T = \frac{\pi}{4}(d_1)^2\ \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f)\ \sigma_{cb}$$

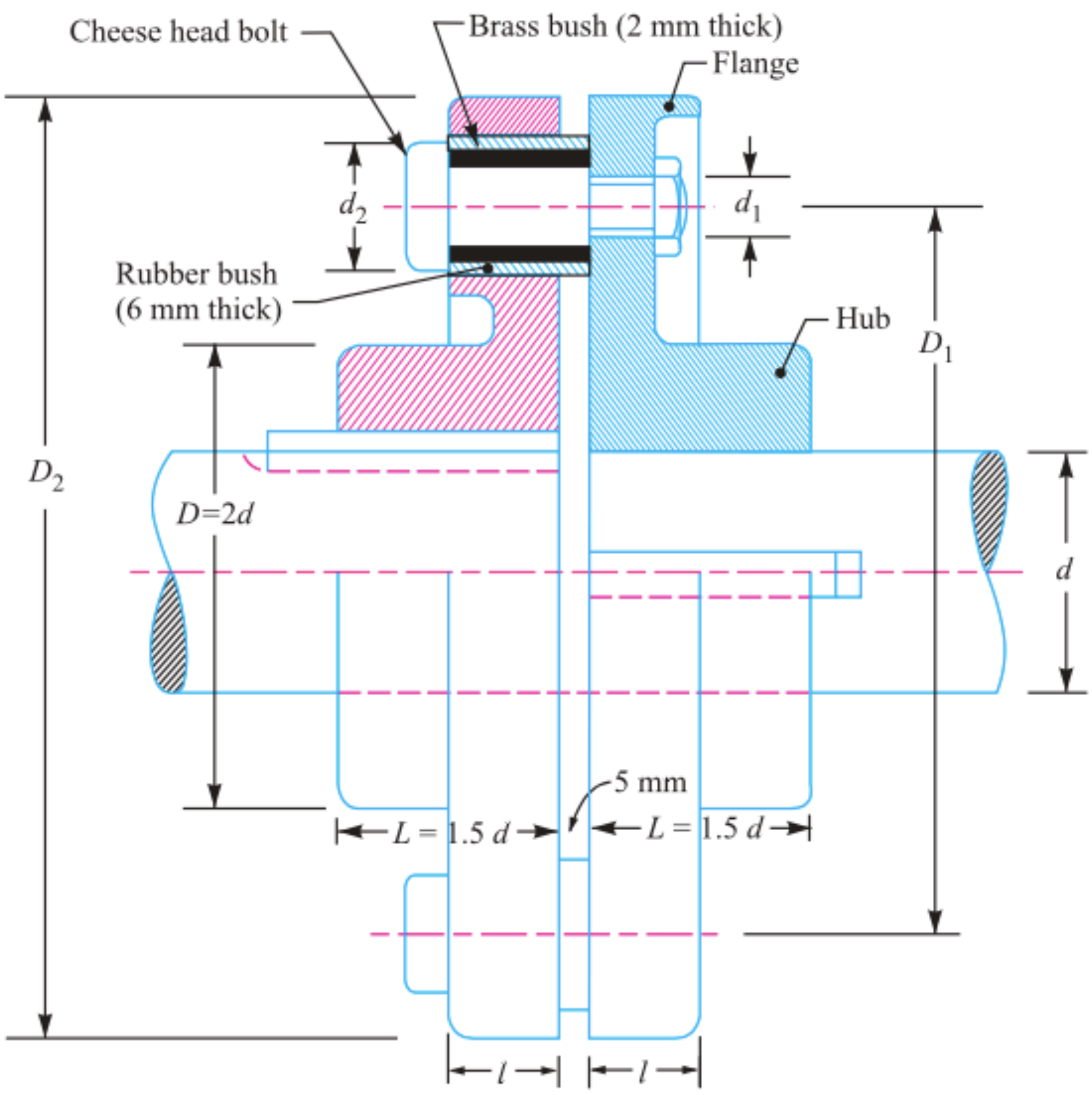
∴ Torque, $$T = (n \times d_1 \times t_f \times \sigma_{cb})\ \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

Flexible Coupling:

Flexible coupling is used so as to permit an axial mis-alignment of the shaft without undue absorption of the power which the shaft are transmitting .

Bushed-pin Flexible Coupling

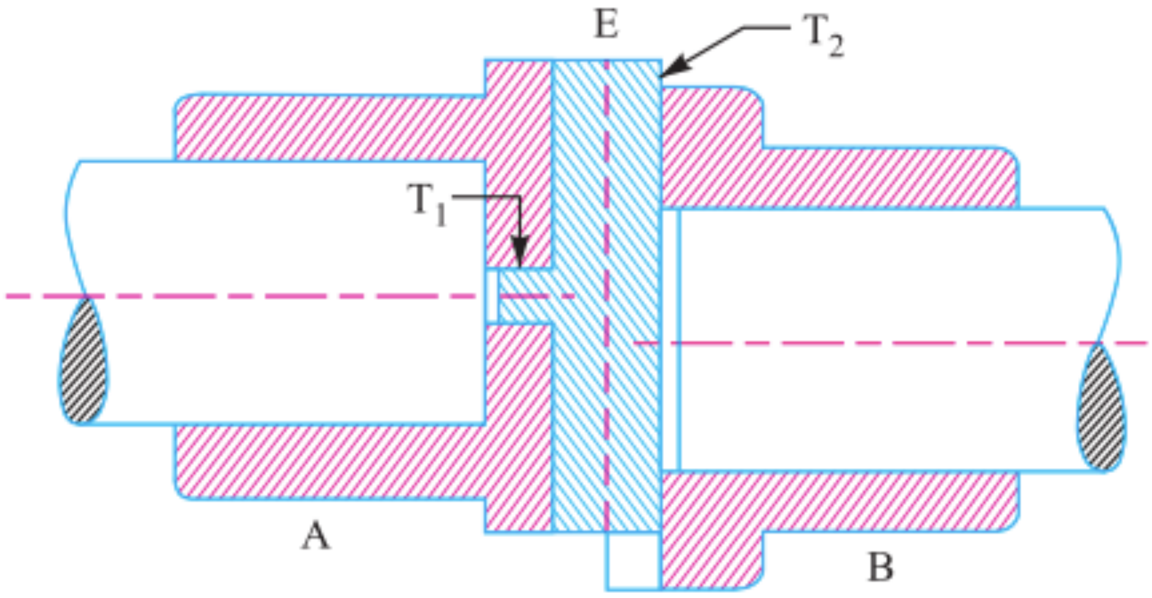


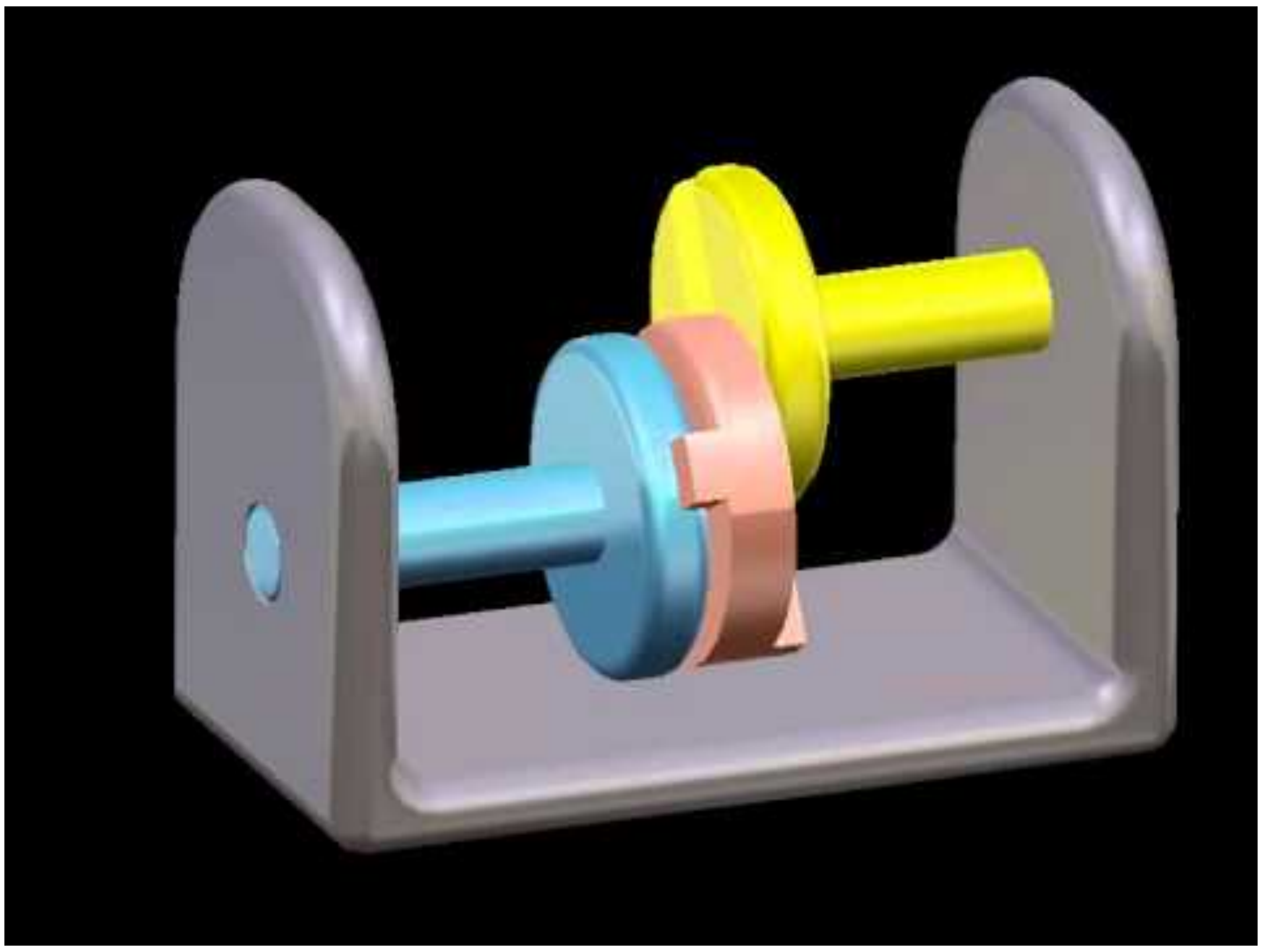
- The coupling bolts are known as pins. The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling.
- There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.

Oldham Coupling

- It is used to join two shafts which have lateral mis-alignment.
- It consists of two flanges A and B with slots and a central floating part E with two tongues T1 and T2 at right angles.
- The central floating part is held by means of a pin passing through the flanges and the floating part. The tongue T1 fits into the slot of flange A and allows for 'to and fro' relative motion of the shafts, while the tongue T2 fits into the slot of the flange B and allows for vertical relative motion of the parts.
- The resultant of these two components of motion will accommodate lateral misalignment of the shaft as they rotate.

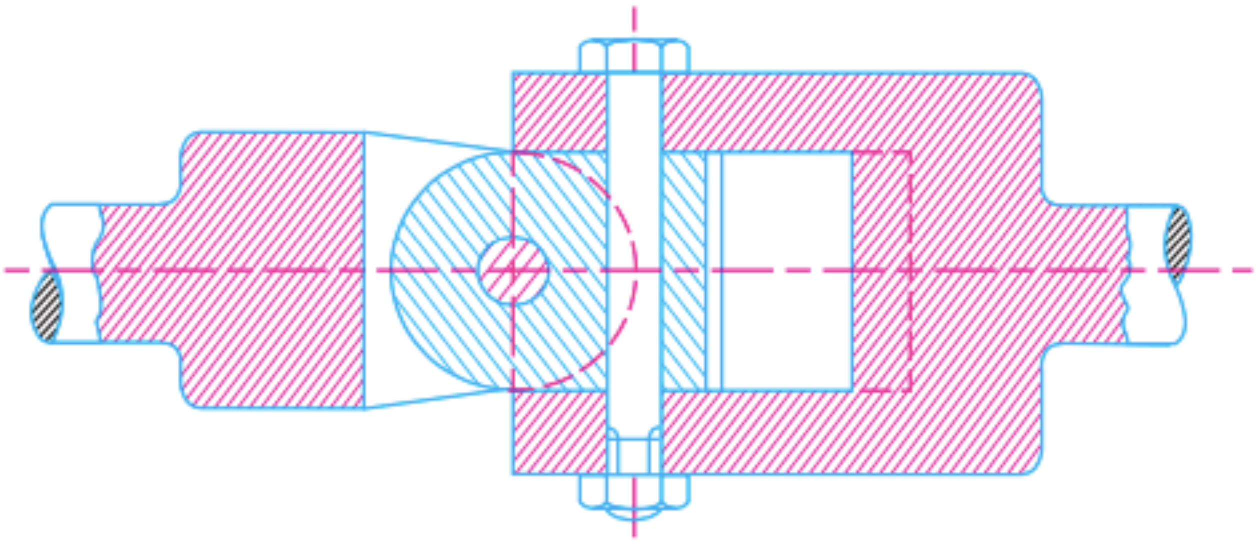
-
-



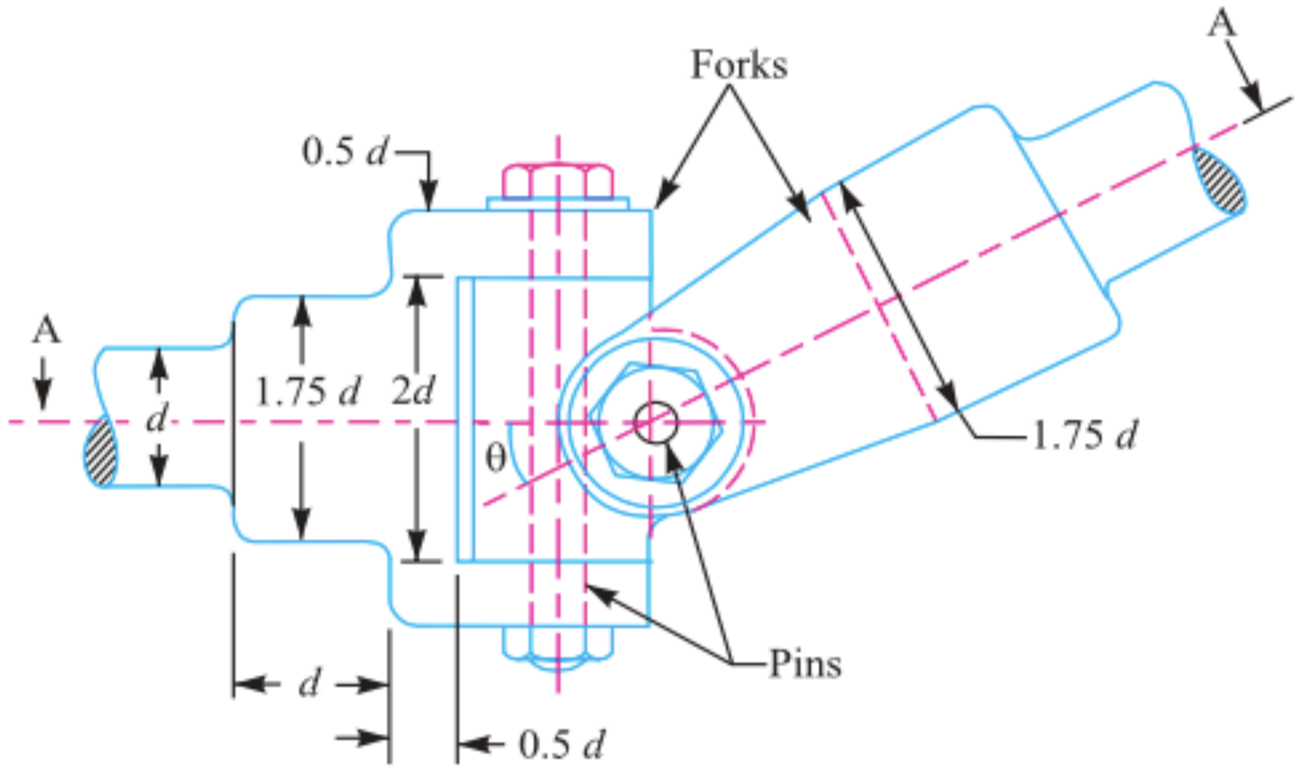


Universal (or Hooke's) Coupling

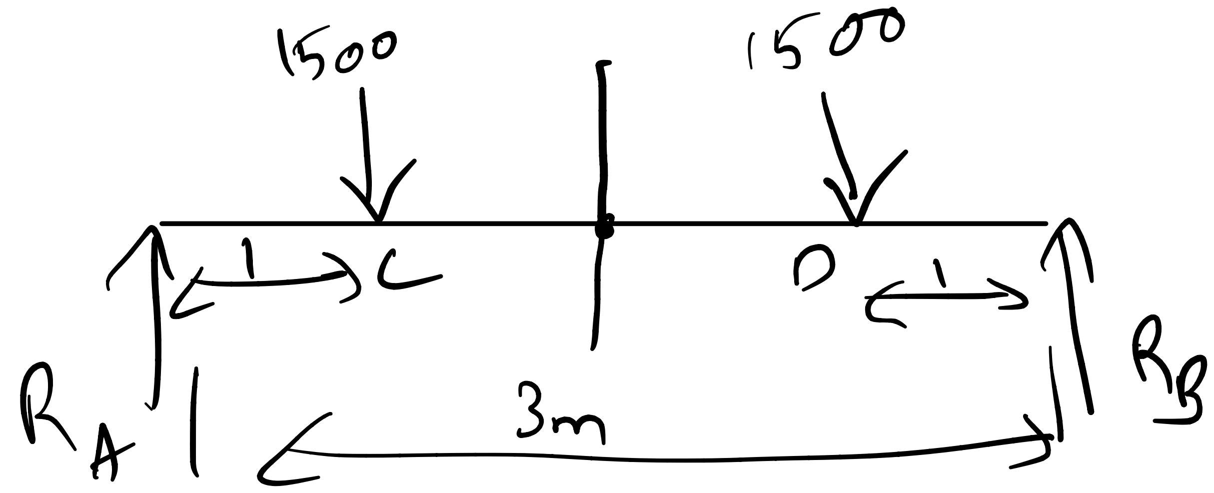
a. A universal or Hooke's coupling is used to connect two shafts whose axes intersect at a small angle.
b. The inclination of the two shafts may be constant, but in actual practice, it varies when the motion is transmitted from one shaft to another.
c. The main application of the universal or Hooke's coupling is found in the transmission from the gear box to the differential or back axle of the automobiles. In such a case, we use two Hooke's coupling, one at each end of the propeller shaft, connecting the gear box at one end and the differential on the other end.
d. A Hooke's coupling is also used for transmission of power to different spindles of multiple drilling machine. It is used as a knee joint in milling machines.



Section A – A



Example A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.



$M_{max} =$

$T = \frac{60P}{2\pi N}$

$P = \frac{2\pi NT}{60}$

$T = \frac{60 \times 100 \times 10^3}{2\pi \times 300}$

$T =$

$T_e =$

assume

$\tau = 60\ N/mm^2$

Example *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

Table 13.1. Proportions of standard parallel, tapered and gib head keys.

Shaft diameter (mm) upto and including	*Key cross-section*		*Shaft diameter (mm) upto and including*	*Key cross-section*	
	Width (mm)	*Thickness (mm)*		*Width (mm)*	*Thickness (mm)*
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.

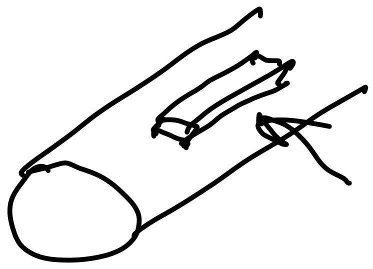
from the table width $w = 16mm$

thickness $t = 10mm$ $\tau_c = 42 mpa$

Shearing Key Torque $T = l \times w \times \tau \times d/2$

Shaft Torque $T = \frac{\pi}{16} \tau d^3 = \frac{\pi}{16} \times 42 \times (50)^3$

Shaft $T = 1.03 \times 10^6$ N-mm



key $T = l \times 16 \times 42 \times 50/2$

$T = 16800\, l$

$1.03 \times 10^6 = 16,800 \times l$

$\underline{l = 61.30 mm}$ → shear

Crushing torque $T = l \times \frac{t}{2} \times \sigma_c \times d/2$

$= l \times \left(\frac{10}{2}\right) \times (70) \times (50/2)$

$= 8750\, l$

$8750\, l = 1.03 \times 10^6$

$\underline{l = 117.74mm}$ → crushing

$\boxed{l = 117.8 mm}$

A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Maximum stress for shaft $\tau_{max} = \dfrac{\sigma_{yt}}{2 \times F.S} = \dfrac{400}{2 \times 2} = 100 \ N/mm^2$

Maximum stress for key $\tau_k = \dfrac{\sigma_{yt}}{2 \times F.S} = \dfrac{340}{2 \times 2} = 85 \ N/mm^2$

$$T = \frac{\pi}{16} \times \tau \times d^3 = L \times w \times \tau \times \frac{d}{2}$$

$$L = 67.2 \text{ mm}$$

$$L_{crushing} = 104.6 \text{ mm}$$

A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

Solution. Given : P = 15 kW = 15 × 10³ W ; N = 960 r.p.m. ; d = 40 mm ; l = 75 mm ; τ = 56 MPa = 56 N/mm² ; σ_c = 112 MPa = 112 N/mm²

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$$\therefore \quad w = 149 \times 10^3 / 84 \times 10^3 = 1.8 \text{ mm}$$

This width of keyway is too small. The width of keyway should be at least $d/4$.

$$\therefore \quad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm}$$ **Ans.**

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \quad \ldots (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\; 0.8125 = 571\,844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47$$ **Ans.**

Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used

Shear stress for shaft, bolt and key material	=	*40 MPa*
Crushing stress for bolt and key	=	*80 MPa*
Shear stress for cast iron	=	*8 MPa*

Draw a neat sketch of the coupling.

Design for hub:

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 900}$$

$$T = 159.15 \text{ N-m}$$

$$T_{max} = T \times 1.35 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

$$T = \frac{\pi}{16} \tau d^3 \Rightarrow 214.85 \times 10^3 = \frac{\pi}{16} \times d^3 \times 40$$

$$d = 30.13 \text{ mm} \simeq 35 \text{ mm}$$

outer dia of hub $D = 2d = 70 \text{ mm}$

Length of hub $l = 1.5d = 52.5 \text{ mm}$

For a hollow shaft

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - d_i^4}{D_o} \right]$$ ~~dt~~ $d_i = 35\,mm$

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \times \left[\frac{(70)^4 - (35)^4}{70} \right]$$

$\tau_c = 3.405\ N/mm^2$ 3.4 mpa //

Induced shear stress < shear stress of C.I

Hub design is safe

Design for key:

Since $\sigma_c = 2\tau_c$, the width & thickness of key will be same

Hence $w = 12mm$

$t = 12mm$

$T = l \times w \times \tau \times \frac{d}{2} \Rightarrow \tau_s = \frac{2 \times T}{l \times w \times d}$

$\tau_s = 19.5 mpa$

$l = 52.5 mm$

$$\tau_c = \frac{215\times10^3\times 2}{52.5\times 12\times 35} = 19.5\ \text{mpa}$$

In terms of crushing

$$T = l\times\frac{t}{2}\times\sigma_c\times\frac{d}{2}$$

$$\boxed{(\sigma_c)_{induced} = 39\ \text{mpa}}$$

Design for flange:

$$t_f = 0.5d = 0.5\times35 = 17.5\ \text{mm}$$

Torque transmitted

$$T = \pi D\times t_f\times\tau\times\frac{D}{2} = \frac{\pi D^2}{2}\times t_f\times\tau$$

$$210\times10^3 = \frac{\pi(70)^2}{2}\times17.5\times\tau$$

$$\boxed{\tau = 1.56\ \text{mpa}} < 8\ \text{mpa}$$

Design for flange is safe

Design for bolts :

$d_1 \rightarrow$ dia of bolts

$n = 3 \rightarrow$ no. of bolts

$D_1 \rightarrow$ pitch circle dia of bolts $= 3d$

$= 105 \text{ mm}$

Torque transmitted

$$T = 210 \times 10^3 \text{ N-mm}$$

$$T = \frac{\pi}{4} (d_1)^2 \times \tau_b \times \frac{D_1}{2} \times n$$

$$210 \times 10^3 = \frac{\pi}{4} (d_1)^2 \times \frac{105}{2} \times 3 \times 40$$

$$d_1 = 6.51 \text{ mm}$$

Assuming coarse threaded.

The bolt size is $\underline{M8}$.

outside dia of flange $D_2 = 4d = 4 \times 35 = 140 \text{ mm}$

Design and draw a cast iron flange coupling for a mild steel shaft transmitting 90 kW at 250 r.p.m. The allowable shear stress in the shaft is 40 MPa and the angle of twist is not to exceed 1° in a length of 20 diameters. The allowable shear stress in the coupling bolts is 30 MPa.

$$T = \frac{\pi}{16} \tau d^3$$

$P = T \times 2\omega$

$$T = \frac{P}{\frac{2\pi N}{60}}$$

$l = 20d$

$\theta = 1^\circ$

$= \frac{\pi}{180}$ rad

$T = 3.43 \times 10^3$ N-m

$= 3437 \times 10^3$ N-mm

$\boxed{180^\circ = \pi \text{ rad}}$

$= 0.0174$ rad

$$3437 \times 10^3 = \frac{\pi}{16} \times 40 \times d^3 \Rightarrow d = 76 \text{ mm}$$

$$\frac{T}{J} = \frac{\cancel{\tau}}{\cancel{R}} = \frac{C \cdot \theta}{l}$$

$$\frac{T}{J} = \frac{C \cdot \theta}{l}$$

84×10^3 N/mm²

$\Rightarrow$

$C \rightarrow$ cast iron $\rightarrow 84$ GPa

$$J = \frac{\pi}{32} d^4$$

$$\frac{3437 \times 10^3}{\frac{\pi}{32} d^{4}} = \frac{84 \times 10^3 \times 0.0174}{20 \times d}$$

$$d^3 = \frac{32}{\pi} \times \frac{3437 \times 20}{84 \times 0.0174}$$

$d = 80$ mm

1) Design for hub:

$D = 2d$

$L = 1.5d = 120$ mm

$D_o = D = 2d = 160$ mm

$D_i = d = 80$ mm

$$T = \frac{\pi}{16} \tau_c \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

$$3437 \times 10^3 = \frac{\pi}{16} \times \tau_c \times \left[\frac{(160)^4 - (80)^2}{160} \right] = 4.56 \text{ mpa}$$

Our design is safe

2) Design for Key:

From tables $w = 25$ mm

$t = 14$ mm

l = length of hub = 120 mm

$T = l \times w \times d/2 \times \tau$ $\tau < 40$ mpa then design is safe

$$3437 \times 10^3 = 120 \times 25 \times 40 \times \tau$$

$$\tau = 28.6 \text{ mpa} < 40$$

3) Design for Flange:

$$T = \frac{\pi D^2}{2} \times t_f \times \tau_c$$

$t_f = 0.5d$

$= 40$ mm

Unit 3

Permanent Joints

Part 1: Riveted Joints

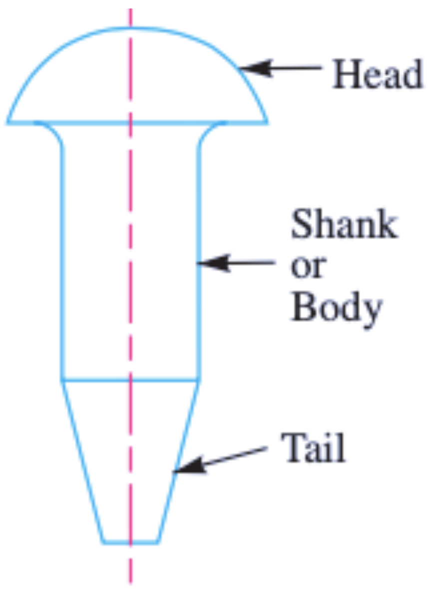
The fastenings (*i.e.* joints) may be classified into the following two groups :

1. **Permanent fastenings, and**
2. **Temporary or detachable fastenings.**

- The ***permanent fastenings*** are those fastenings which cannot be disassembled without destroying the connecting components.
    1. The examples of permanent fastenings in order of strength are **soldered, brazed, welded and riveted joints.**

- The ***temporary*** or ***detachable fastenings*** are those fastenings which can be disassembled without destroying the connecting components.
    1. The examples of temporary fastenings are **screwed, keys, cotters, pins and splined joints.**

A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called **shank** or **body** and lower portion of shank is known as **tail**



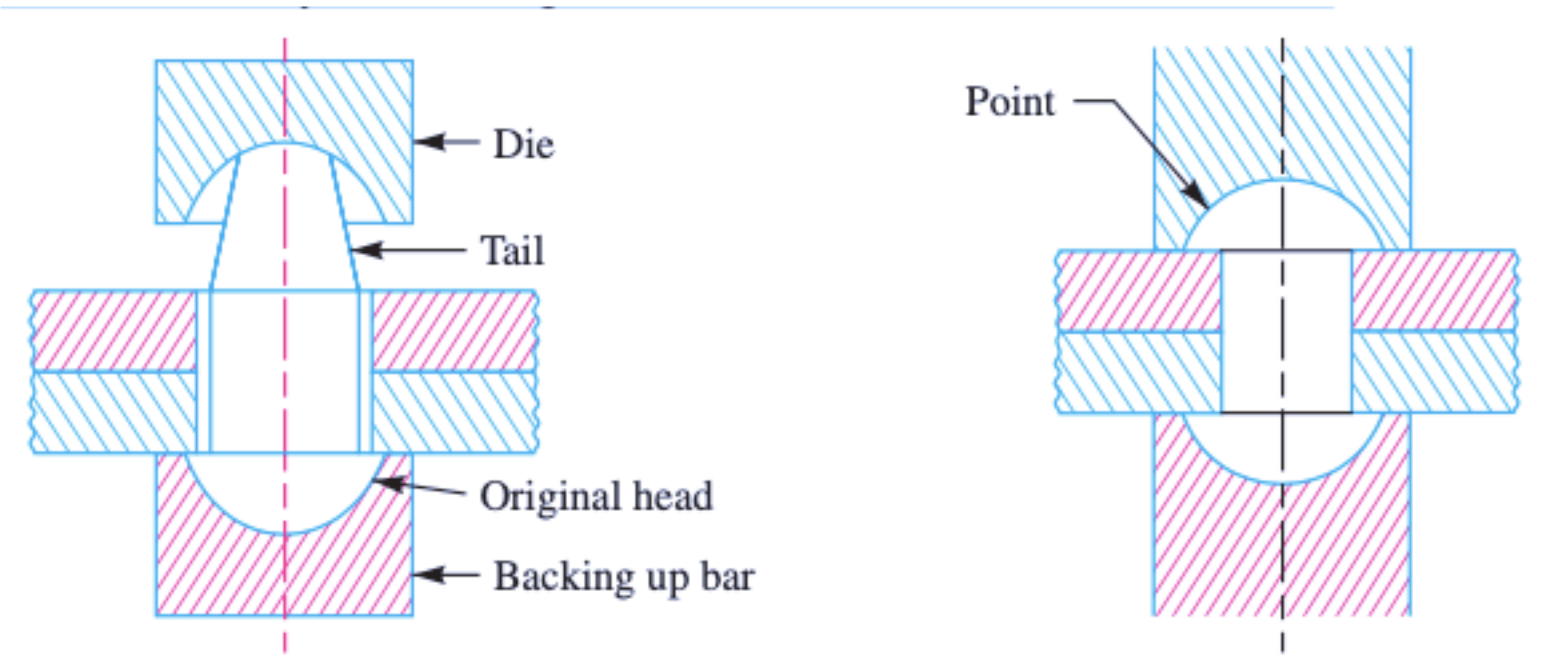
The rivets are used to make permanent fastening between the plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals.

Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the joint. The tightness is necessary in order to contribute to strength and to prevent leakage as in a boiler or in a ship hull.

When two plates are to be fastened together by a rivet as shown in Fig., the holes in the plates are punched and reamed or drilled. **Punching** is the cheapest method and is used for relatively thin plates and in structural work.

Since ***punching injures the material*** around the hole, therefore drilling is used in most pressure-vessel work. In structural and pressure vessel riveting, the diameter of the rivet hole is usually 1.5 mm larger than the nominal diameter of the rivet.



(*a*) Initial position. (*b*) Final position.

A cold rivet or a red hot rivet is introduced into the plates and the ***point*** (*i.e.* second head) is then formed. When a cold rivet is used, the process is known as ***cold riveting*** and when a hot rivet is used, the process is known as ***hot riveting.*** The cold riveting process is used for structural joints while hot riveting is used to make leak proof joints.

Material of Rivets

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminium or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used.

The rivets for general purposes shall be manufactured from steel conforming to the following Indian Standards :

1. (*a*) IS : 1148–1982 (Reaffirmed 1992) – Specification for hot rolled rivet bars (up to 40 mm diameter) for structural purposes; or
2. (*b*) IS : 1149–1982 (Reaffirmed 1992) – Specification for high tensile steel rivet bars for structural purposes.

The rivets for boiler work shall be manufactured from material conforming to IS : 1990 – 1973 (Reaffirmed 1992) – Specification for steel rivets and stay bars for boilers.

- The steel for boiler construction should conform to IS : 2100 – 1970 (Reaffirmed 1992) – Specification for steel billets, bars and sections for boilers.

Essential Qualities of a Rivet

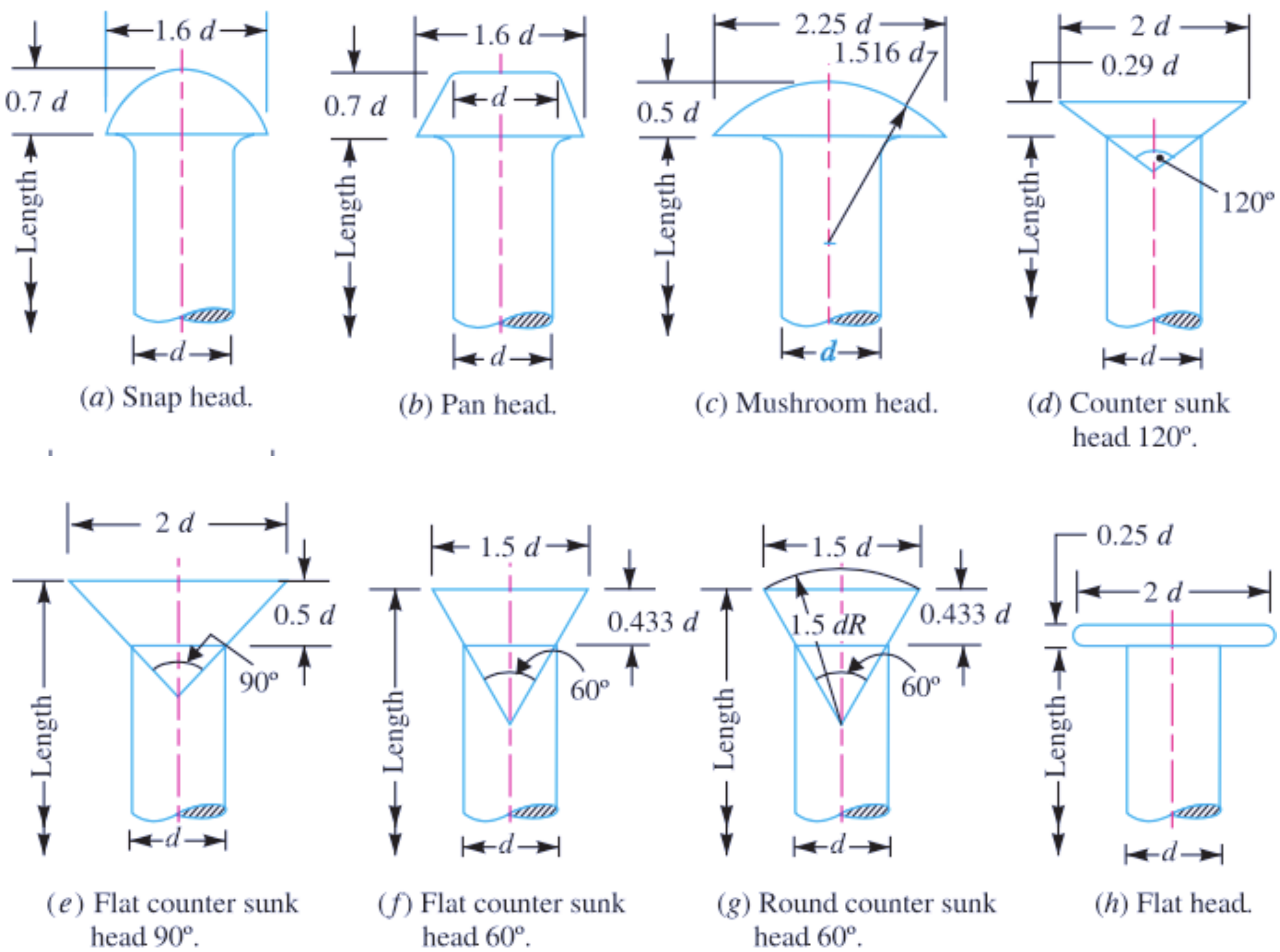
According to Indian standard, IS : 2998 – 1982 (Reaffirmed 1992), the material of a rivet must have

a) a tensile strength not less than 40 N/mm^2 and
b) elongation not less than 26 percent.
c) The material must be of such quality that when in cold condition, the shank shall be bent on itself through 180° without cracking and after being heated to 650°C and quenched, it must pass the same test.
d) The rivet when hot must flatten without cracking to a diameter 2.5 times the diameter of shank.

Manufacture of Rivets

According to Indian standard specifications, the rivets may be made either by cold heading or by hot forging. If rivets are made by the cold heading process, they shall subsequently be adequately heat treated so that the stresses set up in the cold heading process are eliminated. If they are made by hot forging process, care shall be taken to see that the finished rivets cool gradually.

Types of Rivet Heads



The **snap heads** are usually employed for structural work and machine riveting.

The **counter sunk heads** are mainly used for ship building where flush surfaces are necessary.

The **conical heads** (also known as **conoidal heads)** are mainly used in case of hand hammering.

The **pan heads** have maximum strength, but these are difficult to shape.

Types of Riveted Joints

Following are the two types of riveted joints, depending upon the way in which the plates are connected.

1. Lap joint, and 2. Butt joint.

- Lap Joint

A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

- Butt Joint

A butt joint is that in which the main plates are kept in alignment butting (*i.e.* touching) each other and a cover plate (*i.e.* strap) is placed either on one side or on both sides of the main plates.

The cover plate is then riveted together with the main plates. Butt joints are of the following two types :

1. Single strap butt joint, and

2. Double strap butt joint.

In a ***single strap butt joint*,** the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

In a ***double strap butt joint*,** the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

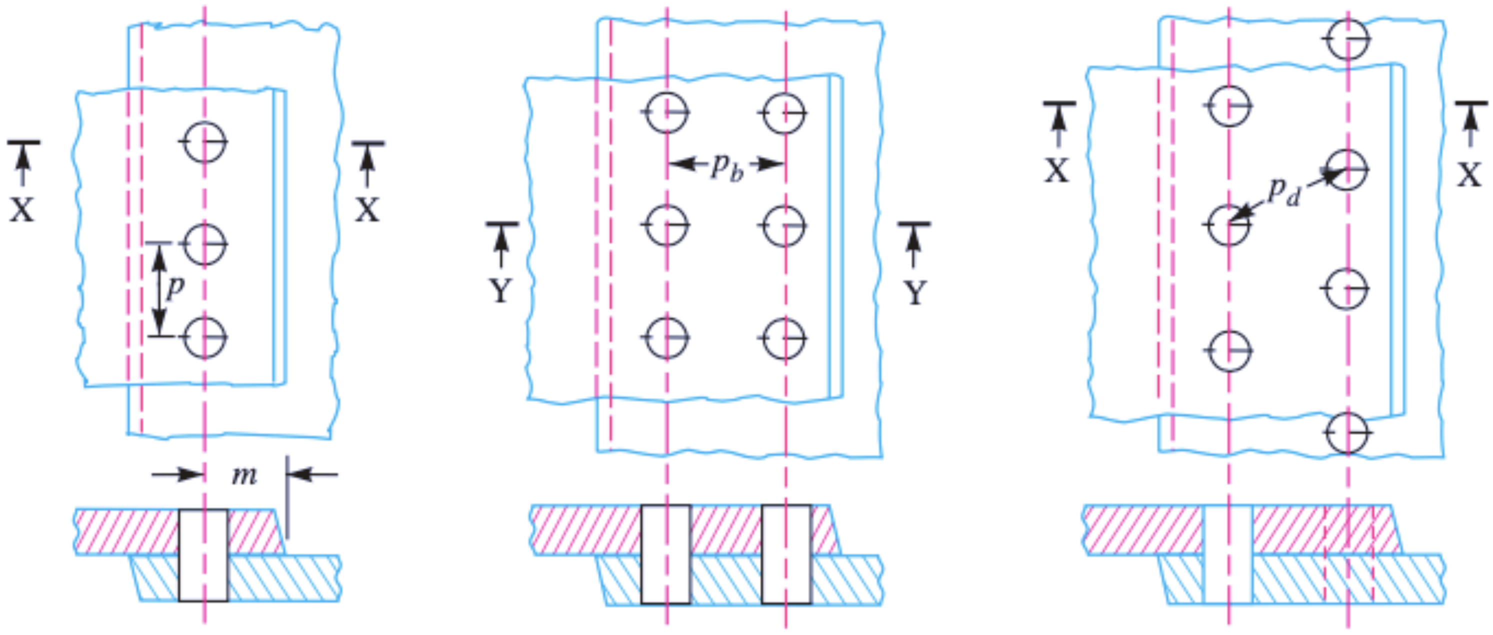
In addition to the above, following are the types of riveted joints depending upon the number of rows of the rivets.

1. Single riveted joint, and

2. Double riveted joint.

A ***single riveted joint*** is that in which there is a single row of rivets in a lap joint as shown in Fig. 9.6 (*a*) and there is a single row of rivets on each side in a butt joint as shown in Fig. 9.8.

A ***double riveted joint*** is that in which there are two rows of rivets in a lap joint as shown in Fig. 9.6 (*b*) and (*c*) and there are two rows of rivets on each side in a butt joint as shown in Fig. 9.9.

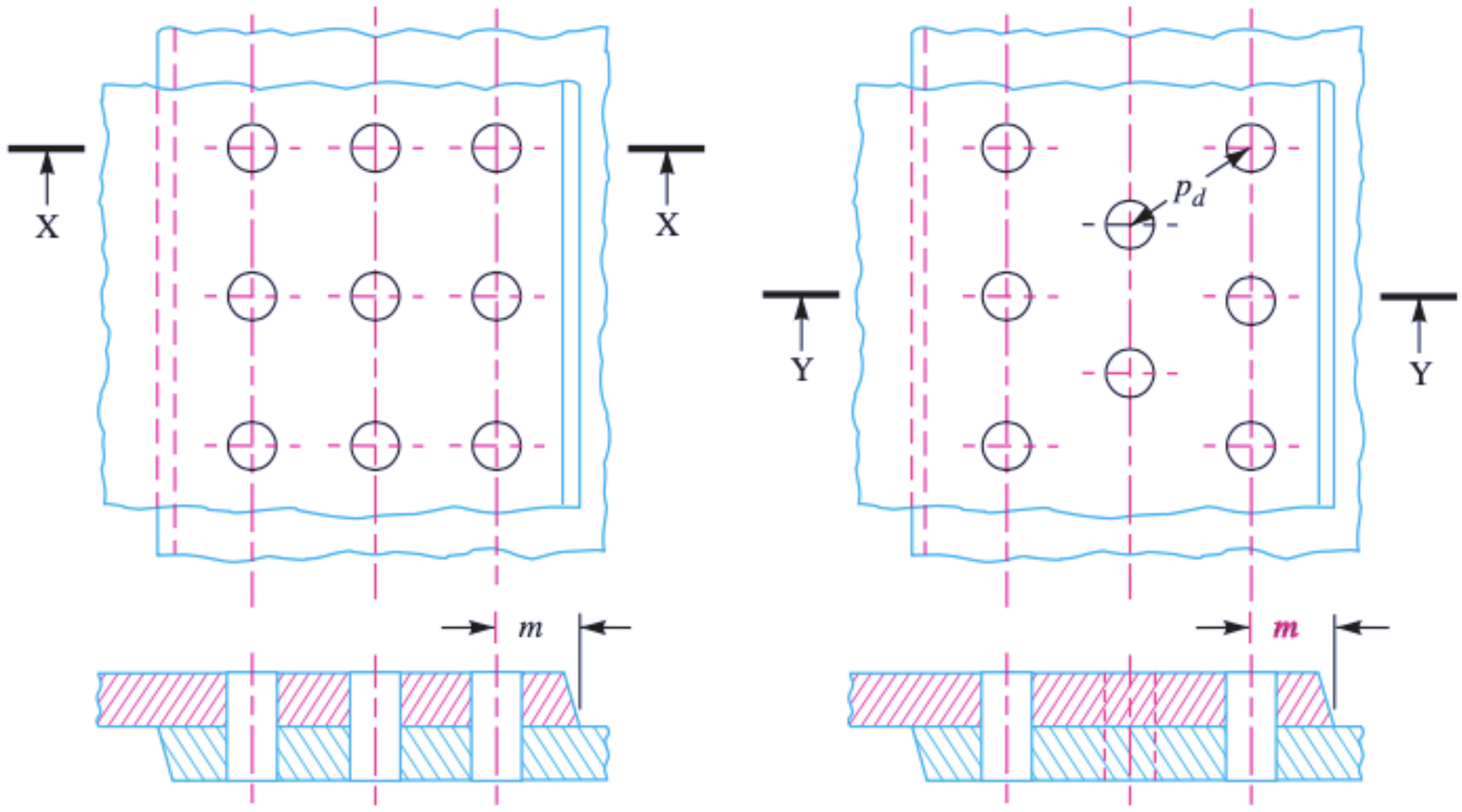


(*a*) Single riveted lap joint.

(*b*) Double riveted lap joint (Chain riveting).

(*c*) Double riveted lap joint (Zig-zag riveting).

Fig. 9.6. Single and double riveted lap joints.



(*a*) Chain riveting.

(*b*) Zig-zag riveting.

Fig. 9.7. Triple riveted lap joint.

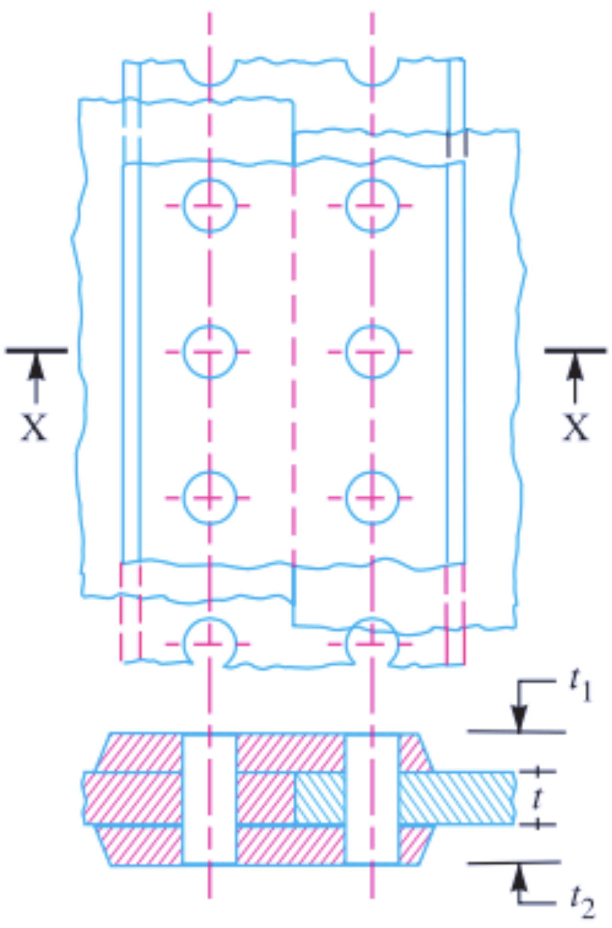
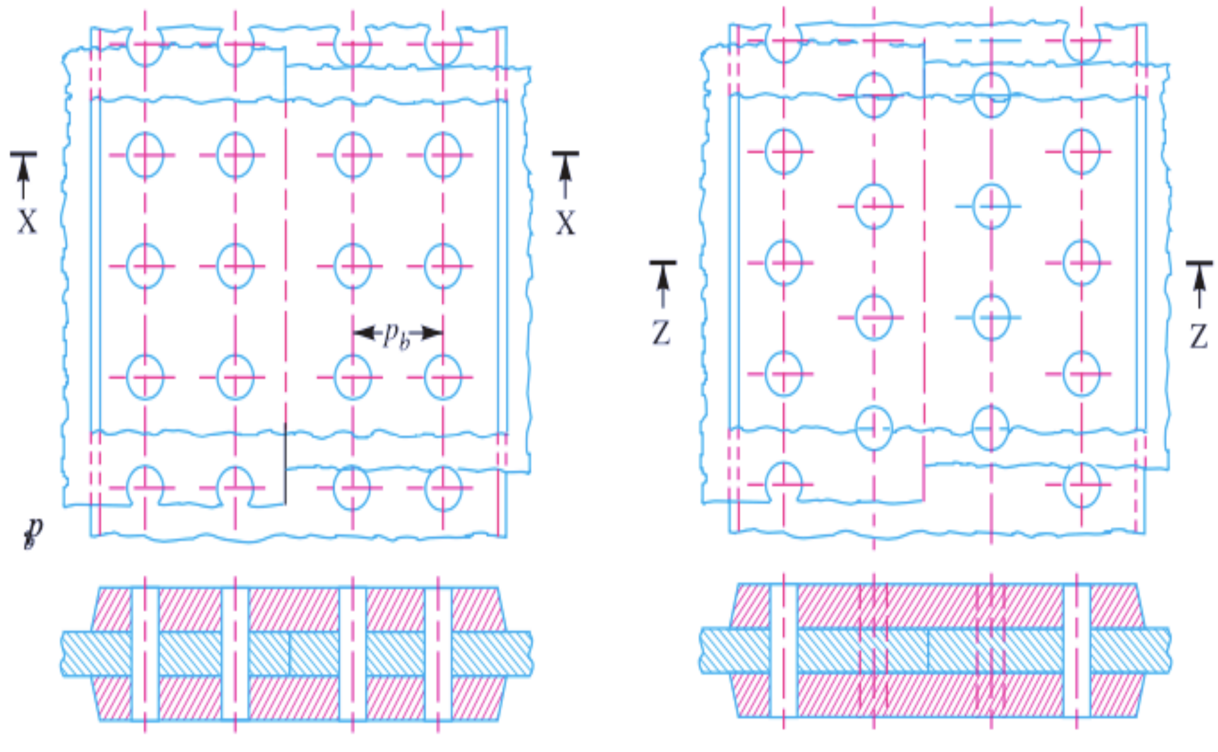


Fig. 9.8. Single riveted double strap butt joint.



(*a*) Chain riveting.

(*b*) Zig-zag riveting.

Fig. 9.9. Double riveted double strap (equal) butt joints.

Important Terms Used in Riveted Joints

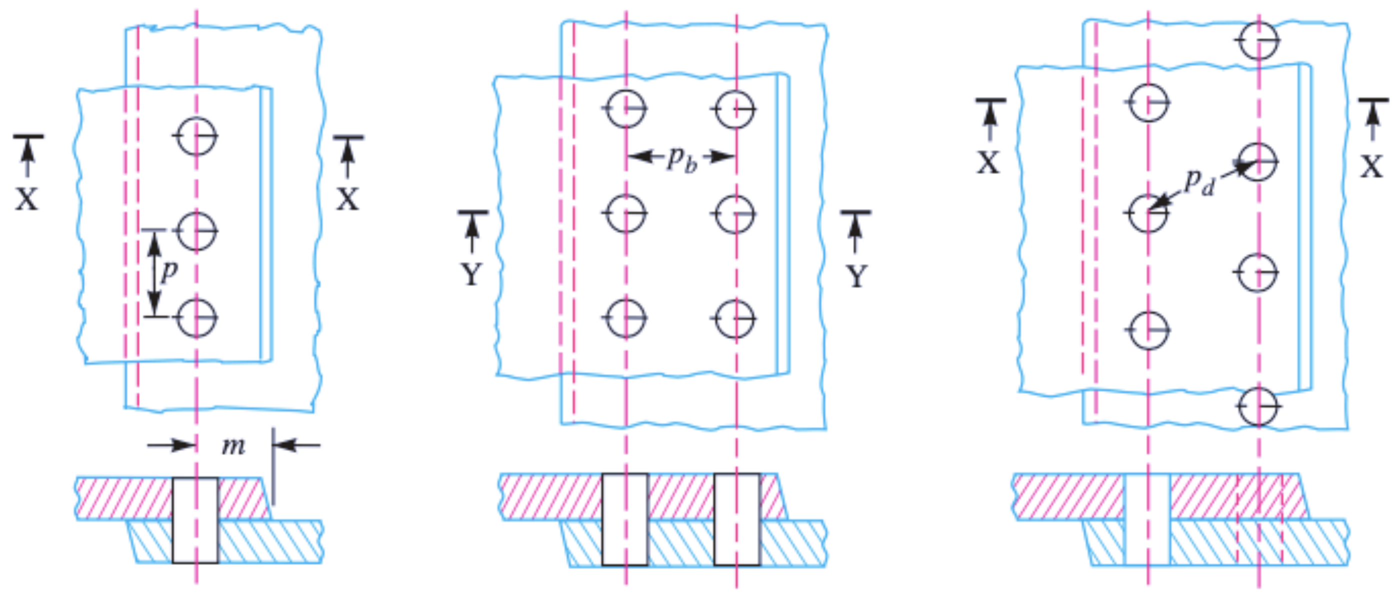
The following terms in connection with the riveted joints are:

1. Pitch. It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the seam. It is usually denoted by p.

2. Back pitch. It is the perpendicular distance between the centre lines of the successive rows. It is usually denoted by p_b.

3. Diagonal pitch. It is the distance between the centres of the rivets in adjacent rows of zig-zag riveted joint. It is usually denoted by p_d.

4. Margin or marginal pitch. It is the distance between the centre of rivet hole to the nearest edge of the plate. It is usually denoted by m.



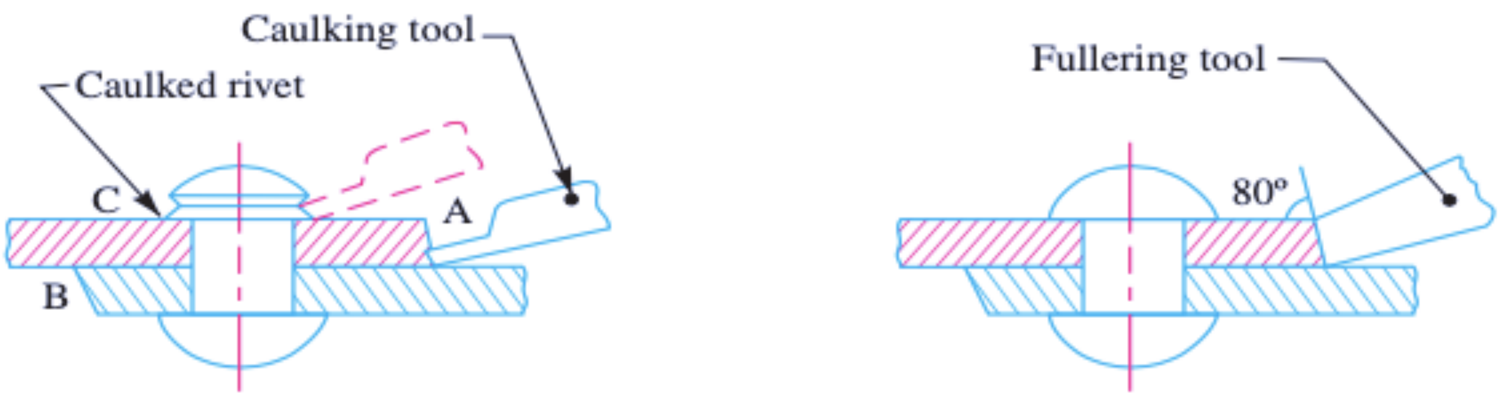
(*a*) Single riveted lap joint. (*b*) Double riveted lap joint (Chain riveting). (*c*) Double riveted lap joint (Zig-zag riveting).

Caulking and Fullering

In order to make the joints leak proof or fluid tight in pressure vessels like steam boilers, air receivers and tanks etc. a process known as **caulking** is employed. In this process, a narrow blunt tool called caulking tool, about 5 mm thick and 38 mm in breadth, is used. The edge of the tool is ground to an angle of 80°. The tool is moved after each blow along the edge of the plate, which is planed to a bevel of 75° to 80° to facilitate the forcing down of edge.

The tool burrs down the plate at A forming a metal to metal joint. In actual practice both edges A & B are caulked. The head of the rivets as shown at C are also turned down with a caulking tool to make a joint steam tight.

A more satisfying way to make the joints air tight is fullering. A fullering tool with a thickness at the end equal to that of the plate is used in such a way that the greatest pressure due to the blows occur near the joint, giving a clean finish, with less risk of damaging the plate



(*a*) Caulking. (*b*) Fullering.

Failures of a Riveted Joint

A riveted joint may fail in the following ways :

1. ***Tearing of the plate at an edge.*** A joint may fail due to tearing of the plate at an edge as shown in Fig. 9.13. This can be avoided by keeping the margin, $m = 1.5d$, where d is the diameter of the rivet hole.

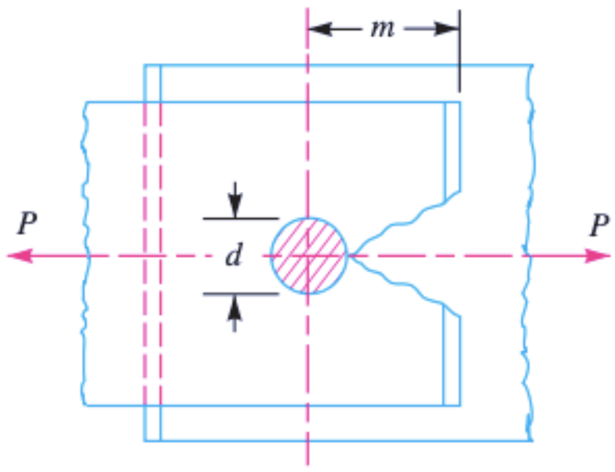


Fig. 9.13. Tearing of the plate at an edge.

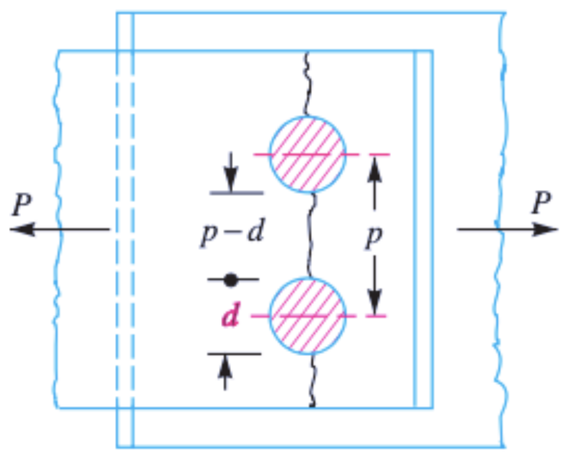


Fig. 9.14. Tearing of the plate across the rows of rivets.

2. ***Tearing of the plate across a row of rivets.*** Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. 9.14. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

The resistance offered by the plate against tearing is known as ***tearing resistance*** or ***tearing strength*** or ***tearing value*** of the plate.

Let p = Pitch of the rivets,

d = Diameter of the rivet hole,

t = Thickness of the plate, and

σ_t = Permissible tensile stress for the plate material.

We know that tearing area per pitch length,

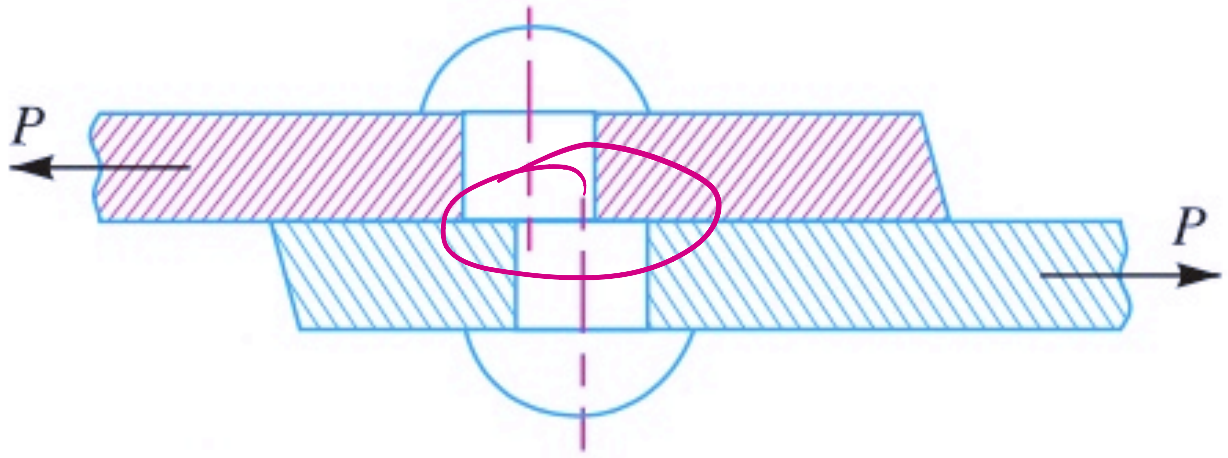
$$A_t = (p - d)\,t$$

Pitch

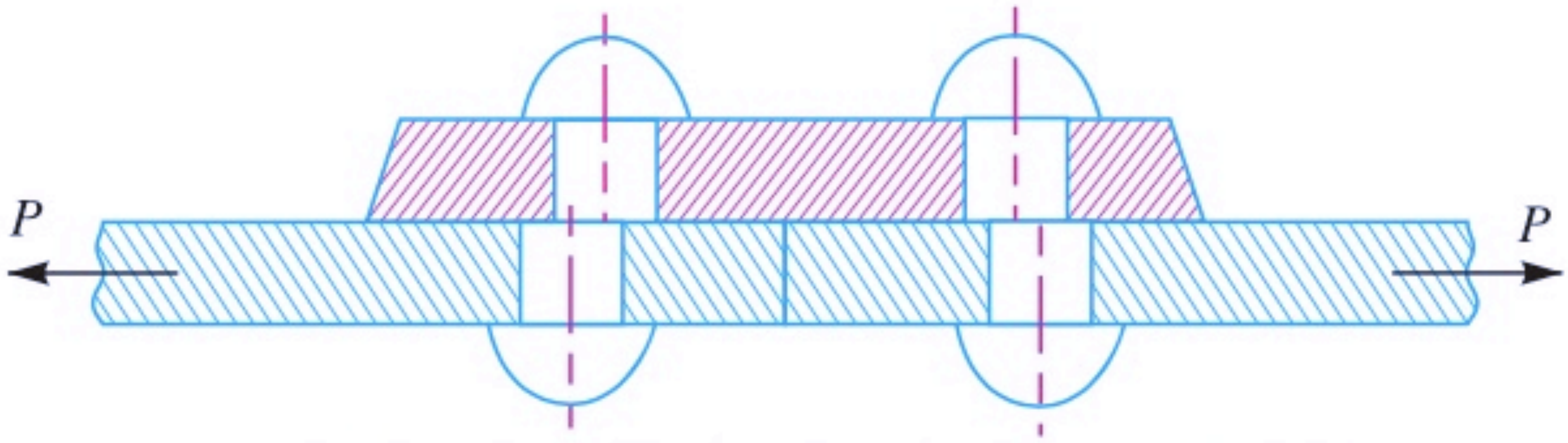
∴ Tearing resistance or pull required to tear off the plate per pitch length,

$$P_t = A_t.\sigma_t = (p - d)t.\sigma_t$$

When the tearing resistance (P_t) is greater than the applied load (P) per pitch length, then this type of failure will not occur.



(*a*) Shearing off a rivet in a lap joint.



(*b*) Shearing off a rivet in a single cover butt joint.

Fig. 9.15. Shearing of rivets.

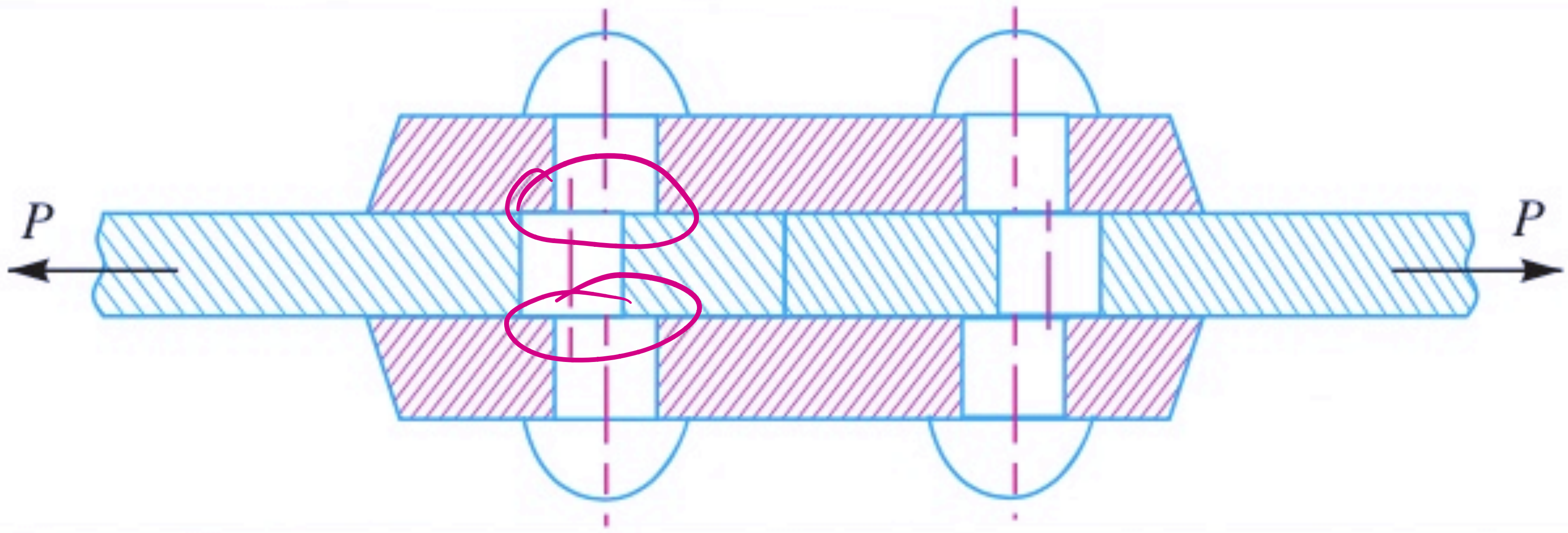


Fig. 9.16. Shearing off a rivet in double cover butt joint.

Let d = Diameter of the rivet hole,

τ = Safe permissible shear stress for the rivet material, and

n = Number of rivets per pitch length.

We know that shearing area,

$$A_s = \frac{\pi}{4} \times d^2 \quad \text{...(In single shear)}$$

$$= 2 \times \frac{\pi}{4} \times d^2 \quad \text{...(Theoretically, in double shear)}$$

$$= 1.875 \times \frac{\pi}{4} \times d^2 \quad \text{...(In double shear, according to Indian Boiler Regulations)}$$

$\therefore$ Shearing resistance or pull required to shear off the rivet per pitch length,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(In single shear)}$$

$$= n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(Theoretically, in double shear)}$$

$$= n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau$$ ...(In double shear, according to Indian Boiler Regulations)

When the shearing resistance (P_s) is greater than the applied load (P) per pitch length, then this type of failure will occur.

4. *Crushing of the plate or rivets.* Sometimes, the rivets do not actually shear off under the tensile stress, but are crushed as shown in Fig. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose. The failure of rivets in such a manner is also known as ***bearing failure***. The area which resists this action is the projected area of the hole or rivet on diametral plane.

The resistance offered by a rivet to be crushed is known as ***crushing resistance*** or ***crushing strength*** or ***bearing value*** of the rivet.

Let d = Diameter of the rivet hole,

t = Thickness of the plate,

σ_c = Safe permissible crushing stress for the rivet or plate material, and

n = Number of rivets per pitch length under crushing.

We know that crushing area per rivet (*i.e.* projected area per rivet),

$$A_c = d.t$$

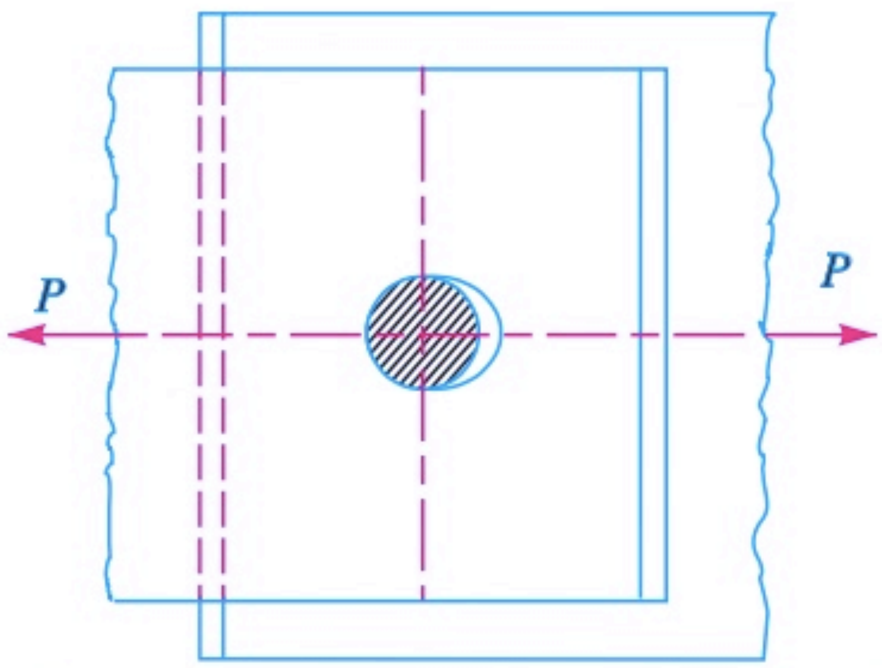
$\therefore$ Total crushing area $= n.d.t$

and crushing resistance or pull required to crush the rivet per pitch length,

$$P_c = n.d.t.\sigma_c$$

When the crushing resistance (P_c) is greater than the applied load (P) per pitch length, then this type of failure will occur.

Note : The number of rivets under shear shall be equal to the number of rivets under crushing.



Strength of a Riveted Joint

The strength of a joint may be defined as the maximum force, which it can transmit, without causing it to fail

P_t, P_s, P_c → tearing, shearing & crushing strengths

If the joint is continuous as in case of boilers, the strength is calculated per pitch length. But if the joint is small, the strength is calculated for the whole length of the plate.

Efficiency of a Riveted Joint

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

We have already discussed that strength of the riveted joint

$$= \text{Least of } P_t, P_s \text{ and } P_c$$

Strength of the un-riveted or solid plate per pitch length,

$$P = p \times t \times \sigma_t$$

$\therefore$ Efficiency of the riveted joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{p \times t \times \sigma_t}$$

where p = Pitch of the rivets,

t = Thickness of the plate, and

σ_t = Permissible tensile stress of the plate material.

Design of Boiler Joints

The boiler has a longitudinal joint as well as circumferential joint. The ***longitudinal joint*** is used to join the ends of the plate to get the required diameter of a boiler. For this purpose, a butt joint with two cover plates is used. The ***circumferential joint*** is used to get the required length of the boiler. For this purpose, a lap joint with one ring overlapping the other alternately is used.

Assumptions in Designing Boiler Joints

The following assumptions are made while designing a joint for boilers :

1. The load on the joint is equally shared by all the rivets. The assumption implies that the shell and plate are rigid and that all the deformation of the joint takes place in the rivets themselves.
2. The tensile stress is equally distributed over the section of metal between the rivets.
3. The shearing stress in all the rivets is uniform.
4. The crushing stress is uniform.
5. There is no bending stress in the rivets.
6. The holes into which the rivets are driven do not weaken the member.
7. The rivet fills the hole after it is driven.
8. The friction between the surfaces of the plate is neglected.

Design of Longitudinal Butt Joint for a Boiler

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.

1. *Thickness of boiler shell.* First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, *i.e.*

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm as corrosion allowance}$$

where
t = Thickness of the boiler shell,
P = Steam pressure in boiler,
D = Internal diameter of boiler shell,
σ_t = Permissible tensile stress, and
η_l = Efficiency of the longitudinal joint.

The following points may be noted :

(*a*) The thickness of the boiler shell should not be less than 7 mm.

(*b*) The efficiency of the joint may be taken from the following table.

Table 9.1. Efficiencies of commercial boiler joints.

Lap joints	*Efficiency (%)*	**Maximum efficiency*	*Butt joints (Double strap)*	*Efficiency (%)*	**Maximum efficiency*
Single riveted	45 to 60	63.3	Single riveted	55 to 60	63.3
Double riveted	63 to 70	77.5	Double riveted	70 to 83	86.6
Triple riveted	72 to 80	86.6	Triple riveted (5 rivets per pitch with unequal width of straps)	80 to 90	95.0
			Quadruple riveted	85 to 94	98.1

(*c*) According to I.B.R., the factor of safety should not be less than 4. The following table shows the values of factor of safety for various kind of joints in boilers.

Table 9.2. Factor of safety for boiler joints.

Type of joint	*Factor of safety*	
	Hand riveting	*Machine riveting*
Lap joint	4.75	4.5
Single strap butt joint	4.75	4.5
Single riveted butt joint with two equal cover straps	4.75	4.5
Double riveted butt joint with two equal cover straps	4.25	4.0

2. *Diameter of rivets.* After finding out the thickness of the boiler shell (t), the diameter of the rivet hole (d) may be determined by using Unwin's empirical formula, *i.e.*

$$d = 6\sqrt{t} \qquad \text{(when } t \text{ is greater than 8 mm)}$$

But if the thickness of plate is less than 8 mm, then the diameter of the rivet hole may be calculated by equating the shearing resistance of the rivets to crushing resistance. In no case, the diameter of rivet hole should not be less than the thickness of the plate, because there will be danger of punch crushing. The following table gives the rivet diameter corresponding to the diameter of rivet hole as per IS : 1928 – 1961 (Reaffirmed 1996).

Table 9.3. Size of rivet diameters for rivet hole diameter as per IS : 1928 – 1961 (Reaffirmed 1996).

Basic size of rivet mm	12	14	16	18	20	22	24	27	30	33	36	39	42	48
Rivet hole diameter (min) mm	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44	50

According to IS : 1928 – 1961 (Reaffirmed 1996), the table on the next page (Table 9.4) gives the preferred length and diameter combination for rivets.

3. *Pitch of rivets.* The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that

(*a*) The pitch of the rivets should not be less than $2d$, which is necessary for the formation of head.

(*b*) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is

$$p_{max} = C \times t + 41.28 \text{ mm}$$

where t = Thickness of the shell plate in mm, and

C = Constant.

$(p-d)t\sigma_t = \frac{\pi}{4}d^2 \times \tau$

The value of the constant C is given in Table 9.5.

Table 9.5. Values of constant *C*.

Number of rivets per pitch length	*Lap joint*	*Butt joint (single strap)*	*Butt joint (double strap)*
1	1.31	1.53	1.75
2	2.62	3.06	3.50
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.00

4. ***Distance between the rows of rivets.*** The distance between the rows of rivets as specified by Indian Boiler Regulations is as follows :

(*a*) For equal number of rivets in more than one row for lap joint or butt joint, the distance between the rows of rivets (p_b) should not be less than

$0.33\,p + 0.67\,d$, for zig-zig riveting, and

$2\,d$, for chain riveting.

(*b*) For joints in which the number of rivets in outer rows is ***half*** the number of rivets in inner rows and if the inner rows are chain riveted, the distance between the outer rows and the next rows should not be less than

$0.33\,p + 0.67$ or $2\,d$, whichever is greater.

The distance between the rows in which there are full number of rivets shall not be less than $2d$.

(*c*) For joints in which the number of rivets in outer rows is ***half*** the number of rivets in inner rows and if the inner rows are zig-zig riveted, the distance between the outer rows and the next rows shall not be less than $0.2\,p + 1.15\,d$. The distance between the rows in which there are full number of rivets (zig-zag) shall not be less than $0.165\,p + 0.67\,d$.

Note : In the above discussion, p is the pitch of the rivets in the outer rows.

5. ***Thickness of butt strap.*** According to I.B.R., the thicknesses for butt strap (t_1) are as given below :

(*a*) The thickness of butt strap, in no case, shall be less than 10 mm.

(*b*) $t_1 = 1.125\,t$, for ordinary (chain riveting) single butt strap.

$t_1 = 1.125\,t\left(\dfrac{p-d}{p-2d}\right)$, for single butt straps, every alternate rivet in outer rows being omitted.

$t_1 = 0.625\,t$, for double butt-straps of equal width having ordinary riveting (chain riveting).

$t_1 = 0.625\,t\left(\dfrac{p-d}{p-2d}\right)$, for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

(*c*) For unequal width of butt straps, the thicknesses of butt strap are

$t_1 = 0.75\,t$, for wide strap on the inside, and

$t_2 = 0.625\,t$, for narrow strap on the outside.

6. ***Margin.*** The margin (m) is taken as $1.5\,d$.

9.18 Design of Circumferential Lap Joint for a Boiler

The following procedure is adopted for the design of circumferential lap joint for a boiler.

1. ***Thickness of the shell and diameter of rivets.*** The thickness of the boiler shell and the diameter of the rivet will be same as for longitudinal joint.

2. ***Number of rivets.*** Since it is a lap joint, therefore the rivets will be in single shear.

∴ Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \qquad \text{...}(i)$$

where n = Total number of rivets.

Knowing the inner diameter of the boiler shell (D), and the pressure of steam (P), the total shearing load acting on the circumferential joint,

$$W_s = \frac{\pi}{4} \times D^2 \times P \qquad ...(ii)$$

From equations (i) and (ii), we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$\therefore \qquad \boxed{n = \left(\frac{D}{d}\right)^2 \frac{P}{\tau}}$$

3. ***Pitch of rivets.*** If the efficiency of the longitudinal joint is known, then the efficiency of the circumferential joint may be obtained. It is generally taken as 50% of tearing efficiency in longitudinal joint, but if more than one circumferential joints is used, then it is 62% for the intermediate joints. Knowing the efficiency of the circumferential lap joint (η_c), the pitch of the rivets for the lap joint (p_1) may be obtained by using the relation :

$$\boxed{\eta_c = \frac{p_1 - d}{p_1}}$$

4. ***Number of rows.*** The number of rows of rivets for the circumferential joint may be obtained from the following relation :

$$\text{Number of rows} = \frac{\text{Total number of rivets}}{\text{Number of rivets in one row}}$$

and the number of rivets in one row

$$= \frac{\pi (D + t)}{p_1}$$

where D = Inner diameter of shell.

5. After finding out the number of rows, the type of the joint (*i.e.* single riveted or double riveted etc.) may be decided. Then the number of rivets in a row and pitch may be re-adjusted. In order to have a leak-proof joint, the pitch for the joint should be checked from Indian Boiler Regulations.

6. The distance between the rows of rivets (*i.e.* back pitch) is calculated by using the relations as discussed in the previous article.

7. After knowing the distance between the rows of rivets (p_b), the overlap of the plate may be fixed by using the relation,

$$\text{Overlap} = (\text{No. of rows of rivets} - 1)\, p_b + m$$

where m = Margin.

Problems

Example 9.1. *A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint.*

If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

To find:

P_t, P_c, P_s

$t = 15$ mm

$d = 25$ mm

$p = 75$ mm

$\sigma_t = 400$ mpa

$\tau = 320$ mpa

$\sigma_c = 640$ mpa

$P_t = (p-d)t\sigma_t \rightarrow$ ultimate tensile stress

$P_c = n d t \sigma_c$

$n = 2$

$P_s = n \times \frac{\pi}{4} d^2 \times \tau$

$P_t = 300 \times 10^3$ N

$P_c = 480 \times 10^3$ N

$P_s = 314.15 \times 10^3$ N

Strength $= 300 \times 10^3$ N

Case 2:

Load applied $= \frac{\text{max load}}{\text{Fos}} = \frac{\text{Strength}}{\text{Fos}}$

$= \frac{300{,}000}{4}$

$P_t = 75{,}000$ N

$(P_c)_{all} = 75{,}000$

$(P_s)_{all} = 75{,}000$

$(P_t)_{all} = (p-d)t(\sigma_t)_a$

$(\sigma_t)_a = 100$ mpa

$(\sigma_c)_a = 100$ mpa $\quad (P_c)_{all} = n d t (\sigma_c)_a$

$(\tau)_a = 76.39$ mpa $\quad (P_s)_{all} = n \times \frac{\pi}{4} d^2 (\tau)_a$

Example 9.2. *Find the efficiency of the following riveted joints :*

1. *Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm.*
2. *Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm.*

Assume

Permissible tensile stress in plate = *120 MPa*

Permissible shearing stress in rivets = *90 MPa*

Permissible crushing stress in rivets = *180 MPa*

$$\eta = \frac{\text{Strength of riveted Joint}}{\text{Strength of non-riveted Joint}}$$

$\sigma_t = 120$ mpa $\qquad \tau = 90$ mpa

$\sigma_c = 180$ mpa

Case (i):

$$P_t = (p-d)t\sigma_t = 21.6 \times 10^3 \text{ N}$$

$$P_c = ndt\sigma_c = 21.6 \times 10^3 \text{ N} \qquad n = 1$$

$$P_s = n \times \frac{\pi}{4} d^2 \times \tau = 28.27 \times 10^3 \text{ N}$$

$$P = p \times t \times \sigma_t = 36 \times 10^3 \text{ N}$$

$$\eta = \frac{21,600}{36,000}$$

$$\boxed{\eta = 60\%}$$

Case (ii):

$$\eta = \frac{32,400}{46,800}$$

$$\boxed{\eta = 69\,\%}$$

$n = 2 \quad \sigma_t = 120$

$p = 65\,mm \quad \sigma_c = 180$

$d = 20\,mm \quad \tau = 90$

$t = 6\,mm$

$P_t = 32,400\ N$

$P_s = 56,548\ N$

$P_c = 43,200\ N$

$P = 46,800\ N$

Example 9.4. *A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume*

$\sigma_t = 80$ MPa ; $\tau = 60$ MPa ; and $\sigma_c = 120$ MPa

State how the joint will fail and find the efficiency of the joint.

1) Thickness of plate = 13mm

2) dia of rivet (d) = $6\sqrt{t}$ m

$= 21.63$ mm

$= 22$ mm

From the standards table;

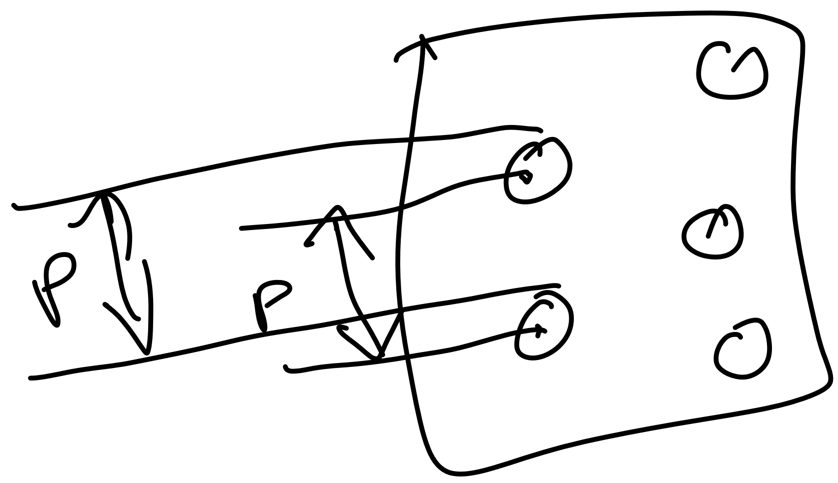
rivet hole dia = 23mm

rivet dia = 22 mm //

3) pitch of rivets : (P)

n = 2

Equating



$$P_t = (p-d)t\sigma_t \qquad P_s = n\frac{\pi}{4}d^2\tau$$

$$P_t = P_s \Rightarrow (P-23)13\times 80 = 2\times\frac{\pi}{4}\times(23)^2\times 60$$

$$P = 71 \text{ mm}$$

$P_{max} = (C \times t) + 41.28$ $\qquad C = 2.62$ [from table]

$P_{max} = 75.34\,mm$

Finally $\quad P = 71\,mm$ //

4) Dist b/n rows of rivets!

$P_b = 0.33P + 0.67d$ [for this zag] from table

$P_b = 38.84 \approx 40\,mm$ //

5) margin (m)

$m = 1.5d = 1.5 \times 23 \simeq 35\,mm$ //

$P_t = (P-d)t\sigma_t = 49{,}920\,N$ Failure happens due to shear load.

$P_s = 49{,}867\,N$

$P_c = 71{,}760\,N$

$$\text{Efficiency} = \frac{\text{least of } P_t, P_c, P_s}{P \times t \times \sigma_t}$$

$$\boxed{\eta = 67.5\%}$$

Example 9.7. *Design a double riveted butt joint with two cover plates for the longitudinal seam of a boiler shell 1.5 m in diameter subjected to a steam pressure of 0.95 N/mm². Assume joint efficiency as 75%, allowable tensile stress in the plate 90 MPa ; compressive stress 140 MPa ; and shear stress in the rivet 56 MPa.*

$P = 0.95 \, N/mm^2$

$D = 1500 \, mm$

$\sigma_t = 90 \, MPa$

$\eta = 0.75$

$\sigma_c = 140 \, MPa$

$\tau = 56 \, MPa$

1) Thickness of plate (t)

$$t = \frac{P \cdot D}{2 \sigma_t \cdot \eta} + 1$$

$$t = 12 \, mm$$

2) Dia of rivet hole (d)

$$d = 6\sqrt{t} \, mm$$

$$d = 20.78 \, mm$$

From table $d = 21 \, mm$ (rivet hole)

$d_r = 20 \, mm$ (rivet)

3) Pitch of rivets (p)

$$(p - d) t \sigma_t = n \times \frac{\pi}{4} d^2 \tau$$

$n = 2$

Since it is a butt joint with 2 cover plate.

The plates are subjected to double shear for double shear,

$$P_s = n \times 1.875 \times \frac{\pi}{4} d^2 \tau$$

$$P_s = 72{,}735 \text{ N}$$

$$P_s = P_t \Rightarrow 72{,}735 = (P - 21)\, 12 \times 90$$

$$\text{pitch } p = 88.34 \text{ mm}$$

$$P_{max} = C \times t + 41.28$$

$$C = 3.5$$

$$P_{max} = 83.28 \text{ mm}$$

$$\text{pitch } \boxed{P = P_{max} = 83.28 \text{ mm}}$$

4) Dist b/n row of rivets:

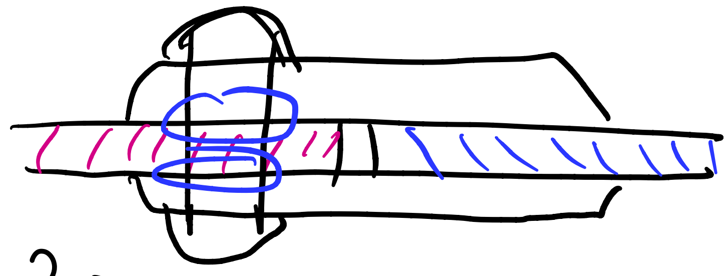
Assume zig zag riveting

$$P_b = 0.33p + 0.67d$$

$$\boxed{P_b = 41.65 \simeq 42 \text{ mm}}$$

5) margin (m)

$$m = 32 \text{ mm}$$



6) thickness of cover plate

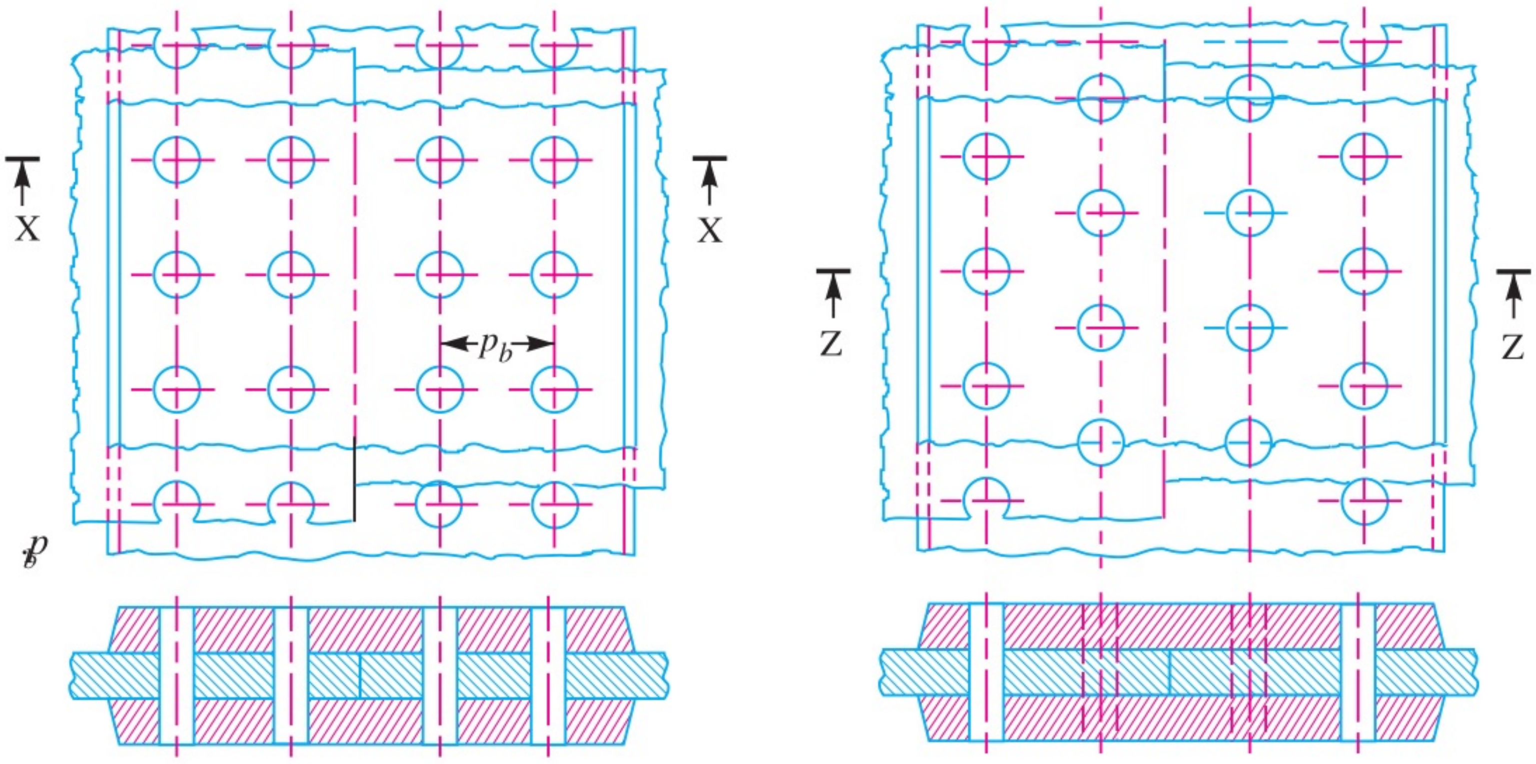
$$t_1 = 0.625t = 7.5\,mm \simeq 8\,mm$$

$P_t = \underline{67262}$ N

$P_c = \underline{70560}$ N

$P_s = \underline{72735}$ N

$\eta = \underline{74.8\,\%}$



(*a*) Chain riveting. (*b*) Zig-zag riveting.

Fig. 9.9. Double riveted double strap (equal) butt joints.

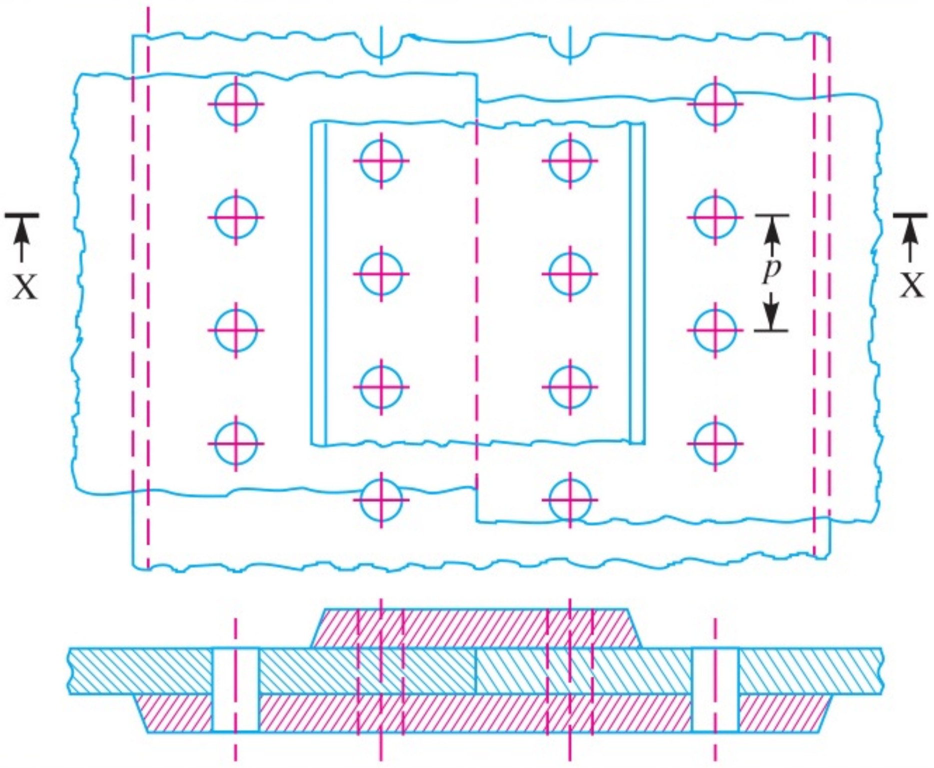


Fig. 9.10. Double riveted double strap (unequal) butt joint with zig-zag riveting.

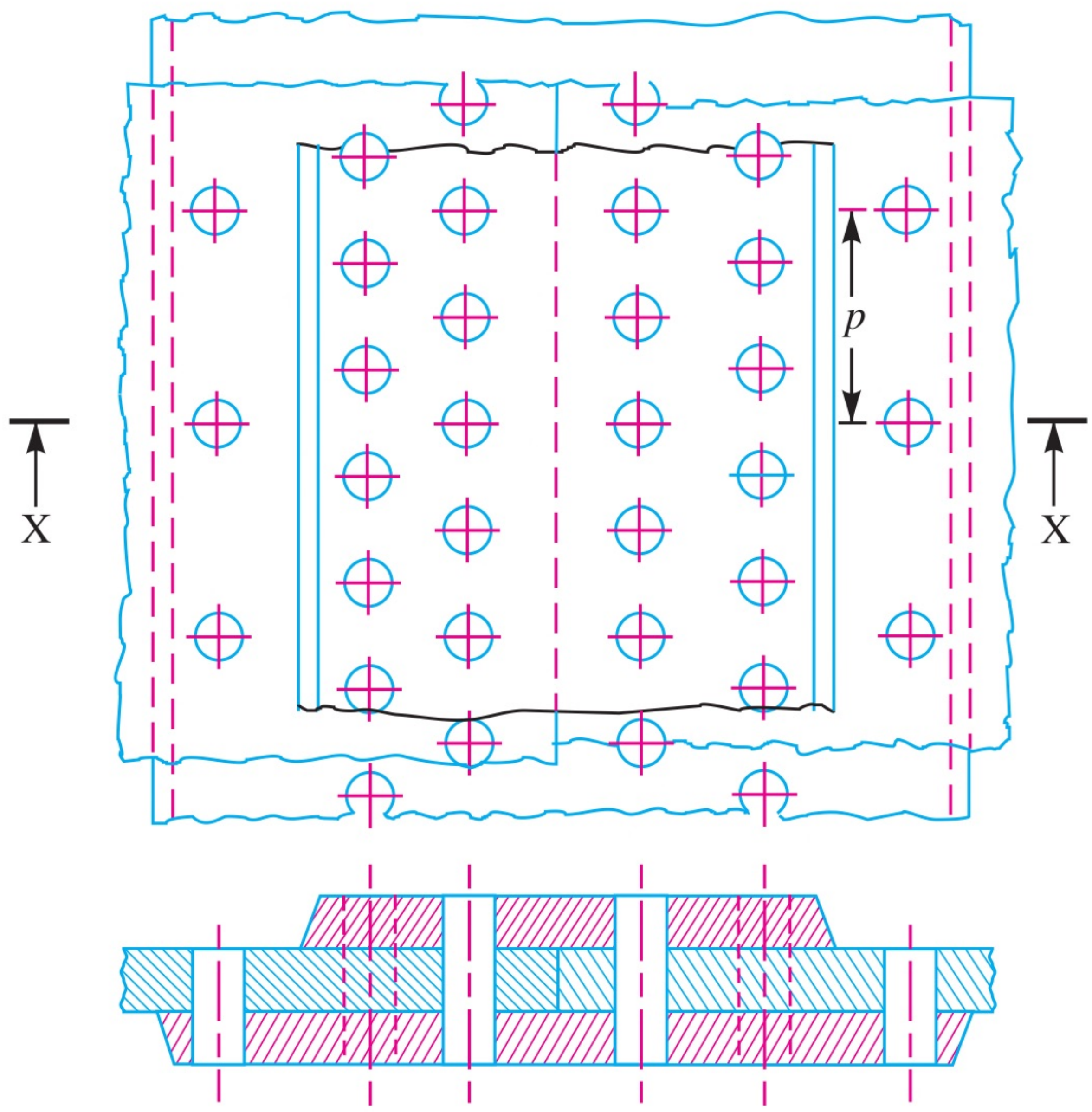


Fig. 9.11. Triple riveted double strap (unequal) butt joint.

A steam boiler is to be designed for a working pressure of 2.5 N/mm2 with its inside diameter 1.6 m. Give the design calculations for the longitudinal and circumferential joints for the following working stresses for steel plates and rivets :

In tension = 75 MPa ; In shear = 60 MPa; In crushing = 125 MPa. Draw the joints to a suitable scale.

Assume the joint to be triple riveted double strap butt joint with unequal cover straps

p = Pitch of the rivet in the outer most row

n = 5

Assignment

Example 9.8. *A pressure vessel has an internal diameter of 1 m and is to be subjected to an internal pressure of 2.75 N/mm² above the atmospheric pressure. Considering it as a thin cylinder and assuming efficiency of its riveted joint to be 79%, calculate the plate thickness if the tensile stress in the material is not to exceed 88 MPa.*

Design a longitudinal double riveted double strap butt joint with equal straps for this vessel. The pitch of the rivets in the outer row is to be double the pitch in the inner row and zig-zag riveting is proposed. The maximum allowable shear stress in the rivets is 64 MPa. You may assume that the rivets in double shear are 1.8 times stronger than in single shear and the joint does not fail by crushing.

Make a sketch of the joint showing all calculated values. Calculate the efficiency of the joint.

Pressure vessel.

Given data:

$D = 1\,m = 1000\,mm$

$p = 2.75\,N/mm^2$

$\eta = 79\,\%$

$\sigma_t = 88\,N/mm^2$

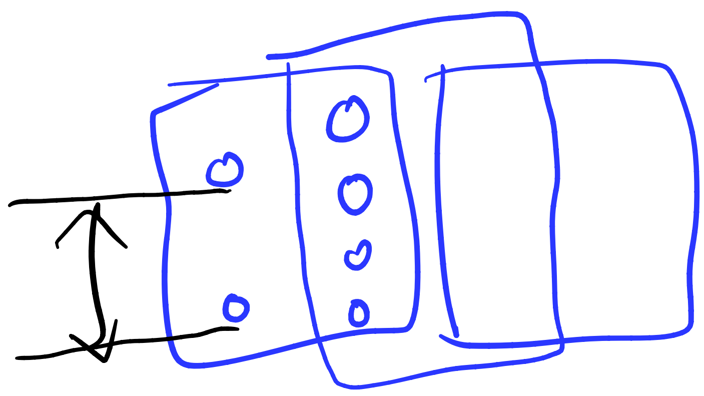
$\tau = 64\,mPa$

1) Thickness of plate

$$t = \frac{PD}{2\sigma_t \times \eta} + 1 = \frac{2.75 \times 1000}{2 \times 88 \times 0.79} + 1$$

$t = 21\,mm$

2) dia of rivet hole:



$d = 6\sqrt{t}\,mm$

$d = 27.5\,mm$

From standards table

dia of rivet hole $(d) = 28.5$ mm

dia of rivet $= 27$ mm

3) pitch of the rivets:

tearing resistance = shearing resistance

p = pitch of the outer row

$$(p-d)t\sigma_t = n \times 1.8 \times \frac{\pi}{4} d^2 \times \tau$$

$$(p-28.5)21 \times 88 = 3 \times 1.8 \times \frac{\pi}{4}(28.5)^2 \times 64$$

$p = 147.8$ mm

$P_{max} = C \times t + 41.28$ mm

$P_{max} = 138.5 \simeq 140$ mm $\quad C = 4.63$

pitch $p = P_{max} = 140$ mm → outer row

Inner row pitch $P_i = P/2 = 70\,mm$

4) Dist b/n the rows of rivets:

$$P_b = 0.2P + 1.15d$$

$$P_b = (0.2 \times 140) + (1.15 \times 28.5) = 61\,mm$$

5) Thickness of butt strap:

$$t = 0.625\,t\left(\frac{p-d}{p-2d}\right)$$

$$= 18\,mm$$

6) margin (m)

$$m = 1.5d = 43\,mm$$

Efficiency $\eta = \dfrac{\text{least of } P_t, P_s}{P \times t \times \sigma_t}$

$P_t = 0.2 \times 10^6\,N$ $\quad P_s = n \times 1.8 \times \frac{\pi}{4} d^2 \times \tau$

$P_s = 0.22 \times 10^6 \, N$

$$\eta = \frac{P_t}{P \times t \times \sigma_t}$$

$$\boxed{\eta = 79.6\,\%}$$

$$= \frac{206.05 \times 10^3}{258,720}$$

Riveted Joint for Structural Use–Joints of Uniform Strength (Lozenge Joint)

A riveted joint known as ***Lozenge joint*** used for roof, bridge work or girders etc. is shown in Fig. In such a joint, diamond riveting is employed so that the joint is made of uniform strength.

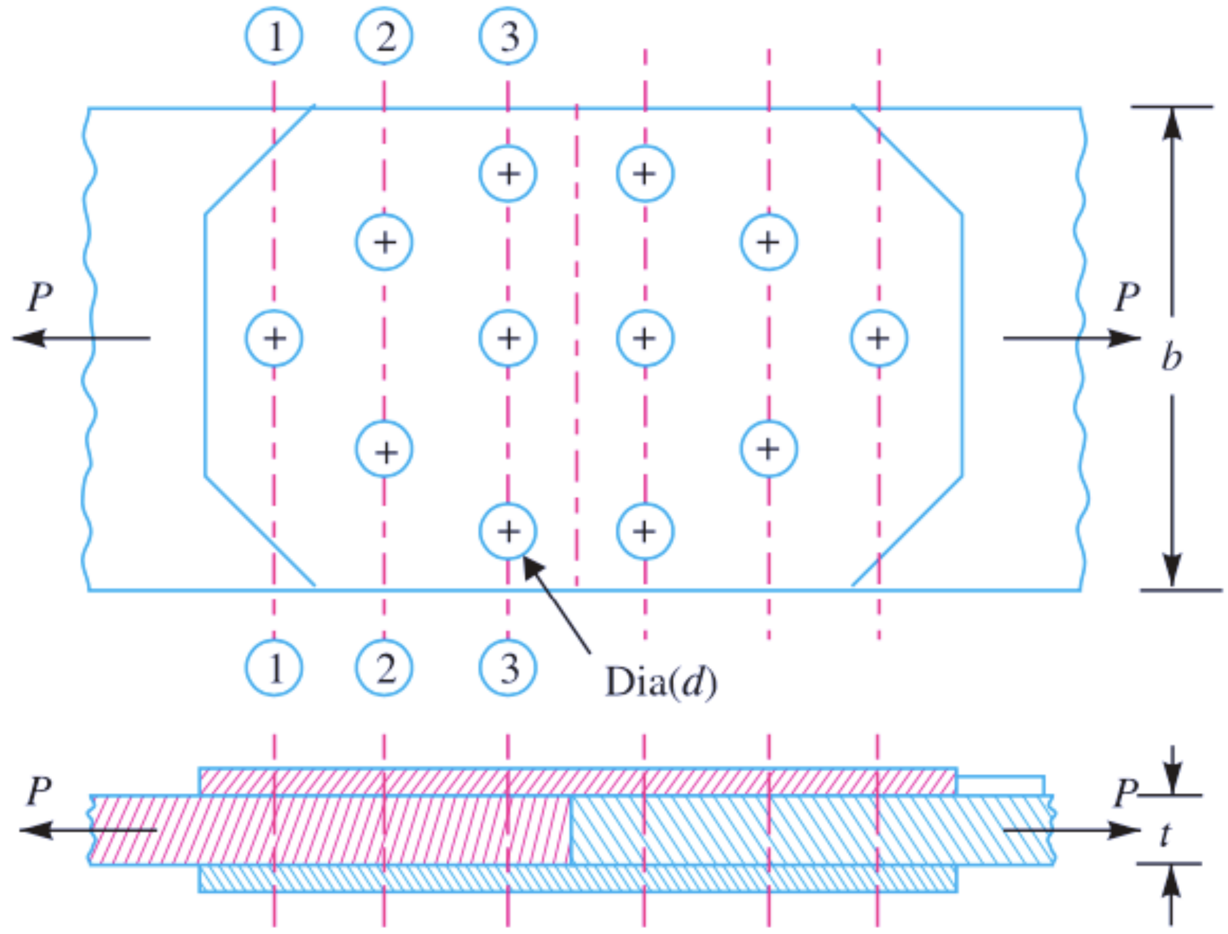


Fig. 9.19. Riveted joint for structural use.

Let b = Width of the plate,

t = Thickness of the plate, and

d = Diameter of the rivet hole.

In designing a Lozenge joint, the following procedure is adopted.

1. *Diameter of rivet*

The diameter of the rivet hole is obtained by using Unwin's formula, *i.e.*

$$d = 6\sqrt{t}$$

Table 9.7. Sizes of rivets for general purposes, according to IS : 1929 – 1982 (Reaffirmed 1996).

Diameter of rivet hole (mm)	13.5	15.5	17.5	19.5	21.5	23.5	25.5	29	32	35	38	41	44	50
Diameter of rivet (mm)	12	14	16	18	20	22	24	27	30	33	36	39	42	48

2. *Number of rivets*

The number of rivets required for the joint may be obtained by the shearing or crushing resistance of the rivets.

Let P_t = Maximum pull acting on the joint. This is the tearing resistance of the plate at the outer row which has only one rivet.

$= (b - d)\, t \times \sigma_t$

and n = Number of rivets.

Since the joint is double strap butt joint, therefore the rivets are in double shear. It is assumed that resistance of a rivet in double shear is 1.75 times than in single shear in order to allow for possible eccentricity of load and defective workmanship.

$\therefore$ Shearing resistance of one rivet,

$$P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau$$

and crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c$$

$\therefore$ Number of rivets required for the joint,

$$n = \frac{P_t}{\text{Least of } P_s \text{ or } P_c}$$

3. From the number of rivets, the number of rows and the number of rivets in each row is decided.

4. *Thickness of the butt straps*

The thickness of the butt strap,

$$t_1 = 1.25\, t, \text{ for single cover strap}$$
$$= 0.75\, t, \text{ for double cover strap}$$

5. *Efficiency of the joint*

First of all, calculate the resistances along the sections 1-1, 2-2 and 3-3.

At section 1-1, there is only one rivet hole.

$\therefore$ Resistance of the joint in tearing along 1-1,

$$P_{t1} = (b - d)\, t \times \sigma_t$$

At section 2-2, there are two rivet holes.

$\therefore$ Resistance of the joint in tearing along 2-2,

$$P_{t2} = (b - 2d)\, t \times \sigma_t + \text{Strength of one rivet in front of section 2-2}$$

(This is due to the fact that for tearing off the plate at section 2-2, the rivet in front of section 2-2 *i.e.* at section 1-1 must first fracture).

Similarly at section 3-3 there are three rivet holes.

$\therefore$ Resistance of the joint in tearing along 3-3,

$$P_{t3} = (b - 3d)\, t \times \sigma_t + \text{Strength of 3 rivets in front of section 3-3}$$

The least value of $P_{t1}, P_{t2}, P_{t3}, P_s$ or P_c is the strength of the joint.

We know that the strength of unriveted plate,

$$P = b \times t \times \sigma_t$$

$\therefore$ Efficiency of the joint,

$$\boxed{\eta = \frac{\text{Least of } P_{t1}, P_{t2}, P_{t3}, P_s \text{ or } P_c}{P}}$$

Eccentric Loaded Riveted Joint

When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the joint is said to be an ***eccentric loaded riveted joint,*** as shown in Fig. The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or primary shear.

Let P = Eccentric load on the joint, and

e = Eccentricity of the load *i.e.* the distance between the line of action of the load and the centroid of the rivet system *i.e.* G

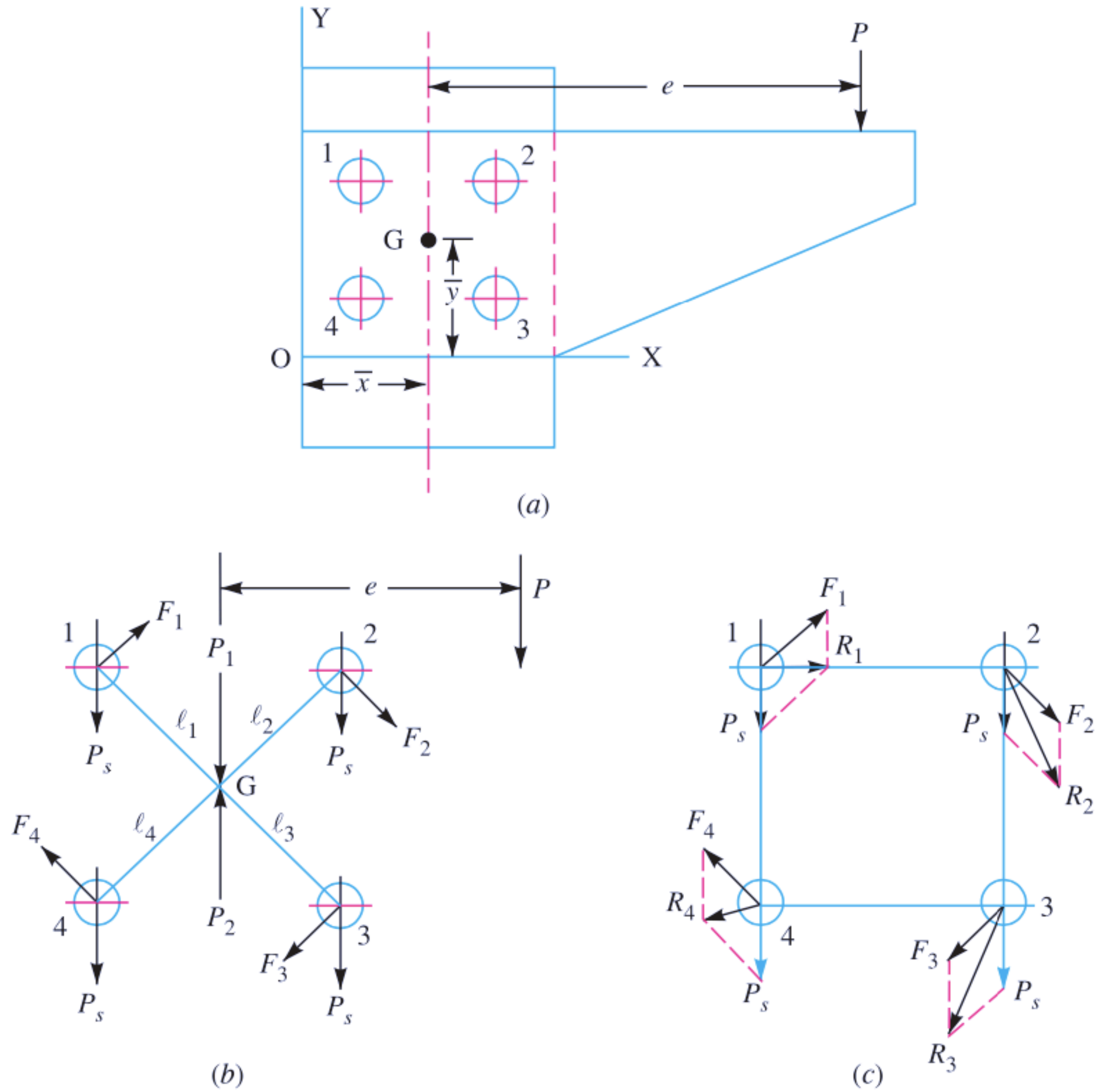


Fig. 9.23. Eccentric loaded riveted joint.

Example 9.14. *An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown in Fig. 9.24.*

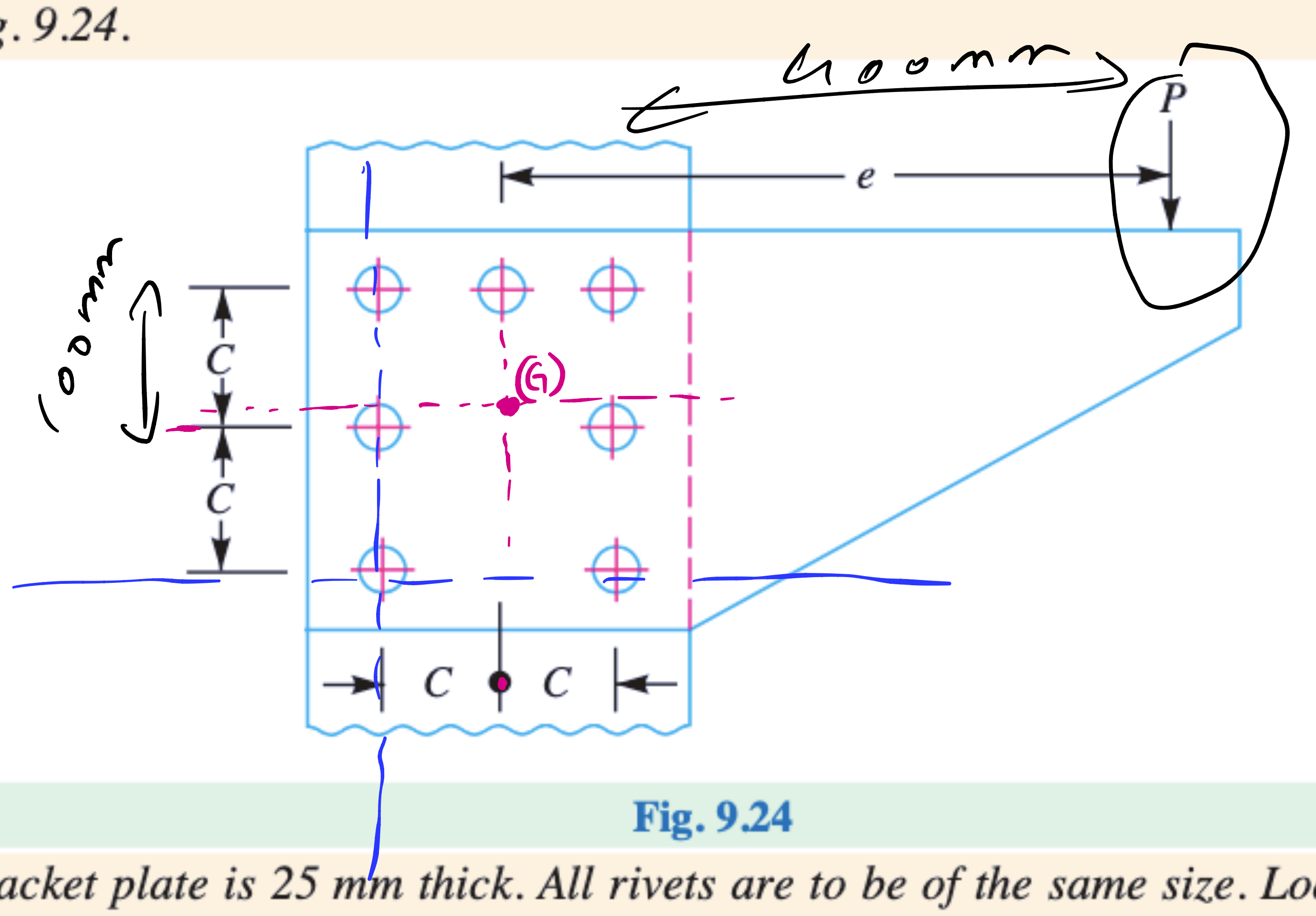


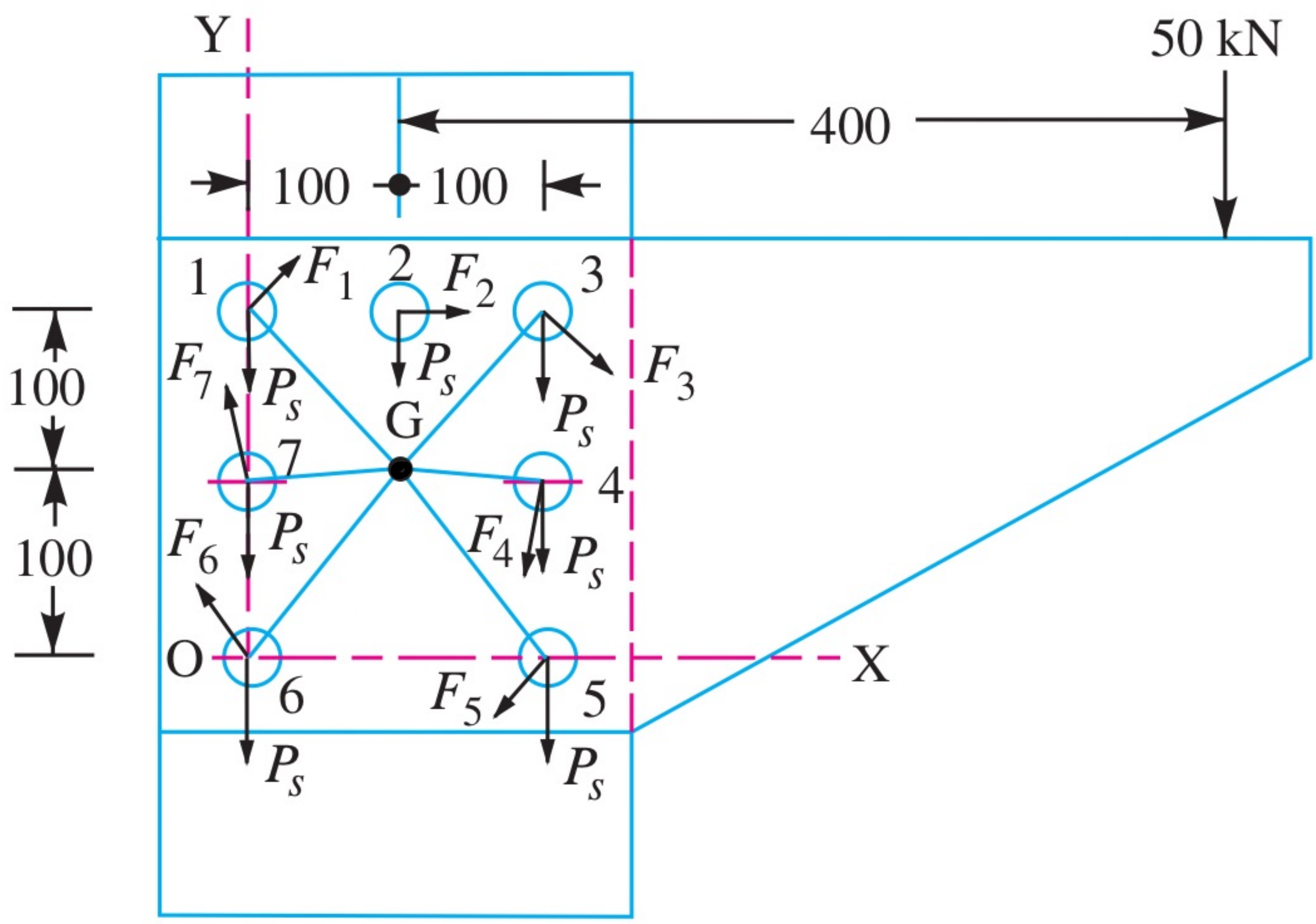
Fig. 9.24

The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 kN ; rivet spacing, C = 100 mm; load arm, e = 400 mm.

Permissible shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the joint.

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n} = \frac{0 + 100 + 200 + 200 + 200}{7} = 100\text{mm}$$

$$\bar{y} = \frac{y_1 + \text{—} \; y_7}{7} = 114.28\text{ mm}$$



P_s (direct shear) $= \dfrac{\text{Force}}{\text{no of rivets}} = \dfrac{50\times10^3}{7} = 7143\ N$

Turning moment $= P\times e$

$= 50\times10^3 \times 400 = 20\times10^6\ N\text{-mm}$

$l_1 = \sqrt{100^2+(85.7)^2} = 131.69\text{mm} = l_3$

$l_2 = 85.7\text{mm}$

$l_4 = l_7 = \sqrt{(14.3)^2+(100)^2} = 101\text{mm}$

$l_5 = l_6 = 152\text{ mm}$

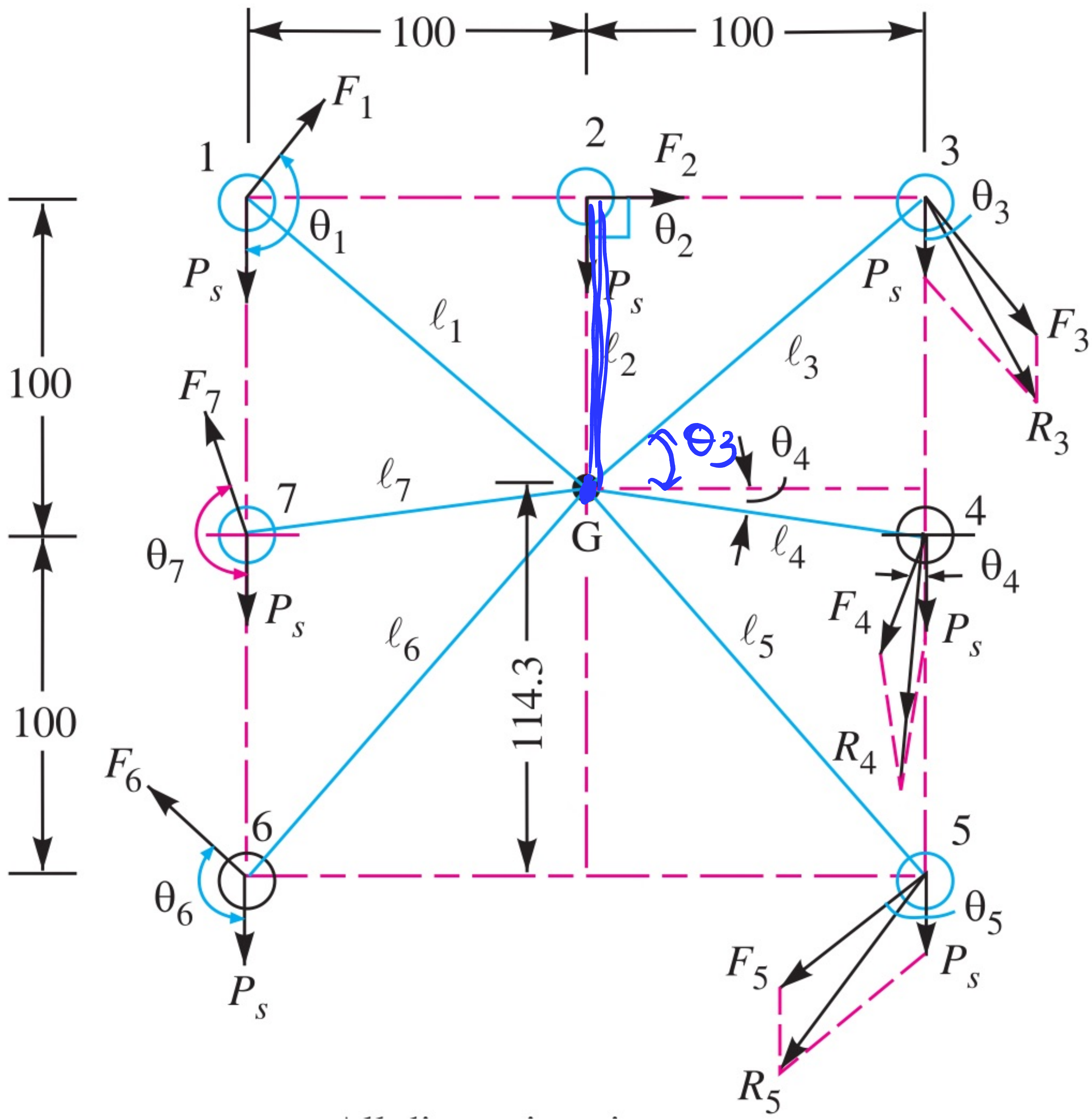
$F_1 \propto l_1 \qquad F_3 \propto l_3$

$F_2 \propto l_2$

$\dfrac{F_1}{l_1} = \dfrac{F_2}{l_2} = \dfrac{F_3}{l_3} --- \quad (1)$

$F_2 = \left(\dfrac{F_1}{l_1}\right) l_2$

$F_3 = \left(\dfrac{F_1}{l_1}\right) l_3$



$$P \times e = F_1 l_1 + F_2 l_2 + F_3 l_3 + F_4 l_4 + F_5 l_5 + F_6 l_6 + F_7 l_7$$

$$= F_1 l_1 + F_1 \frac{l_2}{l_1} l_2 + F_1 \frac{l_3}{l_1} \times l_3 + ---$$

$$P \times e = \frac{F_1}{l_1}\left[l_1^2 + l_2^2 + l_3^2 + l_4^2 + - - \right]$$

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2((31.7)^2 + (85.7)^2 + 2(100)^2 + 2(152)^2 \right]$$

$$\boxed{f_1 = 24,334 \ N}$$

$$F_2 = \frac{f_1 l_2}{l_1} = \frac{24,334}{131.7} \times 85.7 = 15,776 \ N$$

$$F_3 = \frac{F_1 l_3}{l_1} = 24,334 \ N$$

$F_4 = 18,593$ N

$F_5 = 27,981$ N

$F_6 = 27,981$ N

$F_7 = 18,593$ N

The rivets 3, 4, 5 are the max loaded

So $\cos\theta_3 = \frac{100}{l_3} \Rightarrow \cos\theta_3 = 0.76$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

$$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = 0.65$$

$$R_3 = \sqrt{(P_5)^2 + (F_3)^2 + 2F_3 P_5 \cos\theta_3}$$

$R_3 = 30,116.79\ N$

$R_4 = 25,684\ N$

$\Rightarrow$ $\boxed{d = 25.5mm}$

$\boxed{d_4 = 24mm}$

$R_5 = 33,200\ N$

$$\tau = \frac{Force}{Area} \Rightarrow F = \frac{\pi}{4} \times d^2 \times \tau$$

$$33,200 = \frac{\pi}{4} \times 65 \times d^2$$

$$\text{Crushing stress} = \frac{\text{max load}}{\text{Crushing area}} = \frac{33,120}{25.5 \times 25}$$

$\sigma_c = 52\ MPa$ < 120 ✓

Welded Joints

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material.

Advantages and Disadvantages of Welded Joints over Riveted Joints

Following are the advantages and disadvantages of welded joints over riveted joints.

Advantages

1. The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.
6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (*i.e.* circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

Disadvantages

1. Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
2. It requires a highly skilled labour and supervision.
3. Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
4. The inspection of welding work is more difficult than riveting work.

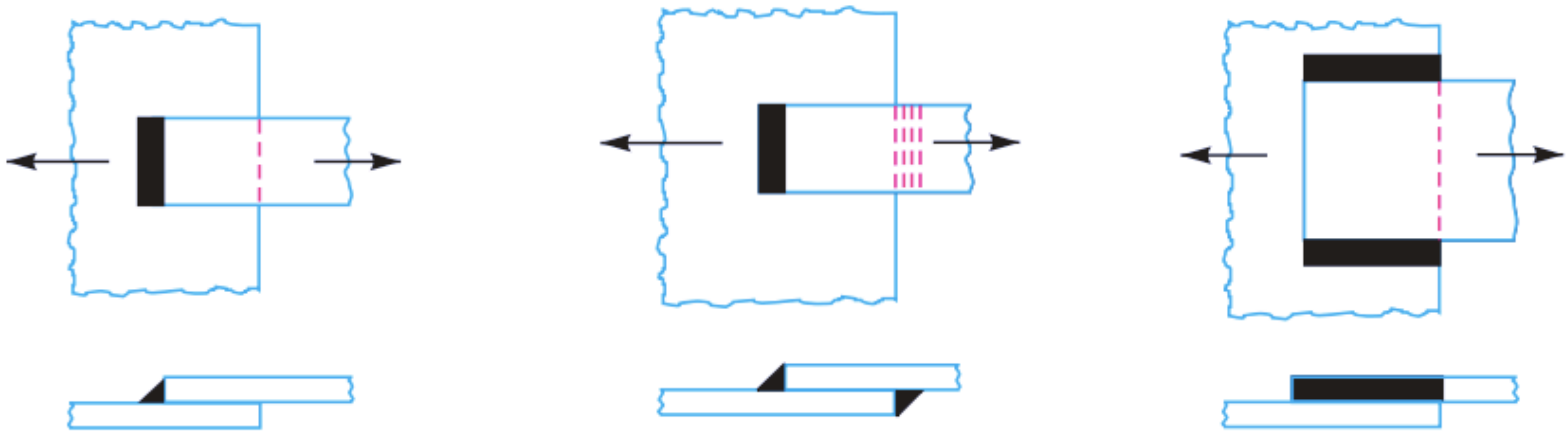
Welding Processes

The welding processes may be broadly classified into the following two groups:

1. Welding processes that use heat alone *e.g.* fusion welding.
2. Welding processes that use a combination of heat and pressure *e.g.* forge welding.

Types of Welded Joints

1. Lap joint or fillet joint, and 2. Butt joint.



(*a*) Single transverse. (*b*) Double transverse. (*c*) Parallel fillet.

Fig. 10.2. Types of lap or fillet joints.

Lap Joint

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be

1. Single transverse fillet, 2. Double transverse fillet, and 3. Parallel fillet joints.

Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides.

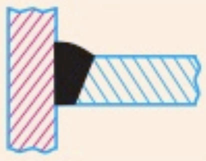
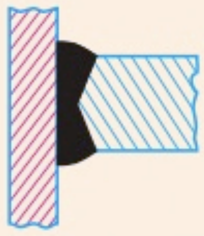
(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

Fig. 10.3. Types of butt joints.

The butt joints may be

1. Square butt joint,
2. Single V-butt joint
3. Single U-butt joint,
4. Double V-butt joint, and
5. Double U-butt joint.

Table 10.1. Basic weld symbols.

S. No.	*Form of weld*	*Sectional representation*	*Symbol*
1.	Fillet		
2.	Square butt		
3.	Single-*V* butt		
4.	Double-*V* butt		
5.	Single-*U* butt		
6.	Double-*U* butt		
7.	Single bevel butt		
8.	Double bevel butt		

10.14 Elements of a Welding Symbol

A welding symbol consists of the following eight elements:

1. Reference line,
2. Arrow,
3. Basic weld symbols,
4. Dimensions and other data,
5. Supplementary symbols,
6. Finish symbols,
7. Tail, and
8. Specification, process or other references.

10.15 Standard Location of Elements of a Welding Symbol

According to Indian Standards, IS: 813 – 1961 (Reaffirmed 1991), the elements of a welding symbol shall have standard locations with respect to each other.

The arrow points to the location of weld, the basic symbols with dimensions are located on one or both sides of reference line. The specification if any is placed in the tail of arrow. Fig. 10.5 shows the standard locations of welding symbols represented on drawing.

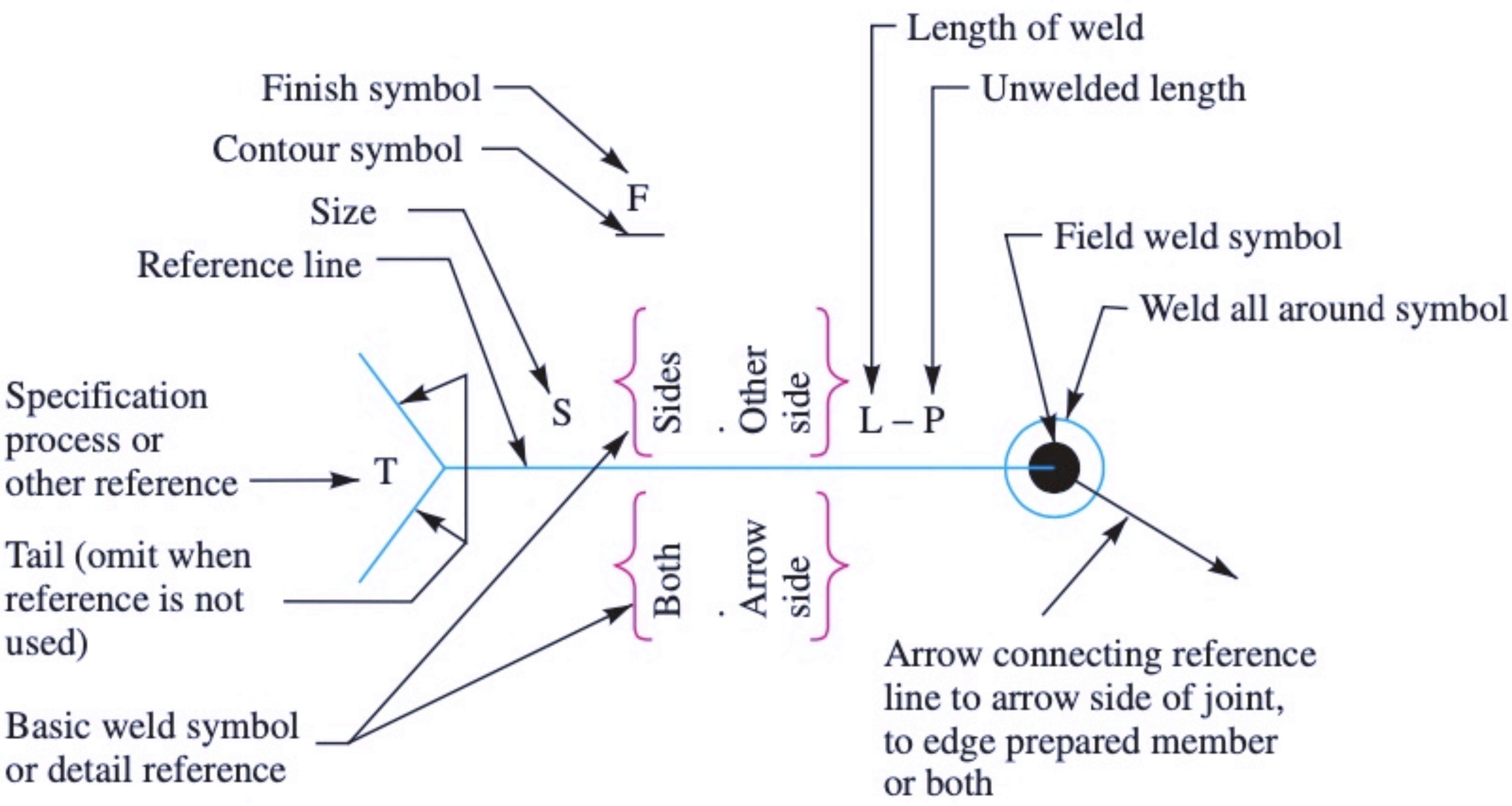
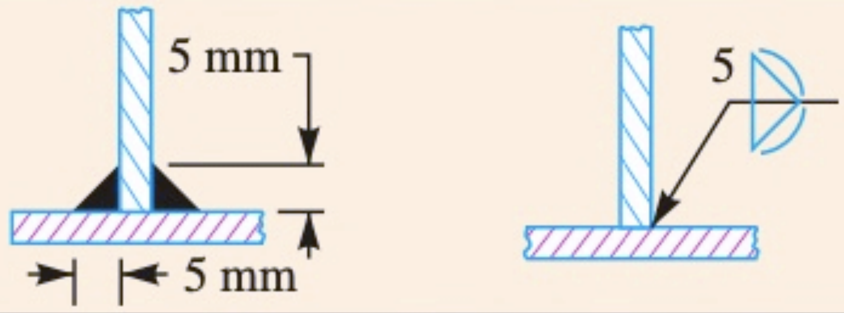
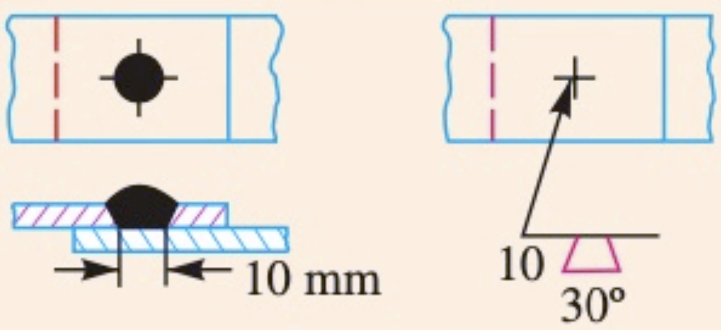
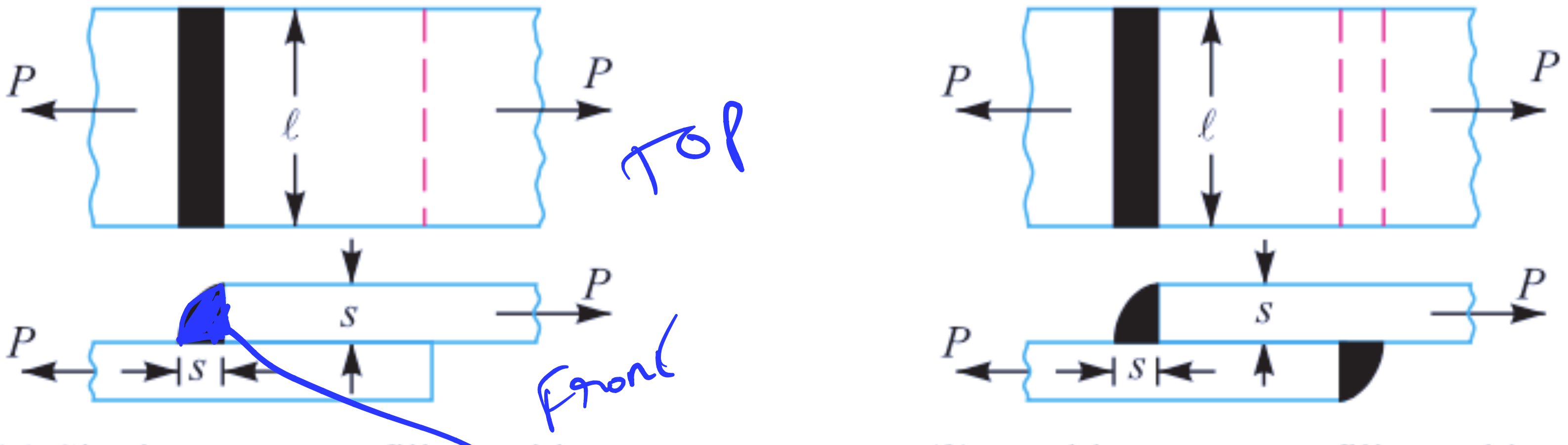


Table 10.3. Representation of welding symbols.

S. No.	*Desired weld*	*Representation on drawing*
1.	Fillet-weld each side of Tee- convex contour	
2.	Single V-butt weld -machining finish	
3.	Double V- butt weld	
4.	Plug weld - 30° Groove-angle-flush contour	

Strength of Transverse Fillet Welded Joints



(*a*) Single transverse fillet weld. (*b*) Double transverse fillet weld.

The transverse fillet welds are designed for tensile strength

It is assumed that the section of fillet is a right angled triangle *ABC* with hypotenuse *AC* making equal angles with other two sides *AB* and *BC*.

$l \rightarrow$ length (or) width

$t = S \sin 45°$

Min Area = throat thickness (t) × length of weld

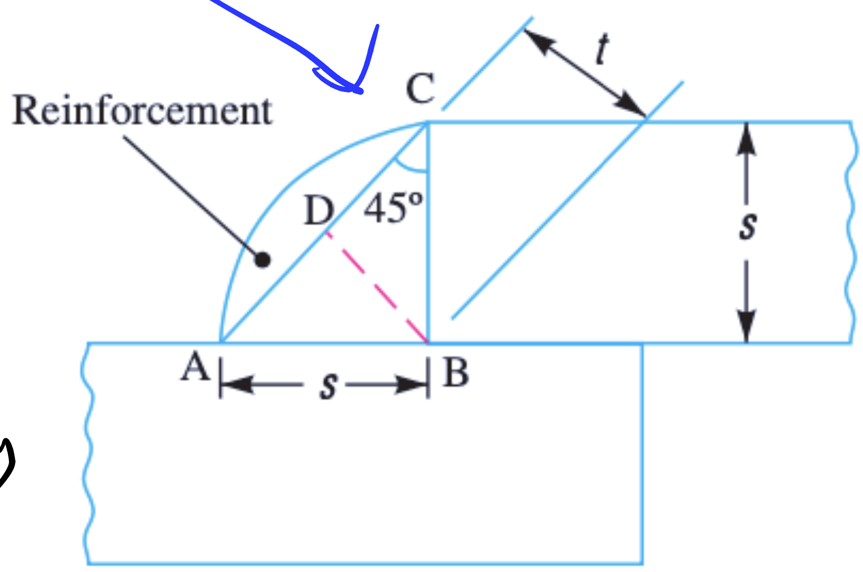


Fig. 10.7. Enlarged view of a fillet weld.

The length of each side is known as **leg or size** of the weld.

The perpendicular distance of the hypotenuse from the intersection of legs (i.e. BD) is known as **throat thickness.**

The minimum area of the weld is obtained at the throat BD, which is given by the **product of the throat thickness and length of weld.**

$\sigma_t \rightarrow$ tensile strength

$A = t \times l$

$= S \sin 45° \times l$

$A = 0.707\, Sl$

Strength of transverse fillet joint $= \sigma_t \times \text{Area}$

$$\boxed{P = 0.707\, s l \times \sigma_t}$$

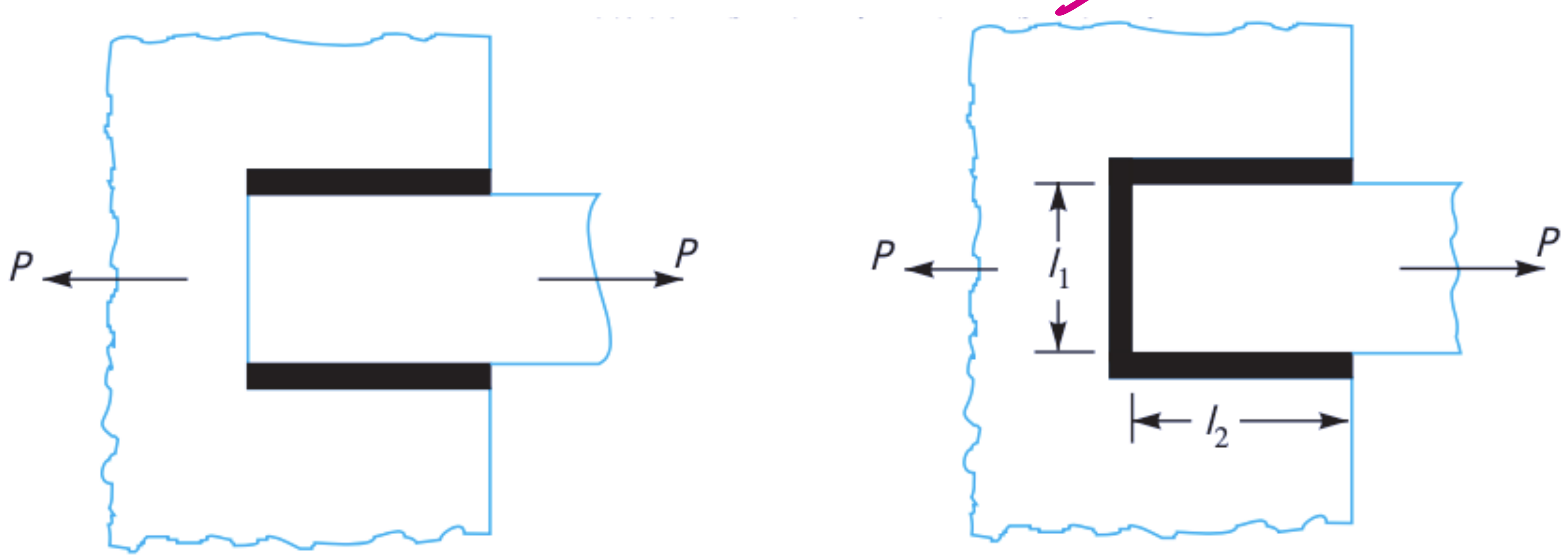
Strength of double transverse fillet joint $= 2 \times 0.707\, s l \sigma_t$

$$\boxed{P_{2t} = 1.414\, s l \sigma_t}$$

Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength

$0.707\, S l \tau_t$
$+ 1.414\, S l \tau$



(*a*) Double parallel fillet weld.

(*b*) Combination of transverse and parallel fillet weld.

Area @ throat is same as transverse Joint

$$A = 0.707\, S l$$

Strength of parallel fillet weld $= 0.707\, S l \times \tau$

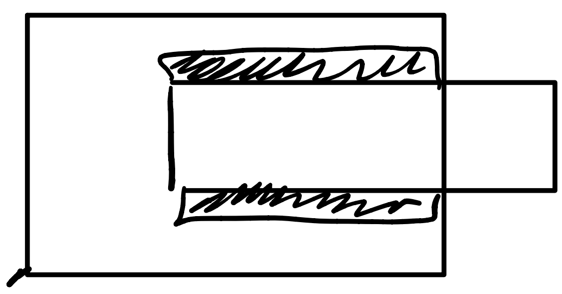
For double ||el fillet weld $\boxed{P = 1.414\, S l \times \tau}$

A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

$l = 100\text{mm}$ load $P = 80\text{ KN}$

$S = 10\text{mm}$

$\tau = 55\text{ mpa}$



$80 \times 10^3 = 1.414\, s \times l\, (55)$

$l = 103\,mm$

Adding 12.5 mm for starting & stopping of weld run

$l = 103 + 12.5 \simeq 116\ mm$

Special Cases of Fillet Welded Joints:

Circular fillet weld subjected to torsion

Let

d = Diameter of rod,

r = Radius of rod,

T = Torque acting on the rod,

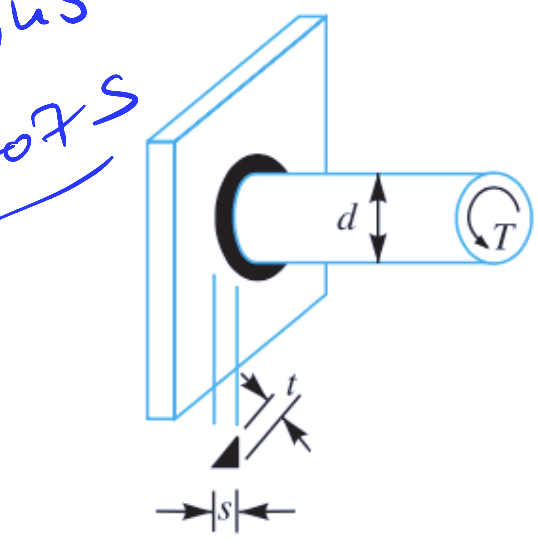
s = Size (or leg) of weld,

t = Throat thickness,

*J = Polar moment of inertia of the weld section $= \dfrac{\pi t d^3}{4}$

$t = s \sin 45$

$t = 0.707 s$



$$\frac{T}{J} = \frac{\tau}{R}$$

$$\tau = \frac{TR}{J} = \frac{T \times \cancel{d}/\cancel{2}}{\pi t d^{\cancel{3}\,2}/\cancel{4}\,2}$$

$$\boxed{\tau = \frac{2T}{\pi t d^2}}$$

@ throat $t = s \sin 45$

$$\tau_{max} = \frac{2T}{\pi\, 0.707 s d^2} = \frac{2.83T}{\pi s d^2}$$

Circular fillet weld subjected to bending moment.

Let

d = Diameter of rod,

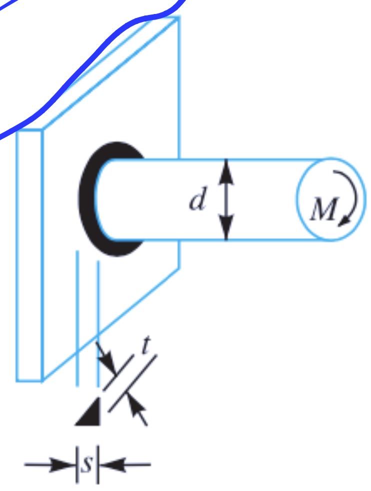
M = Bending moment acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness,

**Z = Section modulus of the weld section $= \dfrac{\pi t d^2}{4}$

$$\frac{I}{y} = Z$$



$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\frac{M}{I/y} = \sigma_b$$

$$\frac{I}{y} = Z$$ Section modulus

$$\sigma_b = \frac{M}{Z}$$

$$\boxed{\sigma_b = \frac{4M}{\pi s d^2}}$$

$$\boxed{\sigma_{b_{max}} = \frac{5.66M}{\pi s d^2}}$$

Long fillet weld subjected to torsion

Let T = Torque acting on the vertical plate,

l = Length of weld,

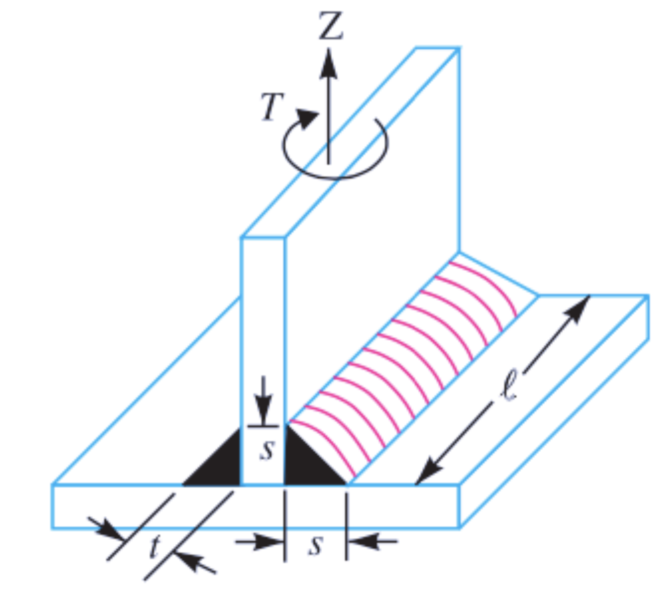
s = Size (or leg) of weld,

t = Throat thickness, and

J = Polar moment of inertia of the weld section

$$= 2 \times \frac{t \times l^3}{12} = \frac{t \times l^3}{6} \quad \ldots$$

($\because$ of both sides weld)



$$\tau = \frac{T \times R}{J}$$

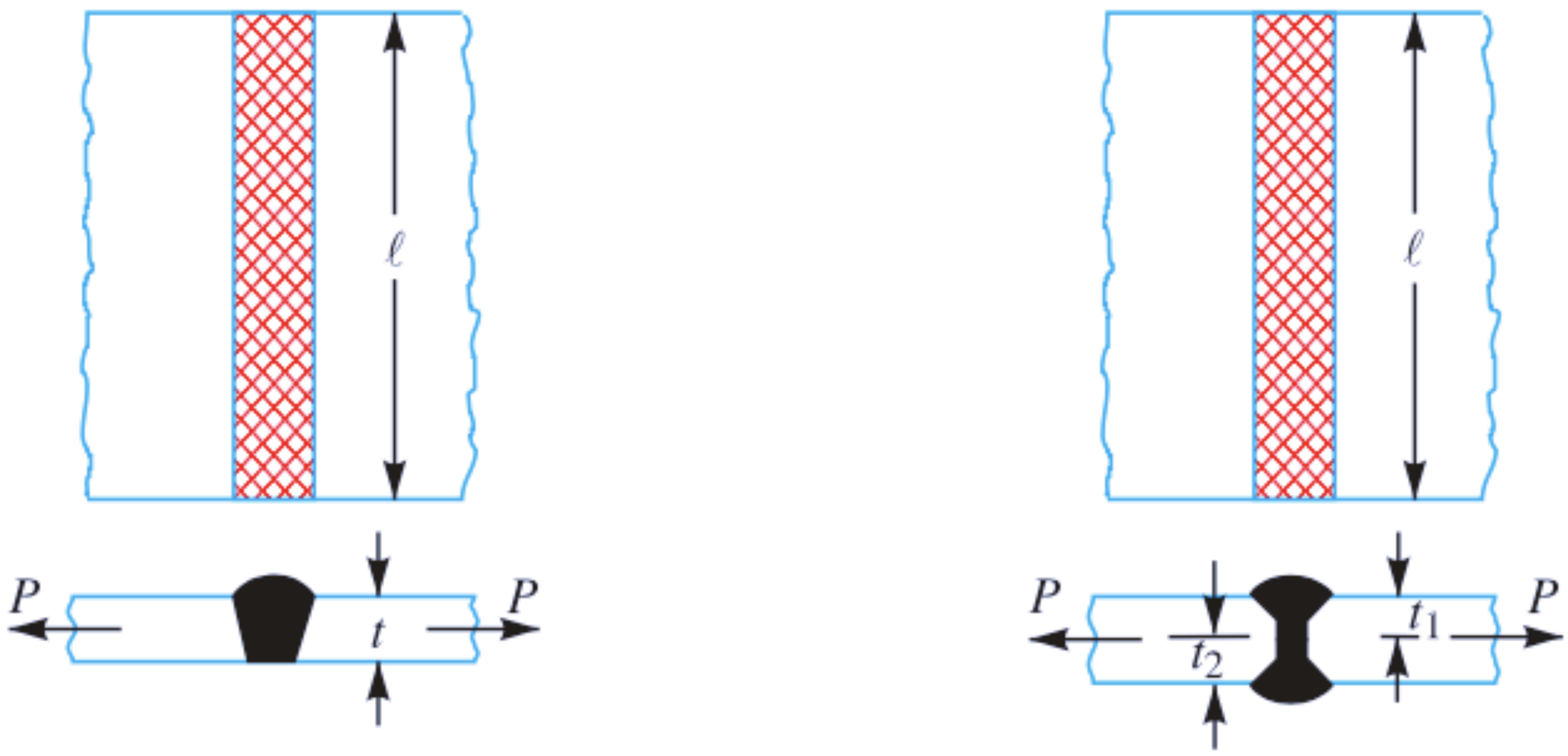
$$R = l/2$$

$$\boxed{\tau = \frac{3T}{t l^2}}$$

$$t = 0.707s$$

$$\tau_{max} = \frac{4.242T}{s l^2}$$

Strength of Butt Joints



(*a*) Single *V*-butt joint. (*b*) Double *V*-butt joint.

The butt joints are designed for tension or compression

In case of butt joint, the length of leg or size of weld is equal to the throat thickness which is equal to thickness of plates.

Tensile strength $= \sigma_t \times t \times l$
Single v-butt

For double v-butt Joint $= \sigma_t \times l \times (t_1 + t_2)$

Table 10.4. Recommended minimum size of welds.

Thickness of plate (mm)	3 – 5	6 – 8	10 – 16	18 – 24	26 – 55	Over 58
Minimum size of weld (mm) (S)	3	5	6	10	14	20

Stresses for Welded Joints

The stresses in welded joints are difficult to determine because of the variable and unpredictable parameters like homogeneity of the weld metal, thermal stresses in the welds, changes of physical properties due to high rate of cooling etc. The stresses are obtained, on the following assumptions:

1. The load is distributed uniformly along the entire length of the weld, and
2. The stress is spread uniformly over its effective section.

Table 10.5. Stresses for welded joints.

Type of weld	*Bare electrode*		*Coated electrode*	
	Steady load (MPa)	*Fatigue load (MPa)*	*Steady load (MPa)*	*Fatigue load (MPa)*
1. Fillet welds (All types)	80	21	98	35
2. Butt welds				
Tension	90	35	110	55
Compression	100	35	125	55
Shear	55	21	70	35

Stress Concentration Factor for Welded Joints

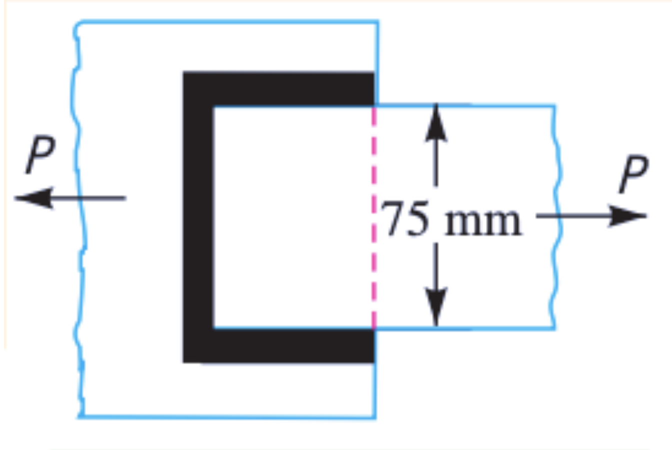
The reinforcement provided to the weld produces stress concentration at the junction of the weld and the parent metal

Table 10.6. Stress concentration factor for welded joints.

Type of joint	*Stress concentration factor*
1. Reinforced butt welds	1.2
2. Toe of transverse fillet welds	1.5
3. End of parallel fillet weld	2.7
4. T-butt joint with sharp corner	2.0

Note : For static loading and any type of joint, stress concentration factor is 1.0.

A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.



$l = 75mm$ $\sigma_t = 70 mpa$

$t = 12.5mm$ $\tau = 56 mpa$

l_1 = Effective length of transverse fillet

$= 75 - 12.5 = 62.5 mm$

$l_2 \rightarrow$ length of parallel fillet weld.

Max load the plate can carry = Area x stress

$= 75 \times 12.5 \times 70$

$P = 65625 N$

Load carried by transverse fillet weld

$= 0.707 s l_1 \times \sigma_t$ $s = 12.5mm$

$P_1 = 38664 N$

Load carried by double ||el fillet weld

$= 1.414 s l_2 \times \tau$

$P_2 = 1.414 \times 12.5 \times l_2 \times 56$ $P_2 = 990 l_2$

$P = P_1 + P_2$

$P_2 = P - P_1 = 65625 - 38664$

$= 26961$

$26961 = 990\, l_2 \Rightarrow \underline{l_2 = 27.23\,mm}$

Total length $= l_2 + 12.5\,mm$

$= 40\,mm$ //

For fatigue loading:

S.C.F
- transverse $= 1.5$
- parallel $= 2.7$

Permissible tensile stress $(\sigma_t)_a = \dfrac{70}{1.5} = 46.66\,mpa$

" shear " $(\tau)_a = \dfrac{56}{2.7} = 20.74\,mpa$

Load carried by transverse $= 0.707 \times 12.5 \times 62.5 \times 46.66$

$= 25772$

parallel $= 1.414 \times 12.5 \times l_2 \times 20.74$

$= 366\, l_2$

$65625 = 25772 + 366\, l_2 \Rightarrow \boxed{l_2 = 109\,mm}$

final $= \underline{121.5\,mm}$

Example 10.4. *A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld so that the maximum stress does not exceed 56 MPa. Consider the joint first under static loading and then under fatigue loading.*

Length of weld for static loading

Let l = Length of weld, and

s = Size of weld = Plate thickness

= 12.5 mm ... (Given)

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414\, s \times l \times \tau$$

$$= 1.414 \times 12.5 \times l \times 56 = 990\, l$$

$$\therefore \quad l = 50 \times 10^3 / 990 = 50.5 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 50.5 + 12.5 = 63 \text{ mm}$$ **Ans.**

Length of weld for fatigue loading

From Table 10.6, we find that the stress concentration factor for parallel fillet welding is 2.7.

$\therefore$ Permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50 \times 10^3 = 1.414\, s \times l \times \tau = 1.414 \times 12.5 \times l \times 20.74 = 367\, l$$

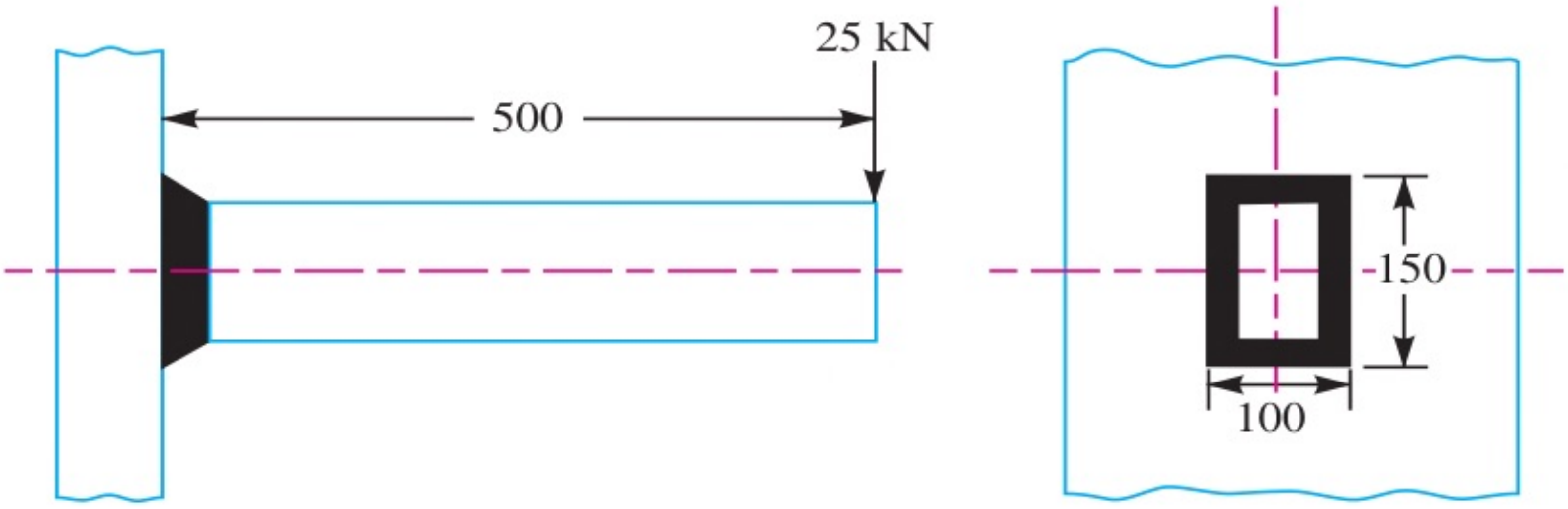
$$\therefore \quad l = 50 \times 10^3 / 367 = 136.2 \text{ mm}$$

Adding 12.5 for starting and stopping of weld run, we have

$$l = 136.2 + 12.5 = 148.7 \text{ mm}$$ **Ans.**

Example 10.11. *A rectangular cross-section bar is welded to a support by means of fillet welds as shown in Fig. 10.26.*

Determine the size of the welds, if the permissible shear stress in the weld is limited to 75 MPa.



All dimensions in mm

Fig. 10.26

Solution. Given : P = 25 kN = 25×10^3N ; τ_{max} = 75 MPa = 75N/mm^2 ; l = 100 mm ; b = 150 mm ; e = 500 mm

Let s = Size of the weld, and

t = Throat thickness.

The joint, as shown in Fig. 10.26, is subjected to direct shear stress and the bending stress. We know that the throat area for a rectangular fillet weld,

$$A = t\,(2b + 2l) = 0.707\,s\,(2b + 2l)$$

$$= 0.707s\,(2 \times 150 + 2 \times 100) = 353.5\,s \text{ mm}^2 \qquad \ldots (\because\ t = 0.707s)$$

$$\therefore \text{ Direct shear stress, } \tau = \frac{P}{A} = \frac{25 \times 10^3}{353.5\,s} = \frac{70.72}{s} \text{ N/mm}^2$$

We know that bending moment,

$$M = P \times e = 25 \times 10^3 \times 500 = 12.5 \times 10^6 \text{ N-mm}$$

From Table 10.7, we find that for a rectangular section, section modulus,

$$Z = t\left(b.l + \frac{b^2}{3}\right) = 0.707\,s\left[150 \times 100 + \frac{(150)^2}{3}\right] = 15\,907.5\,s \text{ mm}^3$$

$$\therefore \text{ Bending stress, } \sigma_b = \frac{M}{Z} = \frac{12.5 \times 10^6}{15\,907.5\,s} = \frac{785.8}{s} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$75 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{785.8}{s}\right)^2 + 4\left(\frac{70.72}{s}\right)^2} = \frac{399.2}{s}$$

$\therefore \qquad s = 399.2 / 75 = 5.32$ mm **Ans.**

Axially Loaded Unsymmetrical Welded Sections

Let l_a = Length of weld at the top,

l_b = Length of weld at the bottom,

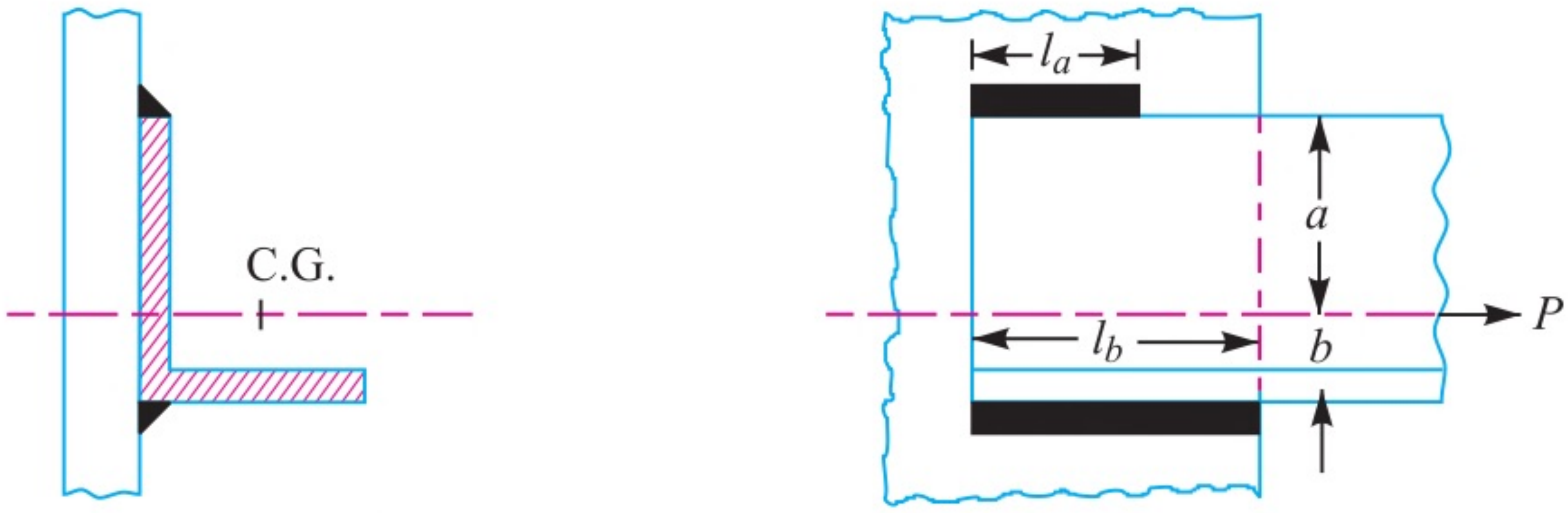
l = Total length of weld = $l_a + l_b$

P = Axial load,

a = Distance of top weld from gravity axis,

b = Distance of bottom weld from gravity axis, and

f = Resistance offered by the weld per unit length.



Moment of top weld w.r.t C.G:

$$= f \times l_a \times a$$

Similarly for bottom weld

$$= f \times l_b \times b$$

$$l_a a f = l_b b f$$

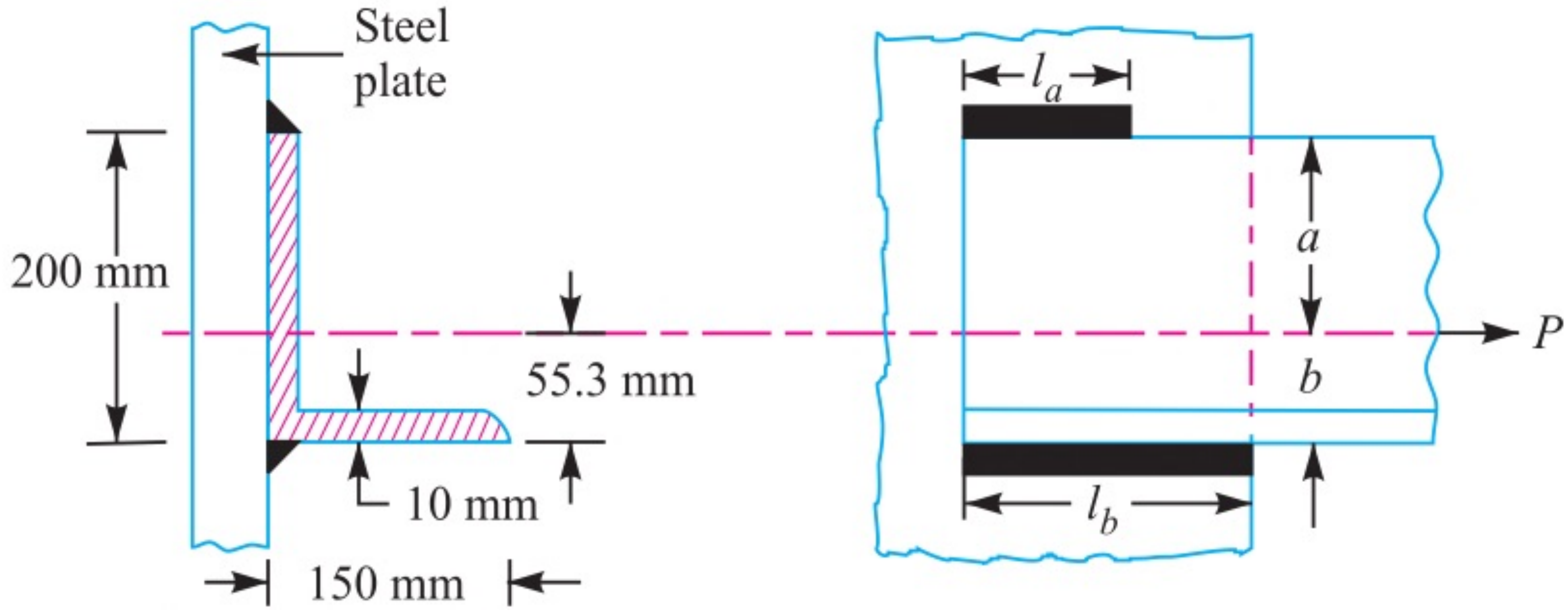
$$\Rightarrow \quad l_a \times a = l_b \times b \quad \text{— (1)}$$

$$l = l_a + l_b \quad \text{— (2)}$$

$$\boxed{l_a = \frac{l b}{a+b}}$$

$$\boxed{l_b = \frac{l a}{a+b}}$$

Example 10.8. *A 200 × 150 × 10 mm angle is to be welded to a steel plate by fillet welds as shown in Fig. 10.21. If the angle is subjected to a static load of 200 kN, find the length of weld at the top and bottom. The allowable shear stress for static loading may be taken as 75 MPa.*



$P = 200\ \text{KN}$ $\quad \tau = 75\ \text{MPa}$ $\quad l_a = ?$

$l_b = ?$

$a + b = 200\ \text{mm}$

$l = l_a + l_b$

$s = 10\ \text{mm}$

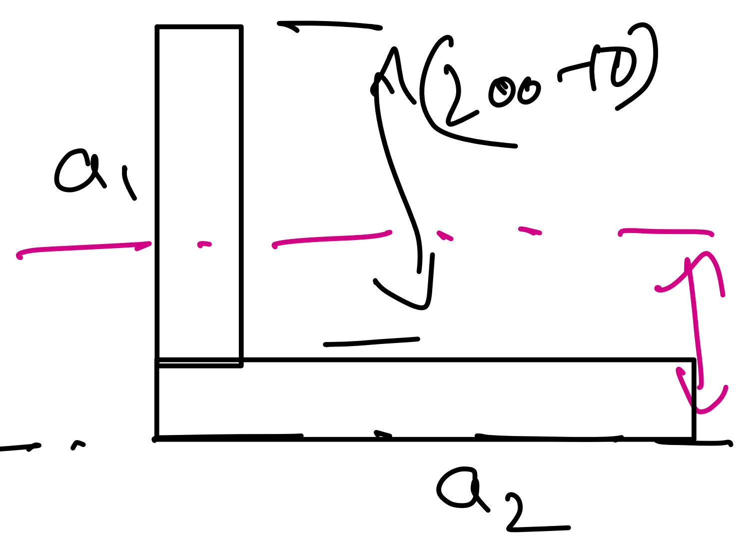
For single ||el weld

$$P = 0.707\, s\, l\, \tau$$

$$200 \times 10^3 = 0.707 \times 10 \times l \times 75$$

$$l = 377\ \text{mm} = l_a + l_b$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$



$$= \frac{(200-10)10 \times 95 + 150 \times 10 \times 5}{(190 \times 10) + (150 \times 10)}$$

$b = 55.3\ \text{mm}$ $\quad a = 200 - 55.3$

$= 144.7\ \text{mm}$

$l = 377 mm$

$a = 144.7$

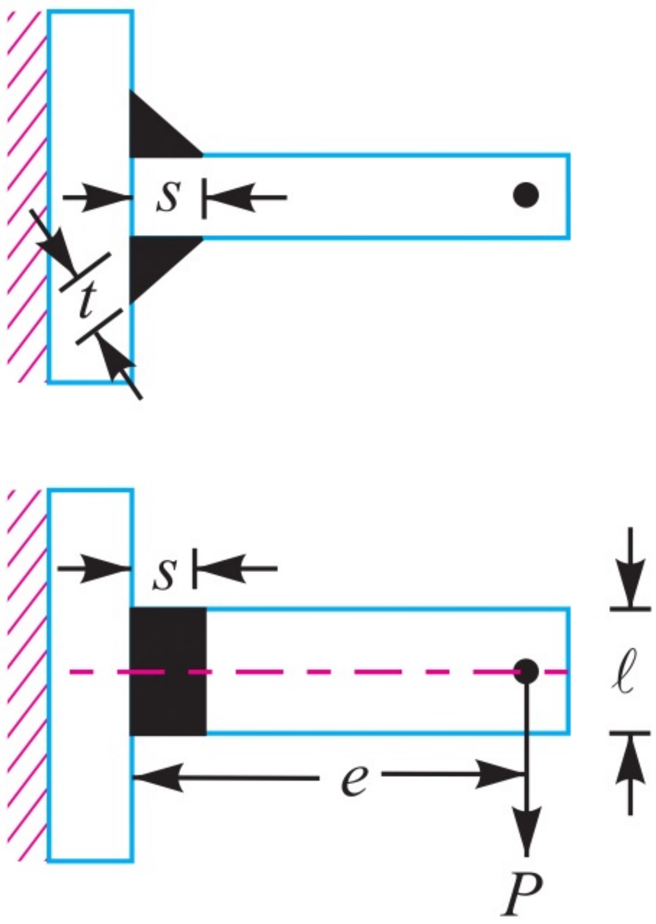
$b = 55.3$

$$l_a = \frac{lb}{a+b}$$

$l_a = 104.3 mm$

$l_b = 272.75 mm$

Eccentrically Loaded Welded Joints



Case (i):

s = Size of weld

l = length of weld

t = Throat thickness

$$\tau = \frac{P}{2 \times t \times l}$$

$$= \frac{P}{2 \times 0.707 s \times l}$$

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force P acting at the welds, and
2. Bending stress due to the bending moment $P \times e$.

$$\tau = \frac{P}{1.414\, s \times l}$$

$$\sigma_b = \frac{M}{I/y} = \frac{M}{Z}$$

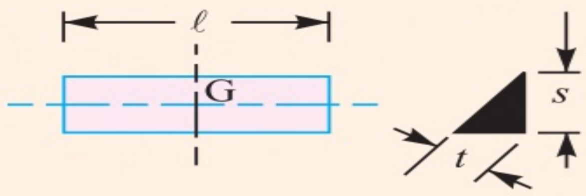
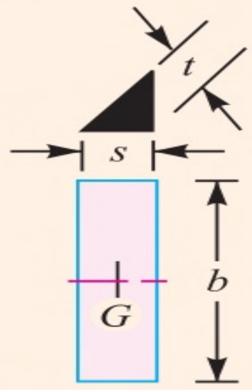
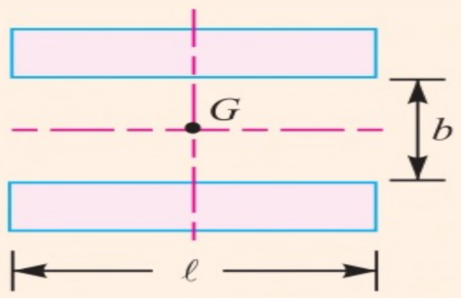
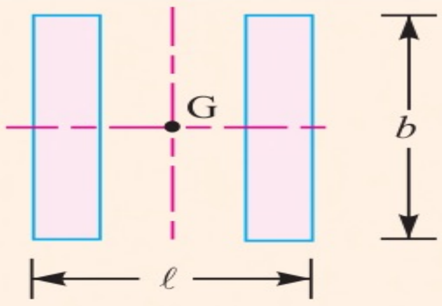
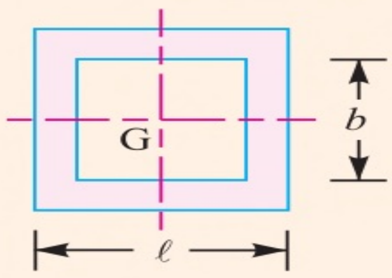
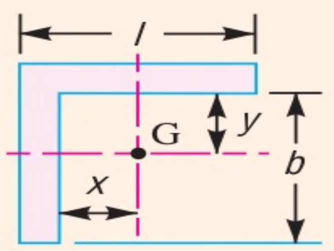
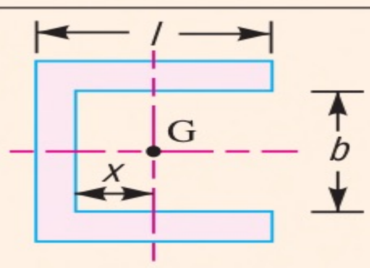
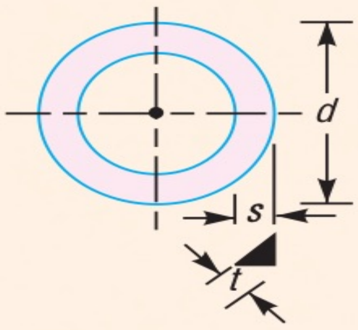
$$\sigma_b = \frac{P \times e}{\frac{t b^2}{3}}$$

$$\sigma_b = \frac{4.242 \times P \times e}{s \times l^2}$$

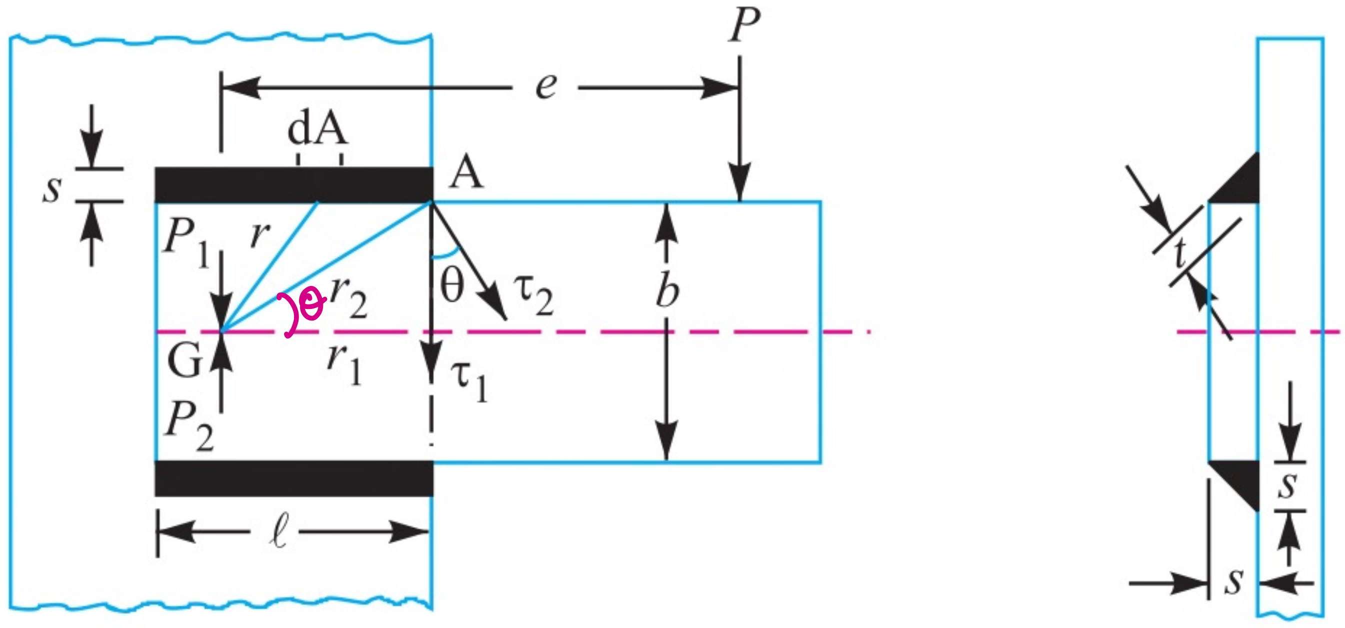
$$\sigma_{t\,max} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

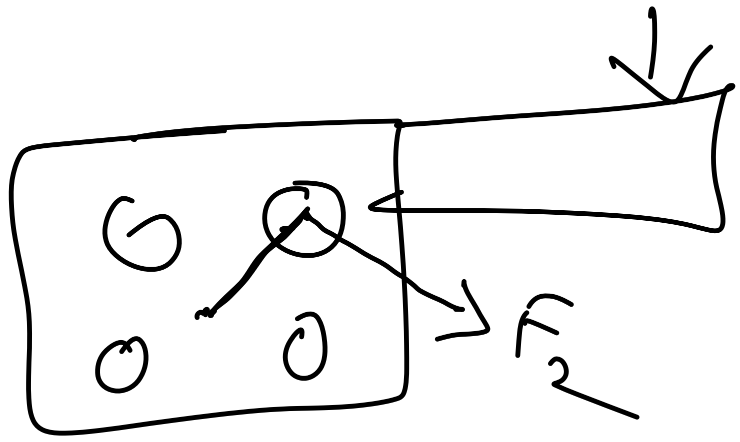
Table 10.7. Polar moment of inertia and section modulus of welds.

S.No	Type of weld	Polar moment of inertia (J)	Section modulus (Z)
1.		$\frac{t.l^3}{12}$	—
2.		$\frac{t.b^3}{12}$	$\frac{t.b^2}{6}$
3.		$\frac{t.l\,(3b^2 + l^2)}{6}$	$t.b.l$
4.		$\frac{t.b\,(b^2 + 3l^2)}{6}$	$\frac{t.b^2}{3}$
5.		$\frac{t\,(b + l)^3}{6}$	$t\left(b.l + \frac{b^2}{3}\right)$
6.	 $x = \frac{l^2}{2\,(l + b)},\ y = \frac{b^2}{2\,(l + b)}$	$t\left[\frac{(b + l)^4 - 6b^2l^2}{12\,(l + b)}\right]$	$t\left(\frac{4l.b + b^2}{6}\right)$ (Top) $t\left[\frac{b^2\,(4lb + b)}{6\,(2l + b)}\right]$ (Bottom)
7.	 $x = \frac{l^2}{2l + b}$	$t\left[\frac{(b + 2l)^3}{12} - \frac{l^2\,(b + l)^2}{b + 2l}\right]$	$t\left(l.b + \frac{b^2}{6}\right)$
8.		$\frac{\pi t d^3}{4}$	$\frac{\pi t d^2}{4}$

Case (ii):



Direct shear stress



$$\tau_1 = \frac{\text{Load}}{\text{shear area}} = \frac{P}{2 \times t \times l}$$

$$\boxed{\tau_1 = \frac{P}{1.414 s \times l}}$$

$$\tau = \frac{T \times r}{J}$$

Secondary shear stress :

$$\tau_2 \propto r_2 \qquad \frac{\tau_2}{r_2} = \text{constant} = \frac{\tau}{r}$$

$$\boxed{\tau = \frac{\tau_2}{r_2} \times r}$$

Considering a small area dA

Shear force = $\tau \times dA$

Elemental torque = $\tau \times dA \times r$

$$dT = \frac{\tau_2}{r_2} \times dA \times r^2$$

$$\int dx = x + c$$

$$\int y^2 dA$$

Total torque = $\int dT$

$$T = \int \frac{\tau_2}{r_2} \times dA \times r^2$$

$$T = \frac{\tau_2}{r_2} \int r^2 \, dA$$

$$\boxed{T = \frac{\tau_2}{r_2} \times J}$$ → polar moment of inertia

$$\boxed{\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}}$$

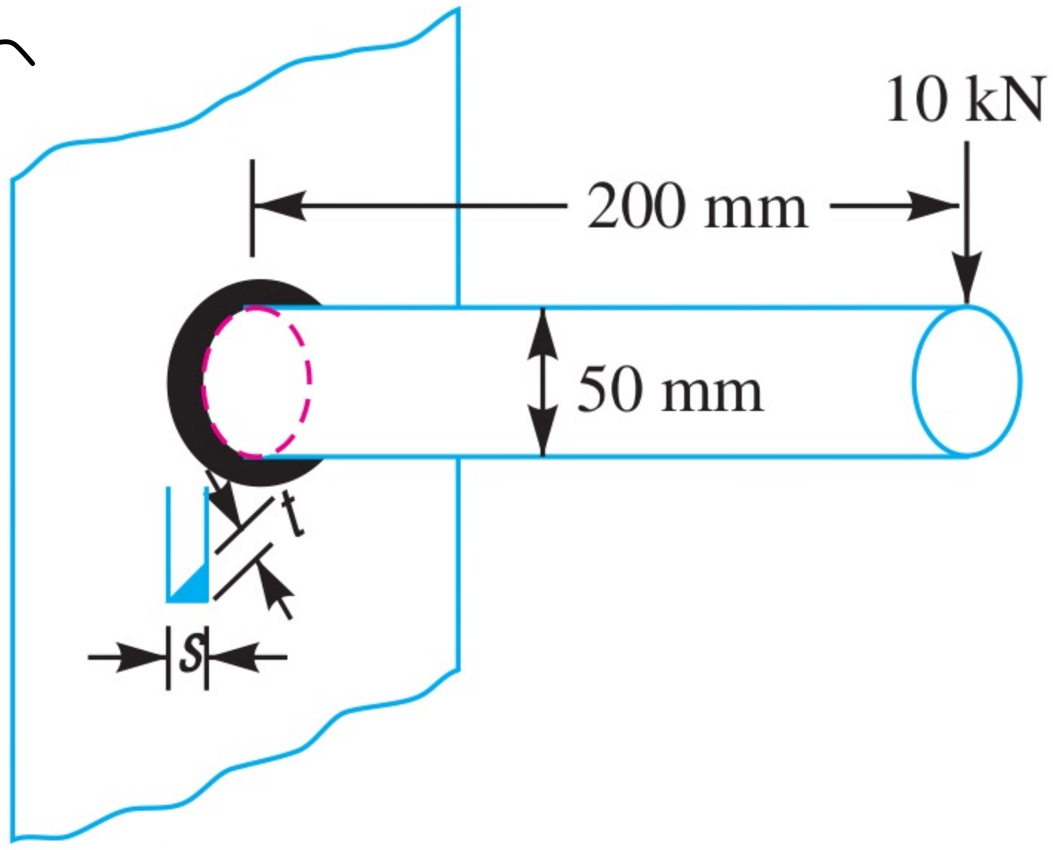
Resultant shear stress

$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \tau_2 \cos\theta}$$

$$\cos\theta = \frac{r_1}{r_2}$$

Example 10.10. *A 50 mm diameter solid shaft is welded to a flat plate as shown in Fig. 10.25. If the size of the weld is 15 mm, find the maximum normal and shear stress in the weld.*

This is a B.M problem



Area @ throat

$= t \times l$

$= t \times \pi D$

$= 0.707 \times S \times \pi \times 50$

$A = 1666 \text{ mm}^2$

Direct shear Stress $= \dfrac{P}{A} = \dfrac{10 \times 10^3}{1666} = 6 \text{ mpa}$

Moment $M = 2 \times 10^6$ N-mm

Section modulus $Z = \dfrac{\pi t d^2}{4} = 20823 \text{ mm}^3$

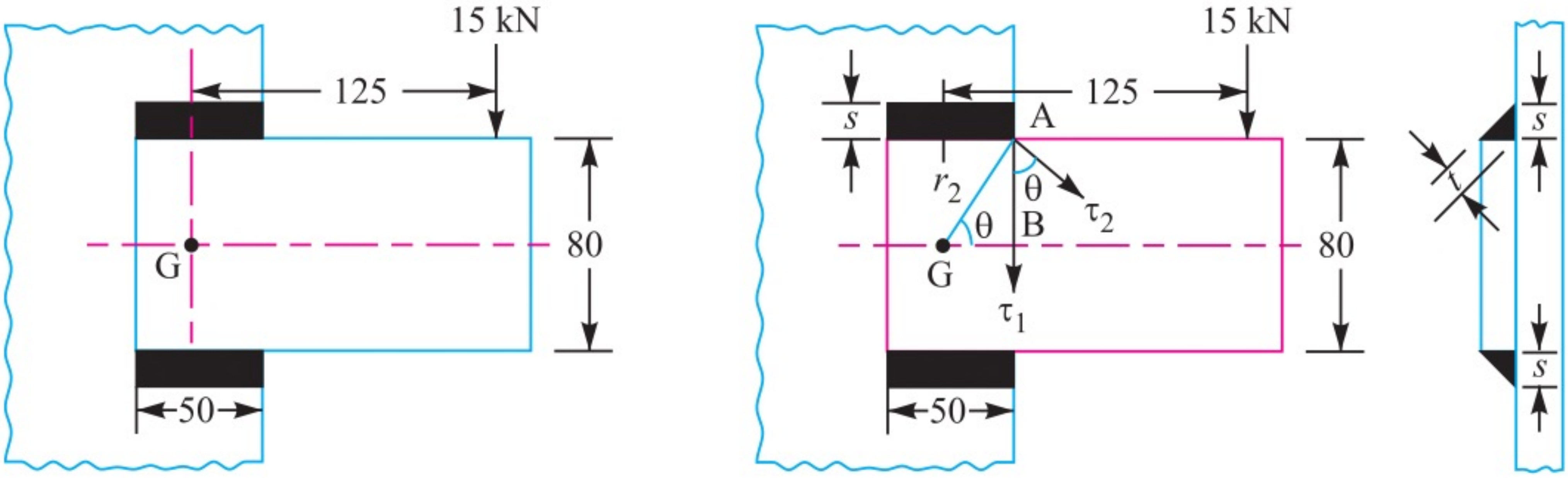
Bending stress $\sigma_b = \dfrac{M}{Z} = 96 \text{ mpa}$

Max normal stress $= \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$

$= 96.4 \text{ mpa}$

Max shear Stress $= \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} = 48.4 \text{ mpa}$

Example 10.13. *A bracket carrying a load of 15 kN is to be welded as shown in Fig. 10.28. Find the size of weld required if the allowable shear stress is not to exceed 80 MPa.*



S → size of weld

t → throat thickness

$$\tau_1 = \frac{\text{Load}}{\text{Shear area}}$$

Shear area
$= 2 \times t \times l$
$= 1.414\, S \times l$

$$\tau_1 = \frac{15 \times 10^3}{1.414 \times S \times 50}$$

$$\tau_1 = \frac{212.16}{S} \ N/mm^2$$

From table

$$J = \frac{t\,l\,(3b^2 + l^2)}{6}$$

$t = 0.707S$

$b = 80$

$l = 50$

$$J = 127{,}8495\ mm^4$$

$$r^2 = (AB)^2 + (BG)^2$$

$$= (40)^2 + (25)^2$$

$$e = 125\,mm$$

$$r_2 = 47.16\ mm$$

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{692}{S}\ N/mm^2$$

$$\cos\theta = r_1/r_2 = \frac{25}{47.16} = 0.53$$

Resultant shear stress

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1\tau_2\cos\theta}$$

$$\tau_1 = \frac{212.16}{S} \quad \tau_2 = \frac{692}{S}$$

$$80 = \sqrt{\left(\frac{212.16}{S}\right)^2 + \left(\frac{692}{S}\right)^2 + 2 \times \frac{212.16}{S} \times \frac{692}{S} \times 0.53}$$

$$S = \begin{matrix} 9.32 \times \\ \cancel{11.64} \end{matrix}$$

$$\boxed{S = 10.3\ mm}$$

Temporary Joints

Section 1 – Screwed Joints

A screw thread is formed by cutting a continuous helical groove on a cylindrical surface. A screw made by cutting a single helical groove on the cylinder is known as *single threaded* (or single-start) screw and if a second thread is cut in the space between the grooves of the first, a *double threaded* (or double-start) screw is formed. Similarly, triple and quadruple (*i.e.* multiple-start) threads may be formed. The helical grooves may be cut either *right hand* or *left hand*.

Advantages and Disadvantages of Screwed Joints

Advantages

1. Screwed joints are highly reliable in operation.
2. Screwed joints are convenient to assemble and disassemble.
3. A wide range of screwed joints may be adopted to various operating conditions.
4. Screws are relatively cheap to produce due to standardisation and highly efficient manufacturing processes.

Disadvantages

The main disadvantage of the screwed joints is the stress concentration in the threaded portions which are vulnerable points under variable load conditions.

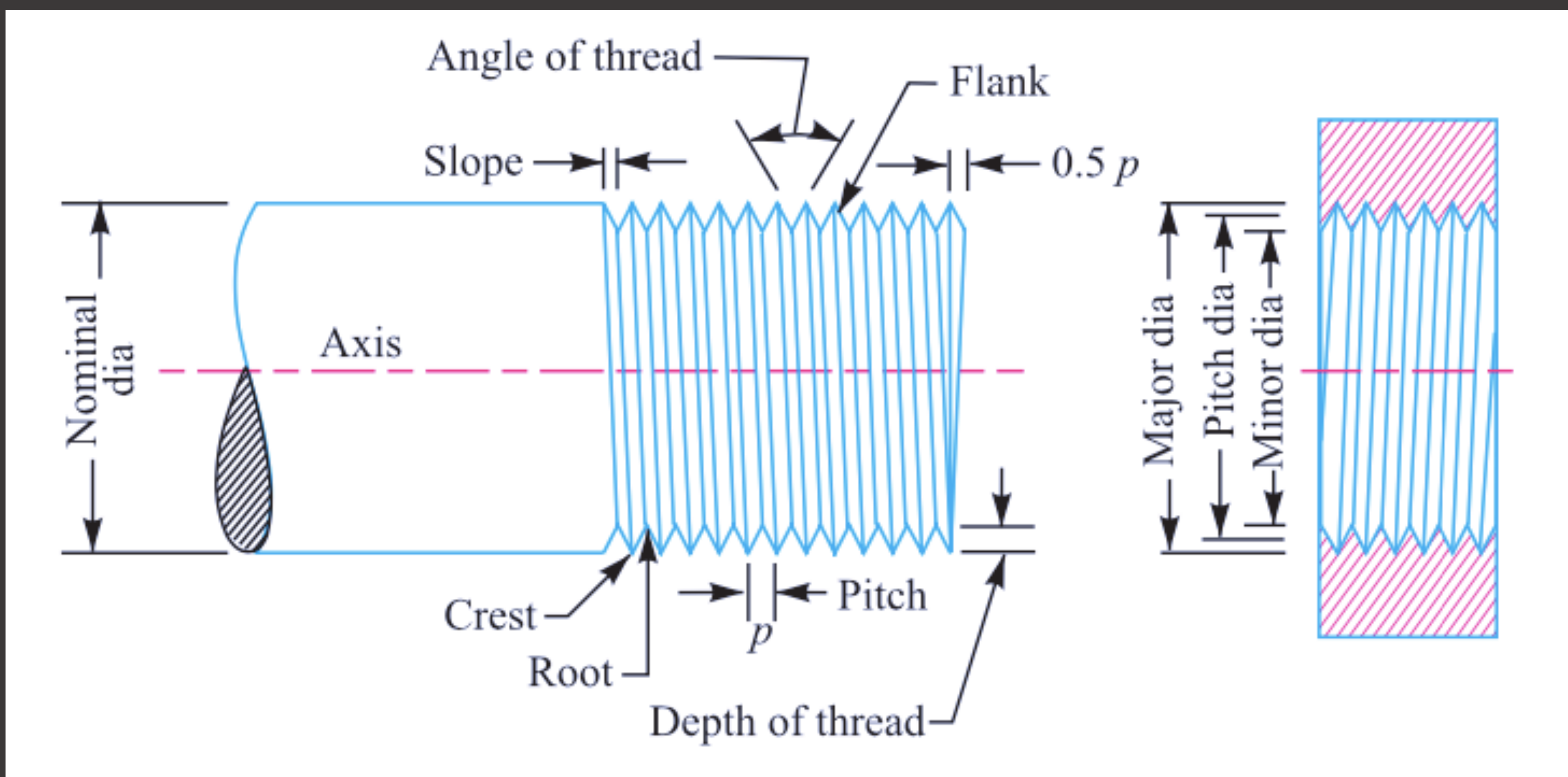
Important Terms Used in Screw Threads

1. ***Major diameter*.** It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as ***outside* or *nominal diameter***.
2. ***Minor diameter*.** It is the smallest diameter of an external or internal screw thread. It is also known as ***core* or *root diameter*.**
3. ***Pitch diameter*.** It is the diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an ***effective diameter***. In a nut and bolt assembly, it is the diameter at which the ridges on the bolt are in complete touch with the ridges of the corresponding nut.

4. **Pitch**. It is the distance from a point on one thread to the corresponding point on the next. This is measured in an axial direction between corresponding points in the same axial plane. Mathematically,

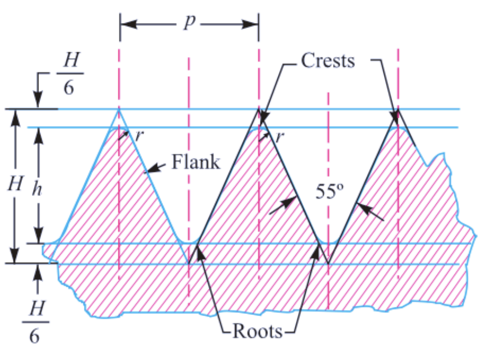
$$\text{Pitch} = \frac{1}{\text{No. of threads per unit length of screw}}$$

5. Lead. It is the distance between two corresponding points on the same helix. It may also be defined as the distance which a screw thread advances axially in one rotation of the nut. Lead is equal to the pitch in case of single start threads, it is twice the pitch in double start, thrice the pitch in triple start and so on.
6. Crest. It is the top surface of the thread.
7. Root. It is the bottom surface created by the two adjacent flanks of the thread.
8. Depth of thread. It is the perpendicular distance between the crest and root.
9. Flank. It is the surface joining the crest and root.
10. Angle of thread. It is the angle included by the flanks of the thread.
11. Slope. It is half the pitch of the thread.



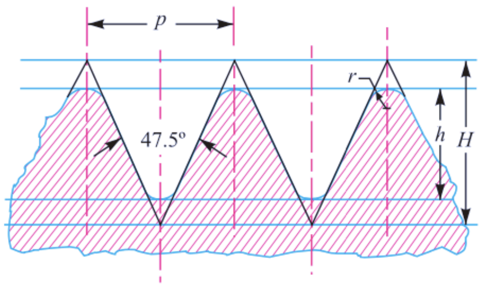
Forms of Screw Threads

1. British standard whitworth (B.S.W.) thread.
2. British association (B.A.)thread.
3. American national standard thread
4. Unified standard thread
5. Square thread.
6. Acme thread
7. Knuckle thread.
8. Buttress thread
9. Metric thread



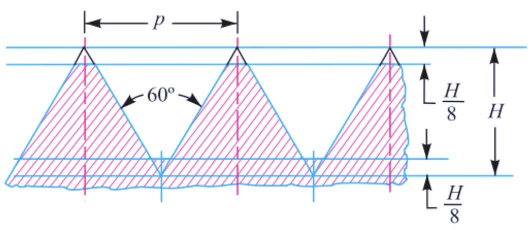
$H = 0.96\,p$; $h = 0.64\,p$; $r = 0.1373\,p$

Fig. 11.2. British standard whitworth (B.S.W) thread.



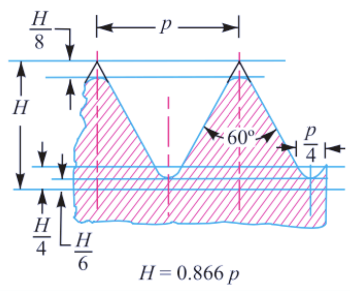
$H = 1.13634\,p$; $h = 0.6\,p$; $r = 0.18083\,p$

Fig. 11.3. British association (B.A.) thread.



$H = 0.866\,p$

Fig. 11.4. American national standard thread.



$H = 0.866\,p$

Fig. 11.5. Unified standard thread.

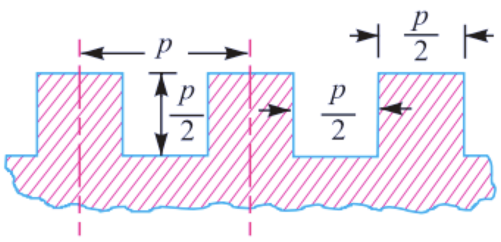


Fig. 11.6. Square thread.

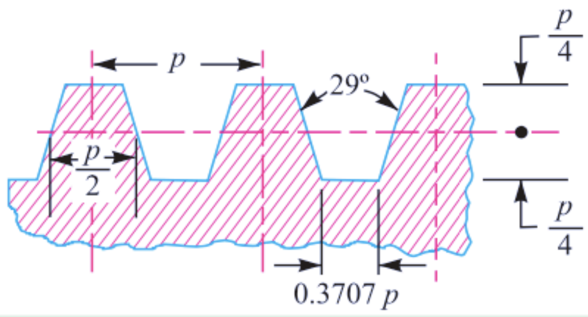


Fig. 11.7. Acme thread.

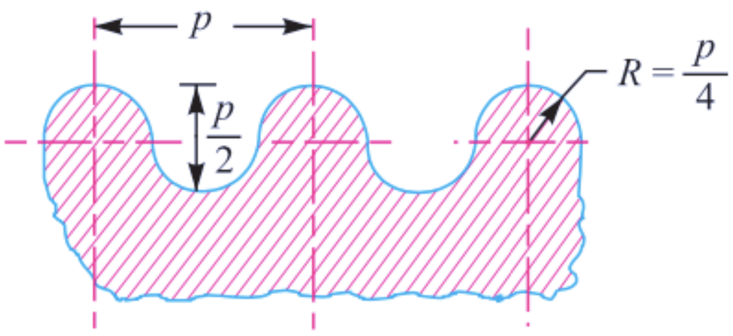
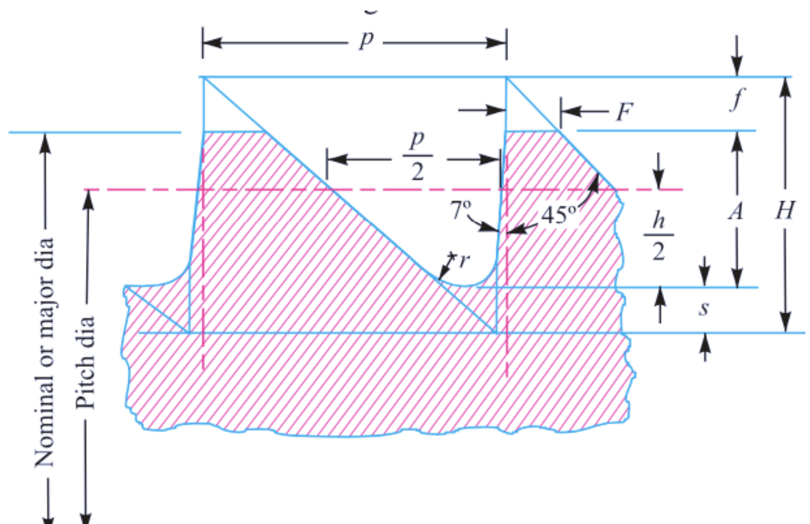


Fig. 11.8. Knuckle thread.



$H = 0.89064\,p$; $A = 0.50286\,p$; $f = 0.24532\,p$;
$s = 0.13946\,p$; $F = 0.27544\,p$; $r = 0.12055\,p$.

Fig. 11.9. Buttress thread.

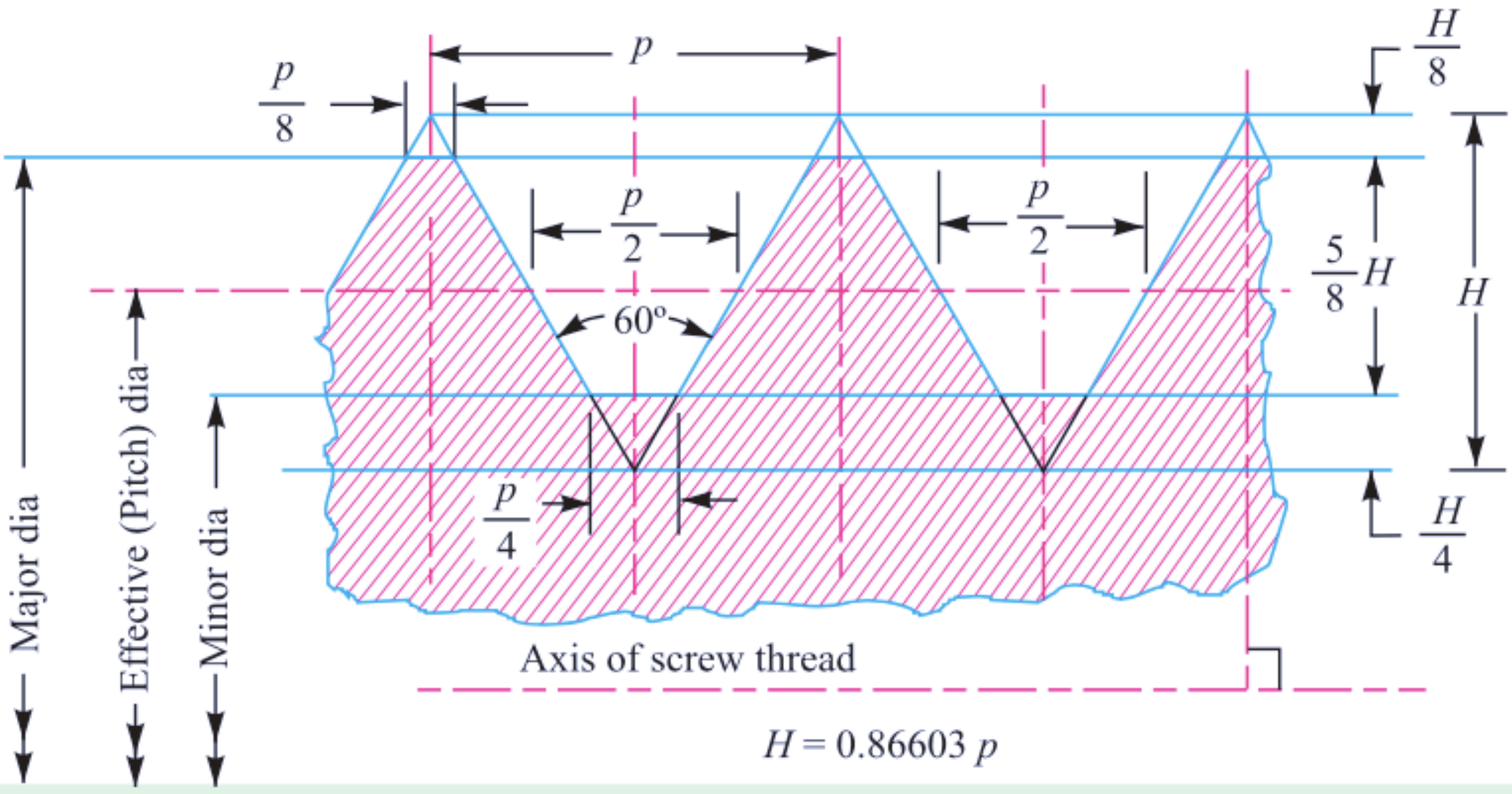
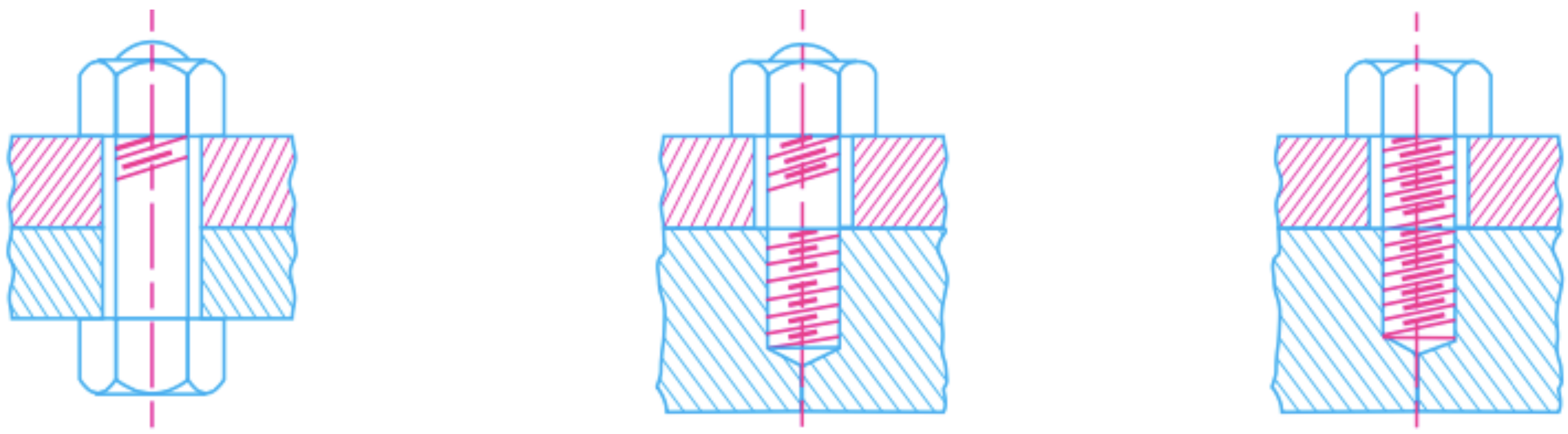


Fig. 11.10. Basic profile of the thread.

Common Types of Screw Fastenings

1. *Through bolts*

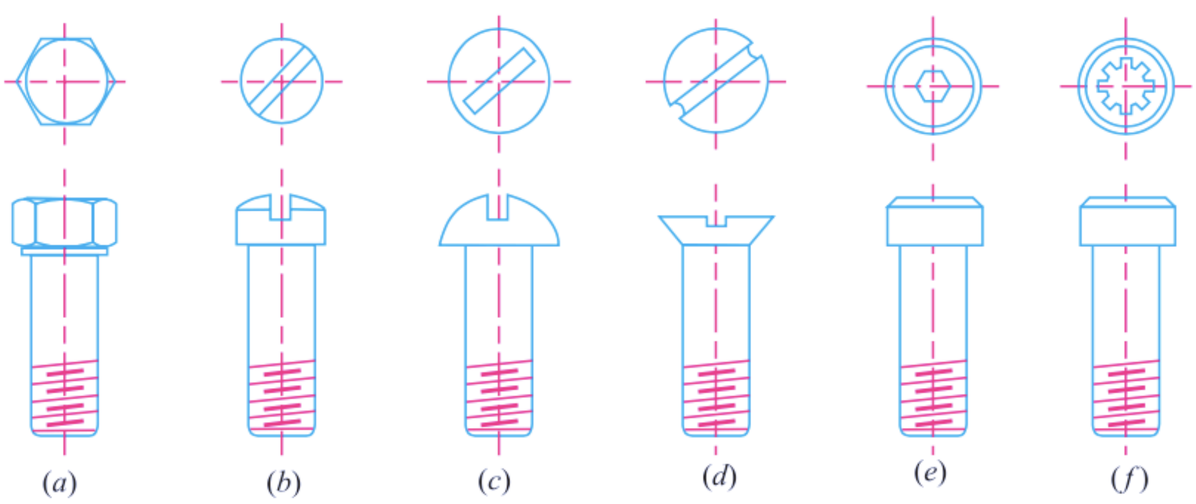


(*a*) Through bolt. (*b*) Tap bolt. (*c*) Stud.

2. *Tap bolts*

3. *Studs.*

4. *Cap screws*



(*a*) Hexagonal head; (*b*) Fillister head; (*c*) Round head; (*d*) Flat head; (*e*) Hexagonal socket; (*f*) Fluted socket.

Fig. 11.13. Types of cap screws.

5. Machine screws

6. Set screws

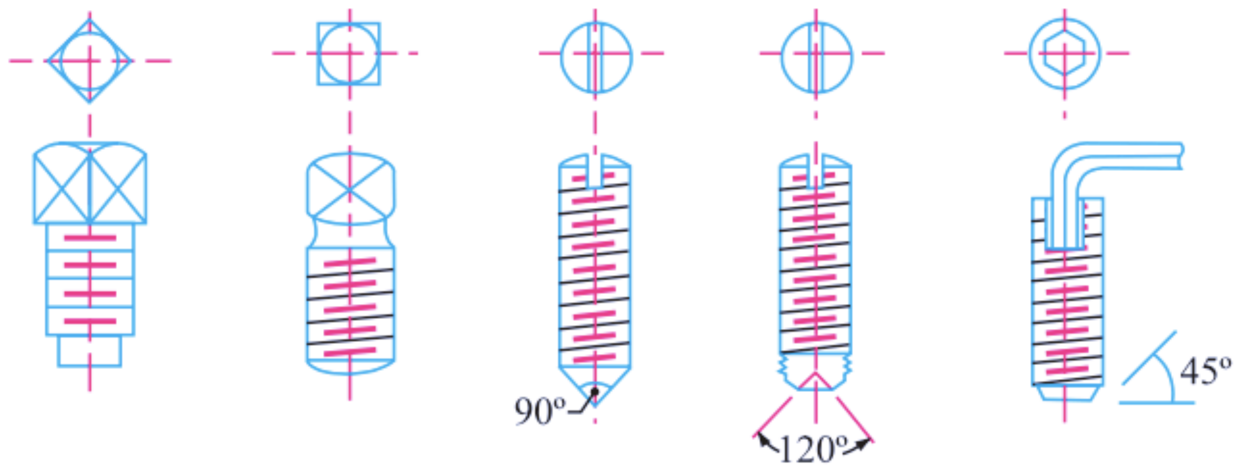


Fig. 11.14. Set screws.

Locking Devices:

Ordinary thread fastenings, generally, remain tight under static loads, but many of these fastenings become loose under the action of variable loads or when machine is subjected to vibrations. The loosening of fastening is very dangerous and must be prevented

1. ***Jam nut or lock nut.*** A most common locking device is a jam, lock or check nut. It has about one-half to two-third thickness of the standard nut. The thin lock nut is first tightened down with ordinary force, and then the upper nut (*i.e.* thicker nut) is tightened down upon it, as shown in Fig. 11.15 (*a*). The upper nut is then held tightly while the lower one is slackened back against it.

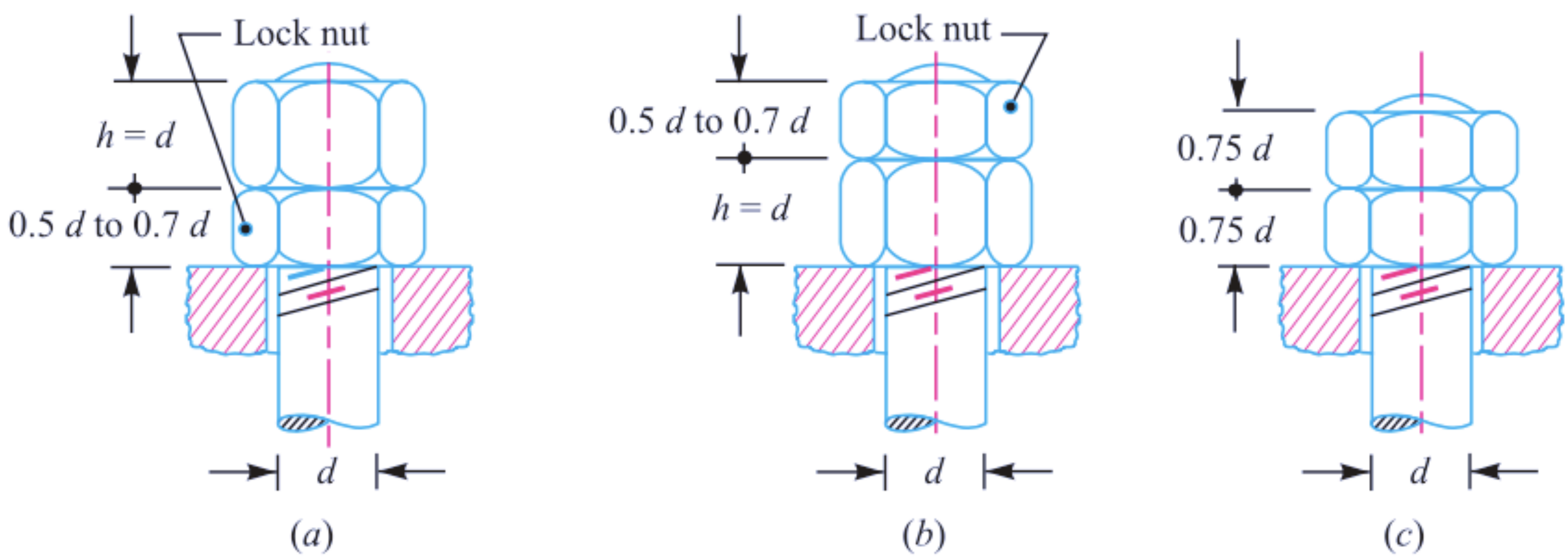


Fig. 11.15. Jam nut or lock nut.

In slackening back the lock nut, a thin spanner is required which is difficult to find in many shops. Therefore to overcome this difficulty, a thin nut is placed on the top as shown in Fig. 11.15 (*b*).

If the nuts are really tightened down as they should be, the upper nut carries a greater tensile load than the bottom one. Therefore, the top nut should be thicker one with a thin nut below it because it is desirable to put whole of the load on the thin nut. In order to overcome both the difficulties, both the nuts are made of the same thickness as shown in Fig. 11.15 (*c*).

2. *Castle nut.* It consists of a hexagonal portion with a cylindrical upper part which is slotted in line with the centre of each face, as shown in Fig. 11.16. The split pin passes through two slots in the nut and a hole in the bolt, so that a positive lock is obtained unless the pin shears. It is extensively used on jobs subjected to sudden shocks and considerable vibration such as in automobile industry.

3. *Sawn nut.* It has a slot sawed about half way through, as shown in Fig. 11.17. After the nut is screwed down, the small screw is tightened which produces more friction between the nut and the bolt. This prevents the loosening of nut.

4. *Penn, ring or grooved nut.* It has a upper portion hexagonal and a lower part cylindrical as shown in Fig. 11.18. It is largely used where bolts pass through connected pieces reasonably near their edges such as in marine type connecting rod ends. The bottom portion is cylindrical and is recessed to receive the tip of the locking set screw. The bolt hole requires counter-boring to receive the cylindrical portion of the nut. In order to prevent bruising of the latter by the case hardened tip of the set screw, it is recessed.

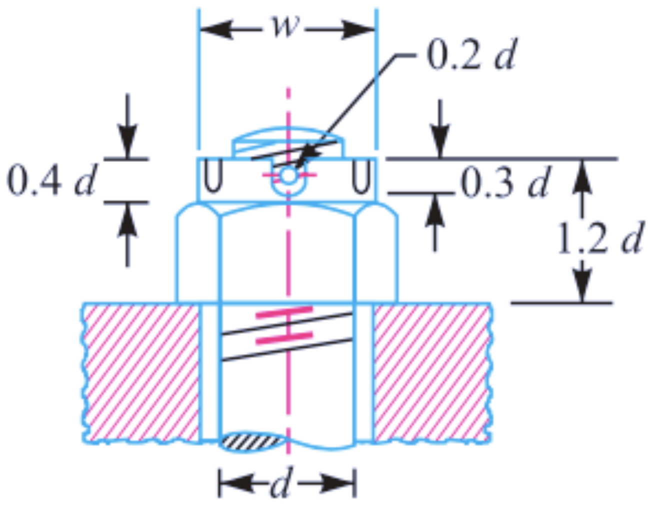


Fig. 11.16. Castle nut.

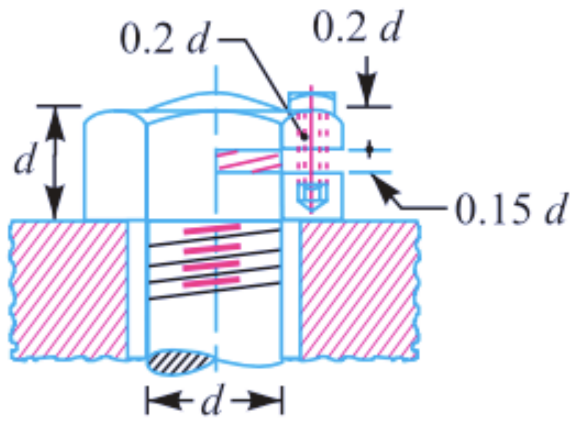


Fig. 11.17. Sawn nut.

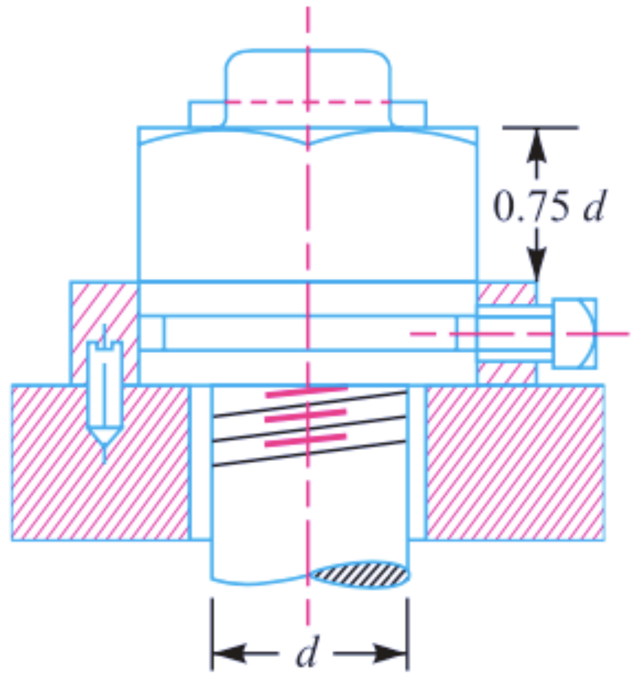


Fig. 11.18. Penn, ring or grooved nut.

5. *Locking with pin.* The nuts may be locked by means of a taper pin or cotter pin passing through the middle of the nut as shown in Fig. 11.19 (*a*). But a split pin is often driven through the bolt above the nut, as shown in Fig. 11.19 (*b*).

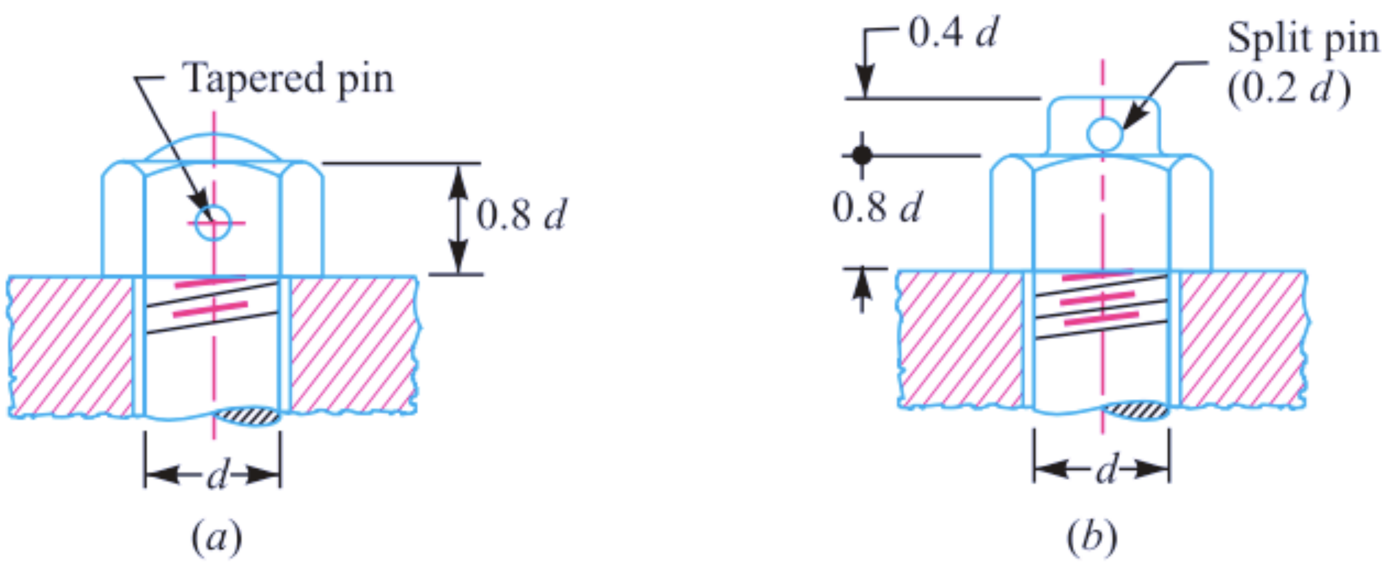


Fig. 11.19. Locking with pin.

6. ***Locking with plate.*** A form of stop plate or locking plate is shown in Fig. 11.20. The nut can be adjusted and subsequently locked through angular intervals of 30° by using these plates.

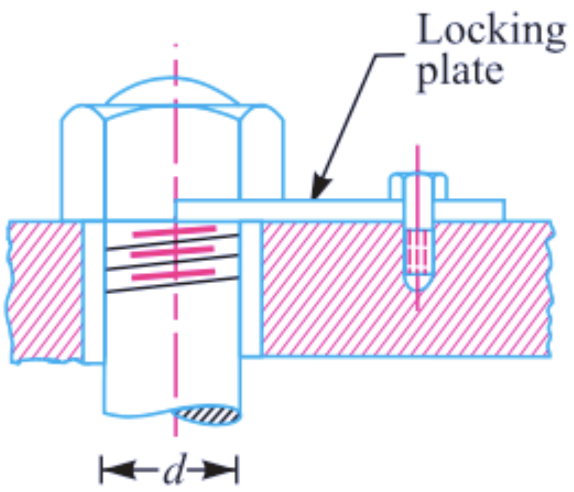


Fig. 11.20. Locking with plate.

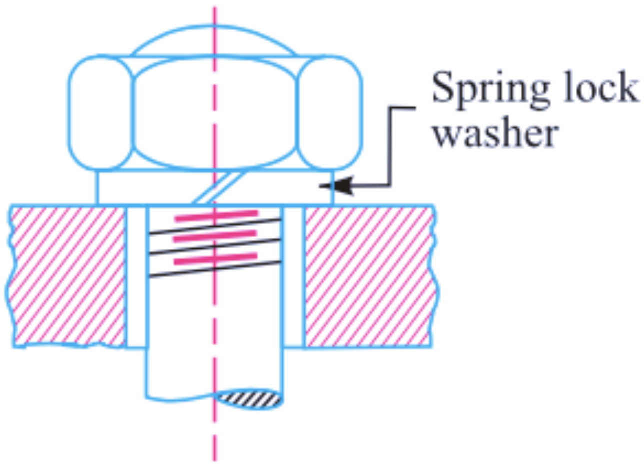


Fig. 11.21. Locking with washer.

7. ***Spring lock washer.*** A spring lock washer is shown in Fig. 11.21. As the nut tightens the washer against the piece below, one edge of the washer is caused to dig itself into that piece, thus increasing the resistance so that the nut will not loosen so easily. There are many kinds of spring lock washers manufactured, some of which are fairly effective.

Designation of Screw Threads

1. ***Size designation***. The size of the screw thread is designated by the letter `*M*' followed by the diameter and pitch, the two being separated by the sign ×. When there is no indication of the pitch, it shall mean that a coarse pitch is implied.

2. ***Tolerance designation***.

a) A figure designating tolerance grade as indicated below:

‘7’ for fine grade, ‘8’ for normal (medium) grade, and ‘9’ for coarse grade.

b) A letter designating the tolerance position as indicated below :

‘*H*’ for unit thread, ‘*d*’ for bolt thread with allowance, and ‘*h*’ for bolt thread without allowance.

For example, A bolt thread of 6 mm size of coarse pitch and with allowance on the threads and normal (medium) tolerance grade is designated as *M*6-8*d*.

11.10 Stresses in Screwed Fastening due to Static Loading

The following stresses in screwed fastening due to static loading are important from the subject point of view :

1. Internal stresses due to screwing up forces,
2. Stresses due to external forces, and
3. Stress due to combination of stresses at (1) and (2).

We shall now discuss these stresses, in detail, in the following articles.

11.11 Initial Stresses due to Screwing up Forces

The following stresses are induced in a bolt, screw or stud when it is screwed up tightly.

1. ***Tensile stress due to stretching of bolt.*** Since none of the above mentioned stresses are accurately determined, therefore bolts are designed on the basis of direct tensile stress with a large factor of safety in order to account for the indeterminate stresses. The initial tension in a bolt, based on experiments, may be found by the relation

$$P_i = 2840\ d\ \text{N}$$

where P_i = Initial tension in a bolt, and

d = Nominal diameter of bolt, in mm.

The above relation is used for making a joint fluid tight like steam engine cylinder cover joints etc. When the joint is not required as tight as fluid-tight joint, then the initial tension in a bolt may be reduced to half of the above value. In such cases

$$P_i = 1420\ d\ \text{N}$$

The small diameter bolts may fail during tightening, therefore bolts of smaller diameter (less than M 16 or M 18) are not permitted in making fluid tight joints.

If the bolt is not initially stressed, then the maximum safe axial load which may be applied to it, is given by

P = Permissible stress × Cross-sectional area at bottom of the thread (*i.e.* stress area)

The stress area may be obtained from Table 11.1 or it may be found by using the relation

$$\text{Stress area} = \frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2$$

where d_p = Pitch diameter, and

d_c = Core or minor diameter.

Simple machine tools.

Note : This picture is given as additional information and is not a direct example of the current chapter.

2. ***Torsional shear stress caused by the frictional resistance of the threads during its tightening.*** The torsional shear stress caused by the frictional resistance of the threads during its tightening may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\therefore \qquad \tau = \frac{T}{J} \times r = \frac{T}{\frac{\pi}{32}(d_c)^4} \times \frac{d_c}{2} = \frac{16\,T}{\pi\,(d_c)^3}$$

where τ = Torsional shear stress,

T = Torque applied, and

d_c = Minor or core diameter of the thread.

It has been shown during experiments that due to repeated unscrewing and tightening of the nut, there is a gradual scoring of the threads, which increases the torsional twisting moment (T).

3. ***Shear stress across the threads.*** The average thread shearing stress for the screw (τ_s) is obtained by using the relation :

$$\tau_s = \frac{P}{\pi d_c \times b \times n}$$

where b = Width of the thread section at the root.

The average thread shearing stress for the nut is

$$\tau_n = \frac{P}{\pi d \times b \times n}$$

where d = Major diameter.

4. ***Compression or crushing stress on threads.*** The compression or crushing stress between the threads (σ_c) may be obtained by using the relation :

$$\sigma_c = \frac{P}{\pi\,[d^2 - (d_c)^2]\,n}$$

where d = Major diameter,

d_c = Minor diameter, and

n = Number of threads in engagement.

5. ***Bending stress if the surfaces under the head or nut are not perfectly parallel to the bolt axis.*** When the outside surfaces of the parts to be connected are not parallel to each other, then the bolt will be subjected to bending action. The bending stress (σ_b) induced in the shank of the bolt is given by

$$\sigma_b = \frac{x \,.\, E}{2l}$$

where x = Difference in height between the extreme corners of the nut or head,

l = Length of the shank of the bolt, and

E = Young's modulus for the material of the bolt.

Example 11.1. *Determine the safe tensile load for a bolt of M 30, assuming a safe tensile stress of 42 MPa.*

Solution. Given : d = 30 mm ; σ_t = 42 MPa = 42 N/mm^2

From Table 11.1 (coarse series), we find that the stress area *i.e.* cross-sectional area at the bottom of the thread corresponding to M 30 is 561 mm^2.

$\therefore$ Safe tensile load = Stress area × σ_t = 561 × 42 = 23 562 N = 23.562 kN **Ans.**

Note: In the above example, we have assumed that the bolt is not initially stressed.

Example 11.2. *Two machine parts are fastened together tightly by means of a 24 mm tap bolt. If the load tending to separate these parts is neglected, find the stress that is set up in the bolt by the initial tightening.*

Solution. Given : d = 24 mm

From Table 11.1 (coarse series), we find that the core diameter of the thread corresponding to M 24 is d_c = 20.32 mm.

Let σ_t = Stress set up in the bolt.

We know that initial tension in the bolt,

$$P = 2840\, d = 2840 \times 24 = 68\ 160 \text{ N}$$

We also know that initial tension in the bolt (P),

$$68\ 160 = \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{\pi}{4}(20.30)^2\,\sigma_t = 324\,\sigma_t$$

$\therefore$ σ_t = 68 160 / 324 = 210 N/mm^2 = 210 MPa **Ans.**

11.12 Stresses due to External Forces

The following stresses are induced in a bolt when it is subjected to an external load.

1. *Tensile stress.* The bolts, studs and screws usually carry a load in the direction of the bolt axis which induces a tensile stress in the bolt.

Let d_c = Root or core diameter of the thread, and

σ_t = Permissible tensile stress for the bolt material.

We know that external load applied,

$$P = \frac{\pi}{4}(d_c)^2\,\sigma_t \quad \text{or} \quad d_c = \sqrt{\frac{4P}{\pi\,\sigma_t}}$$

Now from Table 11.1, the value of the nominal diameter of bolt corresponding to the value of d_c may be obtained or stress area $\left[\frac{\pi}{4}(d_c)^2\right]$ may be fixed.

Notes: (*a*) If the external load is taken up by a number of bolts, then

$$P = \frac{\pi}{4}(d_c)^2\,\sigma_t \times n$$

(*b*) In case the standard table is not available, then for coarse threads, d_c = 0.84 d, where d is the nominal diameter of bolt.

Simple machine tools.

Note : This picture is given as additional information and is not a direct example of the current chapter.

2. ***Shear stress.*** Sometimes, the bolts are used to prevent the relative movement of two or more parts, as in case of flange coupling, then the shear stress is induced in the bolts. The shear stresses should be avoided as far as possible. It should be noted that when the bolts are subjected to direct shearing loads, they should be located in such a way that the shearing load comes upon the body (*i.e.* shank) of the bolt and not upon the threaded portion. In some cases, the bolts may be relieved of shear load by using shear pins. When a number of bolts are used to share the shearing load, the finished bolts should be fitted to the reamed holes.

Let d = Major diameter of the bolt, and

n = Number of bolts.

$\therefore$ Shearing load carried by the bolts,

$$P_s = \frac{\pi}{4} \times d^2 \times \tau \times n \quad \text{or} \quad d = \sqrt{\frac{4\,P_s}{\pi \tau n}}$$

3. ***Combined tension and shear stress.*** When the bolt is subjected to both tension and shear loads, as in case of coupling bolts or bearing, then the diameter of the shank of the bolt is obtained from the shear load and that of threaded part from the tensile load. A diameter slightly larger than that required for either shear or tension may be assumed and stresses due to combined load should be checked for the following principal stresses.

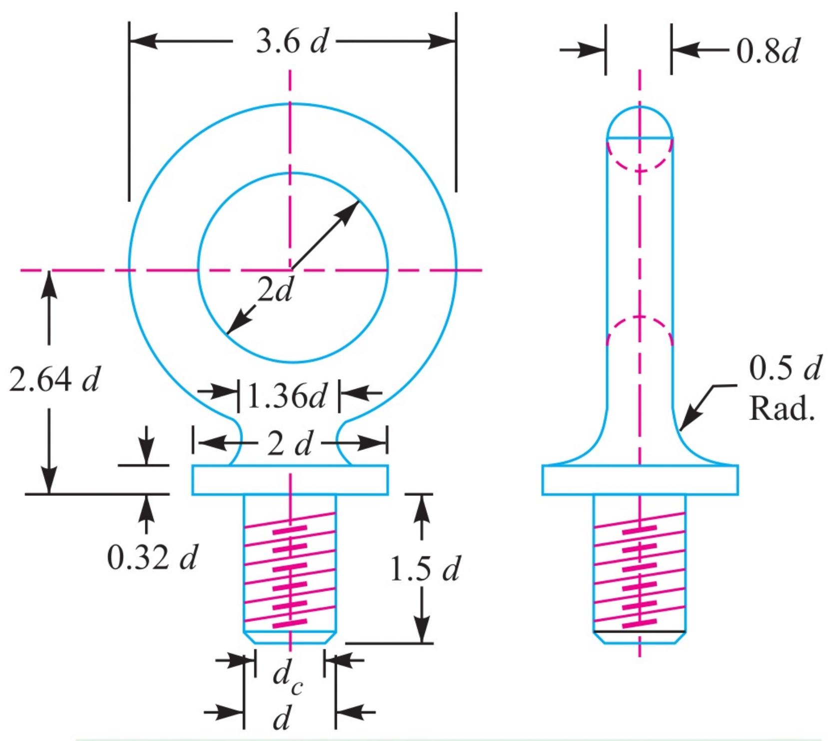
Maximum principal shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

and maximum principal tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

Example 11.3. *An eye bolt is to be used for lifting a load of 60 kN. Find the nominal diameter of the bolt, if the tensile stress is not to exceed 100 MPa. Assume coarse threads.*



$P = 60$ kN

$\sigma_t = 100$ mPa

$60 \times 10^3 = \frac{\pi}{4} \times d_c^2 \times \sigma_t$

$d_c = 27.6$ mm

From table

$d = 33$ mm

$d_c = 28.706$ mm

Example 11.4. *Two shafts are connected by means of a flange coupling to transmit torque of 25 N-m. The flanges of the coupling are fastened by four bolts of the same material at a radius of 30 mm. Find the size of the bolts if the allowable shear stress for the bolt material is 30 MPa.*

$T, n = 4$

$\tau = 30 \, mpa$

$P = \text{stress} \times \frac{\pi}{4} d_c^2$

$T = F \times r$

$F = T/r = 30 \times \frac{\pi}{4} \times d_c^2 \times n$

$d_c =$

11.13 Stress due to Combined Forces

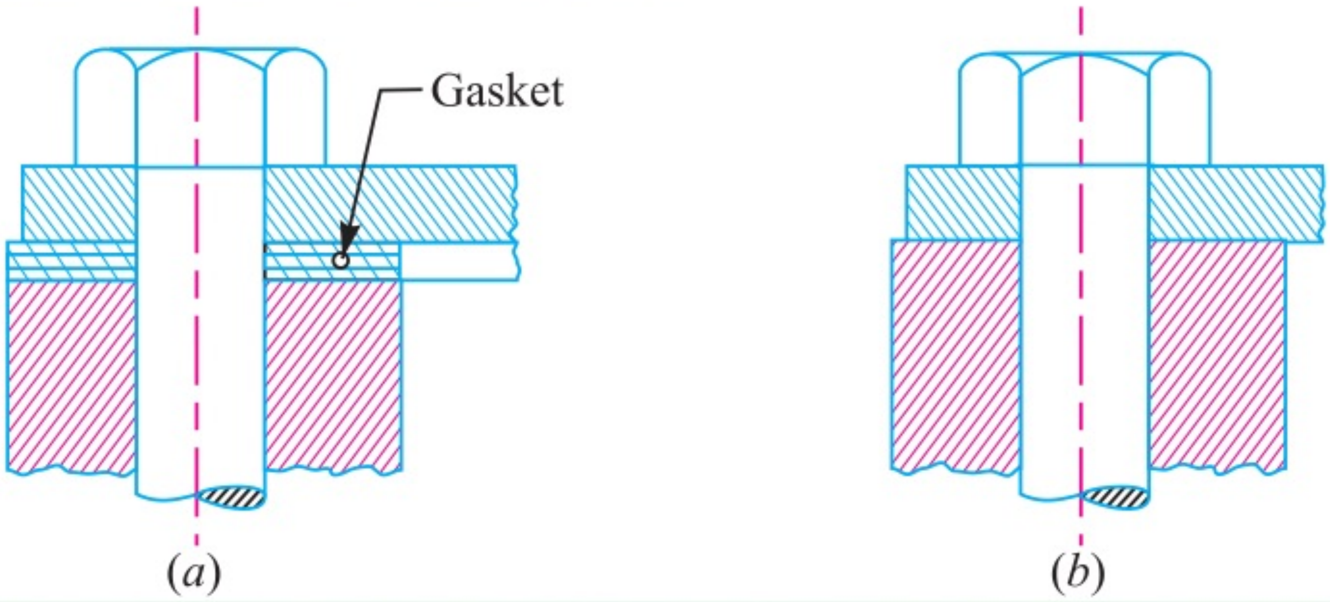


Fig. 11.23

The resultant axial load on a bolt depends upon the following factors :

1. The initial tension due to tightening of the bolt,
2. The extenal load, and
3. The relative elastic yielding (springiness) of the bolt and the connected members.

When the connected members are very yielding as compared with the bolt, which is a soft gasket, as shown in Fig. 11.23 (*a*), then the resultant load on the bolt is approximately equal to the sum of the initial tension and the external load. On the other hand, if the bolt is very yielding as compared with the connected members, as shown in Fig. 11.23 (*b*), then the resultant load will be either the initial tension or the external load, whichever is greater. The actual conditions usually lie between the two extremes. In order to determine the resultant axial load (*P*) on the bolt, the following equation may be used :

$$P = P_1 + \frac{a}{1+a} \times P_2 = P_1 + K.P_2 \qquad \ldots \left(\text{Substituting } \frac{a}{1+a} = K\right)$$

where P_1 = Initial tension due to tightening of the bolt,

P_2 = External load on the bolt, and

a = Ratio of elasticity of connected parts to the elasticity of bolt.

For soft gaskets and large bolts, the value of a is high and the value of $\frac{a}{1+a}$ is approximately equal to unity, so that the resultant load is equal to the sum of the initial tension and the external load.

For hard gaskets or metal to metal contact surfaces and with small bolts, the value of a is small and the resultant load is mainly due to the initial tension (or external load, in rare case it is greater than initial tension).

The value of 'a' may be estimated by the designer to obtain an approximate value for the resultant load. The values of $\frac{a}{1+a}$ (*i.e.* K) for various type of joints are shown in Table 11.2. The designer thus has control over the influence on the resultant load on a bolt by proportioning the sizes of the connected parts and bolts and by specifying initial tension in the bolt.

Table 11.2. Values of *K* for various types of joints.

Type of joint	$K = \frac{a}{1+a}$
Metal to metal joint with through bolts	0.00 to 0.10
Hard copper gasket with long through bolts	0.25 to 0.50
Soft copper gasket with long through bolts	0.50 to 0.75
Soft packing with through bolts	0.75 to 1.00
Soft packing with studs	1.00

$P = P_1$ (or) P_2 whichever is greater

11.14 Design of Cylinder Covers

The cylinder covers may be secured by means of bolts or studs, but studs are preferred. The possible arrangement of securing the cover with bolts and studs is shown in Fig. 11.24 (*a*) and (*b*) respectively. The bolts or studs, cylinder cover plate and cylinder flange may be designed as discussed below:

1. *Design of bolts or studs*

In order to find the size and number of bolts or studs, the following procedure may be adopted.

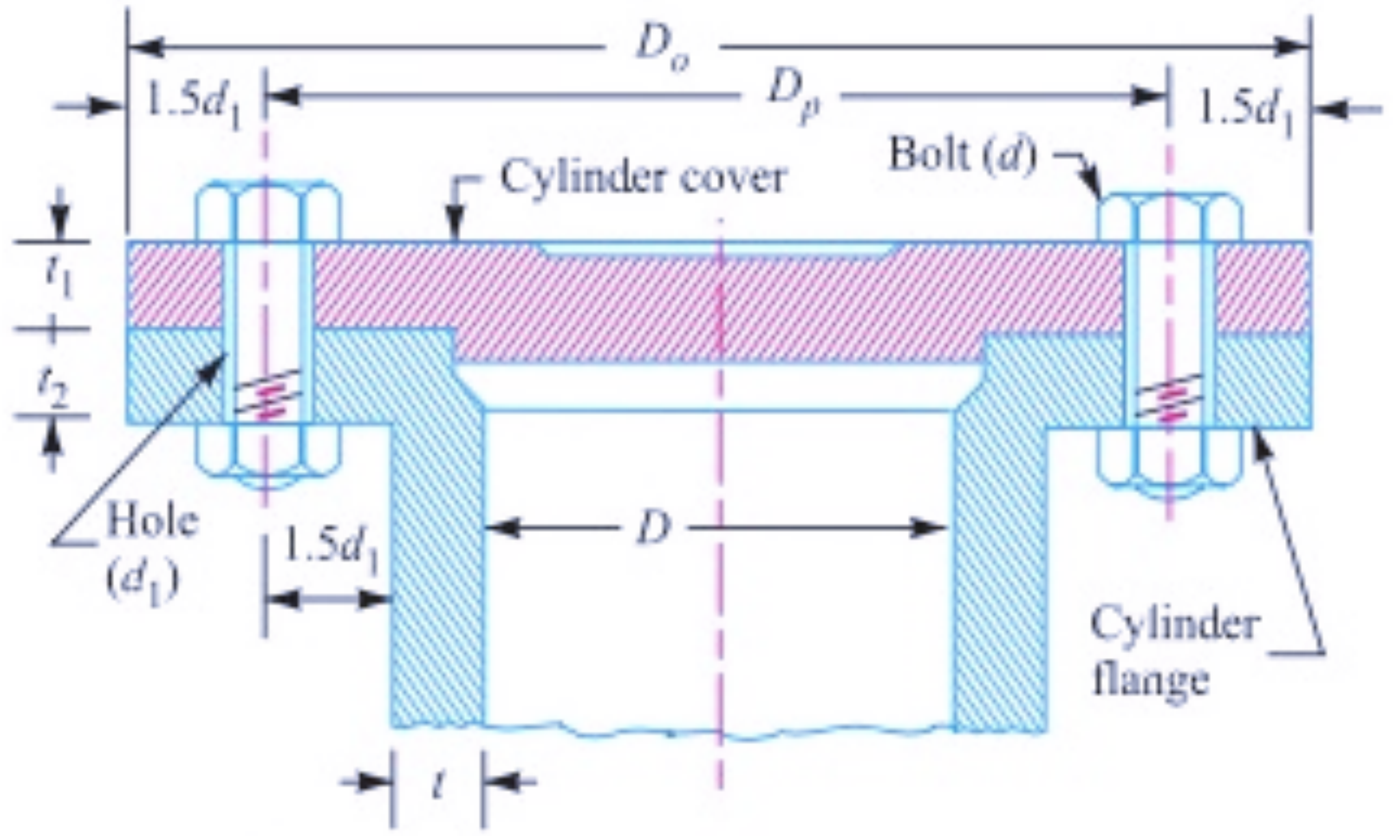
Let D = Diameter of the cylinder,

p = Pressure in the cylinder,

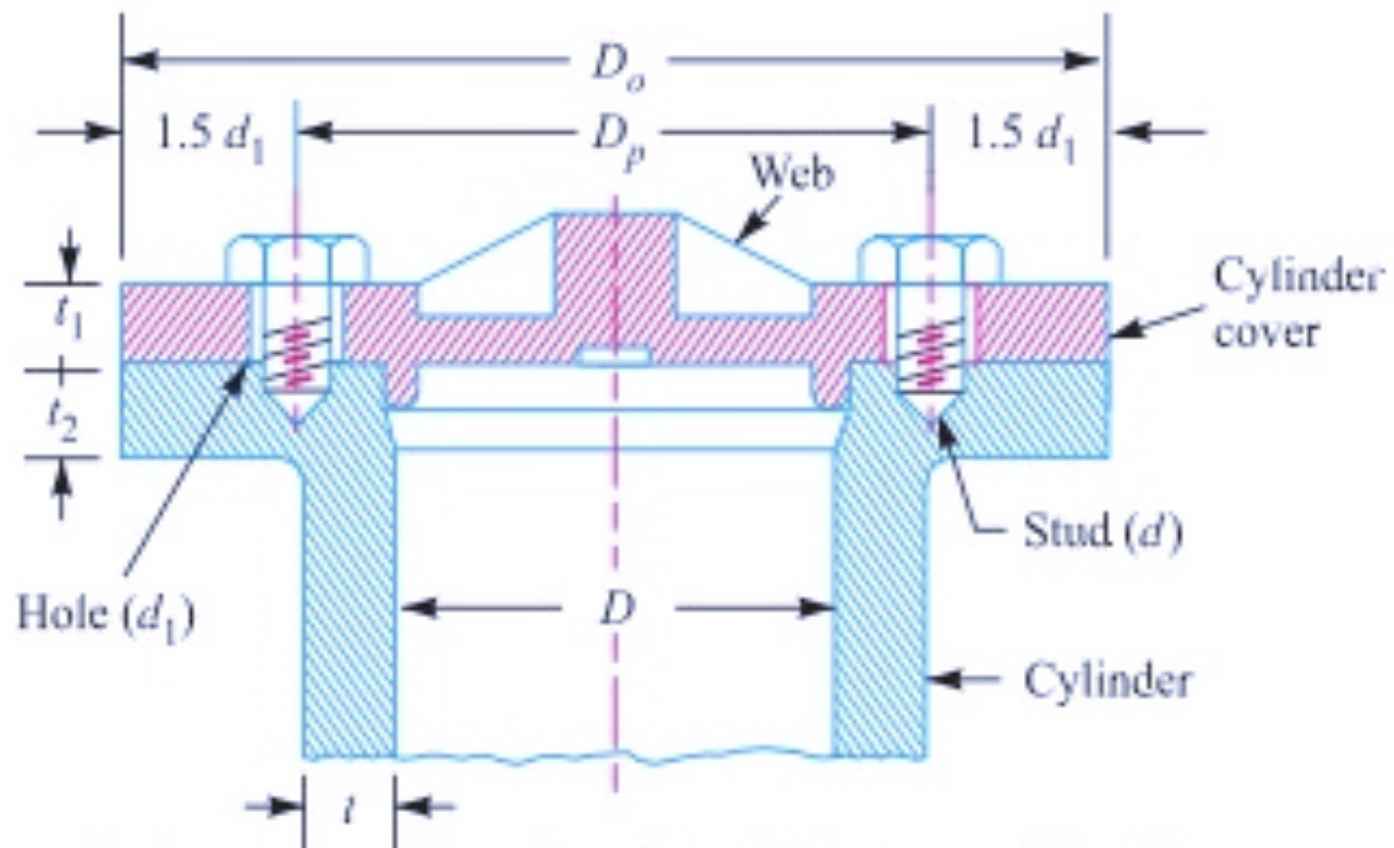
d_c = Core diameter of the bolts or studs,

n = Number of bolts or studs, and

σ_{tb} = Permissible tensile stress for the bolt or stud material.



(*a*) Arrangement of securing the cylinder cover with bolts.



(*b*) Arrangement of securing the cylinder cover with studs.

We know that upward force acting on the cylinder cover,

$$P = \frac{\pi}{4}(D^2)\, p$$

This force is resisted by n number of bolts or studs provided on the cover.

∴ Resisting force offered by n number of bolts or studs,

$$P = \frac{\pi}{4}(d_c)^2\, \sigma_{tb} \times n$$

From equations (*i*) and (*ii*), we have

$$\frac{\pi}{4}(D^2)\, p = \frac{\pi}{4}(d_c)^2\, \sigma_{tb} \times n$$

From this equation, the number of bolts or studs may be obtained, if the size of the bolt or stud is known and *vice-versa*. Usually the size of the bolt is assumed. If the value of n as obtained from the above relation is odd or a fraction, then next higher even number is adopted.

The bolts or studs are screwed up tightly, along with metal gasket or asbestos packing, in order to provide a leak proof joint. We have already discussed that due to the tightening of bolts, sufficient tensile stress is produced in the bolts or studs. This may break the bolts or studs, even before any load due to internal pressure acts upon them. Therefore a bolt or a stud less than 16 mm diameter should never be used.

The tightness of the joint also depends upon the circumferential pitch of the bolts or studs. The circumferential pitch should be between $20\sqrt{d_1}$ and $30\sqrt{d_1}$, where d_1 is the diameter of the hole in mm for bolt or stud. The pitch circle diameter (D_p) is usually taken as $D + 2t + 3d_1$ and outside diameter of the cover is kept as

$$D_o = D_p + 3d_1 = D + 2t + 6d_1$$

where t = Thickness of the cylinder wall.

2. *Design of cylinder cover plate*

The thickness of the cylinder cover plate (t_1) and the thickness of the cylinder flange (t_2) may be determined as discussed below:

Let us consider the semi-cover plate as shown in Fig. 11.25. The internal pressure in the cylinder tries to lift the cylinder cover while the bolts or studs try to retain it in its position. But the centres of pressure of these two loads do not coincide. Hence, the cover plate is subjected to bending stress. The point X is the centre of pressure for bolt load and the point Y is the centre of internal pressure.

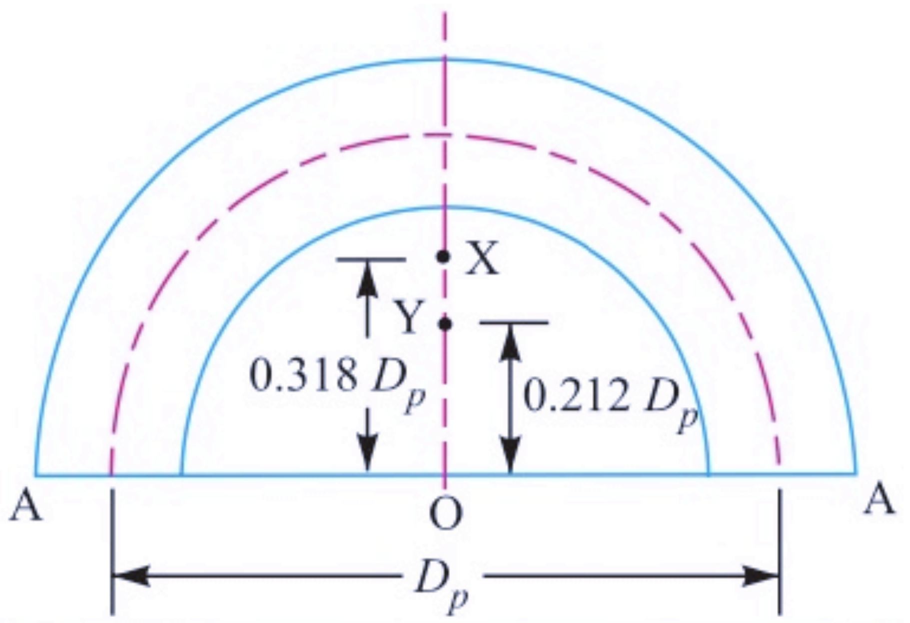


Fig. 11.25. Semi-cover plate of a cylinder.

We know that the bending moment at A-A,

$$M = \frac{\text{Total bolt load}}{2}(OX - OY) = \frac{P}{2}(0.318\ D_p - 0.212\ D_p)$$

$$= \frac{P}{2} \times 0.106\ D_p = 0.053\ P \times D_p$$

Section modulus, $$Z = \frac{1}{6} w\ (t_1)^2$$

where w = Width of plate

= Outside dia. of cover plate – 2 × dia. of bolt hole

= $D_o - 2d_1$

Knowing the tensile stress for the cover plate material, the value of t_1 may be determined by using the bending equation, *i.e.*, $\sigma_t = M / Z$.

3. Design of cylinder flange

The thickness of the cylinder flange (t_2) may be determined from bending consideration. A portion of the cylinder flange under the influence of one bolt is shown in Fig. 11.26.

The load in the bolt produces bending stress in the section X-X. From the geometry of the figure, we find that eccentricity of the load from section X-X is

e = Pitch circle radius – (Radius of bolt hole + Thickness of cylinder wall)

$$= \frac{D_p}{2} - \left(\frac{d_1}{2} + t\right)$$

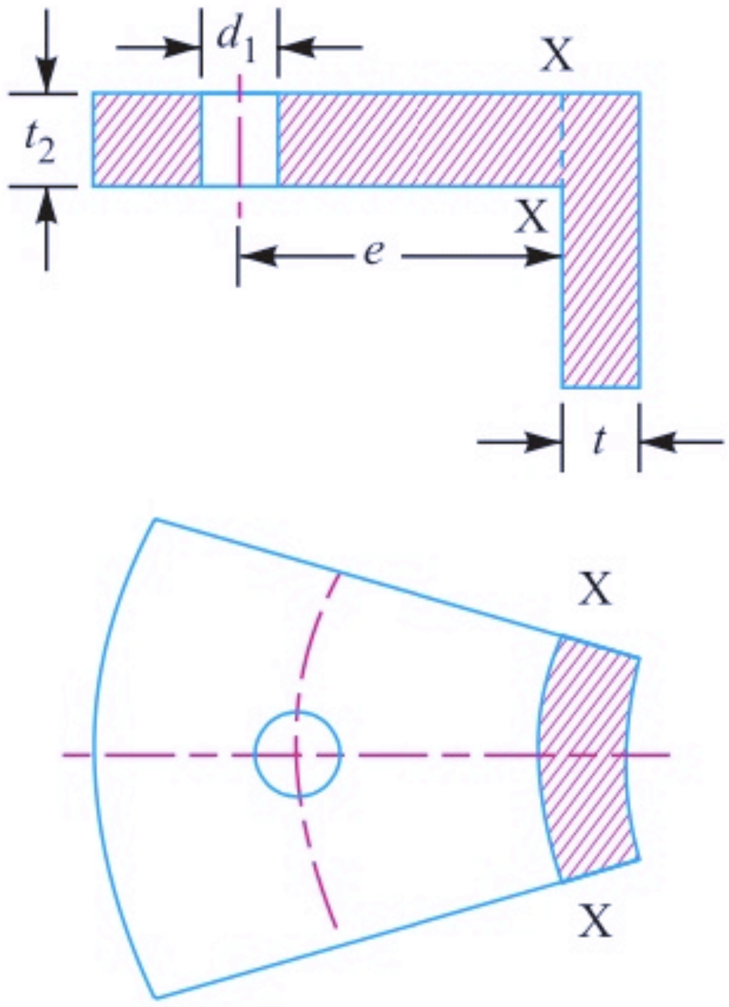


Fig. 11.26. A portion of the cylinder flange.

∴ Bending moment, $$M = \text{Load on each bolt} \times e = \frac{P}{n} \times e$$

Radius of the section X-X,

$$R = \text{Cylinder radius} + \text{Thickness of cylinder wall} = \frac{D}{2} + t$$

Width of the section X-X,

$$w = \frac{2\pi R}{n}\text{, where } n \text{ is the number of bolts.}$$

Section modulus, $$Z = \frac{1}{6} w (t_2)^2$$

Knowing the tensile stress for the cylinder flange material, the value of t_2 may be obtained by using the bending equation *i.e.* $\sigma_t = M / Z$.

Example 11.6. *A steam engine cylinder has an effective diameter of 350 mm and the maximum steam pressure acting on the cylinder cover is 1.25 N/mm². Calculate the number and size of studs required to fix the cylinder cover, assuming the permissible stress in the studs as 33 MPa.*

$D = 350\,mm$ $\quad p = 1.25\,mpa$ $\quad \sigma_t = 33\,mpa$

press force $\quad P = \frac{\pi}{4}(D)^2 \times p$

$P = 120264\,N$

Force (or) Load on Studs

$= \frac{\pi}{4}(d_c)^2 \times \sigma_t \times n$ → no. of studs

Assume $d = 24\,mm$, from table for $d = 24\,mm$,

$d_c = 20.32\,mm$

$n = \frac{120264}{10{,}700} \Rightarrow \boxed{n = 12}$

$d = 24\,mm$ → nominal dia of Studs

$d_1 = 25\,mm$ → Stud hole dia

$D_p = D + 2t + 3d_1$ $\quad$ (assume $t = 10\,mm$)

$= 350 + (2 \times 10) + 3 \times 25$

$\boxed{D_p = 445\,mm}$

Circumferential pitch

$$= \frac{\pi D_p}{n} = \frac{\pi \times 445}{12} = 116.5\,mm$$

Min circum pitch $= 20\sqrt{d_1} = 100\,mm$

Max " " $= 30\sqrt{d_1} = 150\,mm$

Our assumption is correct.

Our Stud size is M24.

Design of cylinder cover:

$t_1 \rightarrow$ thickness of cover plate

$M = 0.053\, P \times D_p$

$M = 2.83 \times 10^6$ N-mm

$$Z = \frac{1}{6} w t_1^2$$

$$w = D_o - 2d_1$$
$$= D_p + 3d_1 - 2d_1$$
$$= D_p + d_1$$

$$Z = 78.33\, t_1^2$$

$\sigma_t = M/Z$ | $33 = \dfrac{2.83 \times 10^6}{78.33\, t_1^2}$

$\boxed{t_1 = 33\ mm} \rightarrow$ thickness of cover plate.

Example 11.9. *A steam engine of effective diameter 300 mm is subjected to a steam pressure of 1.5 N/mm². The cylinder head is connected by 8 bolts having yield point 330 MPa and endurance limit at 240 MPa. The bolts are tightened with an initial preload of 1.5 times the steam load. A soft copper gasket is used to make the joint leak-proof. Assuming a factor of safety 2, find the size of bolt required. The stiffness factor for copper gasket may be taken as 0.5.*

$D = 300\,mm$ $\quad p = 1.5\,N/mm^2$ $\quad n = 8$

$\sigma_y = 330\,mpa$; $\sigma_e = 240\,mpa$

K → Stiffness factor

$$\boxed{P_1 = 1.5 P_2}$$

$$P_{total} \text{ (or) } P_{max} = P_1 + K \cdot P_2$$

$$P_2 = \frac{\pi}{4}(D^2 \times p)$$

$$P_{max} = 1.5P_2 + (0.5)P_2$$
$$= 2P_2$$
$$= 2,12,057\ N$$

Total load is shared by 8 bolts

max Load on each bolt $= \dfrac{P_{max}}{8} = 26,510\ N$

min load on each bolt $= \dfrac{P_{min}}{8} = \dfrac{P_1}{8} = 19,882\ N$

Using Soderberg method:

$$\frac{1}{FOS} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\sigma_m = \frac{F_m}{A} \quad ; \quad \sigma_v = \frac{F_v}{A}$$

$$F_m = \frac{F_{max} + F_{min}}{2} \quad ; \quad F_v = \frac{F_{max} - F_{min}}{2}$$

$$\underline{d_c = \ ?}$$

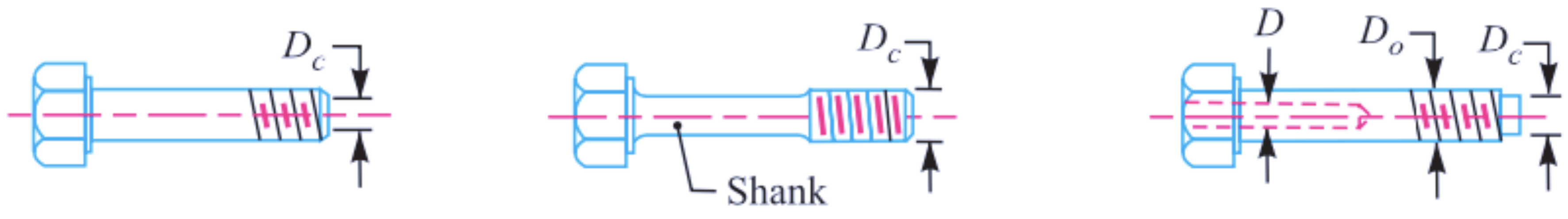
Bolts of Uniform Strength

When a bolt is subjected to shock loading, as in case of a cylinder head bolt of an internal combustion engine, the resilience of the bolt should be considered in order to prevent breakage at the thread.

- In an ordinary bolt shown in Fig. (*a*), the effect of the impulsive loads applied axially is concentrated on the weakest part of the bolt *i.e*. the cross-sectional area at the root of the threads.
- In other words, the stress in the threaded part of the bolt will be higher than that in the shank. Hence a great portion of the energy will be absorbed at the region of the threaded part which may fracture the threaded portion because of its small length.

  - If the shank of the bolt is turned down to a diameter equal or even slightly less than the core diameter of the thread (D_C) as shown in Fig.(*b*), then shank of the bolt will undergo a higher stress.
  - This means that a shank will absorb a large portion of the energy, thus relieving the material at the sections near the thread. The bolt, in this way, becomes stronger and lighter and it increases the shock absorbing capacity of the bolt because of an increased modulus of resilience.

This gives us ***bolts of uniform strength***. The resilience of a bolt may also be increased by increasing its length.



(*a*) (*b*) (*c*)

A second alternative method of obtaining the bolts of uniform strength is shown in Fig (c). In this method, an axial hole is drilled through the head as far as the thread portion such that the area of the shank becomes equal to the root area of the thread.

Let D = Diameter of the hole.

D_o = Outer diameter of the thread, and

D_c = Root or core diameter of the thread.

$$\therefore \quad \frac{\pi}{4} D^2 = \frac{\pi}{4}\left[(D_o)^2 - (D_c)^2\right]$$

or

$$D^2 = (D_o)^2 - (D_c)^2$$

$$\therefore \quad D = \sqrt{(D_o)^2 - (D_c)^2}$$

Design of a Nut

When a bolt and nut is made of mild steel, then the effective height of nut is made equal to the nominal diameter of the bolt.

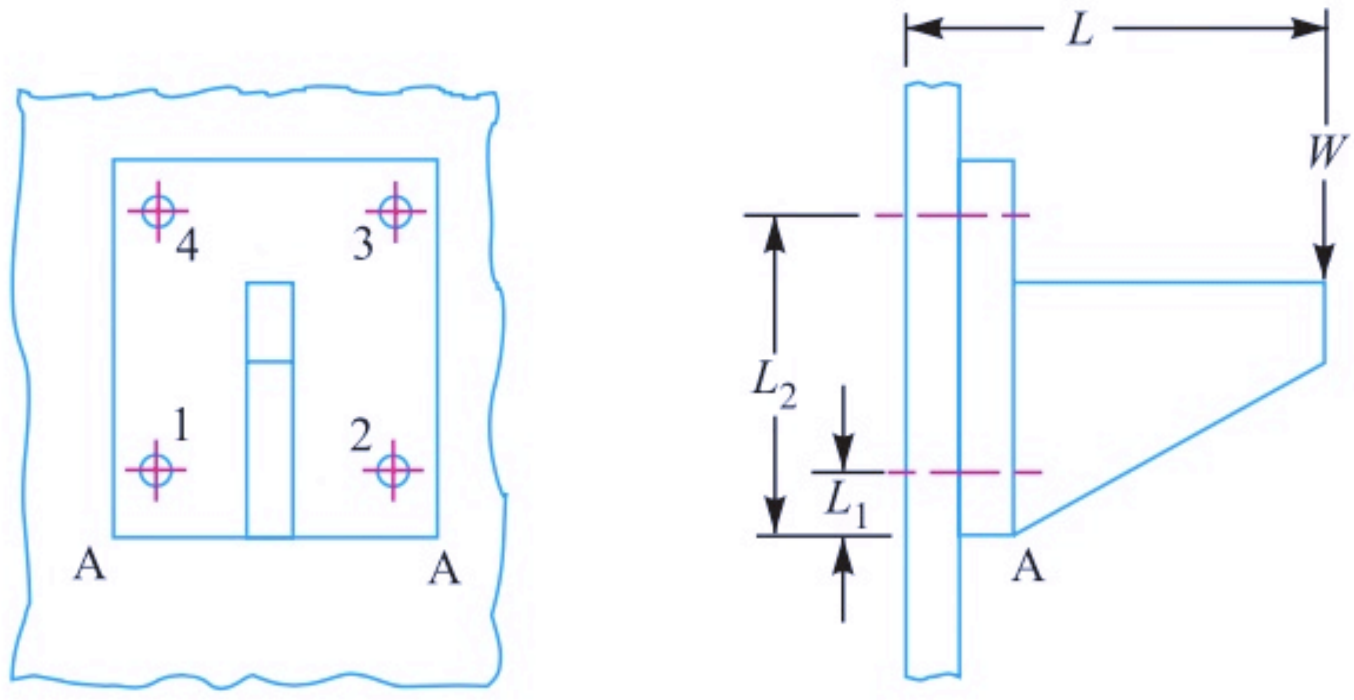
If the nut is made of weaker material than the bolt, then the height of nut should be larger, such as 1.5 d for gun metal, 2 d for cast iron and 2.5 d for aluminium alloys (where d is the nominal diameter of the bolt).

In case cast iron or aluminium nut is used, then V-threads are permissible only for permanent fastenings, because threads in these materials are damaged due to repeated screwing and unscrewing.

When these materials are to be used for parts frequently removed and fastened, a screw in steel bushing for cast iron and cast-in-bronze or monel metal insert should be used for aluminium and should be drilled and tapped in place.

11.20 Eccentric Load Acting Perpendicular to the Axis of Bolts

A wall bracket carrying an eccentric load perpendicular to the axis of the bolts is shown in Fig. 11.34.



$$W_s = \frac{W}{n}$$

$w \rightarrow$ Load per unit length

$W_1 \rightarrow$ Load on each bolt @ a distance L_1

$$W_1 = w \times L_1$$

Moment of the load about "A"

$$M_1 = W_1 \times L_1 = w L_1^2$$

$W_2 \rightarrow$ @ a distance L_2

$$W_2 = w \times L_2$$

$$M_2 = w L_2^2$$

Total moment about tilting edge

$$M_t = 2w\left[L_1^2 + L_2^2\right] \quad \text{—— (1)}$$

Total moment due to load W

$$= W \times L \quad - (2)$$

$(1) = (2) \Rightarrow \quad W \times L = 2w[L_1^2 + L_2^2]$

$$\boxed{w = \frac{W \times L}{2[L_1^2 + L_2^2]}}$$

→ Load per unit length

$$W_1 = w \times L_1 = \frac{W L L_1}{2[L_1^2 + L_2^2]}$$

$$W_2 = \frac{W L L_2}{2[L_1^2 + L_2^2]}$$

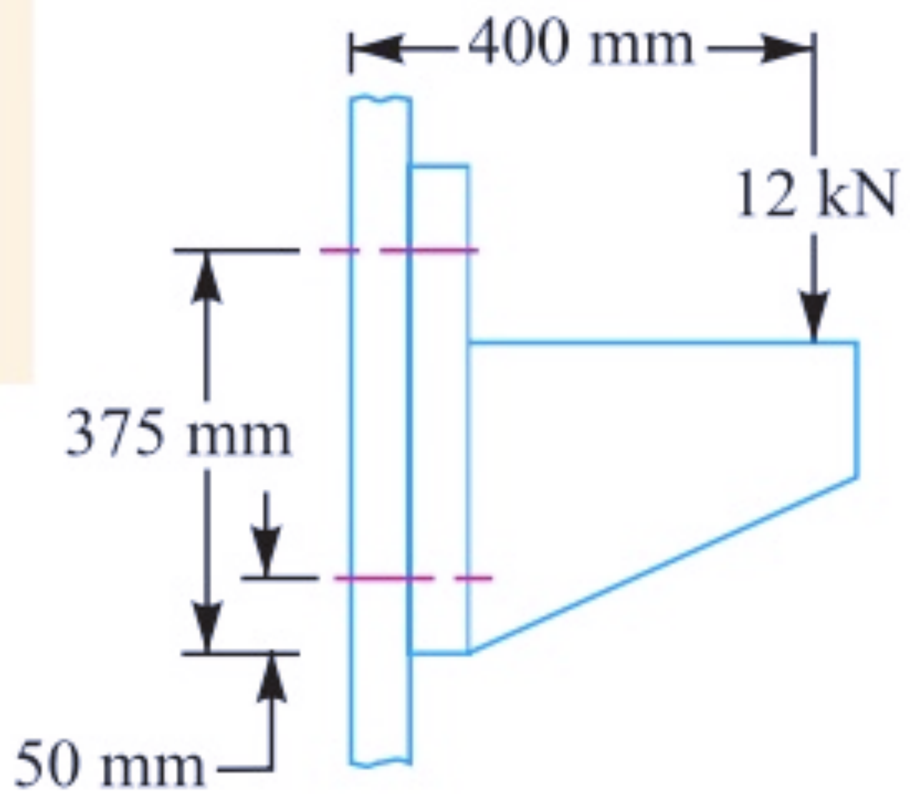
$\underline{W_t = W_2}$ → max tensile load due to moment

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4W_s^2}\right]$$

For supporting the travelling crane in a workshop, the brackets are fixed on steel columns as shown in Fig. The maximum load that comes on the bracket is 12 kN acting vertically at a distance of 400 mm from the face of the column.

The vertical face of the bracket is secured to a column by four bolts, in two rows (two in each row) at a distance of 50 mm from the lower edge of the bracket.

Determine the size of the bolts if the permissible value of the tensile stress for the bolt material is 84 MPa. Also find the cross-section of the arm of the bracket which is rectangular.



$$W_s = \frac{W}{n} = \frac{12}{4} = 3 \text{ KN}$$

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{W_t^2 + 4W_s^2}\right]$$

$$W_t = \frac{WLL_2}{2[L_1^2 + L_2^2]} = 6.28 \text{ KN}$$

$$W_{te} = 7.482 \text{ KN}$$

(i) Size of bolts:

$$\sigma_t = 84 \text{ mpa}$$

$$W_{te} = \frac{\pi}{4} d_c^2 \times \sigma_t$$

$$d_c = 10.64 \text{ mm}$$

$$\left.\begin{array}{l} d = 14 \text{ mm} \\ d_c = 11.456 \text{ m} \end{array}\right\} \text{from table}$$

So size of bolt – M14

(iii) Area of c/s:

t, b

$\sigma_b = 84$ mpa

$$Z = \frac{tb^2}{6}$$

M = max Bending moment
$= 12 \times 10^3 \times 400$

$$\frac{M}{I} = \frac{\sigma_b}{y} = \cancel{\frac{E}{R}}$$

$$\frac{M}{Z} = \sigma_b$$

$$\frac{\frac{tb^3}{12}}{b/2} = \frac{tb^2}{6}$$

$$\frac{12 \times 10^3 \times 400}{tb^2/6} = 84$$

$$tb^2 = 343 \times 10^3$$

b = 5 mm
t = 13,720 mm

t = 20 mm
b = 131 mm

b = 250 mm
t = 5.5 mm