UNIT-I APPLIED PHYSICS

WAVE OPTICS

Introduction

Basically optics is the branch of science which deals with the study of light.

It is also known as the branch of physics, which deals with the study of properties and nature of light. Optics is mainly divided into two parts.

i) **Geometrical optics** which deals with the image formation by optical systems.

That is the Geometrical optics concerns with the formation of images, when light rays passes through an optical system, such as a lens and a prism.

ii) **Physical optics** which deals with the nature of light.

That is the physical optics deals with the nature of light, such as Interference, Diffraction and polarization.

Interference

Interference is that phenomena, in which two wave trains, when superposed at a point, produce collinear oscillations such that the resultant intensity at the point of superposition not only depends on the amplitudes of the component waves but also on their phase difference at the point of interference.

The interfered effect at any point can be observed by the eye, only if the effect is steady over sufficiently long intervals of observation.

The effect is steady only if the phase relations between the interfering waves remain constant over that time interval.

The phase emission of a wave train from a source, change at random. This random change in the emission phase changes the phase of waves train at the given point.

The phase difference between two wave trains at a point of their superposition will vary with time, if their frequencies are not equal.

Thus constant phase relations between the interfering waves requires sources of

i) Same and single frequency and

ii) Constant emission phase difference.

The condition (i) is fulfilled if the sources are monochromatic and of the same frequency.

The condition (ii) requires coherent sources.

Coherent source

Coherent sources are those sources, which maintain their emission phase difference constant for all time although each one may change its emission phase abruptly and at random.

Constructive interference

If two wave trains at the p

oint of superposition produced collinear vibrations interfere in the same phase, then the interference is said to be constructive. This is possible when the phase difference of the two wave trains at the point of superposition is $2n\pi$, where n is an integer.

In that case the resultant amplitude is the sum of the individual amplitudes and the intensity is maximum. The corresponding path difference between the two interfering wave trains is an integral multiple of the wavelength, provided the sources are equiphased.

$\therefore$ Path difference = $n\lambda$, n = 1,2,3,……

Destructive Interference

If the two wave trains interfere in the opposite phase, then the interference is said to be destructive. This is possible when the phase difference of the two wave trains at the point of super position is $(2n-1)\pi$, Where n = an integer.

In this case the resultant amplitude is the difference of the individual amplitudes and the intensity is minimum.

The corresponding path difference between the interfering waves should be an odd multiple of half the wavelength, if the sources are equally phased.

$\therefore$ Path difference = $(2n-1)\dfrac{\lambda}{2}, n = 1,2,3,.....$

Interference in thin films

The colors of thin films, soap bubbles and oil slicks can be explained as due to the phenomena of interference.

Let a plane wave front be allowed to incident normally on a thin film of uniform thickness t.

The plane wave front is obtained with the help of a partially reflecting a glass plate G inclined at an angle 45^0 with the parallel monochromatic beam of light.

The plane wave front is partly reflected at the upper surface of the film and partly transmitted into the film. This is shown in figure (1).

The transmitted wave front is reflected again from the bottom surface of the film and emerges through the first surface.

The wave front reflected from the upper surface and the lower surface interfere with each other. The resultant interference pattern can be observed with eye without obstructing the incident wave front.

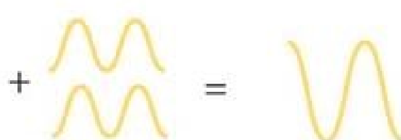
Here the following two points are observed.

i) The wavelength reflected light from the lower surface of the film, traverses an additional path 2 μt.
(μt from upper surface to lower surface and μt from lower surface to upper surface).
Where μ is the refractive index of the film.

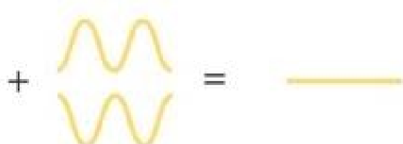
ii) When the film is placed in air, the wave front reflected from the upper surface undergoes an additional phase change of π (Because the reflection takes place at the surface of a denser medium). Here it should be noted that no phase change takes place at lower surface because the reflection takes place at the surface of rarer medium.

Now when the path difference, 2 μt = nλ, Constructive interference takes place and the film appears bright.

Here n = 1, 2, 3 …When the path difference, $2\mu t = (2n+1)\frac{\lambda}{2}$,destructive interference takes place and the film appears dark.
Here n = 0, 1, 2, 3……
Note: μt is the optical thickness of the film.



Constructive interference



Destructive interference

The constructive and destructive interferences are shown

Above.

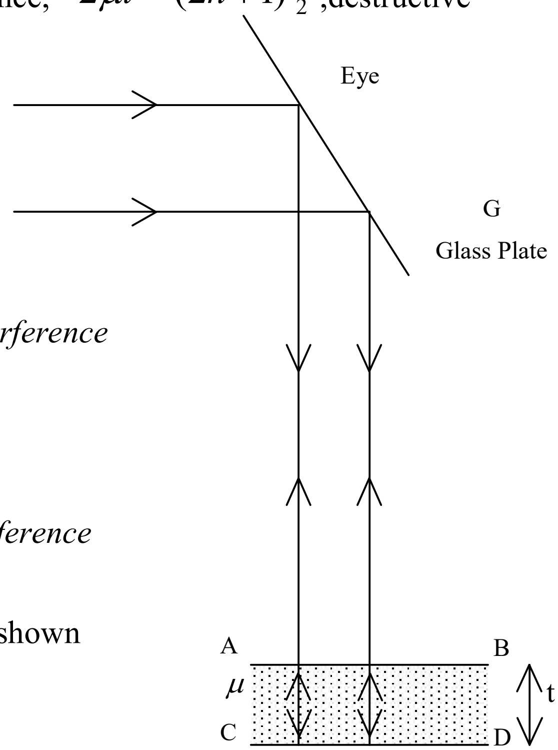


Figure (1) Interference in thin films

Interference in the films by Reflection:

Let us consider a plane parallel film, as shown in figure (4) below.
Let PA be a ray of light incidenting on the upper surface as shown in the figure (4).
PA light ray makes an angle of incidence i.
Now part of the light is reflected into the film in the direction AB and the other part is refracted into film in the direction AC.
The light AC which is refracted is reflected at C and emerges at D. The emerged light DF is parallel to ABH
At the Normal incidence, the path difference between rays ABH and DF is the two times the optical thickness of the film $(2\mu t)$.
The two parallel rays of light AB and DF will interfere in the field of Eye and produce interference pattern.
Now the path difference between the rays AB and DF, for Normal Incidence is given by

$$\Delta = 2\mu t \quad \text{------} \quad (1)$$

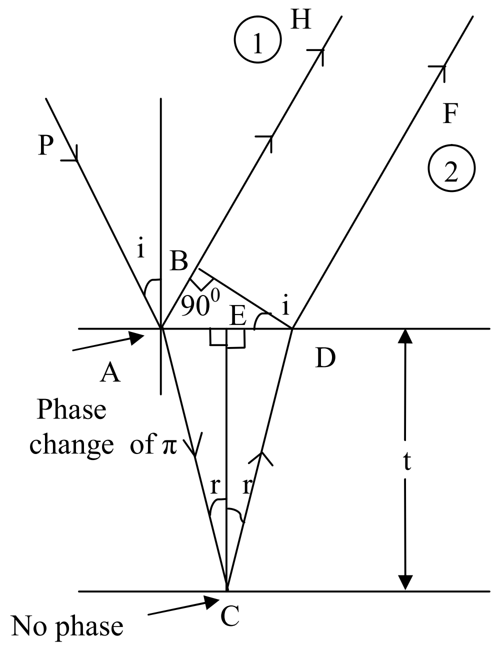


Fig. 4: Interference in thin films (thin parallel films)

At oblique incidence the path difference is given by

$$\Delta = \mu(AC + CD) - AB \quad \text{-----} \quad (2)$$

Now from the figure (4), triangle AEC is a right angled triangle.

$$\therefore \frac{EC}{AC} = \cos r \Rightarrow AC = \frac{EC}{\cos r} \quad \text{-----} \quad (3)$$

Triangle CED and Triangle AEC are similar and are right angled triangles.

$$\therefore \frac{EC}{CD} = \cos r \Rightarrow \therefore CD = \frac{EC}{\cos r} \quad \text{----------} \quad (4)$$

Now $AC + CD = \frac{EC}{\cos r} + \frac{EC}{\cos r}$

$$AC + CD == \frac{2EC}{\cos r}$$

But EC =t, thickness of the film.

$$\therefore AC + CD = \frac{2t}{\cos r} \quad \text{------} \quad (5)$$

Also from the right angled triangle ABD,

$$\sin i = \frac{AB}{AD} \Rightarrow AB = AD\sin i$$

AB= (AE+ED) Sini,

AB= 2AE Sin i (since AE=ED, AE+ED=AD)

Also Tan r = $\frac{AE}{EC}$ [from the right anlged triangle AEC]

$AE = EC\,Tan\,r$

$\therefore AB = 2t\ TanrSini$ ------- (6)

$\therefore$ From equations (2), (5) and (6), we get

$$\Delta = \frac{2\mu t}{\cos r} - 2t\,Tanr\sin i$$

But we know that (Snell's Law) $\frac{\sin i}{\sin r} = \mu$, μ=Refractive index of material of the Film.

$$\sin i = \mu \sin r$$

$$\therefore \Delta = \frac{2\mu t}{\cos r} - 2t\,T\text{an}\,r\,\mu\sin r$$

$$\therefore \Delta = \frac{2\mu t}{\cos r} - 2\mu t\,T\text{an}\,r\sin r$$

$$\Delta = 2\mu t\left[\frac{1}{\cos r} - T\text{an}\,r\sin r\right]$$

$$\Delta = 2\mu t\left[\frac{1-\sin^2 r}{\cos r}\right] = 2\mu t\left[\frac{Cos^2 r}{\cos r}\right]$$

$\Delta = 2\mu t\cos r$ ------------- (7)

Where μ is the refractive index of the medium between the surfaces of the film. For the reflected ray AB, the reflection is occurring in the denser medium, a phase change of π occurs. This phase change π is equivalent to path difference of $\frac{\lambda}{2}$.

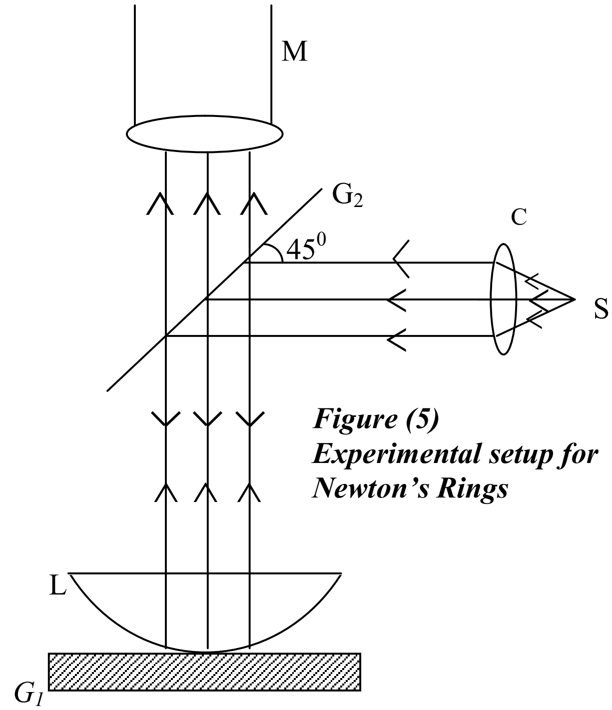


Figure (5) Experimental setup for Newton's Rings

$\therefore$ The condition for maxima for the air film to appear bright is

$$2\mu t\cos r + \frac{\lambda}{2} = n\lambda$$

$$2\mu t\cos r = n\lambda - \frac{\lambda}{2}$$

$$2\mu t\cos r = (2n-1)\frac{\lambda}{2} \qquad \text{------ (8)}$$

For the reflected ray CD and transmitted ray of light DF, No phase change occurs. Because, the reflection of light CD takes place at a surface of lower refractive index.

$\therefore$ The film appear dark in the reflected light

$$2\mu t\cos r+\frac{\lambda}{2}=(2n+1)\frac{\lambda}{2}$$

When $$2\mu t\cos r=(2n+1)\frac{\lambda}{2}-\frac{\lambda}{2}$$

$$2\mu t\cos r=(2n+\cancel{1}-\cancel{1})\frac{\lambda}{2}$$

$$2\mu t\cos r=n\lambda$$

Where $n=0,1,2,3,....$

Newton's Rings

When a Plano convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. The thickness of the film at the point of contact is zero. If a monochromatic light is allowed to fall normally and viewed as shown in figure (5), then alternative dark and bright circular fringes are observed.

The fringes are circular because the air film has a circular symmetry.

Newton's Rings are formed because of the interference between the waves reflected from the top and bottom surfaces of the air film between the curved surface and the glass plate as shown in figure (5).

figure (5) shows the experimental setup for Newton's Rings. In the setup G, is the plane glass plate. L is a Plano convex lens. S is a monochromatic source of light. G_2 is the glass plate inclined at an angle 45^0 with the incident parallel light from the source S. C is a double convex lens. M is the microscope, through which we can observe interference fringes.

Theory: Newton's rings are formed due to interference between the waves reflected from the top and bottom surfaces of the air film formed between the glass plate and curved surface of the plano convex lens. The formation of Newton's Rings can be explained by using the Figure (6).

L is the Plano Convex lens. G is a plane glass, plate. AB is the monochromatic Ray of light, which is incidenting on the system.

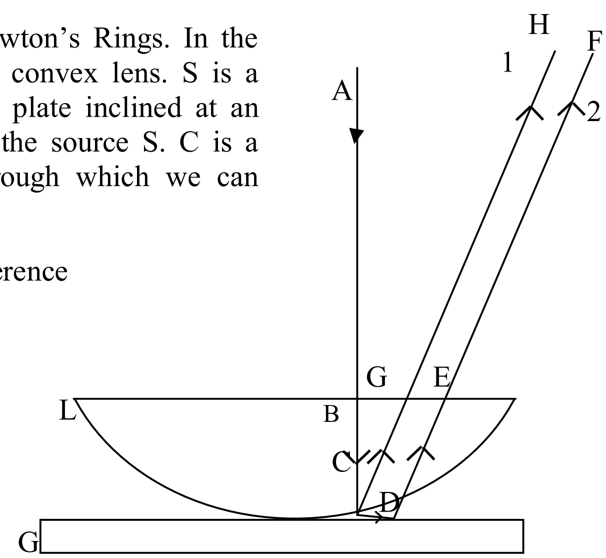


Figure (6): Interference in Newton's rings setup.

A part of the light is reflected at C (glass air boundary), which goes out in the form of rays (1). Without any phase reversal.

This is because at the point 'C' a light ray is reflected from a rarer medium.

The other part is refracted along CD, at the point D it is again reflected and goes out in the form of ray (2). (DEF Ray of light).

The ray (2) suffers a phase reversal of π. This is because at the point D, the light ray is reflected from the denser medium glass.

The reflected rays (1) and (2) [GH and EF] are in a position to produce interference fringes as they have been derived from the same ray AB. Hence they fulfill the condition of interference.

As the rings are formed in the reflected light, the path difference between them is

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2} \qquad \text{---- (1)}$$

Since the interference is taking place because of the air film, for air film $\mu = 1$.

And for Normal incidence, r=0.

Now the path difference $\Delta = 2(1)t\cos(0) + \frac{\lambda}{2}$

$$\therefore \Delta = 2t + \frac{\lambda}{2} \qquad \text{---- (2)}$$

Where t= thickness of the air film.

At the point of contact, t=0, and the path difference $\Delta = \frac{\lambda}{2}$.

This is the condition of minimum intensity. Hence the central spot is dark.

Now the **condition for bright fringe is**

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = n\lambda - \frac{\lambda}{2}$$

$$2t = \frac{2n\lambda - \lambda}{2}$$

$$2t = (2n-1)\frac{\lambda}{2}, \qquad \text{----- (3)}$$

Where n= 1,2,3,……

The condition for dark fringe is

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2t = (2n+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$2t = \left[(2n+1) - 1\right]\frac{\lambda}{2}$$

$$2t = n\lambda \qquad \text{---- (4) Here n= 0, 1, 2…}$$

Relation between t, r and R is given by

$$t = \frac{r^2}{2R} \qquad \text{------- (5)}$$

For a $\frac{\text{Brighter Fringe}}{2t = (2n-1)\frac{\lambda}{2}}$ the condition is

Now substituting the value of t, we get

$$\cancel{2}\left(\frac{r^2}{\cancel{2}R}\right) = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2} \qquad \text{----- (6)}$$

Here $r =$ Radius of the Ring.
If D = diameter of the Brighter Ring, then

$$r = \frac{D}{2}$$

$$\therefore \left(\frac{D}{2}\right)^2 = \frac{(2n-1)\lambda R}{2} \qquad \therefore D^2 = \frac{4(2n-1)\lambda R}{2} \quad , D^2 = 2(2n-1)\lambda R$$

$$D = \sqrt{2\lambda R}\sqrt{2n-1} \qquad \text{---- (7)}$$

From Equation (7) $D\alpha\sqrt{(2n-1)}$

$\therefore$ The diameter of the Bright Ring is proportional to the Square root of odd natural number.
For mth Bright Ring (m is a higher order fringe).

$$D_m = \sqrt{2\lambda R}\sqrt{2m-1}$$

For nth the Bright Ring (n is a lower order fringe).

$$D_n = \sqrt{2\lambda R}\sqrt{2n-1}$$

Similarly $D_m^2 = 2\lambda R(2m-1)$

$$D_n^2 = 2\lambda R(2n-1)$$

$$D_m^2 - D_n^2 = 2\lambda R(2m-1) - 2\lambda R(2n-1)$$

$$D_m^2 - D_n^2 = 2\lambda R\left[2m-1-(2n-1)\right]$$

$$D_m^2 - D_n^2 = 2\lambda R\left[2m-\cancel{1}-2n+\cancel{1}\right]$$

$$D_m^2 - D_n^2 = 4\lambda R[m-n]$$

$$\therefore R = \frac{D_m{}^2 - D_{n^2}}{4\lambda(m-n)} \qquad \text{----- (8)}$$

Also for a dark fringe, the condition is $2t = n\lambda$ ------ (9)

But $t = \frac{r^2}{2R}$

$$\therefore \cancel{2}\frac{r^2}{\cancel{2}R} = n\lambda$$

$r^2 = n\lambda R$

But $r = \frac{D}{2}$

$\therefore$ Diameter of the Ring is given by

$\left(\frac{D}{2}\right)^2 = n\lambda R \Rightarrow \frac{D^2}{4} = n\lambda R$

$D^2 = 4n\lambda R$

$D = 2\sqrt{n\lambda R}$ --- (10)

Thus the diameter of the rings is proportional to the square root of the Natural Numbers.
Now Diameter of the m^{th} Dark Ring is given by

$$D_m^2 = 4m\lambda R \qquad \text{---- (11)}$$

Diameter of the n^{th} Dark ring is given by

$$D_n^2 = 4n\lambda R \qquad \text{----- (12)}$$

By measuring the diameters of the dark rings.
We can calculate the Radius of curvature of the Plano convex lens.
From Equations (11) and (12), we have

$$D_m^2 - D_n^2 = 4m\lambda R - 4n\lambda R$$

$$D_m^2 - D_n^2 = 4\lambda R(m-n)$$

$\therefore$ Radius of curvature of the Plano convex lens

$R = \frac{D_m^2 - D_n^2}{4\lambda(m-n)}$, Here $m > n$ …..(13)

If R is known, the wavelength of the source λ can be calculated as follows..

$\therefore \lambda = \frac{D_m^2 - D_n^2}{4R(m-n)}$ ----- (14)

Note: 1. Determination of wave length of a light source

Let R be the Radius of curvature of a Plano convex lens. Let λ be the wavelength of Monochromatic light used.

Let D_m and D_n are the diameters of m^{th} and n^{th} dark Rings respectively.

Then $D_m^2 = 4m\lambda R$

and $D_n^2 = 4n\lambda R$

Now $D_m^2 - D_n^2 = 4(m-n)\lambda R$

$\therefore \lambda = \frac{D_m^2 - D_n^2}{4R(m-n)}$ $[m > n]$

Newton's Rings are formed with Newton's Rings setup. By using a traveling microscope, the readings of the

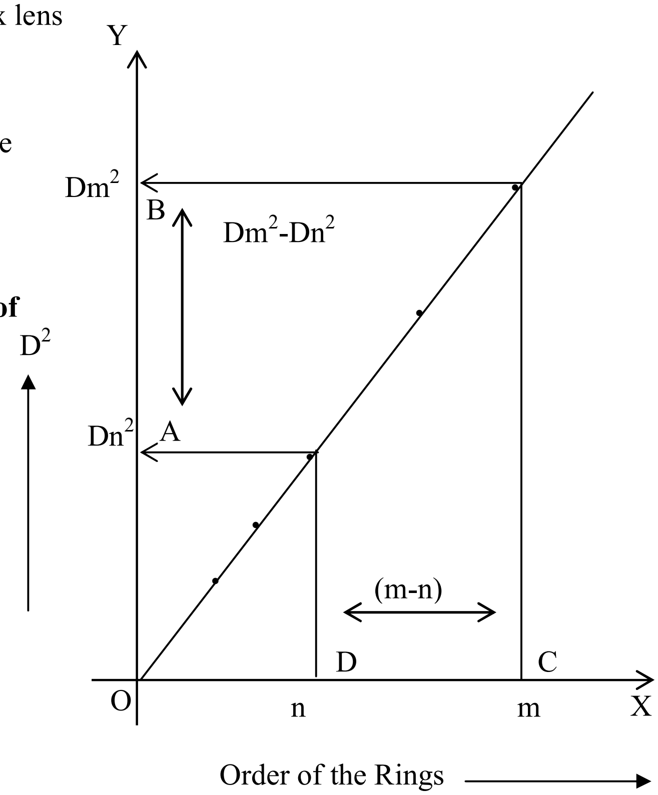


Fig 10: Graph between D² and order of ring

different orders of dark rings were noted from one edge of the Rings to other edge. The diameters of different orders of the Rings are calculated. A graph between D^2 and the order of the Rings is drawn. A straight line graph is obtained as shown in figure (10).

From the graph

$CD = m - n \quad AB = D_m^2 - D_n^2$

$\therefore$ From the graph, the values of $(m-n)$ and $\left(D_m^2 - D_n^2\right)$ are calculated.

The radius of curvature R of the Plano Convex lens can be obtained with the help of the spherometer. Substituting these values in the formulae.

$\lambda = \dfrac{D_m^2 - D_n^2}{4R(m-n)}, \lambda$ Can be calculated.

Note 2: Determine of Refractive Index of a Liquid.

Now the Newton's Rings system is placed into a container containing a liquid of refractive index μ. Now we have to find the value of refractive index of the liquid.

Now the air film is replaced by the liquid film.

Now again the experiment is repeated. The diameters of m^{th} and n^{th} dark Rings are now obtained.

Then we have

$$D_m^{1^2} - D_n^{1^2} = \frac{4(m-n)\lambda R}{\mu} \qquad \text{--- (1)}$$

Also for air film, we have

$$D_m^2 - D_n^2 = 4(m-n)\lambda R \qquad \text{---- (2)}$$

From equations (1) and (2), we get

Using these formulae, we can calculate μ.

$$\mu = \frac{D_m^2 - D_n^2}{D_m^{1^2} - D_n^{1^2}}$$

DIFFRACTION

Introduction

Diffraction confirms the wave nature of light. Usually waves bend round the corner of the obstacles in their path. For example, water waves coming from a small hole spread out in all directions as if they have originated at the hole. Similarly sound waves pass round obstacles of moderate dimensions. Similarly light waves bends round the corners of an obstacle is called diffraction.

Diffraction – Explanation

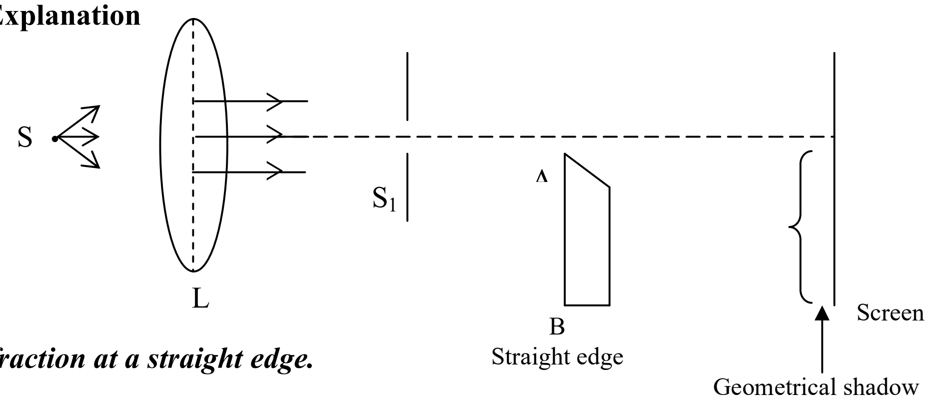


Figure (1) Diffraction at a straight edge.

Light from a monochromatic source's' is allowed to fall on a lens L. Now the light is rendered parallel. S_1 is a slit. AB is a straight edge. The parallel beam of light passes through slit S_1. The light from the slit S_1 falls on the straight edge. Now a geometrical shadow is observed on the screen. The shadow is not a sharp one. Above the shadow, parallel to the edge A, several bright and dark bands are seen due to diffraction. Thus the bending of light waves round the edges of opaque obstacle or narrow slits and spreading of light into geometrical shadow region is known as diffraction of light.

Types of diffraction

Fresnel Diffraction

In this class of diffraction, the source of light and the screen are at finite distance from the aperture or obstacle having sharp edge. The incident wave front on the aperture or obstacle is either spherical or cylindrical. For the study of this diffraction lenses are not required.

Fraunhofer Diffraction: In this class of diffraction the source of light and the screen are at infinite distance from the diffraction aperture or obstacle. Due to this for focusing the light, we need a lens. This diffraction can be studied in any direction. Here the incident wavefront is a plane wave front.

	Fresnel Diffraction		Fraunhofer Diffraction
1.	Point source of light or an illuminated narrow slit is used as light source.	1.	Extended source of light at infinite distance is used as light source.
2.	Light incident on the obstacle or aperture is a spherical wave front.	2.	Light incident on the obstacle or aperture is a plane wave front.
3.	The source and screen are at finite distance from the aperture or obstacle producing diffraction.	3.	The source and screen are at infinite distance from the aperture or obstacle.
4.	Lenses are not used to focus the light rays.	4.	Converging lens is used to focus the light rays.

Fraunhofer Diffraction at a Single Slit:

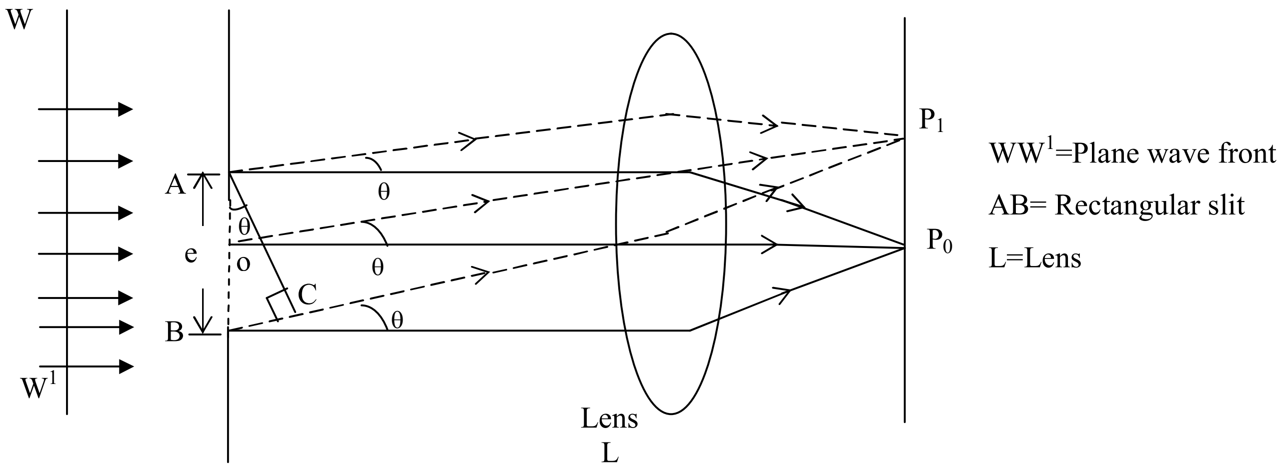


Figure (2) Fraunhofer diffraction at a single slit

Consider a slit AB of width 'e'. ww' is a plane wave front of monochromatic light of wavelength λ is incidenting normally on the slit. The diffracted light through the slit is focused by using a convex lens on to a screen placed in the focal plane of the lens. According to Huygens – Fresnel every point on the wave front in the plane of the slit is a source of secondary wavelet. These secondary wavelets spread out in all directions to the right.

The secondary wavelets traveling normal to the slit, along the direction OP_0 are brought to focus at P_0 by the convex lens L. Thus P_0 is a central bright image.

The central bright image is formed because there is no path difference for the Ray traveling normal to the slit.

The secondary wavelets traveling at an angle θ with the normal are brought to focus at a point p_1 on the screen.

The intensity of point p_1 depends upon the path difference between the secondary waves originating from the corresponding points of the wave front.

To find intensity at p_1, draw a normal AC from A to the light ray at B.

Now the path difference between the secondary wavelets from A and B in the direction θ is given by

Path difference = BC.

From the figure (2) triangle ABC is a right angled triangle.

$$\therefore \sin\theta = \frac{BC}{AB}$$

$\Rightarrow BC = AB\sin\theta$, But AB = e , width of the slit.

$$\therefore BC = e\sin\theta \qquad \text{------ (1)}$$

Now the phase difference $\phi = \frac{2\pi}{\lambda} \times$ path difference.

$$\therefore \phi = \frac{2\pi}{\lambda} \times e\sin\theta \qquad \text{---- (2)}$$

Now let the width of the slit is divided into 'n' equal parts. The amplitude of the wave from each part is 'a'.

The phase difference between any two successive waves from these parts will be given by

$$\frac{1}{n}[\text{total phase}]=\frac{1}{n}\left[\frac{2\pi}{\lambda}e\sin\theta\right]=d \qquad \text{----- (3)}$$

By the method of vector addition of amplitudes, the Resultant amplitude R is given by

$$R=\frac{a\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)} \qquad \text{---- (4)}$$

From equations (3) and (4)

$$R=\frac{a\sin\left(\cancel{n}\frac{1}{\cancel{n}}\frac{\cancel{2}\pi}{\lambda\cancel{2}}e\sin\theta\right)}{\sin\left(\frac{\cancel{2}\pi}{n\lambda}\frac{e\sin\theta}{\cancel{2}}\right)}$$

$$R=\frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{n\lambda}\right)}$$

Now let $\frac{\pi e\sin\theta}{\lambda}=\alpha$ ----- (5)

$$R=\frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

In the above expression $\left(\frac{\alpha}{n}\right)$ is very small

Hence , $\text{Sin}\left(\frac{\alpha}{n}\right)=\frac{\alpha}{n}$.

$$\therefore R=\frac{a\sin\alpha}{\left(\frac{\alpha}{n}\right)}$$

$$R=\frac{na\sin\alpha}{\alpha}$$

$\Rightarrow R=\frac{A\sin\alpha}{\alpha}$, Here $A=na$ ---- (6)

We know that intensity of light is proportional to square of the amplitude.

Intensity $I=R^2$

$$\Rightarrow I=A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{---- (7)}$$

Note: When 'n' no. of S.H.M. are acting at a point simultaneously, having equal amplitude 'a' and same phase difference 'd', then the resultant amplitude is given by vector addition as

$$R=\frac{a\sin\left(\frac{nd}{2}\right)}{\sin(d/2)}$$

Analysis of Intensity Distribution Principal Maximum:
The resultant amplitude is given by

$$R = A\frac{\sin\alpha}{\alpha}$$

$$\Rightarrow R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$\Rightarrow R = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

If the negative terms vanish, the value of R will be maximum i.e. When $\alpha = 0$

$$\therefore \alpha = \frac{\pi e \sin\theta}{\lambda} = 0$$

$$\Rightarrow \sin\theta = 0$$

$$\Rightarrow \theta = 0 \qquad \text{------ (8)}$$

Now the maximum value of R is A, R=A
Now maximum intensity $I_{\max} = R^2 = A^2$
The condition $\theta = 0$ means that the maximum intensity is formed at P_o.
This maximum intensity is known as **Principal maximum.**

Minimum Intensity Positions

Resultant amplitude $R = A\frac{\sin\alpha}{\alpha}$

Intensity I will be minimum when $\sin\alpha = 0$.
i.e. when R=0, I will be minimum
now $\sin\alpha = 0$

$$\Rightarrow \alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, ..., \pm m\pi$$

But $\alpha = \frac{\pi e \sin\theta}{\lambda} = \pm m\pi$

$$\Rightarrow e\sin\theta = \pm m\lambda \qquad \text{-------(9)}$$

Where m=1, 2, 3…
Therefore we get the points of minimum intensity on either side of principal maximum.
For m=0, $\sin\theta = 0$. This corresponds to Principal Maximum.

Secondary maxima

In between these maximum intensity positions, we will have secondary maxima or subsidiary maxima.

The position of secondary maxima can be obtained by differentiating the expression $I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ with respect to α and equating it to zero.

Now $\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$

$$\Rightarrow A^2\frac{2\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

Here either $\sin\alpha = 0$ or $\cos\alpha - \sin\alpha = 0$.
But $\sin\alpha = 0$ gives positions of minima.

Now the positions of secondary maxima are given by

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha\cos\alpha = \sin\alpha$$

$$\Rightarrow \alpha = \tan\alpha \qquad \text{---- (10)}$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $y=\alpha$ and $y=\tan\alpha$ on the same graph. The plots $y=\alpha$ and $y=\tan\alpha$ are shown in fig (2).

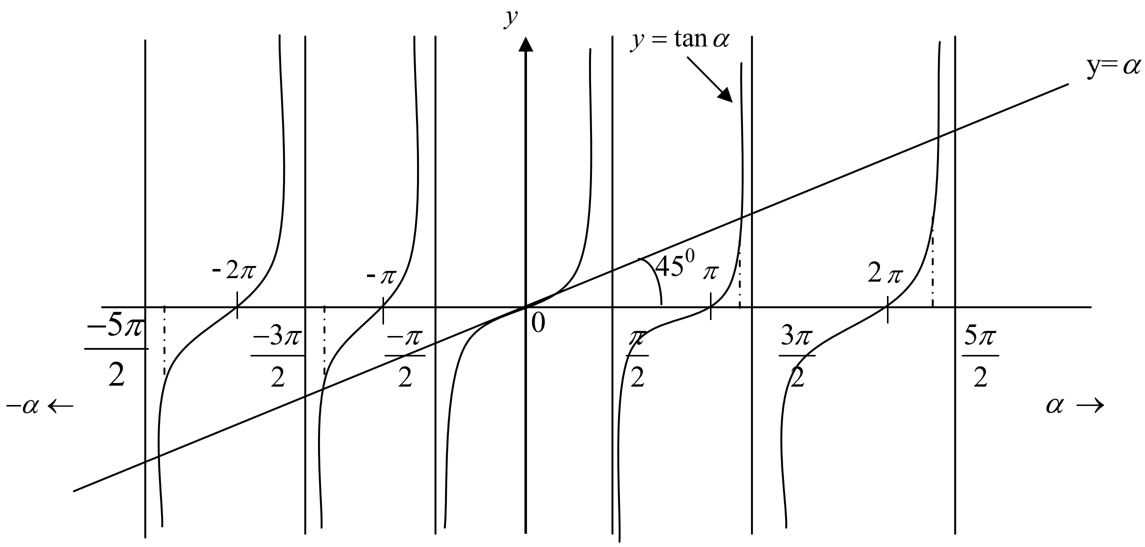


Figure (3) plots of $y=\alpha$ and $y=\tan\alpha$.

From the plots, the points of intersection are given by $\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ...$

Substituting the above values in equation (7), we get the intensities in various maxima.

$\alpha = 0, I_0 = A^2$ (Principal Maximum).

$$\alpha = \frac{3\pi}{2}, I_1 = A^2\left[\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right]^2 = \frac{A^2}{22} \qquad \text{(1}^{\text{st}}\text{ Secondary maximum)}$$

$$\alpha = \frac{5\pi}{2}, I_2 = A^2\left[\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right]^2 = \frac{A^2}{62} \qquad \text{(2}^{\text{nd}}\text{ secondary maximum)}$$

From the above expression it is clear that most of the incident light is concentrated in the principal maximum.

Intensity Distribution: The variation of intensity with respect to α is shown in figure (4).

The diffraction pattern consist of a central principal maximum for $\alpha = 0$

There are secondary maxima of decreasing intensity on either sides of it at positions $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$.

Between secondary maxima there are positions of minima at $\alpha = \pm\pi, \pm2\pi, \pm3\pi, ...$

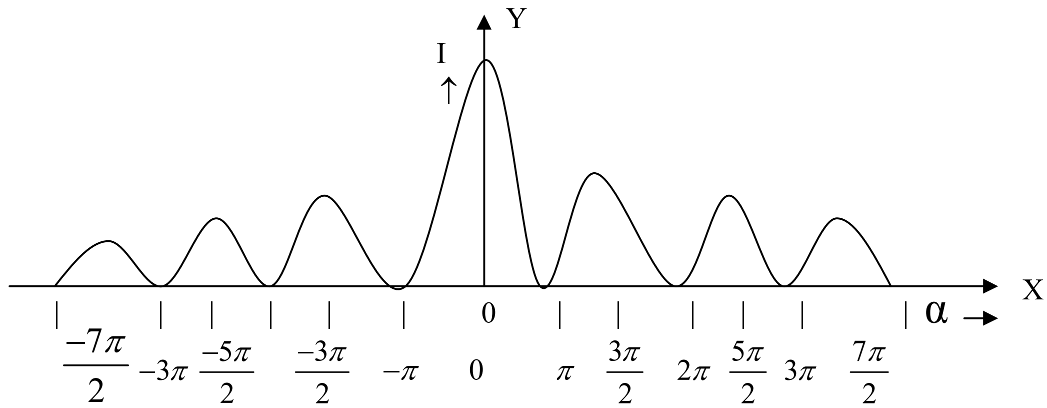


Figure (4): Intensity Distribution

Fraunhofer Diffraction at Double slit.

Let S_1 and S_2 be two slits of equal widths e. the two slits are repeated by a distance d. The distance between the middle points of the two slits is (e+d). A monochromatic light of wavelength λ is incident normally on the two slits. The light diffracted from these slits is focused by a lens of the screen placed in the focal plane of the lens. The diffraction of two slits is the combination of diffraction as well as interference. That is the pattern on the screen is the diffraction pattern due to single slit on which a system of interference fringes is superposed.

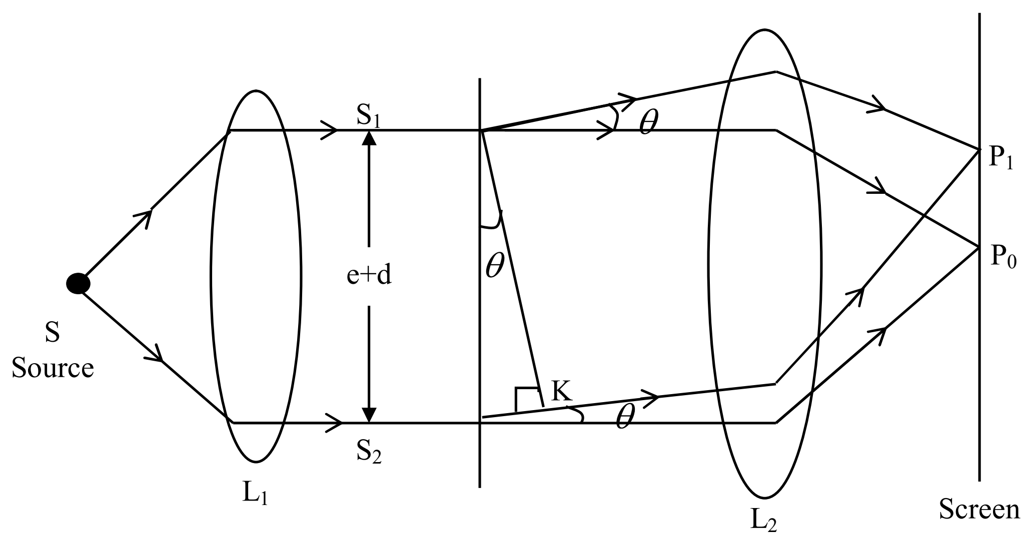


Figure (5) Fraunhofer diffraction at double slit.

Let a plane wave front is incident normally on both slits, all points within the slits became the sources of secondary wavelets. These secondary wavelets from the slits travel uniformly in all directions. The secondary wavelets traveling in the direction of incident light come to a focus at P_o. The secondary wavelets traveling in a direction making an angle θ with the incident direction come to a focus at P_1.

From the theory of diffraction due to a single slit, the resultant amplitude R due to all wavelets diffracted from each slit in a direction θ is given by

$$R = \frac{A\sin\alpha}{\alpha}$$

Now let us consider the two slits are equivalent to two coherent sources S_1 and S_2 arranged at mid-points of the slits. Here each source is sending a wavelet of amplitude $\left(\frac{A\sin\alpha}{\alpha}\right)$ in the direction θ.

$\therefore$ The resultant amplitudes at a point p_1 on the screen will be the result of interference between two waves of amplitude $\left(\frac{A\sin\alpha}{\alpha}\right)$ and having a phase δ.

For calculating δ, draw a perpendicular S_1K on to the S_2 secondary wavelet making an angle θ with the normal.

Now the path difference between the wavelets from S_1 and S_2 in the direction θ is given by S_2K.

From the figure triangle S_1KS_2 is a right angled triangle.

$$\therefore \sin\theta = \frac{S_2k}{S_1S_2} \Rightarrow S_2k = S_1S_2\sin\theta$$

$$S_2k = (e+d)\sin\theta \qquad \text{---(1)}$$

$$\therefore \text{ Phase difference } \delta = \frac{2\pi}{\lambda}\times\text{(path difference)}$$

$$\delta = \frac{2\pi}{\lambda}\times(e+d)\sin\theta \qquad \text{---- (2)}$$

To find the resultant amplitude at P_1, vector addition method is used. Here the two sides of the triangle represents amplitudes of wavelets from S_1 and S_2. This is shown in figure (6). The third side represents the resultant amplitude.

$$(OH)^2 = (OG)^2 + (GH)^2 + 2(OG)(GH)\cos\delta$$

$$R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2 + \left(A\frac{\sin\alpha}{\alpha}\right)^2$$

$$+2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\alpha$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2[1+1+2\cos\delta]$$

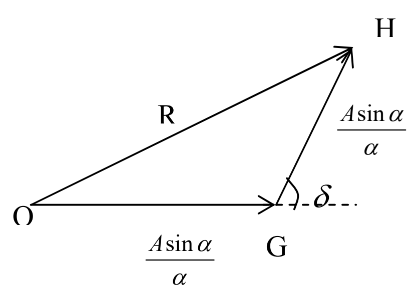


Figure (6) Vector Addition

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2[2+2\cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 2[1+\cos\delta]$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 2[\cancel{1}+2\cos^2(\delta/2)-\cancel{1}]$$

$$R^2 = 4\left(\frac{A\sin\alpha}{\alpha}\right)^2\cos^2(\delta/2) \qquad \text{---- (3)}$$

Now from equations (2) and (3), we have

$$R^2 = \frac{4A^2\sin^2\alpha}{\alpha^2}\cos^2\left[\frac{\pi}{\lambda}(e+d)\sin\theta\right]$$

Now let $\beta = \dfrac{\pi(e+d)\sin\theta}{\lambda}$

$$\therefore R^2 = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \cos^2\beta \qquad \text{------------------- (4)}$$

Now the resultant intensity is given by

$$I = R^2 = 4A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \cos^2\beta \qquad \text{--------------------(5)}$$

From the expression (5) it is clear that the resultant intensity is the product of two factors.

1. $A^2\left(\dfrac{\sin\alpha}{\alpha^2}\right)$, This represents the intensity distribution in the diffraction pattern due to a single slit.
2. $\cos^2\beta$ which gives the interference pattern due to wavelets from two parallel slits (double slits)

The resultant intensity is due to both diffraction and Interference effects.

Intensity due to Diffraction effects

The diffraction $A^2\left(\dfrac{\sin\alpha}{\alpha}\right)^2$ gives the principal maximum at the centre of the screen with alternative minima and secondary maxima of decreasing intensity. The principal maximum occurs at $\theta = 0$. The minima occurs when $\sin\alpha = 0$

$\alpha = \pm m\pi$, where m=1,2,3,....

$$\frac{\pi e\sin\theta}{\lambda} = m\pi \Rightarrow e\sin\theta = \pm m\lambda$$

The positions of secondary maxima occurs for

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, ...$$

Intensity due to interference effects

The interferences term $\cos^2\beta$ gives a set of equivalent bright and dark fringes.

The maxima will occurs for $\cos^2\beta$ =1.

i.e. $\beta = \pm n\pi$, where $n = 0,1,2,3,.....$

i.e. $\beta = 0, \pm\pi, \pm 2\pi, \pm 3\pi, ...$

Now $\dfrac{\pi(e+d)\sin\theta}{\lambda} = \pm n\pi$

$\pi(e+d)\sin\theta = \pm n\lambda$

The minima will occurs for $\cos^2\beta$ =0.

i.e $\beta = \pm(2n+1)\dfrac{\pi}{2}$ where $n = 0,1,2,3,......$

$$\Rightarrow \frac{\pi(e+d)\sin\theta}{\lambda} = \pm(2n+1)\frac{\pi}{2}$$

$$\Rightarrow (e+d)\sin\theta = \pm(2n+1)\frac{\lambda}{2}$$

Intensity Distribution

Figure 7(a) represents the intensity variations due to diffraction, having maxima and secondary maxima of decreasing intensity on either side.

Figure 7(b) shows the intensity variations due to interference.

When the diffraction and interference effects are combined then we get the resultant variation, as shown in figure (7) c. From fig (7)c it is clear that the resultant intensity of minima are not equal to zero, but they have some minimum intensity due to interference effect.

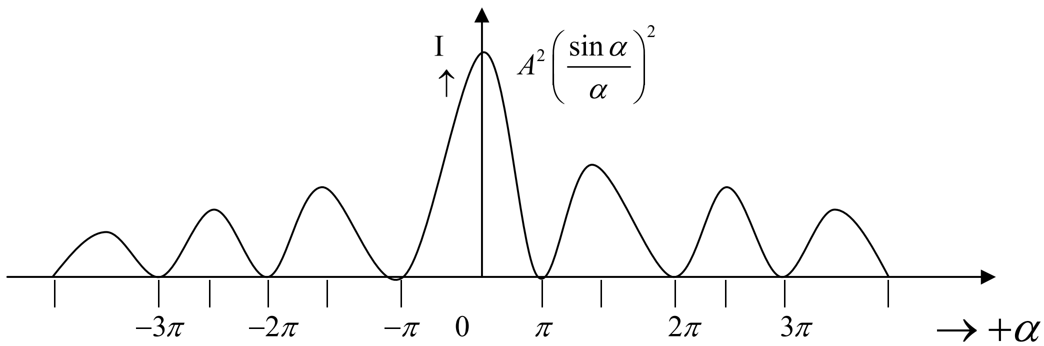


Figure (7a): Diffraction effect

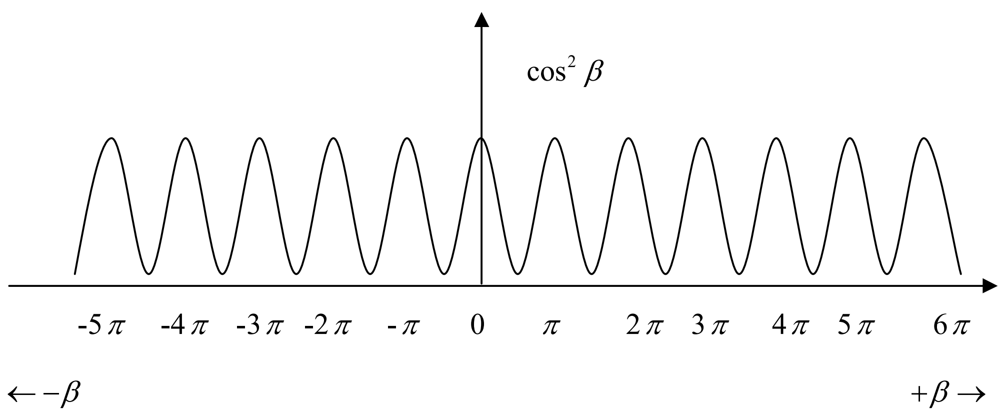


Figure (7b): Interference effect

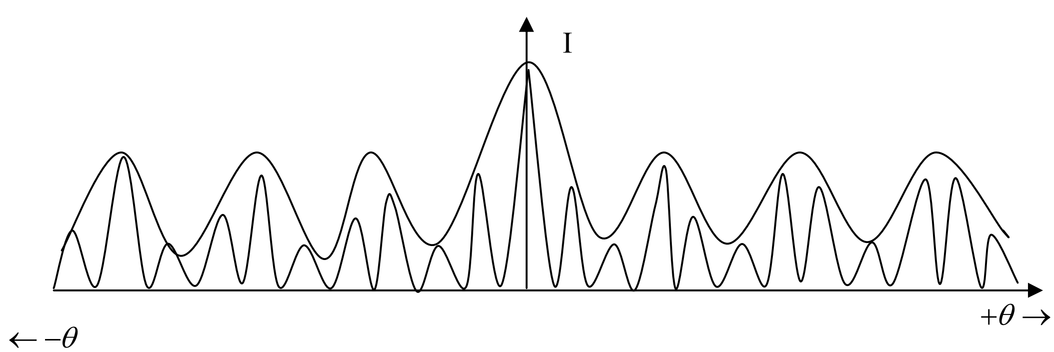


Figure (7c): Resultant intensity

Diffraction Grating

Diffraction grating is an arrangement which consists of a large number of parallel slits of the same width. These parallel slits are separated by equal and opaque spacings.
This arrangement is known as diffraction grating.

Fraunhofer used the first grating consisting of large number of parallel wires placed side by side very closely at regular intervals.

The gratings are designed by ruling equidistant parallel lines on a transparent material such as Glass with a fine diamond tip.

The ruled lines are opaque to light while the space between any two lines is transparent to light and act as a slit. This is shown in figure (8).

Usually gratings are designed by taking the cost of an actual grating on a transparent film like that of cellulose acetate.

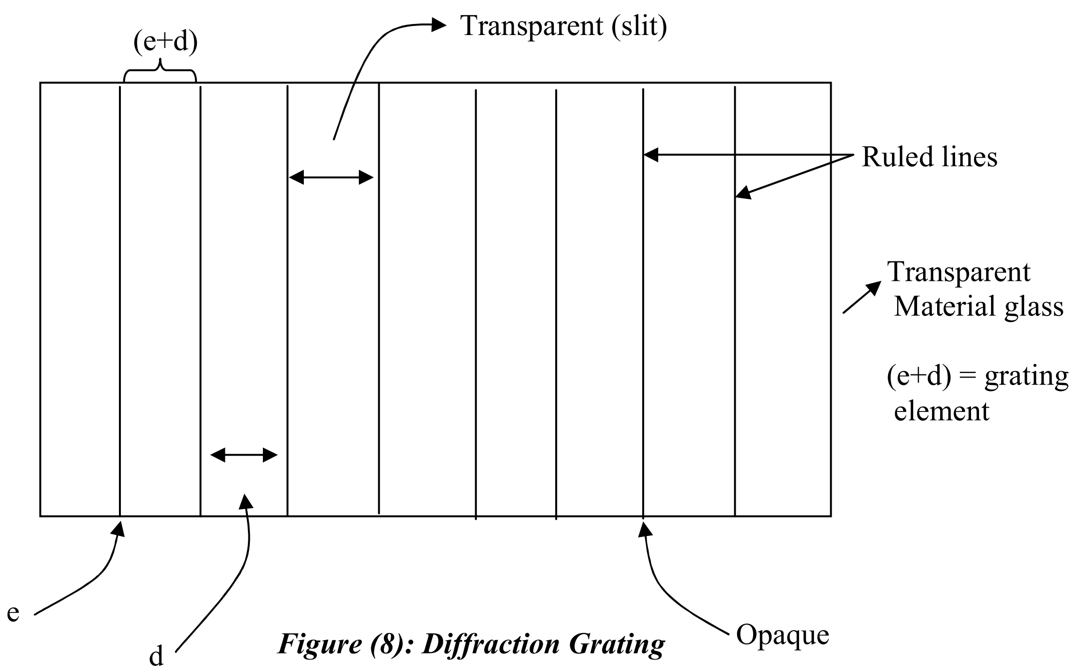


Figure (8): Diffraction Grating

Now solution of cellulose acetate is poured on the ruled surface and allowed to dry, for the formation of a thin film. This thin film is easily detachable from the surface. These impressions of a grating are preserved by mounting the film between two glass plate thin.

Let e be the width of each line.

Let d be the width of the slit.

Now $(e+d)$ is known as grating element.

If 'N' is the number of lines per inch on the grating, then

$N(e+d)$ grating elements are there per inch.

i.e. $N(e+d) = 1'' = 2.54 cms$

$$(e+d) = \frac{2.54}{N} cm$$

Usually there will be 15,000 lines per inch (or) 30,000 lines per inch on the grating. Due to the narrow width of the slit, it is comparable to wavelength of light.

When light falls on the grating, the light is diffracted through each slit.

As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as Diffraction pattern.

Grating Spectrum

In a grating the condition for the formation of principal maxima is given by

$$(e+d)\sin\theta = n\lambda \qquad \text{---- (1)}$$

Here $(e+d)$ is the grating element. The equation (1) is known as grating equation. From the grating equation, the following points may be observed.

1. For particular wavelength λ, the angle of diffraction θ is different, for principal maxima of different orders.
2. Since the number of lines in the grating is large, maxima appear as sharp and bright parallel lines. These lines are known as spectral lines.
3. For white light incidenting on the grating, the light of different wavelength will be diffracted in different directions for a particular order 'n'.

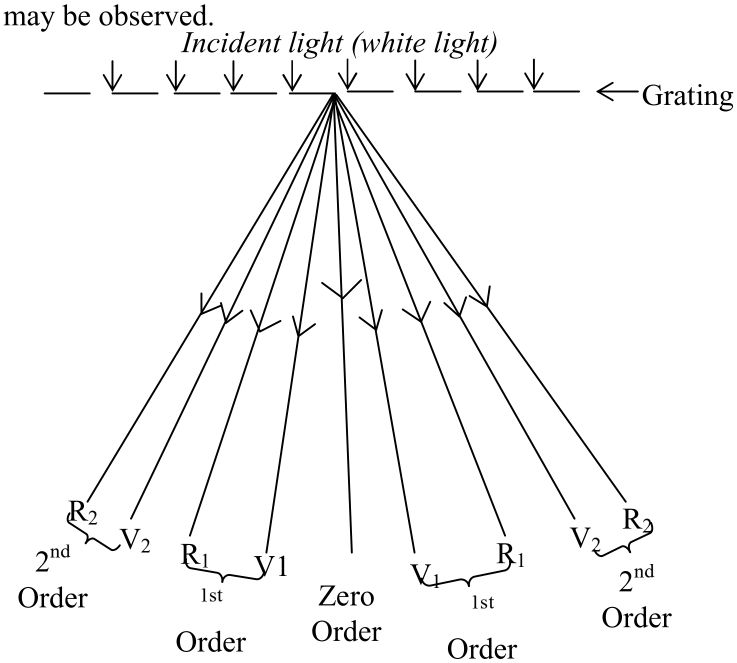


Figure (9) Grating spectrum

4. At the centre, $\theta = 0$, correspondents to maxima of all wavelengths. This maximum will coincide with the central image of the same colour as that of the light source. This forms zero order of the spectrum. This is shown in figure 8.
5. The principal maxima of all wavelengths form the 1st order, 2nd order, spectra for n=1, 2…
6. Larger the wave length, greater is the angle of diffraction. Thus in the spectrum, violet lies inner most and red lies outermost.
7. Intensity is maximum in the zero order and the rest of the orders will have distributed intensities.
8. Spectra of different orders are situated symmetrically on both sides of zero order.
9. The maximum number of orders possible with the grating is given by

$$n_{\max} = \frac{(e+d)}{\lambda}$$

Prepared By

P.V.Ramana Moorthy

Associate Professor,

SITAMS, Chittoor