Power Screws

- *A power screw is a mechanical device used for converting rotary motion into linear motion and transmitting power.*
- A power screw is also called a *translation screw*.
- It uses helical translatory motion of the screw thread in transmitting power

The main applications of power screws are as follows:

(i) to raise the load, e.g., screw-jack;
(ii) to obtain accurate motion in machining operations, e.g., lead-screw of lathe;
(iii) to clamp a work piece, e.g., a vice; and
(iv) to load a specimen, e.g., universal testing machine.

There are three essential parts of the power screw, viz., screw, nut and a part to hold either the screw or the nut in its place.

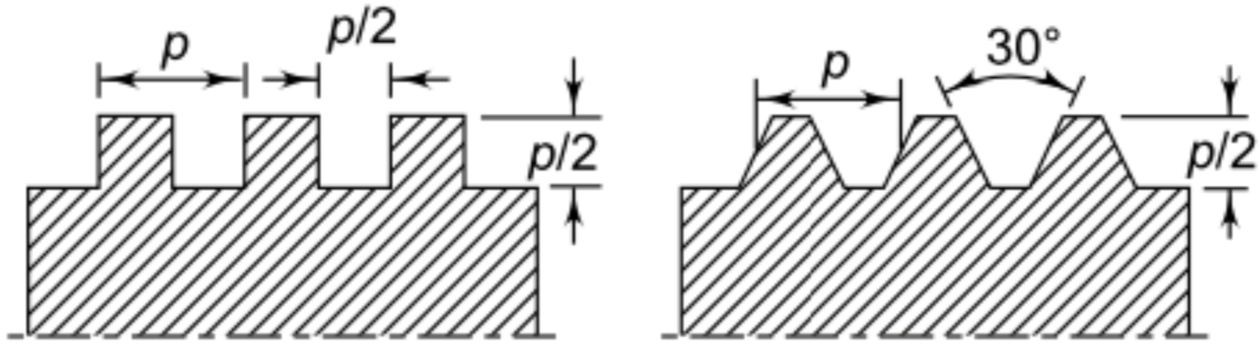
Forms of Threads

There are two popular types of threads used for power screws, viz., *square* and *ISO metric trapezoidal*

The advantages of square threads over trapezoidal threads are as follows:

(i) The efficiency of square threads is more than that of trapezoidal threads.

(ii) There is no radial pressure or side thrust on the nut. This radial pressure is called '*bursting*' pressure on the nut. Since there is no side thrust, the motion of the nut is uniform. The life of the nut is also increased.



(a) Square threads (b) Trapezoidal threads

The disadvantages of square threads are as follows:

(i) Square threads are difficult to manufacture. They are usually turned on a lathe with a single-point cutting tool. Machining with a single-point cutting tool is an expensive operation compared with machining with a multi-point cutting tool.

(ii) The strength of a screw depends upon the thread thickness at the core diameter. As shown in Fig. 6.1, square threads have less thickness at the core diameter than trapezoidal threads. This reduces the load carrying capacity of the screw.

(iii) The wear of the thread surface becomes a serious problem in the service life of the power screw. It is not possible to compensate for wear in square threads. Therefore, when worn out, the nut or the screw requires replacement.

The advantages of trapezoidal threads over square threads are as follows:

(i) Trapezoidal threads are manufactured on a thread milling machine. It employs a multi- point cutting tool. Machining with a multi- point cutting tool is an economic operation compared with machining with a single-point cutting tool. Therefore, trapezoidal threads are economical to manufacture.

(ii) A trapezoidal thread has more thickness at the core diameter than a square thread. Therefore, a screw with trapezoidal threads is stronger than an equivalent screw with square threads. Such a screw has a large load carrying capacity.

The disadvantages of trapezoidal threads are as follows:

(i) The efficiency of trapezoidal threads is less than that of square threads.

(ii) Trapezoidal threads result in side thrust or radial pressure on the nut. The radial pressure or bursting pressure on the nut affects its performance.

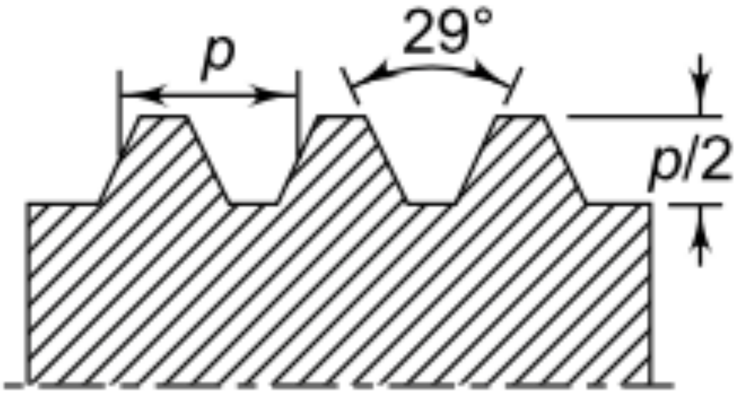


Fig. 6.2 *Acme Threads*

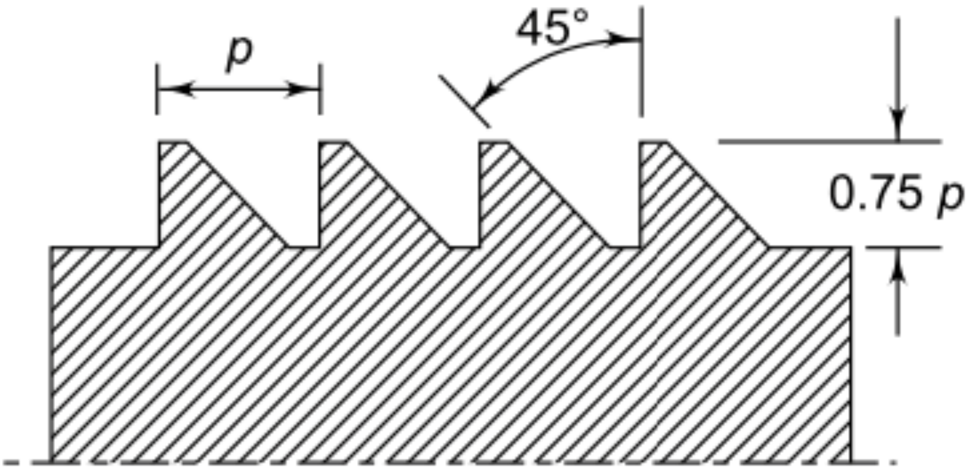


Fig. 6.3 *Buttress Threads*

It combines the advantages of square and trapezoidal threads. Buttress threads are used where a heavy axial force acts along the screw axis in one direction only.

The buttress threads have one disadvantage. It can transmit power and motion only in one direction. On the other hand, square and trapezoidal threads can transmit force and motion in both directions.

1. Square threads are used for screw jacks, presses and clamping devices.
2. Trapezoidal and acme threads are used for lead-screw and other power transmission devices in machine tools.
3. Buttress threads are used in vices, where force is applied only in one direction.

Table 6.1 *Proportions of square threads (normal series)*

Nominal diameter, d (mm)	*Pitch, p (mm)*
22, 24, 26, 28	5
30, 32, 36	6
40, 44	7
48, 50, 52	8
55, 60	9
65, 70, 75, 80	10
85, 90, 95, 100	12

Table 6.2 *Proportions of ISO metric trapezoidal threads*

Nominal diameter, d (mm)	*Pitch, p (mm)*
24, 28	5
32, 36	6
40, 44	7
48, 52	8
60	9
70, 80	10
90, 100	12

Designation of threads:

Sq 30 x 6

30 → nominal dia

6 → pitch

Tr 14 x 7

Tr 40 x 14 (P 7) → Lead

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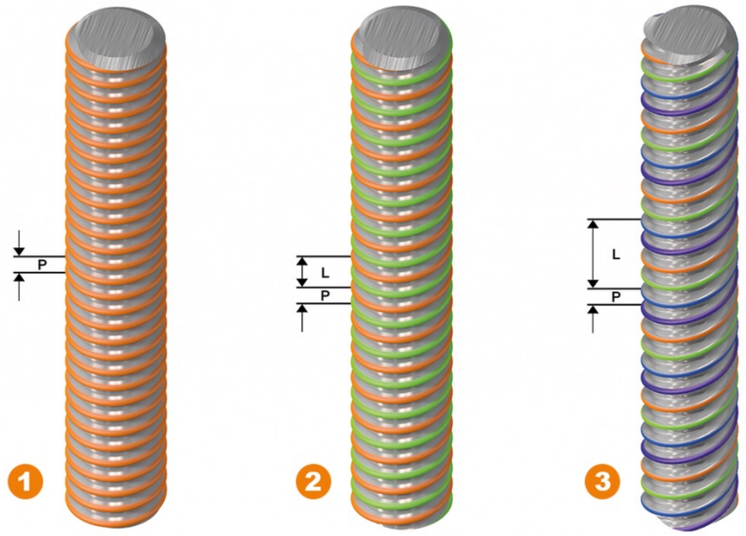
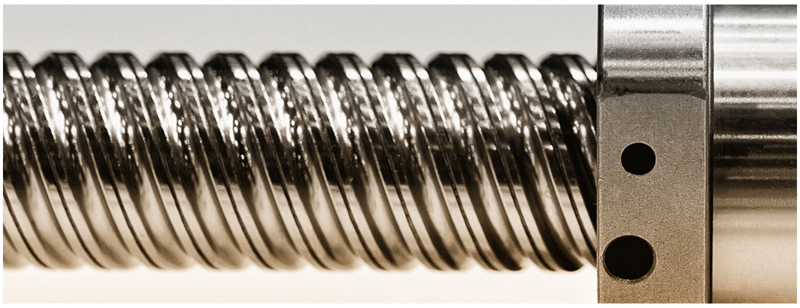
$P = L$ — Single start

$P = L/2$ → Double

$P = L/3$ → Triple


P
L
P
L
P
1
2
3



TERMINOLOGY OF POWER SCREW

(i) **Pitch** The pitch is defined as the distance measured parallel to the axis of the screw from a point on one thread to the corresponding point on the adjacent thread.

(ii) **Lead** The lead is defined as the distance measured parallel to the axis of the screw which the nut will advance in one revolution of the screw. For a single-threaded screw, the lead is same as the pitch. For a double-threaded screw, the lead is twice of the pitch, and so on.

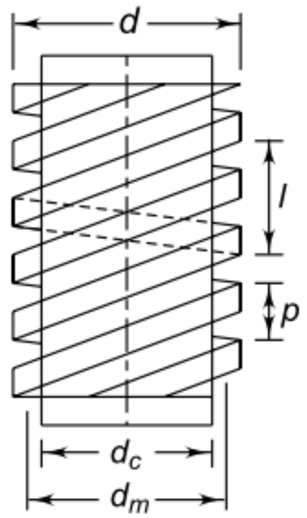


Fig. 6.4 *Terminology of Power Screw*

(iii) **Nominal Diameter** Nominal diameter is the largest diameter of the screw. It is also called major diameter. It is denoted by the letter d.

(iv) **Core Diameter** The core diameter is the smallest diameter of the screw thread. It is also called minor diameter. It is denoted by the letters dc.

(v) **Helix Angle** The helix angle is defined as the angle made by the helix of the thread with a plane perpendicular to the axis of the screw. The helix angle is related to the lead and the mean diameter of the screw. It is also called lead angle. The helix angle is denoted by a.

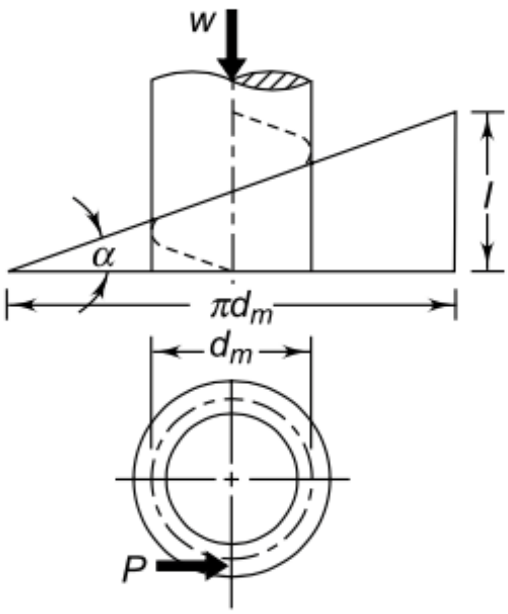


Fig. 6.5 *Development of Thread*

The conclusions that can be drawn on the basis of development of thread,

(i) The screw can be considered as an inclined plane with α as the inclination.

(ii) The load W always acts in a vertically downward direction. When the load W is raised, it moves up the inclined plane. When the load W is lowered, it moves down the inclined plane.

(iii) The load W is raised or lowered by means of an imaginary force P acting at the mean radius of the screw. The force P multiplied by the mean radius (dm/2) gives the torque required to raise or lower the load.

$$\tan\alpha = \frac{l}{\pi d_m}$$

$$\alpha = \tan^{-1}\left(\frac{l}{\pi d_m}\right)$$

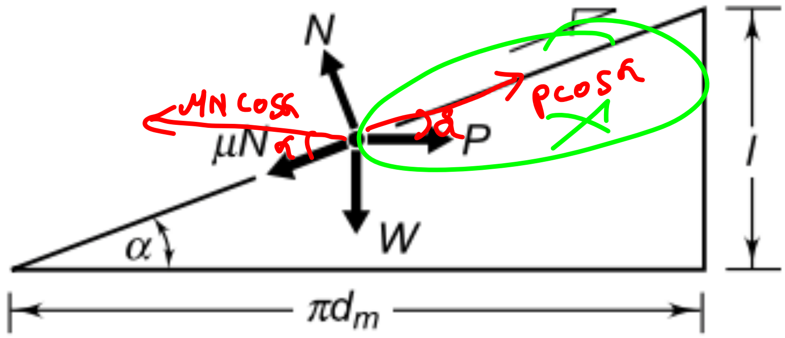
TORQUE REQUIREMENT— LIFTING LOAD

(i) **Load** W It always acts in a vertically downward direction.

(ii) **Normal Reaction** N It acts perpendicular (normal) to the inclined plane.

(iii) **Frictional Force** μN Frictional force acts opposite to the motion. Since the load is moving up the inclined plane, frictional force acts along the inclined plane in the downward direction.

(iv) **Effort** The effort P acts perpendicular to the load W. It may act towards the right or towards the left. It should act towards the right to overcome the friction and raise the load.



$N_V = W + F_V$

Fig. 6.6 *Force Diagram for Lifting Load*

$$P = \mu N \cos\alpha + N\cos(90-\alpha)$$

$$P = \mu N \cos\alpha + N\sin\alpha$$

$$P = N[\mu\cos\alpha + \sin\alpha] \quad (1)$$

$$W = N\cos\alpha - \mu N\sin\alpha \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{P}{W} = \frac{\mu N\cos\alpha + N\sin\alpha}{N\cos\alpha - \mu N\sin\alpha}$$

$$P = \frac{W(\mu N\cos\alpha + N\sin\alpha)}{(N\cos\alpha - \mu N\sin\alpha)} \quad \text{— (3)}$$

$(3)/\cos\alpha$

$$P = \frac{W(\mu N + N\tan\alpha)}{(N - \mu N\tan\alpha)}$$

$\mu = \tan\phi$ $\qquad$ $\phi \rightarrow$ friction angle

$$P = \frac{W(\mu + \tan\alpha)}{1 - \mu\tan\alpha}$$

$$= \frac{W(\tan\phi + \tan\alpha)}{(1 - \tan\phi\tan\alpha)}$$

$$\underline{P = W\tan(\phi + \alpha)}$$

$$\boxed{m_t = \frac{W d_m \tan(\phi+\alpha)}{2}}$$

Torque = Force × distance

$$m_t = P \times d_m/2$$

TORQUE REQUIREMENT— LOWERING LOAD

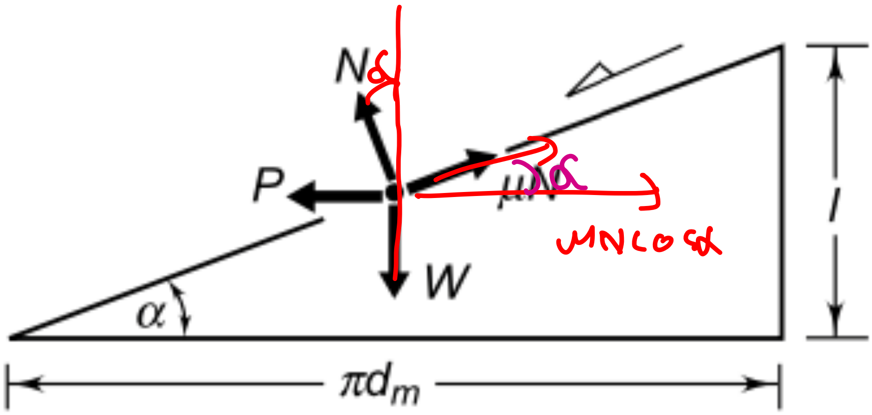


Fig. 6.7 *Force Diagram for Lowering Load*

$$P = \mu N \cos\alpha - N \sin\alpha \quad -(4)$$

$$W = N\cos\alpha + \mu N \sin\alpha \quad -(5)$$

$$P = \frac{W(\mu N\cos\alpha - N\sin\alpha)}{N\cos\alpha + \mu N\sin\alpha}$$

$$P = \frac{W(\mu\cos\alpha - \sin\alpha)}{\cos\alpha + \mu\sin\alpha}$$

$$P = \frac{W(\mu - \tan\alpha)}{1 + \mu\tan\alpha} \qquad \mu = \tan\phi$$

$$P = \frac{W(\tan\phi - \tan\alpha)}{1 + \tan\phi\tan\alpha}$$

$$P = W \tan(\phi - \alpha)$$

Torque $T_2 = P \times \frac{d_m}{2}$

$$T_2 = \frac{W d_m}{2} \tan(\phi - \alpha)$$

SELF-LOCKING SCREW

$P = W \tan(\phi - \alpha)$

when; $\phi < \alpha$ the torque required to lower the load is negative. It indicates a condition that no force is required to lower the load. The load itself will begin to turn the screw and descend down, unless a restraining torque is applied. This condition is called *overhauling* of the screw. This condition is also called *back driving* of screw.

This property is not useful in screw-jack applications. However, it is useful in some other applications like a Yankee screwdriver.

When $\phi \geq \alpha$, then a positive torque is required to lower the load. Under this condition, the load will not turn the screw and will not descend on its own unless an effort P is applied. In this case, the screw is said to be '*self-locking*'. A self-locking screw will hold the load in place without a brake.

"*A screw will be self-locking if the coefficient of friction is equal to or greater than the tangent of the helix angle*".

(i) Self-locking of screw is not possible when the coefficient of friction (μ) is low. The coefficient of friction between the surfaces of the screw and the nut is reduced by lubrication. Excessive lubrication may cause the load to descend on its own.

(ii) Self-locking property of the screw is lost when the lead is large. The lead increases with a number of starts. For a double-start thread, the lead is twice of the pitch and for a triple-threaded screw, it is three times of the pitch. Therefore, single-threaded screw is better than multiple-threaded screws from self-locking considerations.

EFFICIENCY OF SQUARE THREADED SCREW

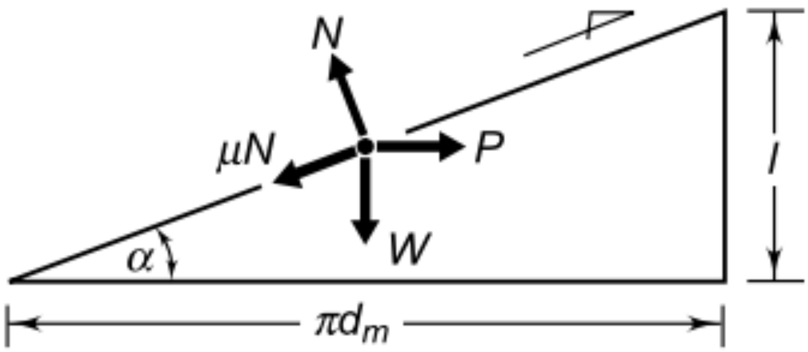


Fig. 6.6 *Force Diagram for Lifting Load*

work output = $W \times l$

work input = $P \times \pi d_m$

Efficiency = $\dfrac{O/P}{I/P} = \dfrac{Wl}{P \pi d_m} = \dfrac{W}{P} \tan\alpha$

$\tan\phi = \mu$

$$\boxed{\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}}$$

Conclusions The efficiency of a square threaded power screw depends upon the following three factors:

(i) Mean diameter of screw

(ii) Lead of the screw

(iii) Coefficient of friction

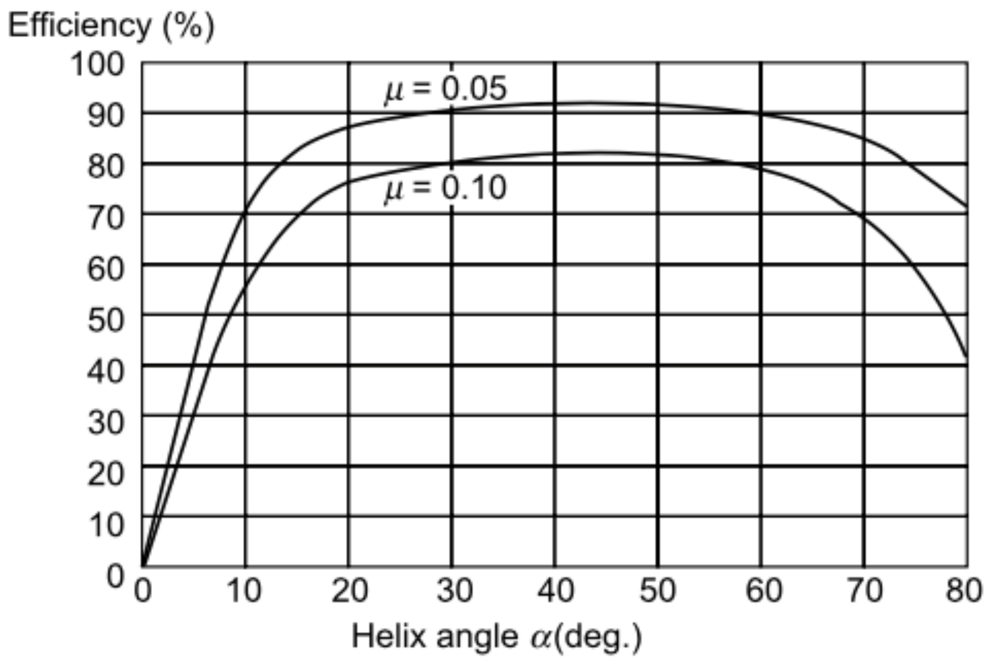


Fig. 6.8 *Efficiency of Square Threaded Screw*

The following conclusions can be derived from the observation of these graphs:

(i) The efficiency of a square threaded screw increases rapidly up to helix angle of 20°.

(ii) The efficiency is maximum, when the helix angle is between 40 to 45°.

(iii) The efficiency decreases after the maximum value is reached.

(iv) The efficiency decreases rapidly when the helix angle exceeds 60°.

(v) The efficiency decreases as the coefficient of friction increases.

$$\eta = \frac{\tan\alpha}{\tan(\phi+\alpha)} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$= \frac{\sin\alpha/\cos\alpha}{\sin(\alpha+\phi)/\cos(\alpha+\phi)}$$

$$= \frac{\sin\alpha\cos(\alpha+\phi)}{\cos\alpha\sin(\alpha+\phi)}$$

$$\eta = \frac{2\sin\alpha\cos(\alpha+\phi)}{2\cos\alpha\sin(\alpha+\phi)}$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$\eta = \frac{\sin(2\alpha+\phi) - \sin(\phi)}{\sin(2\alpha+\phi) + \sin(\phi)}$$

$$2\alpha + \phi = 90^\circ$$

$$\boxed{\alpha = 45 - \frac{\phi}{2}}$$

$$\eta = \frac{W}{P}\tan\alpha$$

$$= \frac{\tan\alpha}{\tan(\phi+\alpha)}$$

$$\boxed{\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}}$$

For self locking screw:

$$\phi \geq \alpha$$

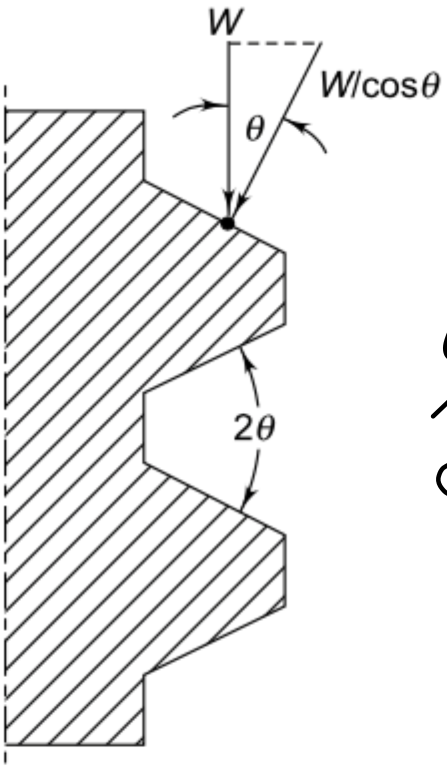
$$\eta = \frac{\tan\alpha}{\tan(\phi+\alpha)}$$

$$\eta \leq \frac{\tan\phi}{\tan(\phi+\phi)} \Rightarrow \eta \leq \frac{\tan\phi}{\tan 2\phi}$$

$$\eta \leq \frac{\cancel{\tan\phi}\,(1-\tan^2\phi)}{2\,\cancel{\tan\phi}}$$

$$\eta \leq \left[\frac{1}{2} - \frac{\tan^2\phi}{2}\right]$$

TRAPEZOIDAL AND ACME THREADS



W

$N = W \rightarrow$ Square

$$\frac{W}{\cos\theta} = \frac{W}{0.678}$$

$$\cancel{\frac{W}{0.678} < W}$$

$$1.47W < W$$

Fig. 6.9 *Force Diagram for Trapezoidal Thread*

For ISO Metric trapezoidal thread, $2\theta = 30°$
For acme thread, $2\theta = 29°$

$$N = \frac{W}{\cos\theta}$$

$$f = \mu N = \frac{\mu W}{\cos\theta} = \mu W \sec\theta = (\mu \sec\theta) W$$

$N = W$

$f = \mu N = \mu W \rightarrow$ Square

Effective co-eff of friction $= \mu \sec\theta$

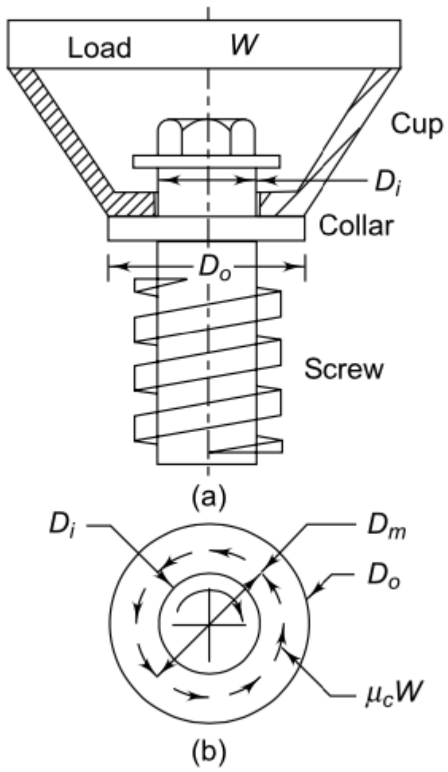
$$P = \frac{W(\mu\sec\theta - \tan\alpha)}{(1 + \mu\sec\theta\tan\alpha)}$$

$$\eta = \frac{W}{P}\tan\alpha$$

$$M_t = \frac{P \times d_m}{2}$$

$$\eta = \frac{\tan\alpha(1-\mu\tan\alpha)}{(\mu+\tan\alpha)}$$

COLLAR FRICTION TORQUE



In certain applications, the collar between the cup and the screw is replaced by thrust ball bearing **to reduce friction**. The advantage of using thrust ball bearing at the collar is that the sliding friction is replaced by rolling friction. The collar friction torque becomes almost negligible in these cases.

$$(M_t)_c = \frac{\mu_c W}{3} \cdot \frac{(D_o^3 - D_i^3)}{(D_o^2 - D_i^2)} \rightarrow \text{uniform pressure theory}$$

$$(M_t)_c = \frac{\mu_c W}{4} (D_o + D_i) \rightarrow \text{uniform wear theory}$$

$D_o \rightarrow$ outer dia of collar

$D_i \rightarrow$ inner " " "

$\mu_c \rightarrow$ co-eff of friction of collar

Overall efficiency

1 → Power screw torque

2 → Collar friction torque

Total torque $T = T_1 + T_2$

$$(M_t)_t = \frac{W d_m}{2} \tan(\phi + \alpha) + \frac{\mu_c W}{4}(D_o + D_i)$$

Overall efficiency $= \frac{O/P}{I/P} = \frac{\text{Total O/P torque}}{\text{Input torque}}$

$$= \frac{\text{Work O/P}}{\text{Work I/P}}$$

$$\boxed{\eta_o = \frac{W \times l}{(M_t)_t \times 2\pi}}$$

Work Input = Torque × Angle rotates

$= (M_t)_t \times 2\pi$

COEFFICIENT OF FRICTION

It has been found that the coefficient of friction (m) at the thread surface depends upon the workmanship in cutting the threads and on the type of the lubricant. It is practically independent of the load, rubbing velocity or materials. An average value of 0.15 can be taken for the coefficient of friction at the thread surface, when the screw is lubricated with mineral oil.

When thrust ball bearing is used at the collar surface, its coefficient of friction is about 1/10th of plain sliding surface. It varies from 0.01 to 0.02.

Table 6.3 *Coefficient of friction for thrust collars*

Material combination	μ_c	
	Starting	*Running*
Soft-steel – cast iron	0.17	0.12
Hardened steel – cast iron	0.15	0.09
Soft steel – bronze	0.10	0.08
Hardened steel – bronze	0.08	0.06

Problem : The nominal diameter of a triple- threaded square screw is 50 mm, while the pitch is 8 mm. It is used with a collar having an outer diameter of 100 mm and inner diameter as 65 mm. The coefficient of friction at the thread surface as well as at the collar surface can be taken as 0.15. The screw is used to raise a load of 15 kN. Using the uniform wear theory for collar friction,

calculate:

(i) torque required to raise the load;

(ii) torque required to lower the load; and

(iii) the force required to raise the load, if applied at a radius of 500 mm.

Given data: 3 threads $l = 3p = 24\,mm$

$d = 50\,mm$, $p = 8\,mm$

$D_o = 100\,mm$, $D_i = 65\,mm$ $\mu_s = \mu_c = 0.15$

$W = 15\,kN$

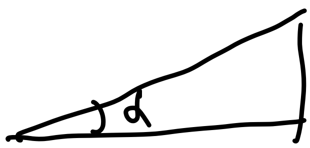
According to uniform wear theory $(M_t)_c = \frac{\mu_c W}{4}(D_o + D_i)$

Total torque $(M_t)_t = (M_t)_s + (M_t)_c$ $d_m = \frac{d_o + d_i}{2}$

$$(M_t)_s = \frac{W d_m}{2}\tan(\phi + \alpha)$$

$$d_m = d - 0.5p = 50 - 0.5 \times 8 = 46\,mm$$

$$\tan\alpha = \frac{l}{\pi d_m} \Rightarrow \alpha = \tan^{-1}\left(\frac{l}{\pi d_m}\right)$$



$$\alpha = 9.429^\circ$$

$\phi = \tan^{-1}(\mu)$ ϕ – friction angle

$\underline{\phi = 8.53^\circ}$

Torque = force × dist

$w = 15\,kN$

$$(M_t)_s = \frac{w d_m}{2} \tan(\phi + \alpha)$$

$$= \frac{15 \times 10^3 \times 46}{2} \times \tan(8.53 + 9.429)$$

$$(M_t)_s = 111824.41 \text{ N-mm}$$

$\mu_c = 0.15$

$D_o = 100\,mm$

$D_i = 65\,mm$

$$(M_t)_c = \frac{\mu_c \cdot w}{4}(D_o + D_i)$$

$$= 92.81 \text{ kN-mm}$$

$$= 92812 \text{ N-mm}$$

$(+)$

$$(M_t)_t = (M_t)_s + (M_t)_c = 204636.9 \text{ N-mm}$$

$$\boxed{(M_t)_t = \underline{204.63} \text{ N-m}}$$

Torque req to lower the load :

$$(M_t)_s = \frac{w d_m}{2} \tan(\phi - \alpha) = \underline{-5413.6} \text{ N-mm}$$

$$(M_t)_c = 92812 \text{ N-mm}$$

$(+)$

$$(M_t)_t = 87404.8 \text{ N-mm} = \underline{87.4 \text{ N-m}}$$

lowering the load

Force req when applied @ 500mm (for raising the load)

Torque = Force × dist

$$\text{Force} = \frac{\text{Torque}}{\text{dist}} = \frac{(Mt)_{\to \text{raising}}}{500}$$

$$= \frac{204 \times 10^3 \ N\text{-}\cancel{mm}}{500 \ \cancel{mm}}$$

$$\boxed{\text{Force } P = 408 \ N}$$

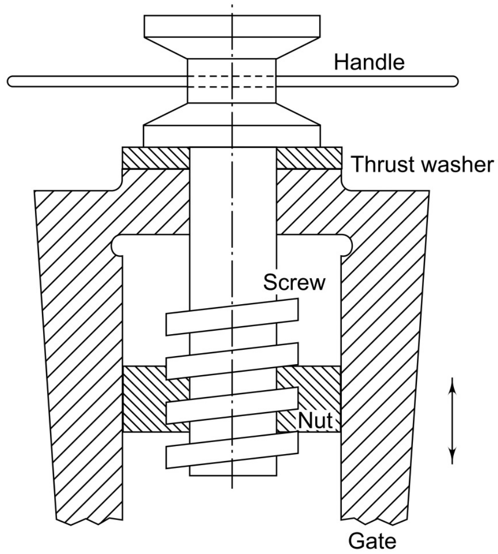
Example 6.4 *The construction of a gate valve used in high-pressure pipeline is shown in Fig. 6.13. The screw is rotated in its place by means of the handle. The nut is fixed to the gate. When the screw rotates, the nut along with the gate moves downward or upward depending upon the direction of rotation of the screw. The screw has single-start square threads of 40 mm outer diameter and 7 mm pitch. The weight of the gate is 5 kN. The water pressure in the pipeline induces frictional resistance between the gate and its seat. The resultant frictional resistance in the axial direction is 2 kN. The inner and outer diameters of thrust washer are 40 and 80 mm respectively. The values of coefficient of friction at the threads and at the washer are 0.15 and 0.12 respectively. The handle is rotated by the two arms, each exerting equal force at a radius of 500 mm from the axis of the screw. Calculate*

(i) the maximum force exerted by each arm when the gate is being raised;

(ii) the maximum force exerted by each arm when the gate is being lowered;

(iii) the efficiency of the gate mechanism; and

(iv) the length of the nut, if the permissible bearing pressure is 5 N/mm².



Solution

Given For screw, $d = 40$ mm $\quad l = p = 7$ mm

$\mu = 0.15$

For collar, $D_o = 80$ mm $\quad D_i = 40$ mm

$\mu = 0.12$

For handle, radius = 500 mm

For nut $S_b = 5$ N/mm²

For gate, weight = 5 kN

frictional resistant = 2 kN

Step I *Force exerted by each arm to raise the gate*

From Eq. (6.2),

$d_m = \frac{d_o + d_i}{2}$

$$d_m = d - 0.5\,p = 40 - 0.5(7) = 36.5 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_m} = \frac{7}{\pi\,(36.5)}$$

$$= 0.061 \quad \text{or} \quad \alpha = 3.493°$$

$$\tan\phi = \mu = 0.15 \quad \text{or} \quad \phi = 8.531°.$$

Frictional resistance acts opposite to the motion. When the gate is being raised the frictional force acts in downward direction. Therefore, axial force on the screw consists of addition of the weight of the gate plus the frictional resistance. Or,

$$W = 5000 + 2000 = 7000 \text{ N}$$

From Eq. (6.6),

$$M_t = \frac{W d_m}{2} \tan\,(\phi + \alpha)$$

$$= \frac{(7000)(36.5)}{2} \tan\,(8.531 + 3.493)$$

$$= 27\,210.04 \text{ N-mm.}$$

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c\,W}{4}\,.\,(D_0 + D_i)$$

$m_t = 2 \times F \times r$

$$= \frac{(0.12)\,(7000)(80 + 40)}{4}$$

$$= 25\,200 \text{ N-mm}$$

$$(M_t)_t = M_t + (M_t)_c = 27\,210.04 + 25\,200$$

$$= 52\,410.04 \text{ N-mm.}$$

There are two arms, each exerting a force P at a radius of 500 mm. Therefore,

$$(M_t)_t = 2P \times 500$$

$$P = \frac{(M_t)_t}{1000} = \frac{52\,410.04}{1000} = 52.41 \text{ N} \qquad \text{(i)}$$

Step II *Force exerted by each arm to lower the gate*

When the gate is being lowered, the frictional resistance acts in a vertically upward direction, while the weight acts in a downward direction. Therefore, the net axial force consists of the difference between the two.

or $\quad W = 5000 - 2000 = 3000$ N

From Eq. (6.8),

$$M_t = \frac{Wd_m}{2}\tan(\phi - \alpha)$$

$$= \frac{(3000)\ (36.5)}{2}\tan(8.531 - 3.493)$$

$$= 4826.60 \text{ N-mm}$$

From Eq. (6.18),

$$(M_t)_c = \frac{\mu_c W}{4}(D_o + D_i)$$

$$= \frac{(0.12)\ (3000)}{4}(80 + 40)$$

$$= 10\,800 \text{ N-mm}$$

$$(M_t)_t = M_t + (M_t)_c = 4826.60 + 10\,800$$

$$= 15\,626.6 \text{ N-mm}$$

The force P exerted by each arm on the handle is given by,

$$(M_t)_t = (2P) \times 500$$

or $$P = \frac{(M_t)_t}{1000} = \frac{15\,626.6}{1000} = 15.63 \text{ N} \qquad \text{(ii)}$$

Step III *Efficiency of gate mechanism*

From Eq. (6.20),

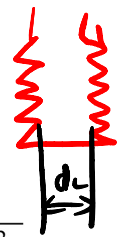
$$\eta_o = \frac{W\,l}{2\pi\ (M_t)_t}$$

Substituting values of W and $(M_t)_t$ obtained in case of raising the gate,

$$\eta_o = \frac{(7000)\ (7)}{2\pi\ (52\,410.04)} = 0.1488 \text{ or } 14.88\% \quad \text{(iii)}$$

Step IV *Length of the nut*

From Eq. (6.11),



$$d_c = d - p = 40 - 7 = 33 \text{ mm}$$

From Eq. (6.23),

$$z = \frac{4W}{\pi S_b(d^2 - d_c^2)} = \frac{4\,(7000)}{\pi\,(5)\,(40^2 - 33^2)}$$

$$= 3.49 \quad \text{or} \quad 4 \text{ threads}$$

$$l = zp = 4 \times 7 = 28 \text{ mm} \qquad \text{(iv)}$$

Problem : A double-threaded power screw, with ISO metric trapezoidal threads is used to raise a load of 300 kN. The nominal diameter is 100 mm and the pitch is 12 mm. The coefficient of friction at the screw threads is 0.15. Neglecting collar friction, calculate

(i) torque required to raise the load;

(ii) torque required to lower the load; and

(iii) efficiency of the screw.

$\}\ P$

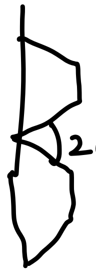
Given data:

$l = 2p$

$\theta = 15^\circ$

$W = 300$ KN

$d = 100$ mm



$2\theta = 30^\circ$

$p = 12$mm $\mu_s = 0.15$

$d_m = d - 0.5p$

$= 100 - 0.5 \times 12$

$= 94$ mm

$(M_t)_s = \frac{W d_m}{2} \tan(\phi + \delta)$ → raising

↳ Square threads

$l = 24$mm

$\tan\delta = \frac{l}{\pi d_m} \Rightarrow \delta = \tan^{-1}\left(\frac{l}{\pi d_m}\right) = \tan^{-1}\left(\frac{24}{\pi d_m}\right)$

$\delta = 4.646^\circ$

Torque req $= P \times \frac{d_m}{2} = \frac{W}{2} \times d_m \left(\frac{\mu \sec\theta + \tan\delta}{1 - \mu\sec\theta \tan\delta}\right)$

$\mu = 0.15$ $\mu\sec\theta = \mu \times \sec(15^\circ) = 0.1553$

$\frac{\mu}{\cos\theta}$

$$(M_t)_s = \frac{(300\times10^3)}{2} \times (94) \left(\frac{0.1553 + \tan(4.626)}{1 - 0.1553\tan(4.626)}\right)$$

$$(M_t)_s = 3378.72 \times 10^3 \text{ N-mm}$$

$$= \underline{3378.72 \text{ N-m}}$$

2) Torque req. to lower the load:

$$M_t = \frac{W d_m}{2} \left(\frac{\mu \sec\theta - \tan\alpha}{1 + \mu \sec\theta \tan\alpha}\right)$$

$$\underline{(M_t)_s = 1030.39 \text{ N-m}}$$

3) Efficiency of Screw:

$$\eta = \frac{\tan\alpha\,(1 - \mu\sec\theta\tan\alpha)}{(\mu\sec\theta + \tan\alpha)}$$

$\tan\alpha$

$\downarrow$

0.081

$$\eta = 0.3379$$

$$\boxed{\eta = 33.8\%}$$

Problem: A machine vice, as shown in Fig. 6.12, has single-start, square threads with 22 mm nominal diameter and 5 mm pitch. The outer and inner diameters of the friction collar are 55 and 45 mm respectively. The coefficients of friction for thread and collar are 0.15 and 0.17 respectively. The machinist can comfortably exert a force of 125 N on the handle at a mean radius of 150 mm. Assuming uniform wear for the collar, calculate

$W \rightarrow P$

$P \rightarrow W$

(i) the clamping force developed between the jaws; and

(ii) the overall efficiency of the clamp.

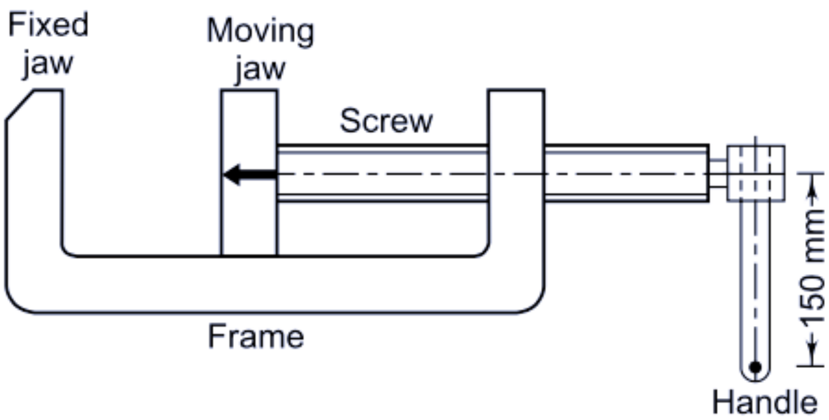


Fig. 6.12

Given data:

$d = 22\,mm$ $\quad l = p = 5\,mm$ $\quad \mu_s = 0.15$

For collar $D_o = 55\,mm$ $\quad \mu_c = 0.17$

$D_i = 45\,mm$

$P = 125\,N$

$r = 150\,mm$

$d_m = d - 0.5p$

$= 22 - 0.5 \times 5$

$= 19.5\,mm$

Torque:

$$M_t = \frac{W d_m}{2} \tan(\phi + \alpha)$$

$$\delta = \tan^{-1}\left(\frac{l}{\pi d_m}\right) \qquad \delta = 4.66^\circ$$

$$\mu = \tan\phi \qquad \phi = \tan^{-1}\mu_s = \tan^{-1}(0.15) = 8.53^\circ$$

$$(M_t)_s = \frac{W \times 19.5}{2}\tan(8.53 + 4.66)$$

$$(M_t)_s = 2.28\,W \quad N\text{-}mm$$

$$(M_t)_c = \frac{\mu_c W}{4}(D_o + D_i)$$

$$= W\left(\frac{0.17}{4}\right)(55 + 45)$$

$$(M_t)_c = 4.25\,W \quad N\text{-}mm$$

Total torque $(M_t)_t = (M_t)_s + (M_t)_c$

$$(M_t)_t = (2.28 + 4.25)\,W$$

Total torque produced by machinist = Force x dist

$$= 125 \times 150$$

$$(2.28 + 4.25)\,W = 125 \times 150$$

$$\underline{W = 2871.3\ N}$$

$$\eta_o = \frac{W \times l}{(M_t)_t \times 2\pi} = \frac{(2871.3) \times 5\,mm}{(125 \times 150) \times 2\pi} = \underline{12.18\%}$$

Belt Drives

Belt, chain and rope drives are called '*flexible*' drives.

There are two types of drives—rigid and flexible.

- Gear drives are called rigid or non-flexible drives. In gear drives, there is direct contact between the driving and driven shafts through the gears.
- In flexible drives, there is an intermediate link such as belt, rope or chain between the driving and driven shafts. Since this link is flexible, the drives are called '*flexible*' drives.
- In gear drives, rotary motion of the driving shaft is directly converted into rotary motion of the driven shaft by means of pinion and gear.
- In flexible drives, the rotary motion of the driving shaft is first converted into translatory motion of the belt or chain and then again converted into rotary motion of the driven shaft.

Thus, a flexible element is **superimposed between the driving and driven elements**.

The advantages of flexible drives over rigid drives are as follows:

(i) Flexible drives transmit power over a comparatively long distance due to an intermediate link between driving and driven shafts.

(ii) Since the intermediate link is long and flexible, it absorbs shock loads and damps vibrations.

The disadvantages of flexible drives are as follows:

(i) They occupy more space.

(ii) The velocity ratio is relatively small.

(iii) In general, the velocity ratio is not constant.

Belts are used to transmit power between two shafts by means of friction. A belt drive consists of three elements—driving and driven pulleys and an endless belt, which envelopes them.

(i) Belt drives can transmit power over considerable distance between the axes of driving and driven shafts.

(ii) The operation of belt drive is smooth and silent.

(iii) They can transmit only a definite load, which if exceeded, will cause the belt to slip over the pulley, thus protecting the parts of the drive against overload.

(iv) They have the ability to absorb the shocks and damp vibration.

(v) They are simple to design.

(vi) They have low initial cost.

Belt drives are mainly used in electric motors, automobiles, machine tools and conveyors.

Depending upon the shape of the cross-section, belts are classified as **flat belts and V-belts.**

Flat belts have a narrow rectangular cross-section, while V-belts have a trapezoidal cross-section.

BELT CONSTRUCTIONS

Belts are made of leather, canvas, rubber or rubberized fabric and synthetic materials.

There are two types of flat belts—leather belt and fabric rubber belt.

The leather belt is made of the best quality leather obtained from either sides of the backbone of a steer.

The fabric rubber belts are made from several layers of canvas or cotton-duck impregnated with rubber

Fabric rubber belts are widely used in engineering industries.

1. There is a specific term ‘ply’ of the belt. In order to make a practical thick belt, the layers of belt material are cemented together as shown in Fig.
2. These layers are called ‘plies’ of belt. Belts are specified according to the number of layers or plies, e.g., single-ply, double-ply or triple-ply belts. The power rating of the belt is also specified per ply of belt, e.g., the power rating of the Dunlop ‘high speed’ belt is 0.0118 kW per ply per mm width of belt.

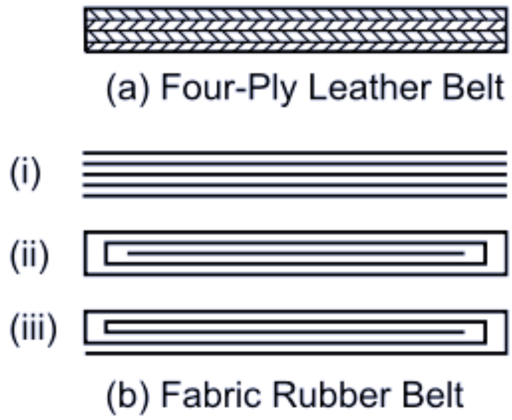


Fig. 13.2 *Flat Belts*

Three types of construction of fabric rubber belts are shown in Fig.13.2.

1. Type (i) is called a raw-edge belt. It consists of plies of fabric cut to the width of the belt.
2. Type (ii) is called layer by layer folded edge belt. It consists of a central ply wrapped around with rectangular plies.
3. Type (iii) is called spirally wrapped folded edge belts. They are made of a single piece of fabric without any rubber layer in between.

Flat belts are produced in the form of long bands and stored in the form of coils. The ends of these belts are joined by methods as

(i) Cemented Joints

(ii) Laced Joint

(iii) Joints with Metal Fasteners

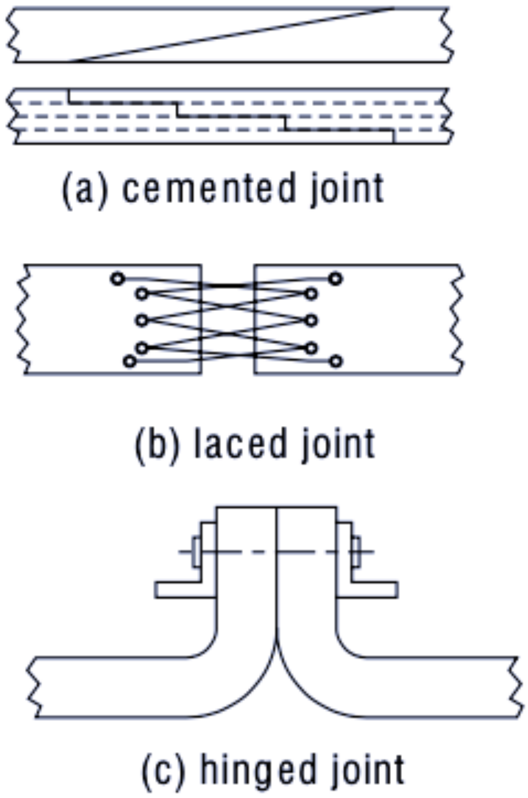


Fig. 13.3 *Flat Belt Joints*

BELT DRIVES

GEOMETRICAL RELATIONSHIPS

There are two types of belt construction—**open and crossed**

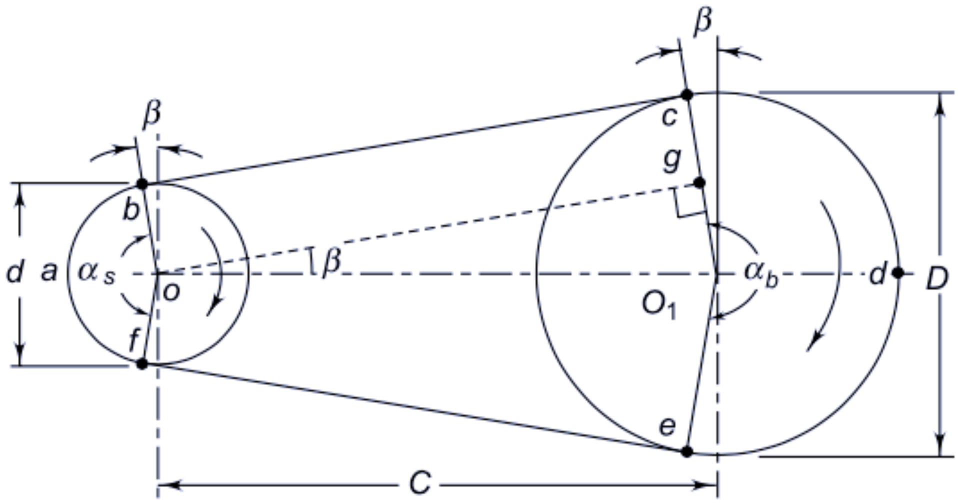


Fig. 13.5 *Open belt drive*

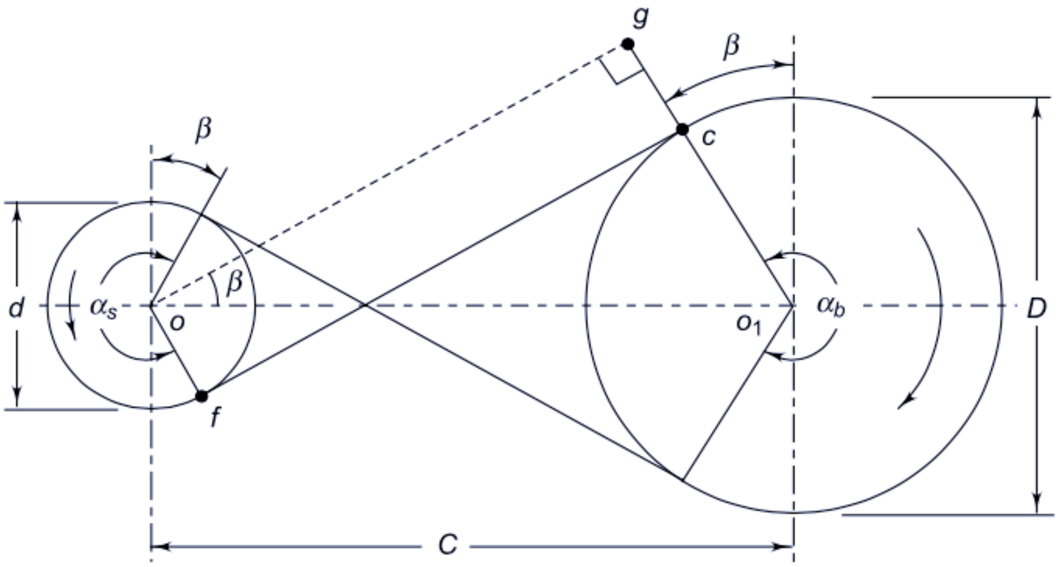
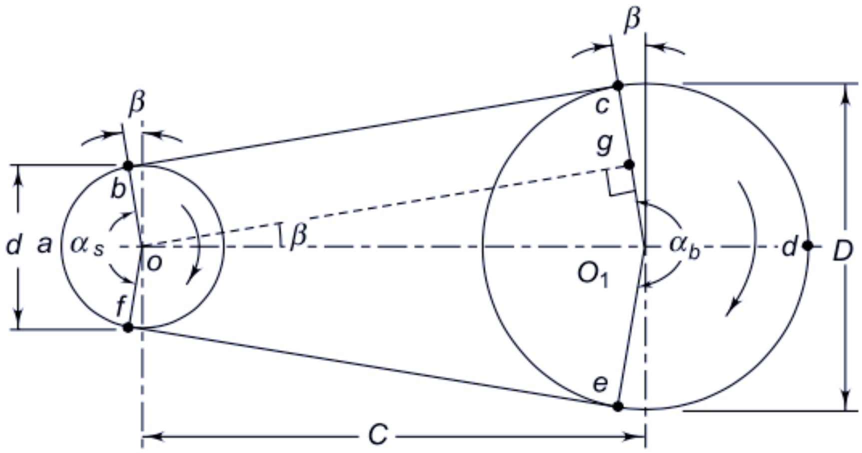


Fig. 13.6 *Crossed Belt Drive*

1. In an open belt drive, both driving and driven pulleys rotate in the same direction. In a crossed belt drive, driving and driven pulleys rotate in the opposite direction.
2. In crossed belt drive, the angle of wrap is more. Therefore, power transmitting capacity of a crossed belt drive is more than that of an open belt drive.
3. In crossed belt drive, the belt rubs against itself while crossing. Also, the belt has to bend in two different planes. These two factors increase the wear and reduce the life of the belt.

4. In open belt drives, when the centre distance is more, the belt whips, i.e., vibrates in a direction perpendicular to the direction of motion. When the centre distance is small, the belt slip increases. Both these factors limit the use of an open belt drive. Crossed belt drives do not have these limitations.

α_s = wrap angle for small pulley (degrees)
α_b = wrap angle for big pulley (degrees)
D = diameter of big pulley (mm)
d = diameter of small pulley (mm)
C = centre distance (mm)



$$\cos\beta = \frac{og}{OO_1}$$

$$og = C\cos\beta$$

Fig. 13.5 *Open belt drive*

From rectangle obcg

$$\boxed{Ob = gc}$$

From Δ^{le} ogO_1

$$\sin\beta = \frac{O_1 g}{OO_1} = \frac{O_1 c - gc}{OO_1}$$

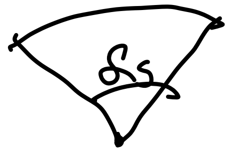
$$\sin\beta = \frac{O_1 c - ob}{OO_1} = \frac{\frac{D}{2} - \frac{d}{2}}{C} = \frac{D-d}{2C}$$

From the figure

$$\alpha_s = 180 - 2\beta$$

$$\alpha_s = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$\delta_b = 180 + 2\beta$$

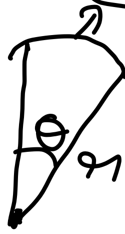


$$\delta_b = 180 + 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

Length of the belt:

$$L = \underline{cb + \text{arc}(baf) + fe + \text{arc}(edc)}$$

$l = r\theta$



$$L = \underline{og} + \frac{d}{2}(\delta_s) + \underline{og} + \frac{D}{2}(\delta_b)$$

$$L = 2C\cos\beta + \frac{d}{2}(\pi - 2\beta) + \frac{D}{2}(\pi + 2\beta)$$

$$L = \frac{\pi}{2}(D+d) + \beta(D-d) + 2C\cos\beta$$

For small values of β, $\underline{\sin\beta = \beta}$

$\sin 0 = 0$

$\sin 0.001 = 0.0013$

$$\beta = \sin\beta = \frac{D-d}{2C}$$

$$\cos\beta = 1 - 2\sin^2\left(\frac{\beta}{2}\right)$$
$$= 1 - 2\left(\frac{\beta}{2}\right)^2$$
$$= 1 - \frac{\beta^2}{2}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= \underline{1 - 2\sin^2\theta}$$

$$\cos\beta = 1 - \frac{(D-d)^2}{4C^2 \times 2} \Rightarrow 1 - \frac{(D-d)^2}{8C^2}$$

$$L = \frac{\pi(D+d)}{2} + \left(\frac{D-d}{2C}\right)(D-d) + 2C\left[1 - \frac{(D-d)^2}{8C^2}\right]$$

$$\boxed{L = 2C + \frac{\pi(D+d)}{2} + \frac{(D-d)^2}{4C}}$$ → Length of open belt

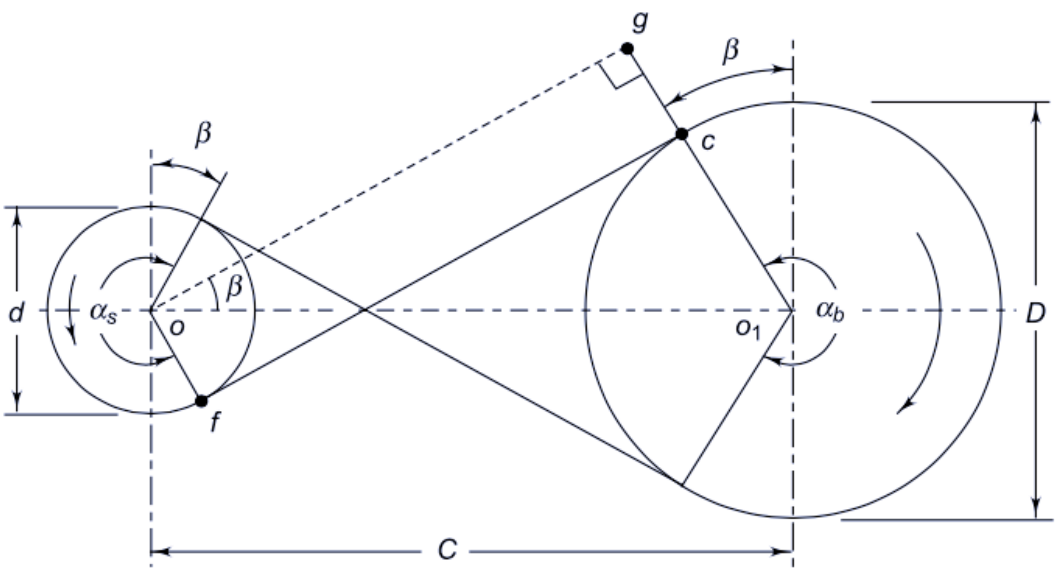


Fig. 13.6 *Crossed Belt Drive*

$$of = gc \qquad \sin\beta = \frac{O_1c + of}{OO_1} = \frac{D+d}{2C}$$

$$\alpha_s = 180 + 2\beta = \alpha_b = 180 + 2\sin^{-1}\left(\frac{D+d}{2c}\right)$$

$$\boxed{L = 2c + \frac{\pi}{2}(D+d) + \frac{(D+d)^2}{4C}}$$

→ Length of crossed belt

ANALYSIS OF BELT TENSIONS

Flat Belts

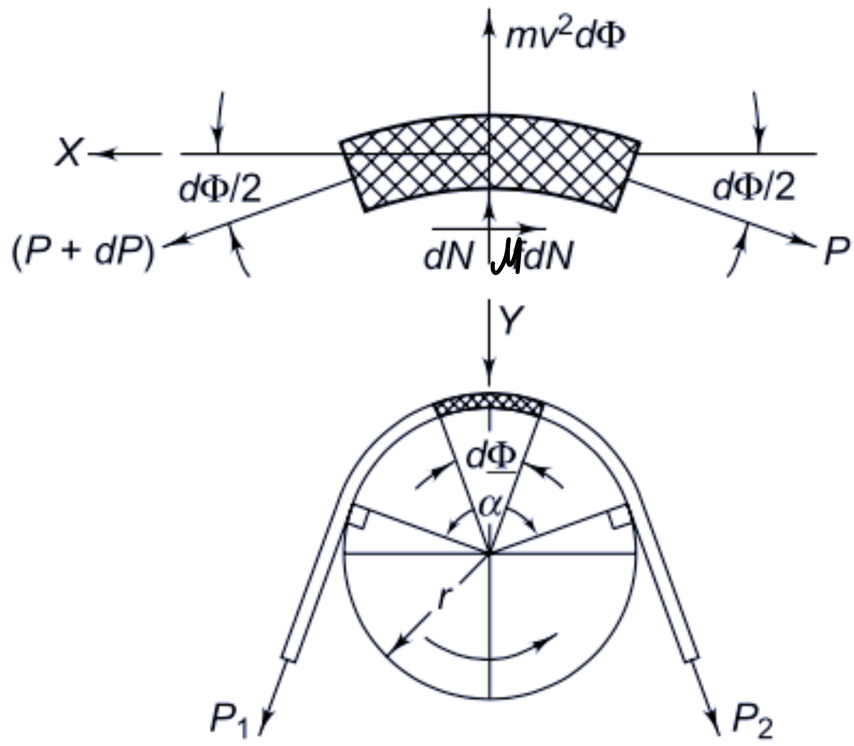


Fig. 13.7 *Forces on Flat Belt*

P_1 = belt tension in the tight side (N)
P_2 = belt tension in the loose side (N)
m = mass of the one meter length of belt (kg/m)
v = belt velocity (m/s)
μ = coefficient of friction
α = angle of wrap for belt (radians)

An element of the belt subtending an angle (df) is in equilibrium under the action of the following forces:

(i) tensions (P) and (P + dP) on the loose and tight sides respectively;

(ii) the normal reaction between the surfaces of the belt and pulley (dN) and the frictional force (μ* dN) at the interface; and

(iii) centrifugal force in radially outward direction.

The length of element is ($rd\phi$)

$m \rightarrow$ mass per unit length

Mass $= M = m \times l$

$$\boxed{M = m \times r\, d\phi}$$

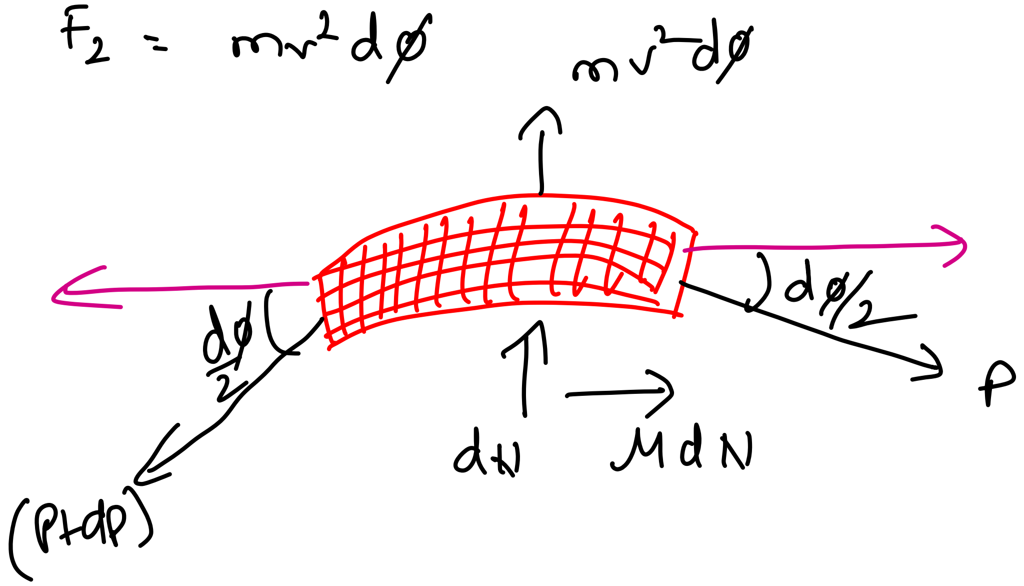
$$a_r = \frac{v^2}{r}$$

Force $= m \times a$

Centrifugal force $= M \times a_r$

$$= m \cancel{r}\, d\phi \times v^2/\cancel{r}$$

$$F_2 = m v^2\, d\phi$$



$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma F_x = 0$

$$(P+dP)\cos\left(\frac{d\phi}{2}\right) - P\cos\left(d\phi/2\right) - \mu\, dN = 0 \quad -(1)$$

$\Sigma F_y = 0$

$$(P+dP)\sin\left(\frac{d\phi}{2}\right) + P\sin\left(d\phi/2\right) - mv^2 d\phi - dN = 0 \quad -(2)$$

For small values of $(d\phi/2)$

$$\cos\left(d\phi/2\right) \simeq 1$$

$$\sin\left(d\phi/2\right) \simeq d\phi/2$$

$$(1) \Rightarrow \cancel{P} + dP - \cancel{P} - \mu dN = 0$$

$$dP = \mu dN$$

$$\boxed{dN = \frac{dP}{\mu}} \rightarrow (3)$$

$$(2) \Rightarrow (P+dP)\left(\frac{d\phi}{2}\right) + P\left(\frac{d\phi}{2}\right) - mv^2 d\phi - dN = 0$$

neglecting $dP \times d\phi$

$$P(d\phi) - mv^2 d\phi - \frac{dP}{\mu} = 0$$

$$d\phi(P - mv^2) = \frac{dP}{\mu}$$

$$\frac{dP}{(P - mv^2)} = \mu d\phi \quad — (4)$$

Integrating (4) with limits $P_2 \rightarrow P_1$

$$\int_{P_2}^{P_1} \frac{dP}{(P - mv^2)} = \int_0^{\delta} \mu d\phi$$

$$\log\left[P - mv^2\right]_{P_2}^{P_1} = \mu(\phi)_0^{\delta}$$

$$\log_e\left(\frac{P_1 - mv^2}{P_2 - mv^2}\right) = \mu\delta$$

$\log a - \log b = \log\left(\frac{a}{b}\right)$

$$\boxed{\frac{P_1 - mv^2}{P_2 - mv^2} = e^{\mu\delta}} \rightarrow$$

Forces experienced by Flat belt

V Belts

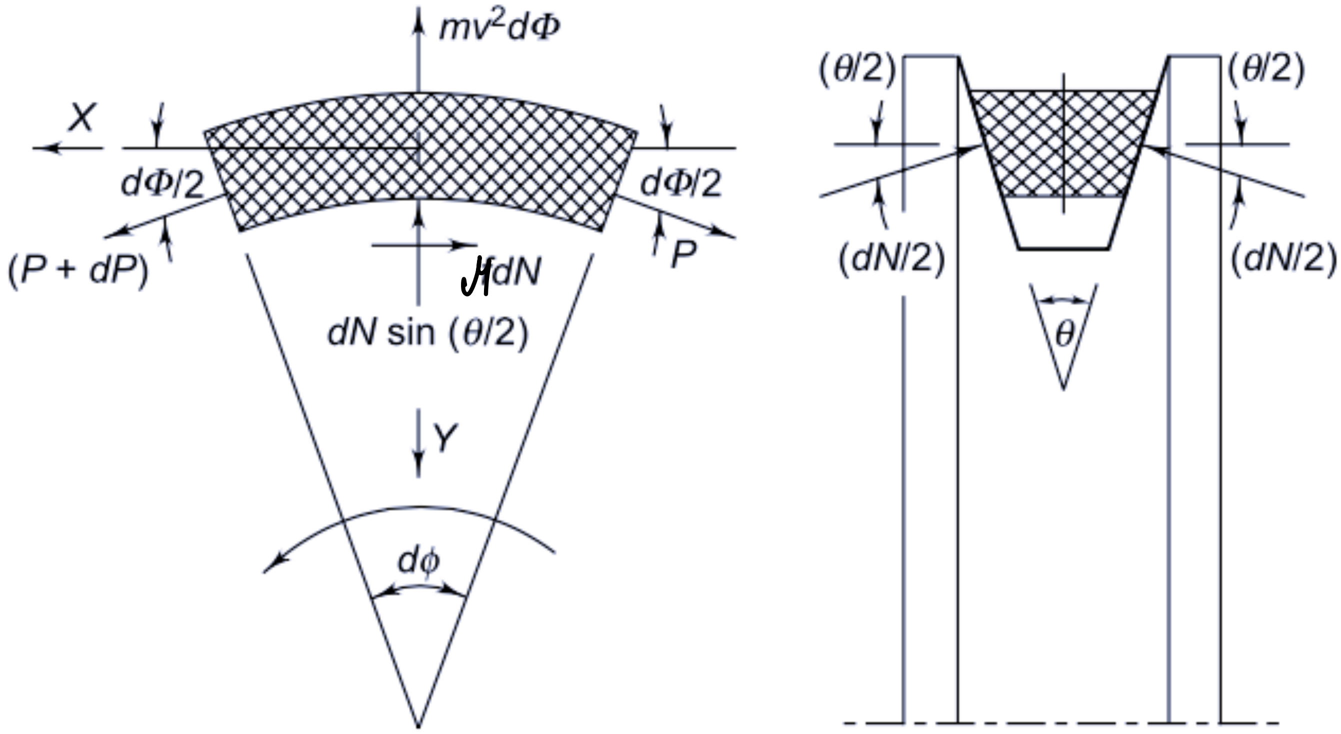


Fig. 13.8 *Forces on V-belt*

(i) The equations of flat and V-belt are identical except that the coefficient of friction μ in flat belt drive is replaced by $\mu/\sin(\theta/2)$ in case of V-belt. In other words, the effective coefficient of friction in V-belt is $[\mu/\sin(\theta/2)$ as compared to $[\mu]$ of flat belt.

(ii) For a V-belt, $\theta = 40°$ or

$[\mu/\sin(\theta/2)] = 2.92\,\mu$

Therefore, for identical materials of belt and pulleys, the coefficient of friction of V-belt is 2.92 times that of flat belt. Consequently, the power-transmitting capacity of V-belt is much more than that of flat belt. Therefore, V-belts are more powerful.

(iii) Due to increased frictional force, the slip is less in V-belt compared with flat belt.

$$(P+dP)\cos\left(\frac{d\phi}{2}\right) - P\cos\left(\frac{d\phi}{2}\right) - \mu\, dN = 0$$

$\cos\frac{d\phi}{2} \simeq 1$

$\sin\frac{d\phi}{2} \simeq \frac{d\phi}{2}$

$$\Rightarrow \boxed{dN = \frac{dP}{\mu}} \quad (5)$$

$$(P+dP)\left(\sin\frac{d\phi}{2}\right) + P\sin\left(\frac{d\phi}{2}\right) - mv^2 d\phi - dN\sin\frac{\theta}{2} = 0$$

$$Pd\phi - mv^2 d\phi - dN\sin\theta/2 = 0$$

$$(P - mv^2)\,d\phi - \frac{dP}{\mu}\sin(\theta/2) = 0$$

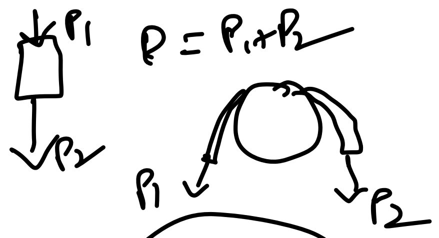
$$\frac{dP}{P-mv^2} = \frac{\mu\, d\phi}{\sin(\theta/2)} \quad \text{—— (6)}$$

Integrating eqn (6)

$$\int_{P_2}^{P_1} \frac{dP}{P-mv^2} = \frac{\mu}{\sin(\theta/2)}\int_0^{\alpha} d\phi$$

$$\log\left(\frac{P_1-mv^2}{P_2-mv^2}\right) = \frac{\mu\alpha}{\sin\theta/2}$$

$$\boxed{\frac{P_1-mv^2}{P_2-mv^2} = e^{\left(\frac{\mu\alpha}{\sin\theta/2}\right)}}$$



$$\frac{2\pi NT}{60}$$

Power transmitted = Force × velocity

$$\boxed{KW = \frac{(P_1-P_2)\times V}{1000}}$$

$$\text{Power} = \frac{\text{w.d}}{\text{time}}$$

$$\text{Power} = \frac{\text{Force}\times\text{dist}}{\text{time}}$$

CONDITION FOR MAXIMUM POWER

When the belt passes over the pulley, the centrifugal force due to its own weight tends to lift the belt from the surface of the pulley.

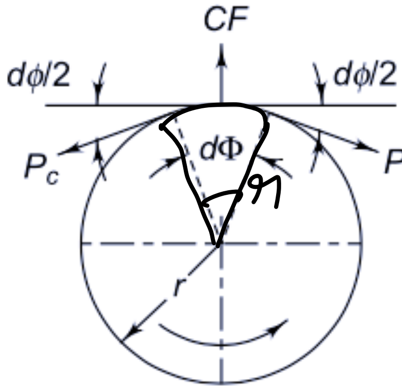


Fig. 13.9

$\underline{\text{Mass } (M)} = m \times l$

$= m r \, d\phi$

$a_r = v^2/r$

For small values of $d\phi/2$

$\sin\left(\frac{d\phi}{2}\right) = d\phi/2$

Force Centrifugal = mass × centrifugal acceleration

$$C \cdot F = m r \, d\phi \times v^2/r = m v^2 d\phi$$

$$C \cdot F = m v^2 d\phi = 2P_c \sin(d\phi/2)$$

The centrifugal force CF induces belt tension P_C.

By symmetry, the centrifugal force induces equal tension on two sides of belt

$$m v^2 \cancel{d\phi} = \cancel{2} P_c \times \cancel{d\phi}/\cancel{2}$$

$$\boxed{P_c = m v^2} \rightarrow (1)$$

$$kW = \frac{(P_1 - P_2) v}{1000}$$

Let b → width of the belt (mm)

t → thickness of the belt (mm)

σ → max. permissible stress (N/mm²)

Max permissible load $= \sigma \times (b \times t)$

$$\boxed{P_{max} = bt\sigma}$$

$P_1 + P_c \leq P_{max}$; $P_1 + P_c = P_{max}$

$$\underline{P_1 = P_{max} - P_c} \quad - (2)$$

$$\frac{P_1}{P_2} = e^{\mu\theta} \qquad P_2 = \frac{P_1}{e^{\mu\theta}}$$

$(P_1 - P_2)v =$ power

$$\left(P_1 - \frac{P_1}{e^{\mu\theta}}\right)v = P_1\left(1 - \frac{1}{e^{\mu\theta}}\right)v$$

Power $= P_1 v \times K$

$= (P_{max} - P_c)\, v \times K$

power $= (P_{max} - mv^2)\, v \times K$

Power $= (P_{max} \times v - mv^3) \times K$

$\frac{dP_o}{dv} = 0 \rightarrow$ then power is max

$$\frac{d}{dv}(P_{max} v - mv^3) = 0$$

$$P_{max} - \underline{3mv^2} = 0$$

$$\boxed{P_{max} = 3mv^2}$$

$$\boxed{P_{max} = 3P_c}$$

$$P_{max} = 3mv^2 \Rightarrow \boxed{v = \sqrt{\frac{P_{max}}{3m}}}$$

$P_1 + P_c \leq P_{max}$; Limiting condition

$$P_1 + P_c = P_{max}$$

$$P_1 + P_c = 3P_c$$

$$\boxed{P_1 = 2P_c}$$

CHARACTERISTICS OF BELT DRIVES

There is a peculiar phenomenon in the belt drive, which is called ‘creep’. Creep is a slight relative motion of the belt as it passes over the pulley.

1. While moving from tight to loose side over the pulley, the belt element is transferred from the zone of higher tension to the zone of lower tension.
2. As the tension in the belt is reduced, the belt becomes shortened and creeps along the surface of the pulley. This causes relative motion between the belt and pulley surface.

Creep results in a decrease in the angular velocity of the driven pulley from that calculated by considering the ratio of diameters of pulleys. The efficiency of the belt drive is reduced by 1 to 2 per cent as a result of creep.

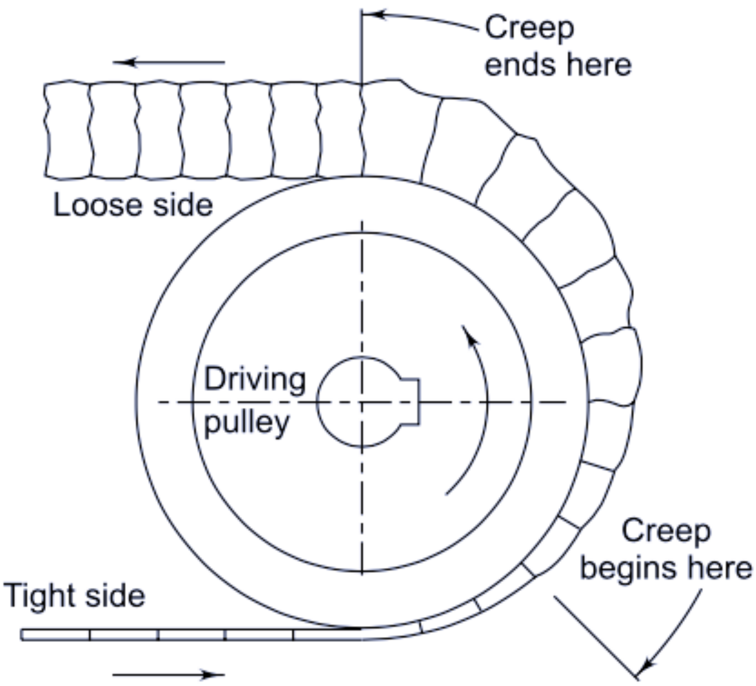


Fig. 13.10 *Belt Creep*

- Both creep and slip lower the expected velocity of the driven member. However, there is a basic difference between the creep and slip.
- Slip is caused by overloads and in this case, the belt slides over the entire arc of contact on the pulley.

The losses in V-belt drives are comparatively more than those of in flat belt drives, because of increased internal friction and creep on the pulleys.

For medium service conditions, the efficiency of flat-belt drives is 96% while the efficiency of V-belt is 95%.

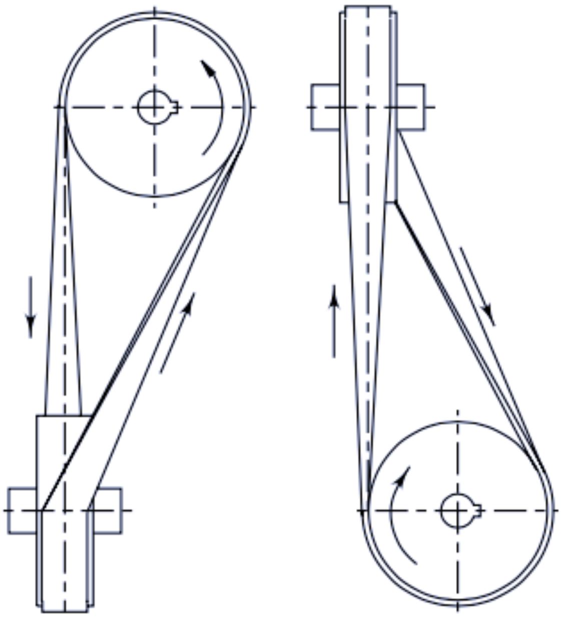


Fig. 13.12 *Quarter-turn Belt Drive*

The **law of belting** states —'The centreline of the belt when it approaches a pulley must lie in the midplane of that pulley'. However, a belt leaving a pulley may be drawn out of the midplane of that pulley.

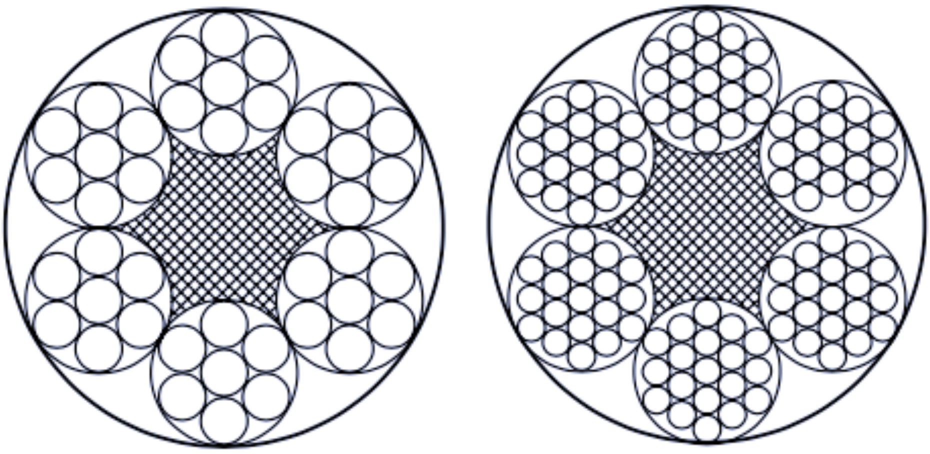
Therefore, for non-parallel shafts, motion is possible only in one direction. Otherwise, the belt is thrown off the pulley.

ROPE DRIVES

Wire ropes are extensively used in hoisting, haulage and material handling equipment. They are also used in stationary applications such as guy wires and stays. The advantages of wire ropes are as follows:

(i) high strength to weight ratio;
(ii) silent operation even at high velocities; and
(iii) greater reliability.

The wire rope consists of a number of strands, each strand comprising several steel wires. The number of wires in each strand is generally 7, 19 or 37, while the number of strands is usually six. The individual wires are first twisted into the strand and then the strands are twisted around a fibre or steel core.



(a) 6×7 (6/1) (b) 6×19 (12/6/1)

Fig. 23.3 *Construction of Wire Rope*

The specification of wire ropes includes two numbers, such as 6 x 7 or 6 x 19. The first number indicates the number of strands in the wire rope, while the second gives the number of steel wires in each strand. The popular constructions of steel wire ropes are as follows:

$$6 \times 7\ (6/1)$$
$$6 \times 19\ (12/6/1)$$

Table 23.3 *Breaking load and mass for 6 × 7 (6/1) construction wire ropes*

Nominal diameter (mm) (d_r)	Approximate mass (kg/100 m)		Minimum breaking load corresponding to tensile designation of wires of (kN)					
			1570		1770		1960	
	Fibre core	Steel core	Fibre core	Steel core	Fibre core	Steel core	Fibre core	Steel core
8	22.9	25.2	33	36	38	41	42	45
9	28.9	31.8	42	46	48	51	53	57
10	35.7	39.1	52	56	59	64	65	70
11	43.2	47.6	63	68	71	77	79	85
12	51.5	56.6	75	81	85	91	94	101

In these tables, the nominal diameter (dr) of the wire rope indicates the diameter of the smallest circle enclosing the wire rope.

The tensile designation of wires, such as 1570 or 1770, indicates2the minimum ultimate tensile strength (in N/mm) of the individual wires used for making the wire rope.

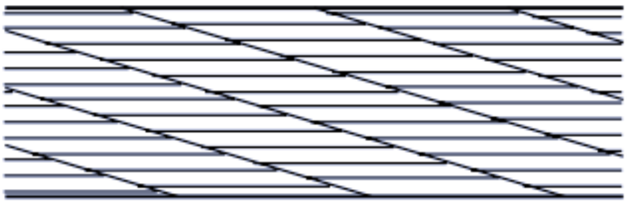
The central portion of the wire rope is called the core.

There are three types of cores—fibre, wire and synthetic material.

- The fibre core consists of natural fibres like sisal, hemp, jute or cotton. The fibre core is flexible and suitable for all conditions except when the rope is subjected to severe crushing, e.g., when working under high load.
- The steel core consists of another strand of fairly soft wires with lower tensile strength.
- The wire core is used where the wire rope is subjected to severe heat or crushing conditions. Plastic cores are used in special purpose wire ropes. It can be a plastic-impregnated fibre core, plastic covered fibre core or a solid plastic core.

There is one more term related to the construction of wire ropes, namely, rope-lay. The lay of the rope refers to the manner in which the wires are helically laid into strands and the strands into the rope.

1. If the wires in the strand are twisted in the same direction as the strands, then the rope is called a Lang's lay rope.
2. When the wires in the strand are twisted in a direction opposite to that of the strands, the rope is said to be regular-lay or ordinary-lay.


(a) Regular lay

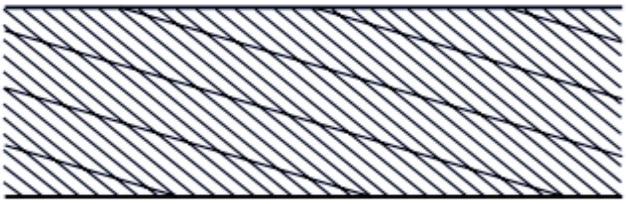
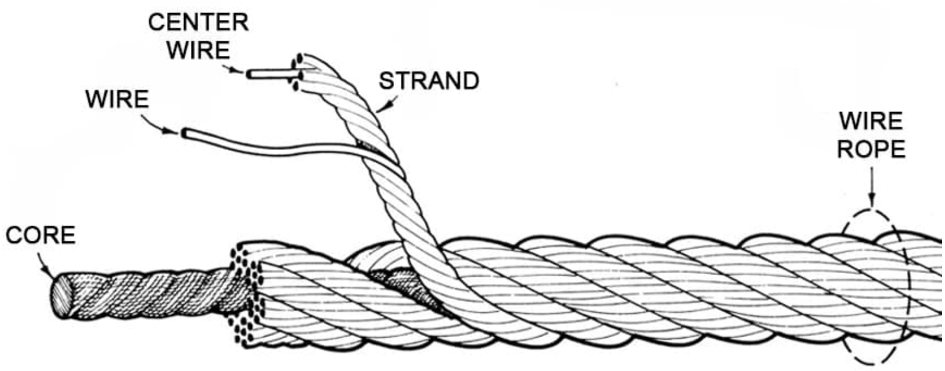
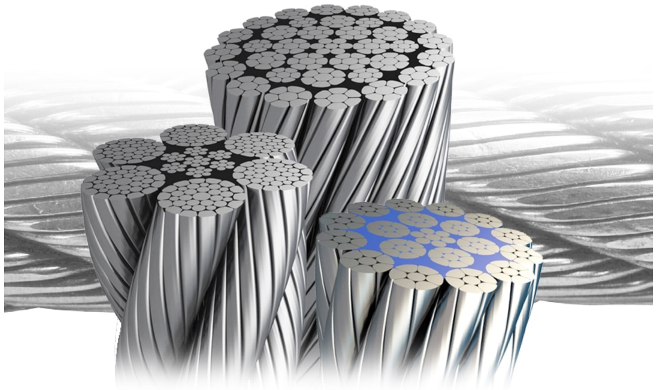

(b) Lang's lay

Fig. 23.4 *Lays of Wire Rope*





STRESSES IN WIRE ROPES

The analysis of stresses in wire rope is complicated, owing to a number of factors.

The individual wires are subjected to direct tensile stress due to the load being raised, as well as to bending stresses.

When the wire rope passes around the periphery of the sheave or the drum, the length of the wires in the outer portion of the rope increases, while that in the inner region decreases. This results in additional tensile stresses in outer wires.

$$\sigma_b = \frac{E d_w}{D}$$

$$\sigma_b = \frac{M_b y}{I} \quad \text{and} \quad y = \frac{d_w}{2}$$

$$\sigma_b = \frac{E_r d_w}{D}$$

Elasticity of the rope

Therefore,

$$\sigma_b = \frac{M_b d_w}{2I} \qquad \text{(a)}$$

where,

d_w = diameter of individual wire (mm)

$$\boxed{P_b = \sigma_b \times A}$$

The elastic-curve equation is given by,

$$\frac{M_b}{EI} = \frac{1}{r}$$

The radius of curvature r in the above equation is equal to the radius of the sheave. Therefore,

$$\frac{M_b}{EI} = \frac{2}{D} \qquad \text{(b)}$$

where,

D = diameter of the sheave (mm)

The failure of the wire rope is mainly due to fatigue or wear, while passing around the sheave. The bending and straightening of the rope as it passes over the sheave results in fluctuating stresses leading to fatigue failure. The individual wires slide on each other and over the sheave resulting in gradual wearing of the load carrying material.

Table 23.7 *Wire-rope data*

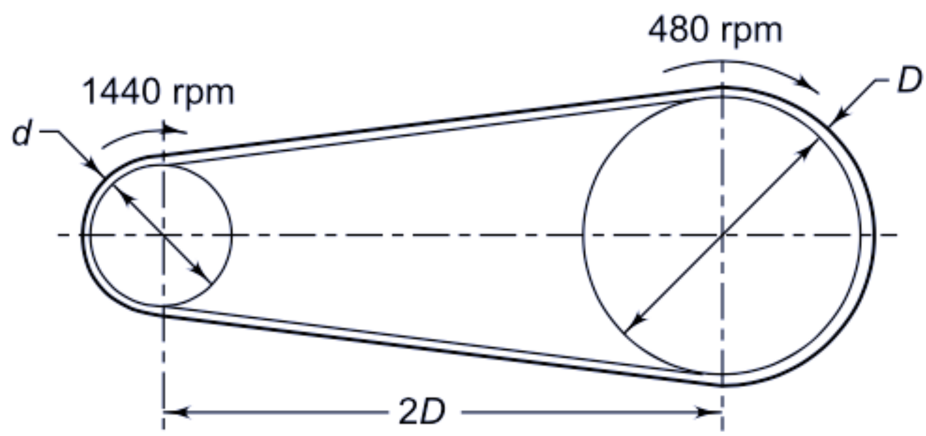
Type of Construction	*Modulus of elasticity of rope (E_r) (N/mm²)*	*Diameter of wire (d_w) (mm)*	*Metallic area of rope (A) (mm²)*	*Sheave diameter (D) (mm)*	
				Minimum	*Recommended*
6 × 7	97 000	0.106 d_r	0.38 d_r^2	42 d_r	72 d_r
6 × 19	83 000	0.063 d_r	0.40 d_r^2	30 d_r	45 d_r
6 × 37	76 000	0.045 d_r	0.40 d_r^2	18 d_r	27 d_r

Problem: The layout of a leather belt drive transmitting 15 kW of power is shown in Fig. The centre distance between the pulleys is twice the diameter of the bigger pulley. The belt should operate at a velocity of 20 m/s approximately and the stresses in the belt should not exceed 2.25 N/mm². The density of leather is 0.95 g/cc and the coefficient of friction is 0.35. The thickness of the belt is 5 mm. Calculate:

(i) the diameter of pulleys;

(ii) the length and width of the belt; and

(iii) the belt tensions.



Given data:

kW = 15 kW $V = 20\, ms^{-1}$ $C = 2D$ $t = 5\,mm$

$\rho = 0.95\, gm/cc$ $\sigma_{max} = 2.25\, N/mm^2$ $\mu = 0.35$

$P = (P_1 - P_2)V$ kW

1) Dia of pulley:

$$v = \frac{\pi d N}{60} \Rightarrow 20 = \frac{\pi \times d \times 1440}{60}$$

$$d = \frac{20 \times 60}{\pi \times 1440} = 0.265\,m = 265\,mm$$

$$\frac{N_2}{N_1} = \frac{D_1}{D_2}$$

$$d = 270\,mm$$

$$d \times N_1 = D \times N_2$$

$$270 \times 1440 = D \times 480$$

$$D = 810\,mm$$

2) Belt length:

$$C = 2D$$
$$= 1620\,mm$$

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$L = 2 \times 1620 + \frac{\pi}{2}(810+270) + \frac{(810-270)^2}{4 \times 1620}$$

$$L = 4981.46\,mm$$

3) Belt width & belt tension:

$v = 20\,ms^{-1}$

$$v = \frac{\pi d n}{60 \times 1000} = \frac{\pi \times 270 \times 1440}{60 \times 1000} = 20.36\,ms^{-1}$$

α_s = Angle of wrap on small pulley

$$\frac{P_1 - mv^2}{P_2 - mv^2} = e^{\mu\alpha}$$

$$\alpha_s = 180 - 2\beta$$

$$= 180 - 2\sin^{-1}\left(\frac{D-d}{2c}\right)$$

$$\alpha_s = 160.8^\circ \times \pi/180 = 2.81 \text{ rad}$$

mass of leather belt

$$m = \rho \times \text{Volume}$$

Volume = (length) x (width) x thickness

$$\text{mass} = (0.95) \times (100)\left(\frac{b}{10}\right)\left(\frac{5}{10}\right) g/m$$

$$= (4.75)\, b \;\; g/m$$

$$1g = 10^{-3} kg$$

$$= (4.75 \times 10^{-3})\, b \;\; kg/m$$

$$mv^2 = (4.75 \times 10^{-3})\, b \times (20.36)^2 = 1.97\, b$$

$$e^{\mu\alpha} = e^{(0.35)(2.81)} = 2.67$$

$$t = 5$$

$$b = b$$

$$\frac{P_1 - 1.97b}{P_2 - 1.97b} = 2.67$$

$$11.25b - 2.67(11.25b - 736.8) + 3.29b = 0$$

$$P_1 - 2.67P_2 + 3.29b = 0 \quad -(1)$$

$$\sigma_{max} = \frac{\text{max Tension}}{\text{Area}} = \frac{P_1}{A}$$

$$P_1 = \sigma A$$

$$= (2.25)(5b)$$

$$P_1 = (11.25b)\, N \quad -(2)$$

Power $\Rightarrow (P_1 - P_2)V \Rightarrow P_1 - P_2 = \dfrac{15 \times 10^3}{20.36} = 736.8$ N — (3)

Solving these three equation?

$b = 130$ mm

$\boxed{P_1 = 1462.5 \text{ N} \ \& \ P_2 = 693 \text{ N}}$

$P_1 - P_2 = 736.8$

$P_1 = 11.25 b$ — (2)

$11.25 b - P_2 = 736.8$

$P_2 = (11.25)b - 736.6$ — (3)

Problem: A temporary elevator is assembled at the construction site to raise building materials, such as cement, to a height of 20 m. It is estimated that the maximum weight of the material to be raised is 5 kN. It is observed that the acceleration in such applications is 1 m/s^2. 10 mm diameter, (6 x 19) construction wire ropes with fibre core are used for this application. The tensile designation of the wire is 1570 and the factor of safety should be 10 for preliminary calculations. Determine the number of wire ropes required for this application. Neglect bending stresses.

Table 23.4 *Breaking load and mass for 6 × 19 (12/6/1) construction wire ropes with fibre core*

Nominal diameter (d_r) (mm)	Approximate mass (kg/100 m)		Minimum breaking load corresponding to tensile designation of (kN)					
			1570		1770		1960	
	Fibre core	Steel core	Fibre core	Steel core	Fibre core	Steel core	Fibre core	Steel core
8	22.1	24.3	31	33	35	37.6	39	41.6
9	28.0	30.8	39	42	44	47.5	49	52.6
10	34.6	38.0	48	52	54	58.7	60	65
11	41.9	46	58	63	66	71.0	73	70.7
12	49.8	54	69	75	78	84.6	87	93.6
13	58.5	64.3	82	88	92	99	102	110
14	67.8	74.5	95	102	107	115	118	127
16	88.6	97.4	124	133	139	150	154	166
18	112	123.0	156	160	176	190	195	210
19	125	137	174	188	196	212	217	234
20	138	152.0	193	208	218	235	241	260
22	167	184.0	234	252	263	204	292	314
24	199	219.0	278	300	318	338	347	375
26	234	257	326	352	368	397	407	439

Given data: $W = 5$ kN ; $h = 20$m $a = 1$ m/s^2

Rope → (6 x 19)

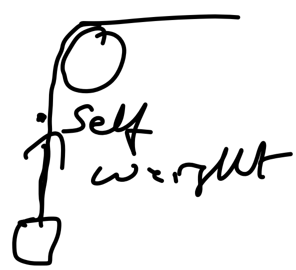
$d_r = 10$ mm tensile designation = 1570. fos = 10

To find z ⇒ no. of wire ropes required

1) weight of material to be lifted

2) weight of individual rope

3) Force due to acceleration



$F = ma$

1) weight of material:

Load to be lifted = 5000 N

Load on individual wire rope $= \left(\frac{5000}{Z}\right)$ N

2) weight of individual wire:

From table;

For $d_r = 10\,mm$ $\frac{mass}{100m} = 34.6$ kg

For 20m length mass $= (34.6) \times 0.2$ kg

weight of rope $= (34.6) \times 0.2 \times 9.81$ N

$= 67.88$ N

3) Force due to acceleration:

mass of material raised $= \frac{5000}{Z} \times \frac{1}{9.81}$

mass of rope $= 34.6 \times 0.2$

Total mass $= \left(\frac{509.68}{Z} + 6.92\right)$ kg

$$\text{Force} = \left(\frac{509.68}{z} + 6.92\right)(1)$$

FOS = 10

From table min load for $d_r = 10mm$

$= 48 \text{ KN}$

$$\frac{48\times10^3}{10} = \left(\frac{5000}{z}\right) + (67.89) + \left[\frac{509.68}{z} + 6.92\right]$$

$\underline{z} = 1.16$

no. of ropes

$\boxed{z = 2 \text{ wire ropes}}$

wear & tear

The amount of wear that occurs depends upon the pressure between the rope and the sheave

The force per unit length of the wire rope $= p \cdot d_r$

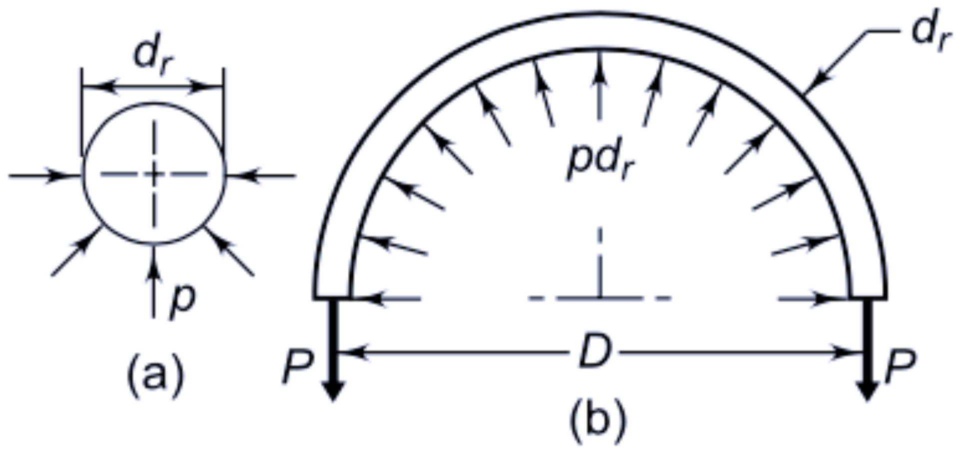


Fig. 23.5 *Forces Acting on Wire Rope around Sheave*

$$2P = p \times d_r \times D$$

$$\boxed{p = \frac{2P}{d_r \cdot D}}$$

Tension

where,

P = tension in the rope (N)

d_r = nominal diameter of wire rope (mm)

D = sheave diameter (mm)

Say σ_{ut} = 700 MPa

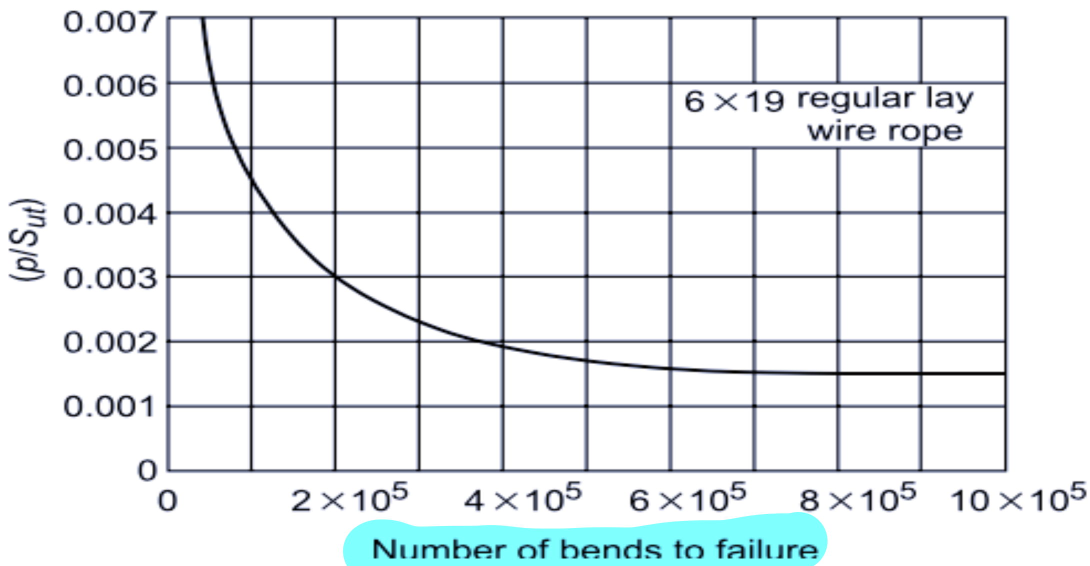


Fig. 23.6 *Relationship for Number of Bends to Failure (Experimental Data)*

Problem: A (6 x 19) wire rope with fibre core and tensile designation of 1570 is used to raise the load of 20 kN as shown in Fig. 23.7. The nominal diameter of the wire rope is 12 mm and the sheave has 500 mm pitch diameter. Determine the expected life of the rope assuming 500 bends per week.

Given data:

tensile desig = 1570 ; W = 20 kN

$d_r = 12$ mm ; D = 500 mm

500 bends/week

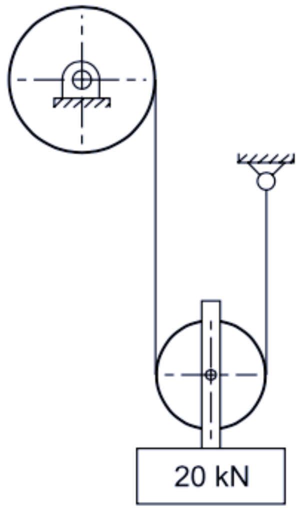


Fig. 23.7

$$\frac{p}{\sigma_{ut}} =$$

$$P = 10\,kN$$

$$p = \frac{2 \times P}{d_r \times D} = \frac{2 \times 10 \times 10^3}{12 \times 500}$$

$$p = 3.33\ N/mm^2$$

$$\sigma_{ut} = 1570\ N/mm^2$$

$$\frac{p}{\sigma_{ut}} = \frac{3.33}{1570} = 0.0021$$

From the curve, for a value of $\frac{p}{\sigma_{ut}} = 0.0021$,

the no. of bends $= 3.3 \times 10^5$

For 1 week = 500 bends

For 1 year $= 500 \times 52$

Life of rope $= \dfrac{3.3 \times 10^5}{500 \times 52}$

Life = 12.69 years

CHAIN DRIVES

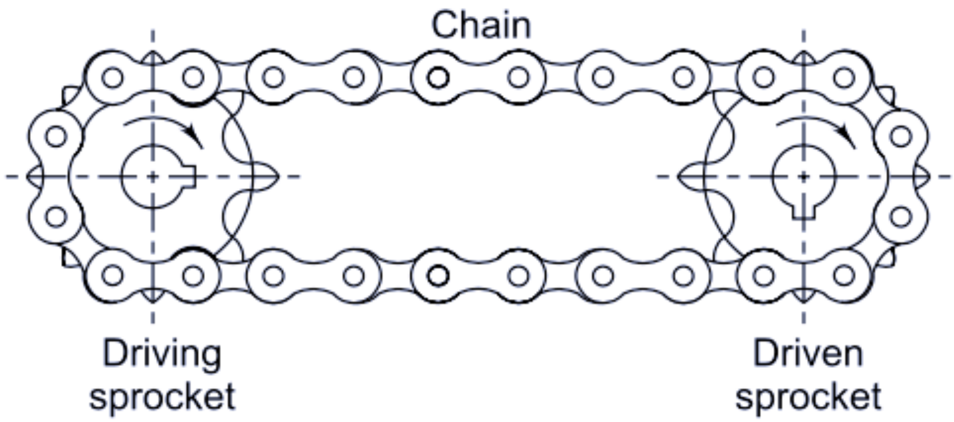


Fig. 14.1 *Chain Drives*

A chain drive consists of an endless chain wrapped around two sprockets as shown in Fig

- A chain can be defined as a series of links connected by pin joints.
- The sprocket is a toothed wheel with a special profile for the teeth. The chain drive is intermediate between belt and gear drives.
- The advantages of chain drives compared with belt and gear drives are as follows:

- Chain drives can be used for long as well as short centre distances. They are particularly suitable for medium centre distance, where gear drives will require additional idler gears. Thus, chain drives can be used over a wide range of centre distances.
- Chain drives have small overall dimensions than belt drives, resulting in compact unit. A chain does not slip and to that extent, chain drive is a positive drive.
- The efficiency of chain drives is high. For properly lubricated chain, the efficiency of chain drive is from 96% to 98%.
  Chain does not require initial tension. Therefore, the forces acting on shafts are reduced.
- Chains are easy to replace.

The disadvantages of chain drives are as follows:

- Chain drives operate without full lubricant film between the joints unlike gears. This results in more wear at the joints. The wear increases the pitch of the chain. The chain is stretched out and may leave the sprocket, if tension is not adjusted from time to time.
- Chain drives are not suitable for non-parallel shafts. Bevel and worm gears and quarter- turn belt drives can be used for non-parallel shafts.
- Chain drive is unsuitable where precise motion is required due to polygonal effect. The velocity of the chain is not constant resulting in non-uniform speed of the driven shaft.
- Chain drives require housing.

Chain drives are popular in the transportation industry, such as bicycle, motorcycle and automobile vehicle. They are used in metal and wood working machinery for the transmission of power.

They are widely used in agricultural machinery, oil-well drilling rigs, building construction and materials handling equipment.

Chain drives are used for velocity ratios less than 10 : 1 and chain velocities of up to 25 m/s. In general, they are recommended to transmit power up to 100 kW.

There are different types of chains. With respect to their purpose, chains are classified into the following three groups:

(i) Load lifting chains
(ii) Hauling chains
(iii) Power transmission chains

Load lifting chains are used for suspending, raising or lowering loads in materials handling equipment. The popular example of this category is a ‘link’ chain

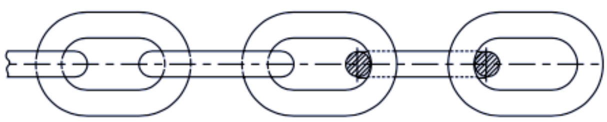


Fig. 14.3 *Link Chain*

Hauling chains are used for carrying materials continuously by sliding, pulling or carrying in conveyors. The popular example of this category is a ‘block’ chain

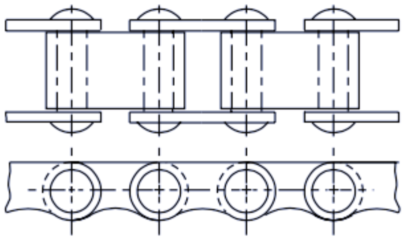


Fig. 14.4 *Block Chain*

ROLLER CHAINS

A roller chain consists of following five parts:

1. Pin
2. Bushing
3. Roller
4. Inner link plate
5. Outer link plate

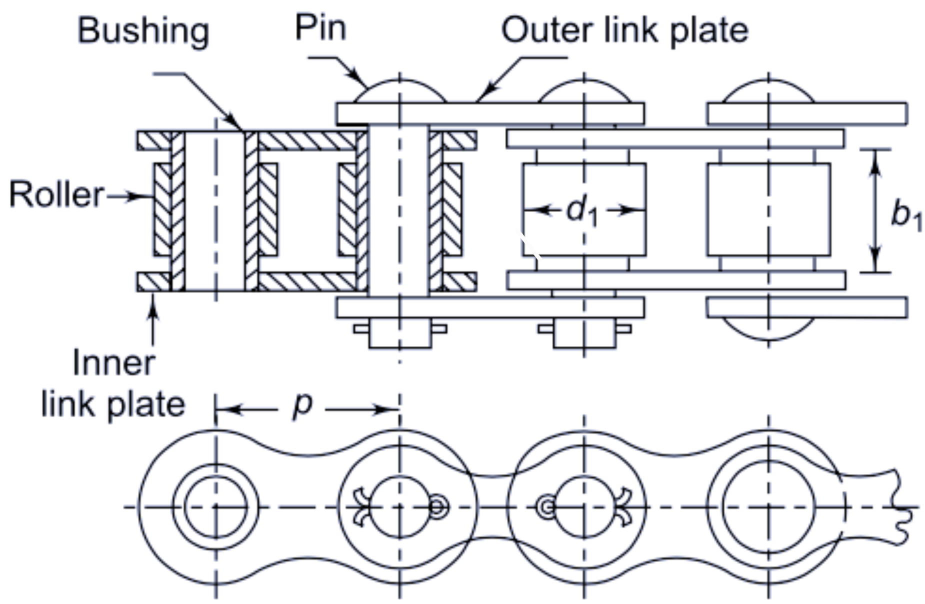


Fig. 14.5 *Construction of Roller Chain*

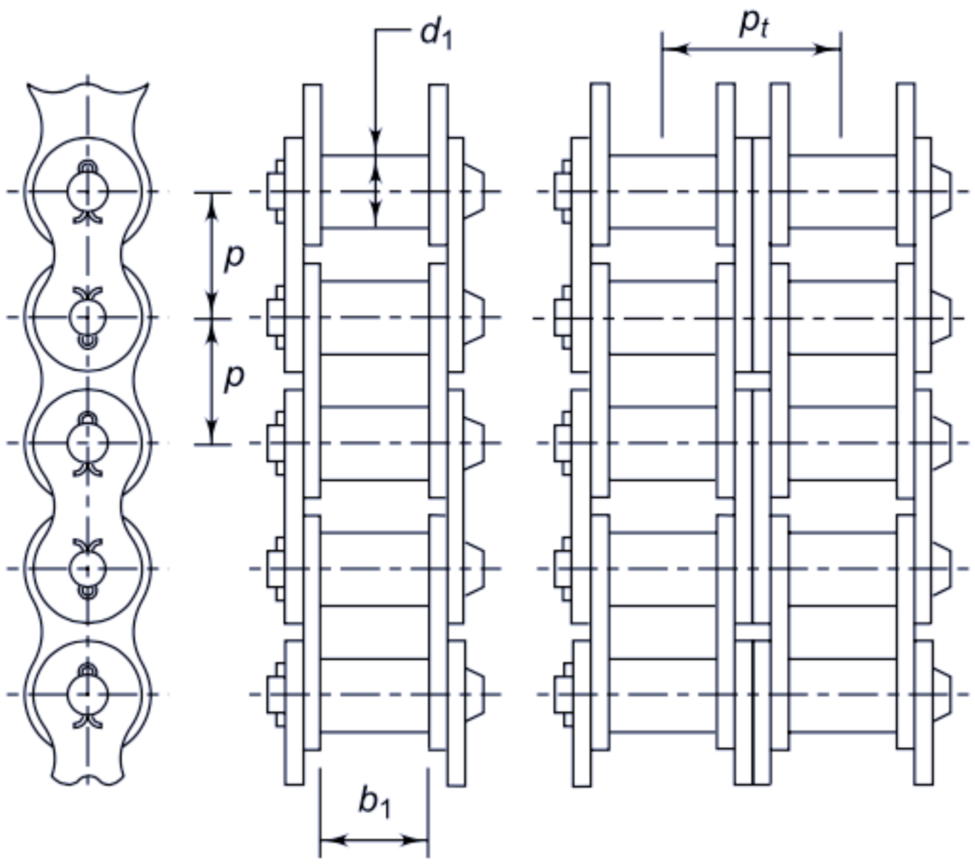
The pin is press fitted to two outer link plates, while the bush is press fitted to inner link plates. The bush and the pin form a swivel joint and the outer link is free to swivel with respect to the inner link.

The rollers are freely fitted on bushes and, during engagement, turn with the teeth of the sprocket wheels.

This results in rolling friction instead of sliding friction between roller and sprocket teeth. The rolling friction reduces wear and frictional power loss and improves the efficiency of the chain drive.

The pitch (p) of the chain is the linear distance between the axes of adjacent rollers.

These chains are available in single- strand or multi-strand constructions such as simple, duplex or triplex chains



Simple Chain Duplex Chain

Designations:

Pitch of chain

08B-2 or 16A-1

Standards

No. of Strands

$$\text{Pitch} = \frac{08}{16}\text{ inch}$$

$$= \frac{08}{16} \times 25.4\text{ mm}$$

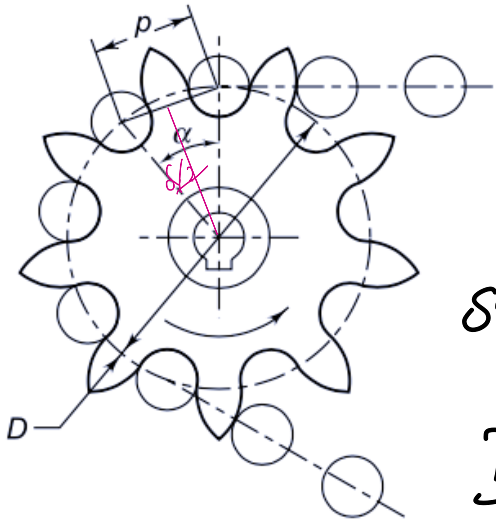
$$= 12.7\text{ mm}$$

$$\frac{16}{16}\text{ inch} = 1\text{ inch} = 25.4\text{mm}$$

Table 14.1 *Dimensions and breaking loads of roller chains*

ISO chain number	*Pitch p (mm)*	*Roller diameter d_1 (mm) (max.)*	*Width b_1 (mm) (min.)*	*Transverse pitch p_t (mm)*	*Breaking load (min) N*		
					Simple	*Duplex*	*Triplex*
05B	8.00	5.00	3.00	5.64	4 400	7 800	11 100
06B	9.525	6.35	5.72	10.24	8 900	16 900	24 900
08A (ANSI-40)	12.70	7.95	7.85	14.38	13 800	27 600	41 400
08B	12.70	8.51	7.75	13.92	17 800	31 100	44 500
10A (ANSI-50)	15.875	10.16	9.4	18.11	21 800	43 600	65 400
10B	15.875	10.16	9.65	16.59	22 200	44 500	66 700
12A (ANSI-60)	19.05	11.91	12.57	22.78	31 100	62 300	93 400
12B	19.05	12.07	11.68	19.46	28 900	57 800	86 700
16A (ANSI-80)	25.40	15.88	15.75	29.29	55 600	111 200	166 800
16B	25.40	15.88	17.02	31.88	42 300	84 500	126 800
20A(ANSI-100)	31.75	19.05	18.90	35.76	86 700	173 500	260 200
20B	31.75	19.05	19.56	36.45	64 500	129 000	193 500
24A (ANSI-120)	38.10	22.23	25.22	45.44	124 600	249 100	373 700
24B	38.10	25.40	25.40	48.36	97 900	195 700	293 600
28A(ANSI-140)	44.45	25.40	25.22	48.87	169 000	338 100	507 100
28B	44.45	27.94	30.99	59.56	129 000	258 000	387 000
32A(ANSI-160)	50.80	28.58	31.55	58.55	222 400	444 800	667 200
32B	50.80	29.21	30.99	58.55	169 000	338 100	507 100
40A(ANSI-200)	63.50	39.68	37.85	71.55	347 000	693 900	1040 900
40B	63.50	39.37	38.10	72.29	262 400	524 900	787 300
48A	76.20	47.63	47.35	87.83	500 400	1000 800	1501 300
48B	76.20	48.26	45.72	91.21	400 300	800 700	1201 000
64B	101.60	63.50	60.96	119.89	711 700	1423 400	—

GEOMETRIC RELATIONSHIPS



$$\delta = \frac{360}{Z}$$

$$\sin\left(\frac{\delta}{2}\right) = \frac{p/2}{D/2}$$

$$D = \frac{p}{\sin(\delta/2)}$$

D is the pitch circle diameter of the sprocket and δ is called the pitch angle. The pitch circle diameter of the sprocket is defined as the diameter of an imaginary circle that passes through the centres of link pins as the chain is wrapped on the sprocket.

$Z \rightarrow$ no. of sprokets

$$D = \frac{p}{\sin\left(\frac{180}{Z}\right)}$$

$$i = \frac{n_1}{n_2} = \frac{Z_2}{Z_1}$$

$$v = \frac{\pi D N}{60 \times 10^3}$$

$$\downarrow$$

$$D = \frac{p}{180/Z}$$

$$\frac{1000}{200} = 5$$

$$v = \frac{\pi \times \frac{Zp}{\pi} \times N}{60 \times 10^3}$$

$$D = \frac{Zp}{180}$$

Smaller value of $\theta \rightarrow \sin\theta = \theta$

$$v = \frac{ZpN}{60 \times 10^3}$$

$$= \frac{Zp}{\pi}$$

$$\boxed{L = L_n \times p}$$

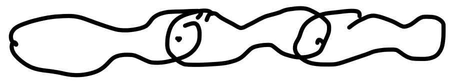
$L_n \rightarrow$ no. of links in chain

$p \rightarrow$ pitch

It is always preferred to have an ‘even’ number of links, since the chain consists of alternate pairs of inner and outer link plates.

When the chain has an odd number of links, an additional link, called ‘offset’ link, is provided. The offset link is, however, weaker than the main links.

Length of chain is expressed in terms of no. of links



$L = L_n \times P$

$L_n \rightarrow$ no. of links in chain

$P \rightarrow$ pitch in mm

$$L_n = 2\left(\frac{a}{P}\right) + \left(\frac{Z_1 + Z_2}{2}\right) + \left(\frac{Z_2 - Z_1}{2\pi}\right)^2 \times \left(\frac{P}{a}\right)$$

$Z_1 \rightarrow$ no. of teeth on smaller sprocket

$a \rightarrow$ centre dist b/n axes of driving & driven wheel

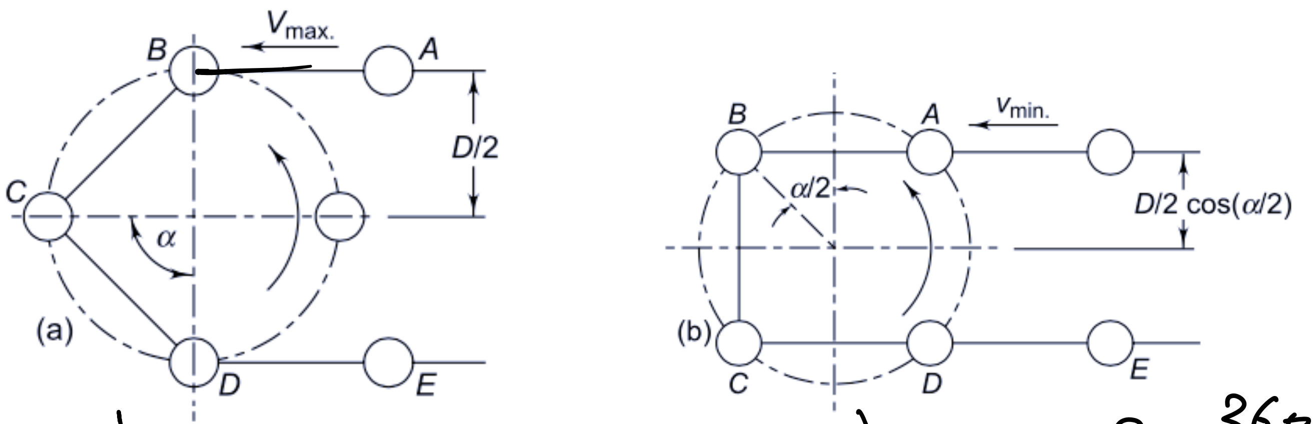
$Z_2 \rightarrow$ on larger sprocket

Corrected centre distance:

$$a = \frac{p}{4}\left\{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right] + \sqrt{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right]^2 - 8\left[\frac{z_2-z_1}{2\pi}\right]^2}\right\}$$

POLYGONAL EFFECT

The chain passes around the sprocket as a series of chordal links. This action is similar to that of a non- slipping belt wrapped around a rotating polygon.



$\delta = \frac{360}{Z}$

Linear velocity $V_{max} = \frac{\pi D n}{60 \times 10^3}$ m/s

$V_{min} = \frac{\pi D \cos(\delta/2) \times n}{60 \times 10^3}$

$(V_{max} - V_{min}) \propto \left[1 - \cos\left(\frac{\delta}{2}\right)\right]$

$(V_{max} - V_{min}) \propto \left[1 - \cos\left(\frac{180}{Z}\right)\right]$

It is evident that the linear speed of the chain is not uniform but varies from $v_{max.}$ to $v_{min.}$ during every cycle of tooth engagement. This results in a pulsating and jerky motion.

$Z = 11 \rightarrow 4\%$

$Z = 17 \rightarrow 1.6\%$

$Z = 24 \rightarrow < 1\%$ 0.8%

$V_{min} = 0.99 \times V_{max}$

$Z \geq 17$ teeth

19 to 21

For smooth operation at moderate and high speeds, it is considered a good practice to use a driving sprocket with at least 17 teeth. From durability and noise considerations, the minimum number of teeth on the driving sprocket should be 19 or 21.

POWER RATING OF ROLLER CHAINS

$$\text{kW} = \frac{P_1 v}{1000}$$

where

P_1 = allowable tension in the chain (N)

v = average velocity of chain (m/s)

Table 14.2 *Power rating of simple roller chain*

Pinion speed (rpm)	*Power (kW)*								
	06 B	*08A*	*08 B*	*10A*	*10 B*	*12A*	*12 B*	*16A*	*16 B*
50	0.14	0.28	0.34	0.53	0.64	0.94	1.07	2.06	2.59
100	0.25	0.53	0.64	0.98	1.18	1.74	2.01	4.03	4.83
200	0.47	0.98	1.18	1.83	2.19	3.40	3.75	7.34	8.94
300	0.61	1.34	1.70	2.68	3.15	4.56	5.43	11.63	13.06
500	1.09	2.24	2.72	4.34	5.01	7.69	8.53	16.99	20.57
700	1.48	2.95	3.66	5.91	6.71	10.73	11.63	23.26	27.73
1000	2.03	3.94	5.09	8.05	8.97	14.32	15.65	28.63	34.89
1400	2.73	5.28	6.81	11.18	11.67	14.32	18.15	18.49	38.47
1800	3.44	6.98	8.10	8.05	13.03	10.44	19.85	—	—
2000	3.80	6.26	8.67	7.16	13.49	8.50	20.57	—	—

For a given application, the kW rating of the chain is determined by the following relationship:

$$\text{kW rating of chain} = \frac{(\text{kW to be transmitted}) \times K_s}{K_1 \times K_2}$$

where

Ks = service factor
K1 = multiple strand factor
K2 = tooth correction factor

Table 14.3 *Service factor (K_s)*

Type of driven load	*Type of input power*		
	IC engine with hydraulic drive	*Electric motor*	*IC engine with mechanical drive*
(i) *Smooth:* agitator, fan, light conveyor	1.0	1.0	1.2
(ii) *Moderate shock:* machine tools, crane, heavy conveyor, food mixer, grinder	1.2	1.3	1.4
(iii) *Heavy shock:* punch press, hammer mill, reciprocating conveyor, rolling mill drive	1.4	1.4	1.7

Table 14.4 *Multiple strand factor (K_1)*

Number of strands	K_1
1	1.0
2	1.7
3	2.5
4	3.3
5	3.9
6	4.6

Table 14.5 *Tooth correction factor (K_2)*

Number of teeth on the driving sprocket	K_2
15	0.85
16	0.92
17	1.00
18	1.05
19	1.11
20	1.18
21	1.26
22	1.29
23	1.35
24	1.41
25	1.46
30	1.73

For a satisfactory performance of roller chains, the centre distance between the sprockets should provide at least a 120° wrap angle on the smaller sprocket. In practice, the recommended centre distance is between 30 to 50 chain pitches.

Therefore, $30p < a < 50p$

The expected service life of these chains is 15,000 hours. The velocity ratio should be kept below 6 : 1 to get a satisfactory performance.

Problem: A single-strand chain No. 12A is used in a mechanical drive. The driving sprocket has 17 teeth and rotates at 1000 rpm. What is the factor of safety used for standard power rating? Neglect centrifugal force acting on the chain.

Given data: pitch $p = \frac{12}{16}$ inch $= \frac{12}{16} \times 25.4 = 19.05$ mm

$Z_1 = 17$ $n = 1000$ rpm $p = 19.05$ mm

$v = \frac{\pi D N}{60}$

velocity $V = \frac{Z_1 p n_1}{60 \times 10^3} = \frac{17 \times 19.05 \times 1000}{60 \times 1000}$

$V = \frac{17 \times 19.05}{60}$ $V = 5.4\ ms^{-1}$

From table 14.2; the power rating for 12A @ 1000 rpm $= 14.32$ kW

$14.32 \times 10^3 = P_1 \times V \rightarrow 5.4$

$P_1 = 2651.85\ N$

$1\ kgf = 9.81\ N$

$P_1 = 2651.85\ N$

Breaking load (or) ultimate load = 3200 kgf

$= 3200 \times 9.81\ N$

$= 31,392\ N$

31,100

$$FOS = \frac{31,392}{2651.85} = 11.83 / 11.73$$

Problem: It is required to design a chain drive to connect 5 kW, 1400 rpm electric motor to a drilling machine. The speed reduction is 3 : 1. The centre distance should be approximately 500 mm.

(i) Select a proper roller chain for the drive.

(ii) Determine the number of chain links.

(iii) Specify the correct centre distance between the axes of sprockets.

Given data:

Power = 5 kw a = 500 mm n_1 = 1400 rpm

velocity ratio i = 3

$Z \geq 17$

1) Power rating of the chain:

19 (or) 21

Teeth in sprocket = 21

Assuming the chain is simple.

$$\text{Power (or) kw rating} = \frac{(\text{Kw to be transmitted}) \times K_s}{K_1 \times K_2}$$

From table 14.3, Service factor for moderate

Shock $K_s = 1.3$

$K_1 = 1.0$ → for single strand

$K_2 = 1.26$

$$(\text{Kw rating}) = \frac{5 \times 1.3}{1 \times 1.26} = 5.15 \text{ Kw}$$

From table 14.2 (Power rating) for 5.15 kW & 1400 RPM The chain chosen is "08A"

No. of chain links:

$$L_n = 2\left(\frac{a}{p}\right) + \left(\frac{z_1+z_2}{2}\right) + \left(\frac{z_2-z_1}{2\pi}\right)^2 \times \left(\frac{p}{a}\right)$$

$$L_n = 2\left(\frac{500}{12.7}\right) + \left(\frac{21+63}{2}\right) + \left(\frac{63-21}{2\pi}\right)^2 \times \left(\frac{12.7}{500}\right)$$

$$L_n = 121.87$$

$$L_n = 122 \text{ links}$$

$a = 500$ mm

$P = \frac{08}{16} \times 25.4$ mm

$= 12.7$ mm

$z_1 = 21$ teeth

$z_2 = 63$ teeth

$i = \frac{z_2}{z_1} \Rightarrow \frac{z_2}{21} = 3$

Correct Centre distance:

$$a = \frac{P}{4}\left\{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right] + \sqrt{\left[L_n - \left(\frac{z_1+z_2}{2}\right)\right]^2 - 8\left(\frac{z_2-z_1}{2\pi}\right)^2}\right\}$$

$$L_n - \left(\frac{z_1+z_2}{2}\right) = 122 - 42 = 80$$

$$a = \frac{12.7}{4}\left\{(80) + \sqrt{(80)^2 - 8\left(\frac{21}{\pi}\right)^2}\right\}$$

$$\boxed{a = 500.8 \text{ mm}}$$

SPRINGS

A spring is defined as an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed.

The important functions and applications of springs are as follows:

1. (i) Springs are used to absorb shocks and vibrations, e.g., vehicle suspension springs, railway buffer springs, buffer springs in elevators and vibration mounts for machinery.
2. (ii) Springs are used to store energy, e.g., springs used in clocks, toys, movie-cameras, circuit breakers and starters.
3. (iii) Springs are used to measure force, e.g., springs used in weighing balances and scales.
4. (iv) Springs are used to apply force and control motion. There are a number of springs used for this purpose. In the cam and follower mechanism, spring is used to maintain contact between the two elements. In engine valve mechanism, spring is used to return the rocker arm to its normal position when the disturbing force is removed. The spring used in clutch provides the required force to engage the clutch.

TYPES OF SPRINGS

Springs are classified according to their shape. The shape can be a helical coil of a wire, a piece of stamping or a flat wound-up strip. The most popular type of spring is the helical spring.

There are two basic types of helical springs—compression spring and extension spring

The external force acts along the axis of the spring and induces torsional shear stress in the spring wire. It should be noted that although the spring is under compression, the wire of helical compression spring is not subjected to compressive stress.

The words ‘compression’ and ‘extension’ are related to the total spring and not the stresses in spring wire.

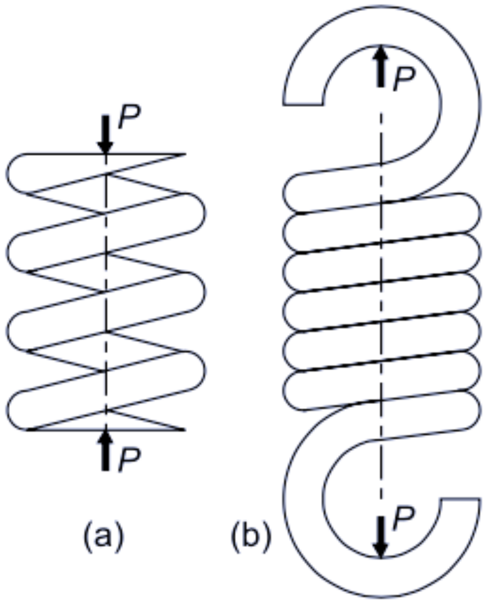


Fig. 10.1 *Helical Springs: (a) Compression Spring (b) Extension Spring*

The helical springs are sometimes classified as closely-coiled helical spring and open-coiled helical spring. The difference between them is as follows:

(i) A helical spring is said to be closely coiled spring, when the spring wire is coiled so close that the plane containing each coil is almost at right angles to the axis of the helix. In other words, the helix angle is very small. It is usually less than 10°.

(ii) A helical spring is said to be open-coiled spring, when the spring wire is coiled in such a way, that there is large gap between adjacent coils. In other words, the helix angle is large. It is usually more than 10°.

Helical springs, compression as well as extension, have the following advantages:

(i) They are easy to manufacture.
(ii) They are cheaper than other types of springs.
(iii) Their reliability is high.
(iv) The deflection of the spring is linearly proportional to the force acting on the spring.

A helical torsion spring is shown in Fig.

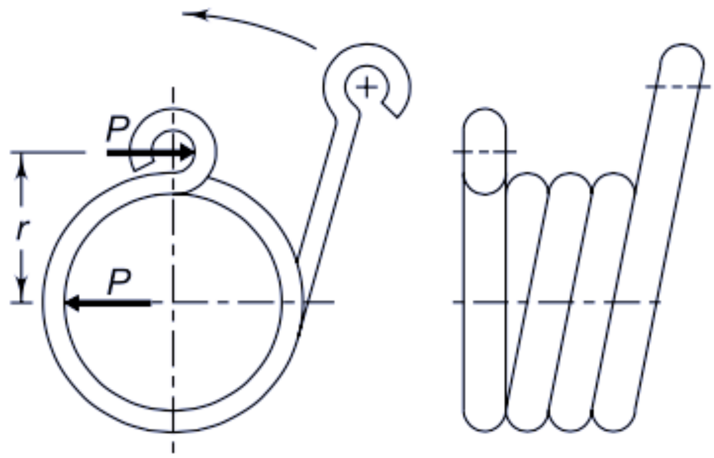


Fig. 10.2 *Helical Torsion Spring*

The ends are formed in such a way that the spring is loaded by a torque about the axis of the coils. Helical torsion spring is used to transmit torque to a particular component in the machine or the mechanism.

Helical torsion spring is used in door-hinges, brush holders, automobile starters and door locks. The helical torsion spring resists the bending moment $(P \times r)$, which tends to wind up the spring. The bending moment induces bending stresses in the spring wire.

A *multi-leaf* or *laminated spring* consists of a series of flat plates, usually of semi-elliptical shape

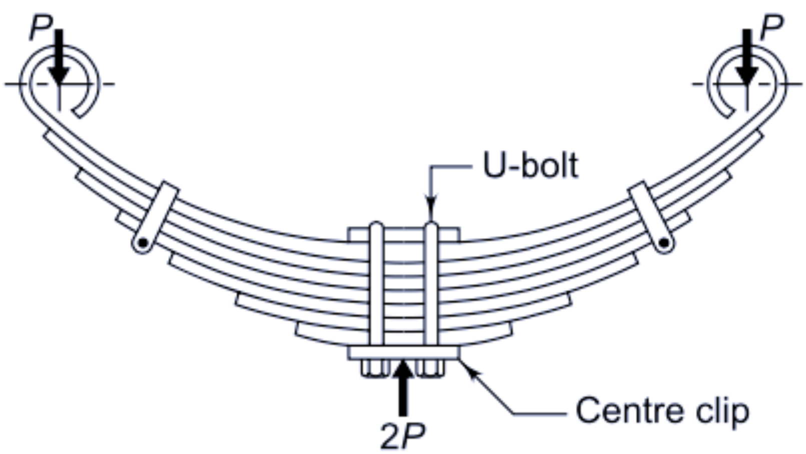


Fig. 10.3 *Semi-elliptic Leaf Spring*

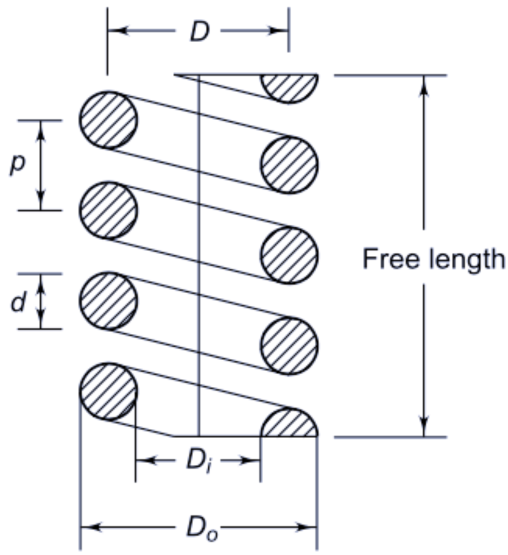
The leaves are held together by means of U-bolts and a centre clip.

The longest leaf, called the master leaf, is bent at the two ends to form spring eyes. The leaves of multi-leaf spring are subjected to bending stresses. Multi-leaf springs are widely used in automobile and railroad suspensions.

Other Types include:

- helical springs of rectangular cross-section
- spiral torsion springs
- disk or belleville springs and
- volute springs.

TERMINOLOGY OF HELICAL SPRINGS



d = wire diameter of spring (mm)

$$D = \frac{D_o + D_i}{2}$$

D_i = inside diameter of spring coil (mm)

D_o = outside diameter of spring coil (mm)

D = mean coil diameter (mm)

There is an important parameter in spring design called *spring index*. It is denoted by the letter *C*. *The spring index is defined as the ratio of mean coil diameter to wire diameter*

$C = D/d$

The spring index indicates the relative sharpness of the curvature of the coil.

1. A low spring index means high sharpness of curvature.
2. When the spring index is low ($C < 3$), the actual stresses in the wire are excessive due to curvature effect.
3. Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires.

I. When the spring index is high ($C > 15$), it results in large variation in the coil diameter.

II. Such a spring is prone to buckling and also tangles easily during handling.

A spring index from 4 to 12 is considered best from manufacturing considerations.

$C = 6$ to 9

There are three terms—*free length, compressed length* and *solid length*,

(i) Solid Length Solid length is defined as the axial length of the spring which is so compressed that the adjacent coils touch each other. In this case, the spring is completely compressed and no further compression is possible. The solid length is given by,

Solid length = N_t*d where,

N_t = total number of coils

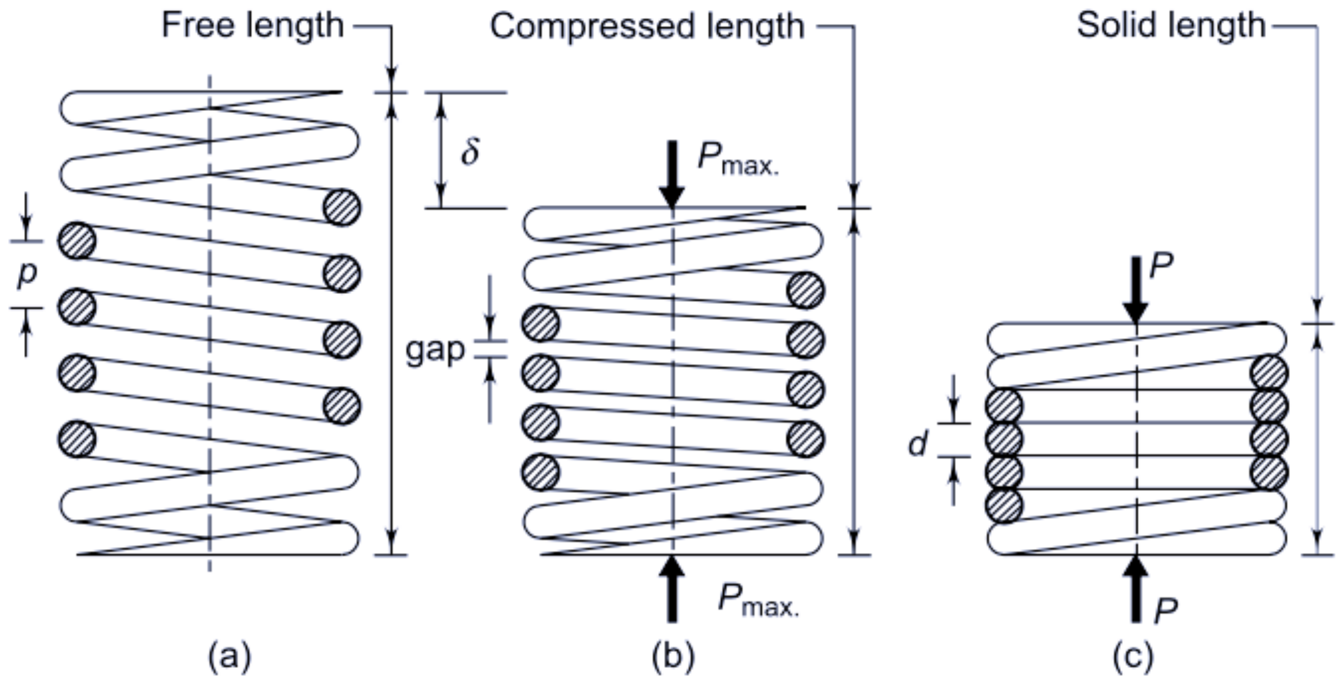
(ii) Compressed Length Compressed length is defined as the axial length of the spring, which is subjected to maximum compressive force. In this case, the spring is subjected to maximum deflection δ. When the spring is subjected to maximum force, there should be some gap or clearance between the adjacent coils. The gap is essential to prevent clashing of the coils. The clashing allowance or the total axial gap is usually taken as 15% of the maximum deflection. Sometimes, an arbitrary decision is taken and it is assumed that there is a gap of 1 or 2 mm between adjacent coils under maximum load condition. In this case, the total axial gap is given by,

Total gap = (*Nt* – 1) * Gap between adjacent coils

(iii) Free Length Free length is defined as the axial length of an unloaded helical compression spring. In this case, no external force acts on the spring. Free length is an important dimension in spring design and manufacture. It is the length of the spring in free condition prior to assembly. Free length is given by,

free length = compressed length + δ

= solid length + total axial gap + δ



Pitch : The *pitch of the coil* is defined as the axial distance between adjacent coils in uncompressed state of spring. It is denoted by *p*. It is given by,

$$p = \frac{\text{free length}}{(N_t - 1)}$$

The *stiffness of the spring* (*k*) is defined as the force required to produce unit deflection.

$$K = P/\delta \; (N/mm)$$

There are various names for stiffness of spring such as rate of spring, gradient of spring, scale of spring or simply spring constant. The stiffness of spring represents the slope of the load-deflection line.

There are two terms related to the spring coils, viz., active coils and inactive coils. *Active coils* are the coils in the spring which contribute to spring action, support the external force and deflect under the action of force. A portion of the end coils, which is in contact with the seat, does not contribute to spring action and are called *inactive coils*. These coils do not support the load and do not deflect under the action of an external force. The number of inactive coils is given by,

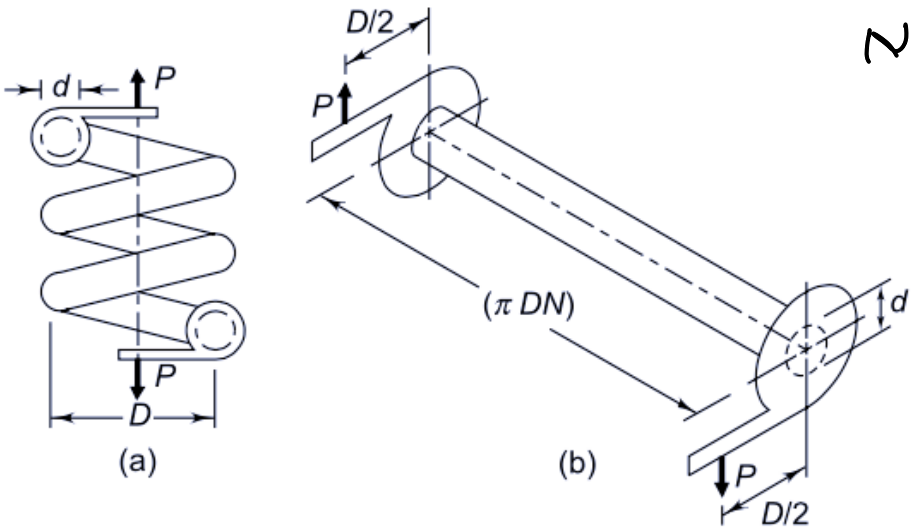
$$\text{inactive coils} = Nt - N$$

$N \rightarrow$ active coils

STRESS AND DEFLECTION EQUATIONS

There are two basic equations for the design of helical springs, viz., load-stress equation and load-deflection equation

- A helical spring made from the wire of circular cross-section
- ***D*** and ***d*** are the mean coil diameter and wire diameter respectively.
- The number of active coils in this spring is *N*.
- The spring is subjected to an axial force *P*.
- When the wire of the helical spring is uncoiled and straightened, it takes the shape of a bar



N, P

Fig. 10.8 *(a) Helical Spring (b) Helical Spring-unbent*

$$T = \frac{\pi}{16} \tau d^3$$

(i) The diameter of the bar is equal to the wire diameter of the spring (d).

(ii) The length of one coil in the spring is (πD). There are N such active coils. Therefore, the length of equivalent bar is (πDN).

(iii) The bar is fitted with a bracket at each end. The length of this bracket is equal to mean coil radius of the spring ($D/2$).

Torsional Shear stress only occur

$$M_t = \frac{PD}{2} \quad \Big| \quad \tau_1 = \frac{16 M_t}{\pi d^3} = \frac{16 \times \left(\frac{PD}{2}\right)}{\pi d^3}$$

$\tau_1 = \frac{8PD}{\pi d^3}$

$C \rightarrow$ spring index

$C = D/d$ $D \rightarrow$ mean coil dia

$\boxed{\tau_1 = \frac{8PC}{\pi d^2}}$

When the equivalent bar is bent in the form of helical coil, there are additional stresses on account of following two factors:

(i) There is direct or transverse shear stress in the spring wire.

(ii) When the bar is bent in the form of coil, the length of the inside fibre is less than the length of the outside fibre. This results in stress concentration at the inside fibre of the coil.

The resultant stress consists of superimposition of torsional shear stress, direct shear stress and additional stresses due to the curvature of the coil.

K_S = factor to account for direct shear stress

K_c = factor to account for stress concentration due to curvature effect

The combined effect of these two factors is

$$K = K_S * K_c$$

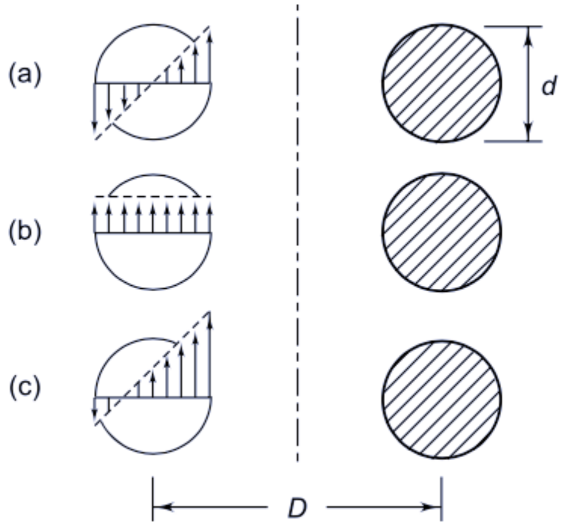


Fig. 10.9 *Stresses in Spring Wire: (a) Pure Torsional Stress (b) Direct Shear Stress (c) Combined Torsional, Direct and Curvature Shear Stresses*

Direct shear stress $\tau_2 = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d^2}$

$$\tau_2 = \frac{4P}{\pi d^2} = \frac{4P}{\pi d^2} \times \frac{2}{2} \times \left(\frac{d}{D}\right) \times \left(\frac{D}{d}\right)$$

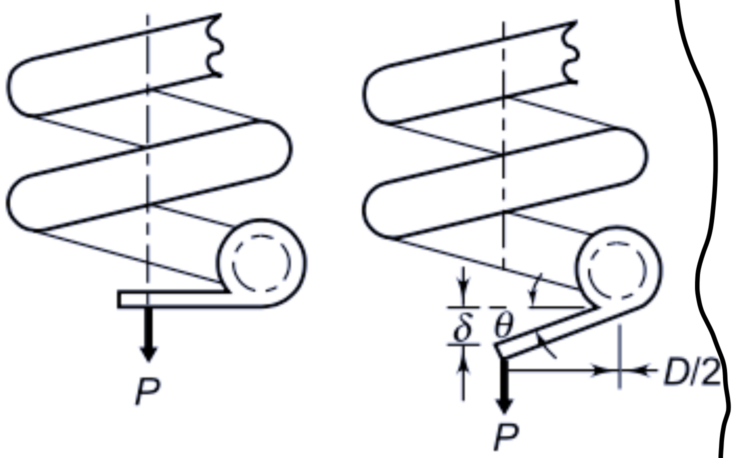
$$\tau_2 = \frac{8PD}{\pi d^3} \left[\frac{0.5d}{D}\right]$$

Total shear stress

$$\tau = \tau_1 + \tau_2 = \frac{8PD}{\pi d^3}\left[1 + \frac{0.5d}{D}\right]$$

$$\tau = \frac{8PD}{\pi d^3}\left[1 + \frac{0.5}{C}\right]$$

$C = D/d$



If θ is small

$\sin\theta = \theta$

$\sin\theta = \frac{\delta}{D/2}$

$\delta = \theta \times D/2$

Fig. 10.10 *Deflection of Spring*

$$\delta = \frac{16PD^2N}{Gd^4} \times D/2 \Rightarrow \delta = \frac{8PD^3N}{Gd^4}$$

Shear stress correction factor $K_s = 1 + \frac{0.5d}{D}$

$$\boxed{\tau = K_s\left(\frac{8PD}{\pi d^3}\right)}$$

$$\tau = K\left(\frac{8PD}{\pi d^3}\right)$$

$K \rightarrow$ Stress factor (or) Wahl factor

$$\boxed{K = \frac{4C-1}{4C-4} + \frac{0.615}{C}}$$

$$\boxed{C = D/d}$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\cdot\theta}{l}$$

$$\theta = \frac{M_t \times l}{G \times J}$$

$G \rightarrow$ modulus of Rigidity

$$= \frac{\left(\frac{PD}{2}\right) \times (\pi D N)}{G \times \frac{\pi}{32} d^4}$$

$\theta \rightarrow$ Angle of twist

$l \rightarrow$ length of bar $= \pi D N$

$J = \frac{\pi}{64} d^4$

$$\theta = \frac{16PD^2N}{Gd^4}$$

$\delta = \theta \times$ length of bracket

$= \theta \times (D/2)$

$$\delta = \frac{16PD^2N}{Gd^4} \times \frac{D}{2}$$

$$\boxed{\delta = \frac{8PD^3N}{Gd^4}}$$

Stiffness of Spring $K = P/\delta$

$$K = \frac{P}{\frac{8PD^3N}{Gd^4}} \Rightarrow \boxed{K = \frac{Gd^4}{8D^3N}}$$

Rate of Spring

G → Same

d → Same

D → Same

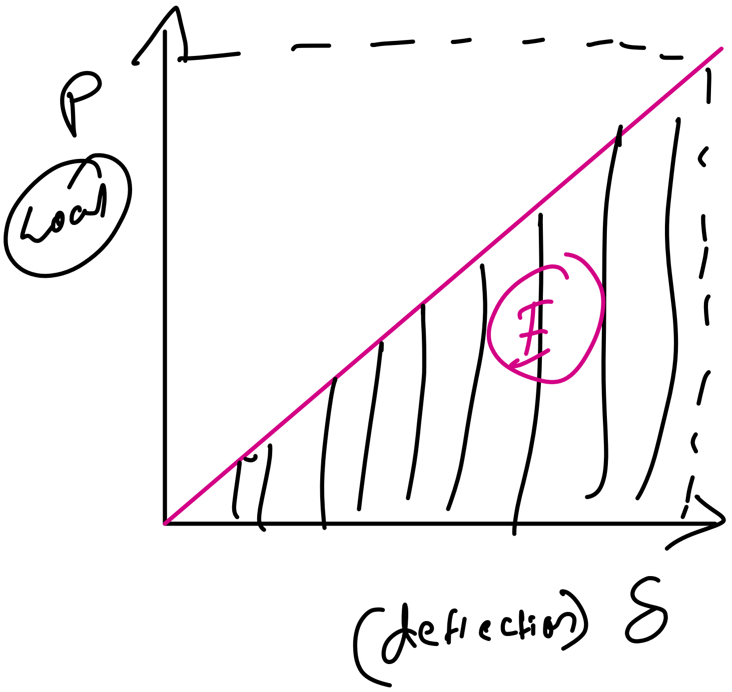
N → Change

$N = N/2$ for spring

$K = P/\delta$

$\delta = P/K$

$\delta \propto P \Rightarrow P \propto \delta$



→ Load deflection diagram

The area of load deflection diagram = Strain Energy

St of energy stored by spring

$$\boxed{E = \frac{1}{2} P \delta}$$

Springs connected in parallel & series:

→ Series connection

1) Force acting on each spring same
2) Total deflection = sum of deflections of individual springs

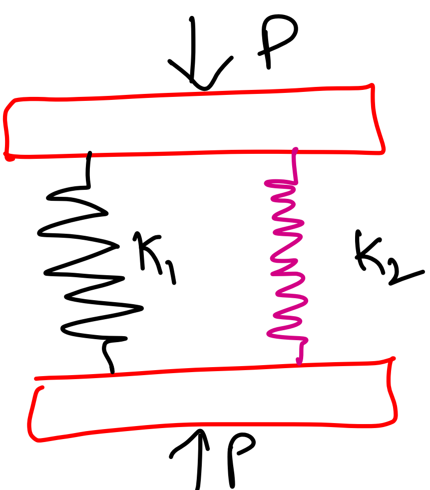
$$\delta = P/K$$

$$P_1 = P_2 = P \quad — (1)$$

$$\delta = \delta_1 + \delta_2$$

$$\frac{\cancel{P}}{K_{eq}} = \frac{\cancel{P_1}}{K_1} + \frac{\cancel{P_2}}{K_2} \Rightarrow \boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \cdots}$$

$$\boxed{K_{eq} = \frac{K_1 K_2}{K_1 + K_2}}$$



→ parallel connection

1) Total force = sum of forces on individual springs
2) Deflection is same for both the springs

$$P = P_1 + P_2 \quad \Big| \quad \delta = \delta_1 = \delta_2$$

$$P = K\delta$$

$$K\cancel{\delta} = K_1\cancel{\delta_1} + K_2\cancel{\delta_2}$$

$$\boxed{K_{eq} = K_1 + K_2} + \cdots$$

Design of helical Springs

δ, P, K

$$\tau = K\left(\frac{8PD}{\pi d^3}\right)$$

D → mean coil

d → dia of wire

$$\tau = \frac{\sigma_{sy}}{(FOS)}$$

FOS > 1.5 → springs

$$\tau = \frac{\sigma_{sy}}{1.5}$$

σ_{sy} → yield stress in shear

$\sigma_{yt} = 0.75\,\sigma_{ut}$

σ_{yt} → tension yield strength

σ_{sy} → yield strength in shear

$\sigma_{sy} = 0.577\,\sigma_{yt}$ → Distortion energy theorem

$$\tau = \frac{(0.577)(0.75)\sigma_{ut}}{1.5}$$

$$\boxed{\tau = 0.3\,\sigma_{ut}}$$

Shear stress is 30–50% of ultimate tensile stress

According to our Indian Standards

$$\tau = 0.5\,\sigma_{ut}$$

1) Force (P) & Stiffness (K) $\Rightarrow$ $\delta = P/K$
$\delta \rightarrow$ deflection

2) $\tau = (0.3 - 0.5)\sigma_{ut}$

3) Spring Index $(C) = D/d \simeq 8 \rightarrow 10$

$C \geq 3$ $\quad$ $C \leq 15$

4) K → wahl's factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

5) wire dia

$$\tau = K\left(\frac{8PC}{\pi d^2}\right)$$

6) $D = C \times d$

7) Solid length, No. of coils, gap b/n coils

8) Free length

9) Pitch

10) Rate (or) actual Stiffness

Problem: It is required to design a helical compression spring subjected to a maximum force of 1250 N. The deflection of the spring corresponding to the maximum force should be approximately 30 mm. The spring index can be taken as 6. The spring is made of patented and cold-drawn steel wire. The ultimate tensile strength and modulus of rigidity of the spring material are 1090 and 81 370 N/mm^2 respectively. The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate:

(i) wire diameter;

(ii) mean coil diameter;

(iii) number of active coils;

(iv) total number of coils;

(v) free length of the spring; and

(vi) pitch of the coil.

Draw a neat sketch of the spring showing various dimensions.

Given data:

$$P = 1250\ N;\quad \delta = 30\,mm;\quad C = 6$$

$$\sigma_{ut} = 1090\ N/mm^2;\quad G = 81{,}370\ N/mm^2$$

$$\tau = 0.5\,\sigma_{ut}$$

i) wire dia:

$$\tau = 0.5\,\sigma_{ut} = 0.5 \times 1090 = 545\ N/mm^2$$

$$\tau = K\left(\frac{8PC}{\pi d^2}\right)$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(6)-1}{4(6)-4} + \frac{0.615}{6}$$

$K = 1.2525 \qquad 545 = 1.2525\left(\frac{8\times 1250\times 6}{\pi\times d^2}\right)$

$$d = \sqrt{\frac{1.2525\times 8\times 1250\times 6}{545\times \pi}}$$

$$d = 6.625\,mm \simeq \underline{7}\,mm$$

2) mean coil dia:

$C = D/d \qquad D = C\times d$

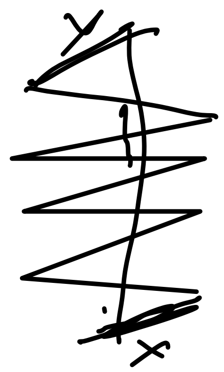
$= 6\times 7 = 42\ mm$

3) No. of active coils:

$$\delta = \frac{8PD^3N}{Gd^4} \qquad 30 = \frac{8\times 1250\times 42^3\times N}{(81,370)\times(7)^4}$$

$N = 7.91 \simeq 8$ coils

Total no. of coils:



$N_t = N+2 = 10$ coils

5) Free length of spring:

Free length = solid length + Total axial gap + deflection

Actual deflection

$$\delta = \frac{8 \times 1250 \times (42)^3 \times 8}{81{,}370 \times (7)^4}$$

$$\delta = 30.34 \text{ mm}$$

Solid length $\quad L_t = N_t \times d$

$$= 70 \text{ mm}$$

Assume there is 1 mm gap b/n coils when max force is applied.

10 coils

Total axial gap $= (N_t - 1) \times 1$

$$= (10 - 1) \times 1$$

$$= 9 \text{ mm}$$

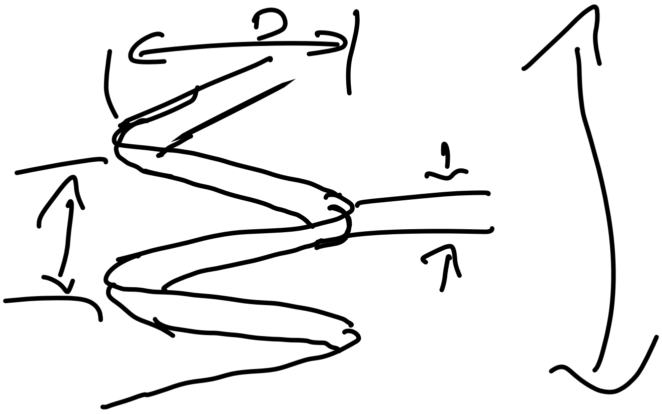
Free length $= 70 \text{ mm} + 9 \text{ mm} + 30.34$

$$= 109.35 \text{ mm}$$

$$\simeq 110 \text{ mm}$$

6) Pitch of the coil:

$$p = \frac{\text{Free length}}{(N_t - 1)} = \frac{110}{(10-1)} = 12.22 \text{ mm}$$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Problem : It is required to design a helical compression spring for the valve mechanism. The axial force acting on the spring is 300 N when the valve is open and 150 N when the valve is closed.
The length of the spring is 30 mm when the valve is open and 35 mm when the valve is closed. The spring is made of oil-hardened and tempered valve spring wire and the ultimate tensile strength is 1370 N/mm2. The permissible shear stress for the spring wire should be taken as 30% of the ultimate tensile strength. The modulus of rigidity is 81 370 N/mm2. The spring is to be fi tted over a valve rod and the minimum inside diameter of the spring should be 20 mm. Design the spring and calculate (i) wire diameter; (ii) mean coil diameter; (iii) number of active coils;

(iv) total number of coils; (v) free length of the spring; and (vi) pitch of the coil. Assume that the clearance between adjacent coils or clash allowance is 15% of the deflection under the maximum load.

Solution

Given $P = 300$ to 150 N

Spring length = 30 to 35 mm $D_i = 20$ mm

$S_{ut} = 1370$ N/mm^2 $G = 81\,370$ N/mm^2

$\tau = 0.3\, S_{ut}$

Step I *Wire diameter*

The spring force and spring length corresponding to closed and open positions of the valve are illustrated in Fig. 10.17. The permissible shear stress is given by,

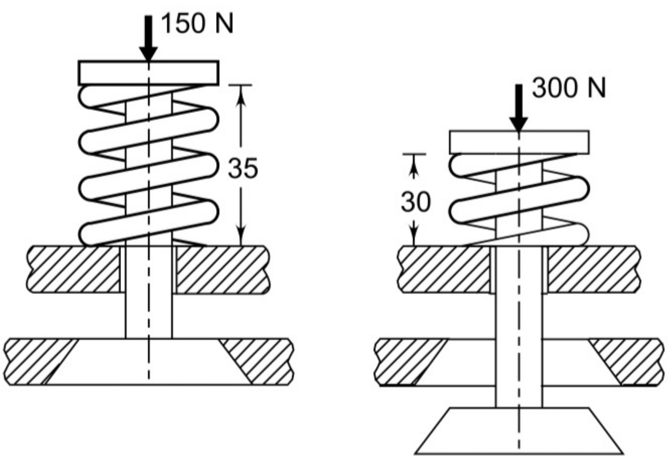
$\tau = 0.3\, S_{ut} = 0.3\,(1370) = 411$ N/mm^2

$D_i = 20$ mm

$D = D_i + d = (20 + d)$ mm

From Eq. (10.6),

$$\tau = K\left(\frac{8PD}{\pi d^3}\right) \quad \text{or} \quad 411 = K\left\{\frac{8(300)(20+d)}{\pi d^3}\right\} \qquad \text{(a)}$$



(a) Closed position (b) Open position

$$\boxed{411 = \frac{8(300)(20+d)}{\pi d^3} \quad \text{or} \quad \frac{d^3}{(20+d)} = 1.8587 \qquad \text{(b)}}$$

The above equation is solved by trial and error method. The values are tabulated in the following way:

d	$d^3/(20 + d)$
5	5
4	2.667
3	1.174

The value of d should be between 3 to 4 mm in order to satisfy Eq. (b). The higher value of d is selected to account for the Wahl correction factor.

Therefore,

$d = 4$ mm $\qquad D = \left(\frac{D_o + D_i}{2}\right)$ (i)

Step II *Mean coil diameter*

$D = D_i + d = 20 + 4 = 24$ mm (ii)

$$C = \frac{D}{d} = \frac{24}{4} = 6$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6 - 1}{4\times 6 - 4} + \frac{0.615}{6}$$

$= 1.2525$

$$\tau = K\left(\frac{8PD}{\pi d^3}\right) = (1.2525)\left\{\frac{8(300)(24)}{\pi (4)^3}\right\}$$

$= 358.81$ N/mm^2

Therefore,

$\boxed{\tau < 411 \text{ N/mm}^2}$

and the design is safe.

Step III *Number of active coils*

Form Eq. (10.8),

$$\delta = \frac{8PD^3 N}{Gd^4} \quad \text{or} \quad (35 - 30) = \frac{8(300-150)(24)^3 N}{(81\,370)(4)^4}$$

$$\therefore \qquad N = 6.28 \text{ or } 7 \text{ coils} \qquad \text{(iii)}$$

Step IV *Total number of coils*

It is assumed that the spring has square and ground ends. The number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 7 + 2 = 9 \text{ coils} \qquad \text{(iv)}$$

Step V *Free length of spring*

The deflection of the spring for the maximum force is given by,

$$\delta = \frac{8PD^3 N}{Gd^4} = \frac{8(300)(24)^3(7)}{(81\,370)(4)^4} = 11.15 \text{ mm}$$

The total gap between the adjacent coils is given by,

Gap = 15% of $\delta = 0.15\ (11.15) = 1.67$ mm

Solid length = $N_t\, d = 9(4) = 36$ mm

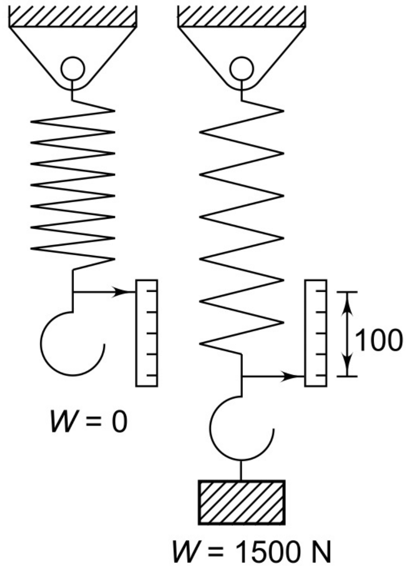
$$\begin{aligned}\text{Free length} &= \text{solid length} + \text{total axial gap} + \delta \\ &= 36 + 1.67 + 11.15 \\ &= 48.82 \text{ or } 50 \text{ mm} \qquad \text{(v)}\end{aligned}$$

Step VI *Pitch of coils*

$$\text{Pitch of coil} = \frac{\text{free length}}{(N_t - 1)} = \frac{50}{(9-1)} = 6.25 \text{ mm} \quad \text{(vi)}$$

Example *A helical tension spring is used in the spring balance to measure the weights. One end of the spring is attached to the rigid support while the other end, which is free, carries the weights to be measured. The maximum weight attached to the spring balance is 1500 N and the length of the scale should be approximately 100 mm. The spring index can be taken as 6. The spring is made of oil-hardened and tempered steel wire with ultimate tensile strength of 1360 N/mm² and modulus of rigidity of 81 370 N/mm². The permissible shear stress in the spring wire should be taken as 50% of the ultimate tensile strength. Design the spring and calculate*

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) required spring rate; and
(v) actual spring rate.



Given data:

$P = 1500\,N$ $\quad \sigma_{ut} = 1360\ N/mm^2$

$\delta = 100\,mm$ $\quad G = 81,370$

$C = 6$ $\quad \tau = 0.5\,\sigma_{ut}$

For $P_{max} = 1500\,N$; $\delta = 100\,mm$

$\tau = 680\ N/mm^2$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.2525$$

$$\tau = K\left[\frac{8PC}{\pi d^2}\right] \Rightarrow \boxed{d^2 = \frac{K \times 8PC}{\pi \tau}} =$$

"D" $C = D/d$

2) $D = C \times d$

3) No. of active coils =

$$\delta = \frac{8PD^3N}{Gd^4}$$

$$100 = \frac{8 \times 1500 \times D^3 \times N}{81370 \times d^4}$$

$$N = \_\_\_\_$$

4) spring rate (or) stiffness

$$K = P/\delta$$

5) Actual Spring Rate!

$$K_{act} = \frac{Gd^4}{8D^3N}$$ //

Problem: It is required to design a helical compression spring subjected to a maximum force of 7.5 kN. The mean coil diameter should be 150 mm from space consideration. The spring rate is 75 N/mm. The spring is made of oil-hardened and tempered steel wire with ultimate tensile strength of 1250 N/mm2. The permissible shear stress for the spring wire is 30% of the ultimate tensile strength (G = 81 370 N/mm2). Calculate

(i) wire diameter; and

(ii) number of active coils.

Given data:

$$P = 7.5\ \text{kN}\ ;\ D = 150\ \text{mm}\ ;\ K = 75\ \text{N/mm}$$

$$\sigma_{ut} = 1250\ \text{N/mm}^2 \qquad \tau = 0.3\,\sigma_{ut} \qquad G = 81{,}370$$

$$\tau = 0.3 \times 1250 = 375\ \text{N/mm}^2$$

$$C \rightarrow \text{Spring Index} = D/d \Rightarrow d = \frac{150}{C}$$

$$\tau = K\left[\frac{8PC}{\pi d^2}\right] \qquad \frac{C}{d^2} = \frac{C}{\left(\frac{150}{C}\right)^2} = K\left[\frac{8PC \times C^2}{\pi \times (150)^2}\right]$$

$$\tau = K\left[\frac{8PC^3}{\pi \times (150)^2}\right]$$

$$KC^3 = \frac{\tau \times \pi \times (150)^2}{8P} = \frac{375 \times \pi \times (150)^2}{8 \times 7500}$$

$$KC^3 = 441.78 \quad \text{---}\ (1)$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad \text{———} (2)$$

C	K	KC^3
5	1.310	163.81 (1.310×5³)
6	1.2525	270.54
7	1.212	416.01
8	1.184	606.20
7.1	1.206	431.64
7.2	1.203	449.01

Lets take $C = 7.2$ $\quad$ 8 (or) K

$$d = \frac{150}{C} = \frac{150}{7.2} = 21 \text{ mm}$$

N $\quad$ 8 (or) K

$$K = \frac{G d^4}{8 D^3 N} \Rightarrow 75 = \frac{(81{,}370) \times (21)^4}{8 \times (150)^3 \times N}$$

$$\boxed{N = 7.81 \simeq 8 \text{ coils}}$$

Problem: A railway wagon moving at a velocity of 1.5 m/s is brought to rest by a bumper consisting of two helical springs arranged in parallel. The mass of the wagon is 1500 kg. The springs are compressed by 150 mm in bringing the wagon to rest. The spring index can be taken as 6. The springs are made of oil-hardened and tempered steel wire with ultimate tensile strength of 1250 N/mm2 and modulus of rigidity of 81 370 N/mm2. The permissible shear stress for the spring wire can be taken as 50% of the ultimate tensile strength. Design the spring and calculate:

(i) wire diameter;
(ii) mean coil diameter;
(iii) number of active coils;
(iv) total number of coils;
(v) solid length;
(vi) free length;
(vii) pitch of the coil;
(viii) required spring rate; and
(ix) actual spring rate.

Given data: $v = 1.5\ m/s$

$m = 1500\ kg$

$\delta = 150\ mm$; $C = 6$;

$\sigma_{ut} = 1250\ N/mm^2$; $G = 81{,}370\ N/mm^2$

$\tau = 0.5\ \sigma_{ut}$

$\tau = 0.5 \times 1250 = 625\ N/mm^2$

i) wire dia:

$$\tau = K\left[\frac{8PC}{\pi d^2}\right]$$

$P = 11{,}250\ N$

$(K.E)_{wagon} = (\text{Strain energy})_{Spring}$

$(K.E)_w = \frac{1}{2} mv^2 = \frac{1}{2}(1500)(1.5)^2 = 1687.5\ N\text{-}m$

$(K.E)_w = 1687.5 \times 10^3\ N\text{-}mm$

$P \rightarrow$ Force experienced by each spring

$150\ mm \rightarrow$ deflect

$\frac{1}{2} P \times \delta \rightarrow$ Strain energy for each Spring

$$E = 2\left[\frac{1}{\cancel{2}} P\delta\right] = (KE)_{w}$$

$$P \times (150) = 16875 \times 10^{3}$$

$$\boxed{P = 11250\ N}$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad C = 6$$

$$K = 1.2525$$

$$\tau \Rightarrow 625\ N/mm^2 = 1.2525\left[\frac{8 \times 11{,}250 \times 6}{\pi \times d^2}\right]$$

$$d = \sqrt{\frac{1.2525 \times 8 \times 11{,}250 \times 6}{\pi \times 625}}$$

$$d = 18.55\,mm$$

$$d = 20\,mm$$

2) Mean Coil dia:

$$D = ? \qquad C = D/d$$

$$D = C \times d$$

$$D = 120\,mm$$

3) No. of active coils:

$$\delta = \frac{8PD^3N}{Gd^4} \Rightarrow N = \frac{150 \times 81{,}370 \times (20)^4}{8 \times 11{,}250 \times (120)^3}$$

$$N = 13$$

4.) Total no. of coils

$$N_t = N + 2 = 15 = \text{coils}$$

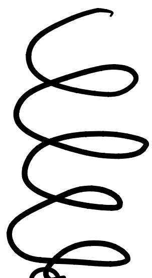
$$\text{Solid length} = N_t \times d$$
$$= 300\ \text{mm}$$

Free length = Solid length + axial gap + δ

$$\text{Actual deflection } \delta = \frac{8PD^3N}{Gd^4} = \frac{8 \times 11{,}250 \times (120)^3 \times 13}{(81{,}370) \times (20)^4}$$

$$\delta = 155.29\ \text{mm}$$

Assuming there is a gap of 2mm b/n adjacent coils when max force is applied.



$$\text{Total axial gap} = (N_t - 1) \times \text{gap}$$
$$= (15 - 1) \times 2 = 28\ \text{mm}$$

Free length = 485 mm

Pitch of the coils:

$$p = \frac{\text{free length}}{(N_t - 1)} = \frac{485}{(15-1)} = 34.64\text{mm}$$

Recu Spring rate:

$$K = P/\delta = \frac{11,250}{150} = 75 \text{ N/mm}$$

Actual spring rate:

$$K = \frac{G d^4}{8 D^3 N} = 72.44 \text{ N/mm}$$

Trial and Error Procedure

Problem: An automotive single-plate clutch, with two pairs of friction surfaces, transmits 300 N-m torque at 1500 rpm. The inner and outer diameters of the friction disk are 170 and 270 mm respectively. The coefficient of friction is 0.35. The normal force on the friction surfaces is exerted by nine helical compression springs, so that the clutch is always engaged. The clutch is disengaged when the external force further compresses the springs. The spring index is 5 and the number of active coils is 6. The springs are made of patented and cold-drawn steel wires of Grade 2. (G = 81 370 N/mm2). The permissible shear stress for the spring wire is 30% of the ultimate tensile strength. Design the springs and specify their dimensions.

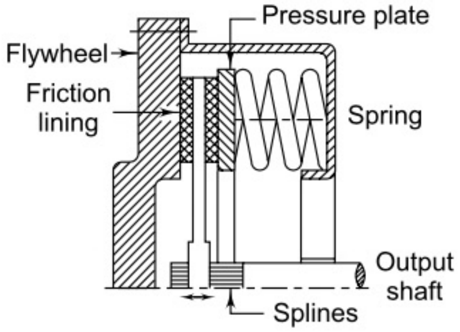


Fig. 10.19 *Clutch Mechanism*

Table 10.1 *Mechanical properties of patented and cold-drawn steel wires*

Wire diameter d (mm)	*Minimum tensile strength (N/mm^2)*			
	Gr.1	*Gr.2*	*Gr.3*	*Gr.4*
0.3	1720	2060	2460	2660
0.4	1700	2040	2430	2620
0.5	1670	2010	2390	2580
0.6	1650	1990	2360	2550
0.7	1630	1970	2320	2530
0.8	1610	1950	2280	2480
0.9	1590	1920	2250	2440
1.0	1570	1900	2240	2400
1.2	1540	1860	2170	2340
1.4	1500	1820	2090	2290
1.6	1470	1780	2080	2250
1.8	1440	1750	2030	2190
2.0	1420	1720	1990	2160
2.5	1370	1640	1890	2050
3.0	1320	1570	1830	1980
3.6	1270	1510	1750	1890
4.0	1250	1480	1700	1840
4.5	1230	1440	1660	1800
5.0	1190	1390	1600	1750
6.0	1130	1320	1530	1670
7.0	1090	1260	1460	1610
8.0	1050	1220	1400	1540

Given data: $M_t = 300$ N-m

2 friction surf

$D_o = 270$mm ; $D_i = 170$mm

$\mu = 0.35$; $C = 5$; $N = 6$

No. of Springs = 9

$\tau = 0.3\, \sigma_{ut}$

1 friction surf $= \frac{300}{2} = 150$ N-m $= 150 \times 10^3$ N-mm

$M_t = \frac{\mu P}{4}(D + d)$

$$P = \frac{(150\times10^3)\times 4}{(0.35)\times(270+170)} = 3896.1\ N$$

Individual spring force $\Rightarrow P_i = \frac{3896.1}{9} = 432.9\ N$

wire dia:

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{20-1}{20-4} + \frac{0.615}{5} = 1.3105$$

$\tau_m = 0.3\times\sigma_{ut}$

$$\tau = K\left[\frac{8PC}{\pi d^2}\right]$$

$$= 1.3105\left[\frac{8\times432.9\times5}{\pi d^2}\right]$$

$$\tau = \frac{7214.28}{d^2}\ N/mm^2$$

For $d = 3\ mm$

$\sigma_{ut} = 1570\ N/mm^2 \Rightarrow \tau_m = 471\ N/mm^2$

$$\tau = \frac{7214.28}{(3)^2} = 801.58\ N/mm^2$$

$\tau_m < \tau \rightarrow$ Design is not safe

For $d = 4\ mm$

$\tau = 450.89\ N/mm^2$ $\quad \tau_m = 0.3\times1480$

$= 444\ N/mm^2$

$\tau > \tau_m \rightarrow$ Not safe

For $d = 4.5\,mm$

$\tau = 356.26\ N/mm^2$ $\quad \tau_m = 0.3 \times 1440$

$= 432\ N/mm^2$

$\tau < \tau_m$ → Design is safe

$d = 4.5\,mm$

Mean coil dia:

$$D = c \times d = 5 \times 4.5 = 22.5\,mm$$

Total no. of coils:

$$N = 6 \quad ; \quad N_t = N + 2 = 8 \text{ coils}$$

Free length:

Solid length $= N_t \times d = 8 \times 4.5 = 36\,mm$

$$\delta = \frac{8 P D^3 N}{G d^4} = 7.09\,mm$$

Assume 1 mm gap b/n coils

Total axial gap $= (N_t - 1) \times 1 = 7\,mm$

Free length $= 36 + 7 + 7.09$

$= 51\,mm$

Square & grounded coils:

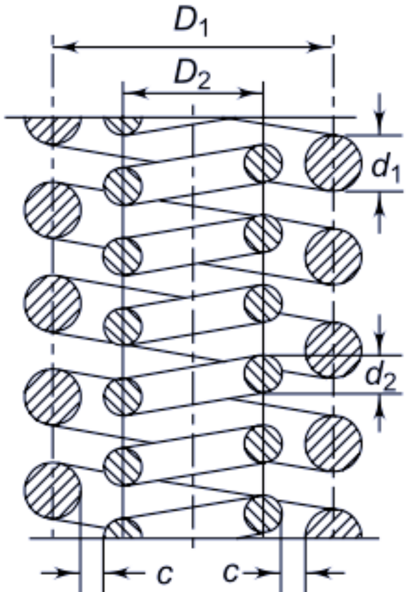
CONCENTRIC SPRINGS

A concentric spring consists of two helical compression springs, one inside the other, having the same axis.

Concentric spring is also called a '*nested*' spring.

If the outer spring has a right-hand helix, the inner spring always has a left-hand helix and vice versa.

Adjacent springs having opposite hands, prevent the locking of coils, in the event of axial misalignment or buckling of springs.



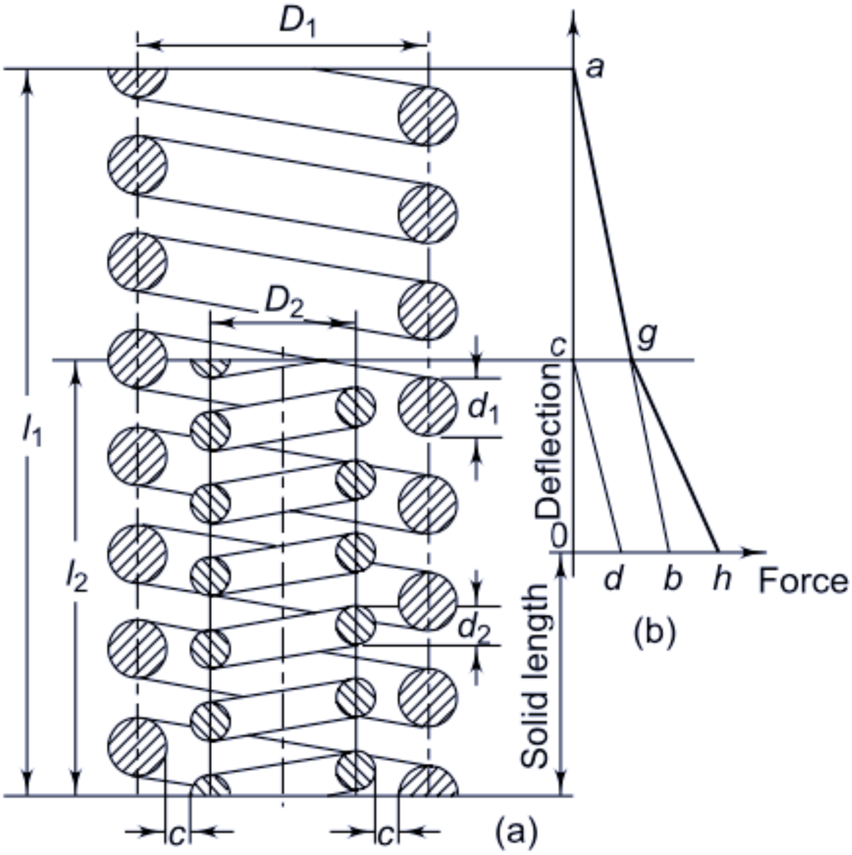
$c \rightarrow$ radial clearance

$(d_1/2)+c+(d_2/2)$ $\quad$ $d_2/2 + c + d_1/2$

Fig. 10.24 *Concentric Springs*

1. Concentric springs are used as valve springs in heavy duty diesel engines, aircraft engines and railroad suspensions.
2. Since there are two springs, the load carrying capacity is increased and heavy load can be transmitted in a restricted space.
3. In concentric spring, the operation of the mechanism continues even if one of the springs breaks. This results in '*fail safe*' system.

In some applications, concentric spring is used to obtain a spring force, which is not directly proportional to its deflection. Such a variable force- deflection characteristic is obtained by nesting two springs, one inside the other, having different free lengths.



This type of concentric spring is used in the governor of variable speed engines to take care of variable centrifugal force.

(i) The springs are made of the same material.

(ii) The maximum torsional shear stresses induced in outer and inner springs are equal.

(iii) They have the same free length.

(iv) Both springs are deflected by the same amount and therefore, have same solid length.

$N_1 \rightarrow$ active in outer spring

$N_2 \rightarrow$ " " inner "

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$\tau_1 = \tau_2$$

$$K_1\left[\frac{8P_1D_1}{\pi d_1^3}\right] = K_2\left[\frac{8P_2D_2}{\pi d_2^3}\right]$$

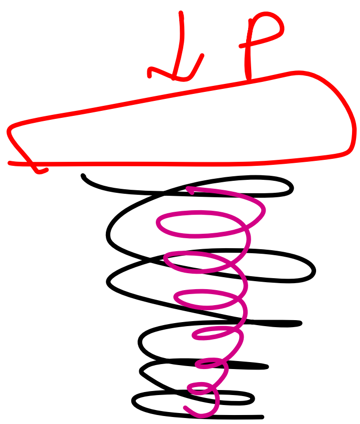
neglecting effect of wahl's factor

$$K_1 = K_2$$

$$\frac{P_1D_1}{d_1^3} = \frac{P_2D_2}{d_2^3} \quad \text{— (1)}$$

$P_1 \rightarrow$ force on outer spring

$P_2 \rightarrow$ inner spring



$$\delta_1 = \delta_2$$

$$\frac{\cancel{8}P_1D_1^3N_1}{\cancel{G}d_1^4} = \frac{\cancel{8}P_2D_2^3N_2}{\cancel{G}d_2^4}$$

$$\frac{P_1D_1^3N_1}{d_1^4} = \frac{P_2D_2^3N_2}{d_2^4} \quad \text{— (2)}$$

$(\text{Solid length})_1 = (\text{Solid length})_2$

$N_t = N$

$N_t = N + x$ → active

$$N_1 d_1 = N_2 d_2 \quad \text{— (3)}$$ (assuming there are no inactive coils)

$N_t d$

$$(2) \Rightarrow \frac{P_1 D_1^3 (\cancel{N_1 d_1})}{d_1^5} = \frac{P_2 D_2^3 (\cancel{N_2 d_2})}{d_2^5}$$

$$\frac{P_1 D_1^3}{d_1^5} = \frac{P_2 D_2^3}{d_2^5} \quad \text{— (4)}$$

$$\frac{P_1 \cancel{D_1}}{\cancel{d_1^3}}\left(\frac{D_1^2}{d_1^2}\right) = \frac{P_2 \cancel{D_2}}{\cancel{d_2^3}}\left(\frac{D_2^2}{d_2^2}\right)$$

$$\boxed{\frac{D_1}{d_1} = \frac{D_2}{d_2} = C} \quad \text{— (5)}$$

$K_1 = K_2$

P

$$(1) \Rightarrow \frac{P_1}{d_1^2}\left(\cancel{\frac{D_1}{d_1}}\right) = \frac{P_2}{d_2^2}\left(\cancel{\frac{D_2}{d_2}}\right)$$

$$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2} \Rightarrow \frac{P_1}{P_2} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2}$$

$$\boxed{\frac{P_1}{P_2} = \frac{a_1}{a_2}}$$

a_1, a_2 → area of cross-section of wire

$$D_1 = D_2 + \left[\frac{d_2}{2} + \frac{d_2}{2}\right] + 2c + \left[\frac{d_1}{2} + \frac{d_1}{2}\right]$$

$$2c = (D_1 - D_2) - (d_1 + d_2)$$

$$C = \frac{(D_1 - D_2) - (d_1 + d_2)}{2}$$

radial Clearance

$$2C = d_1 - d_2 \Rightarrow C = \frac{d_1 - d_2}{2}$$

$$\frac{d_1 - d_2}{2} = \frac{D_1 - D_2}{2} - \frac{(d_1 + d_2)}{2}$$

$$(D_1 - D_2) = 2d_1$$

$$(Cd_1 - Cd_2) = 2d_1 \Rightarrow (C-2)d_1 = Cd_2$$

$$\boxed{\frac{d_1}{d_2} = \frac{C}{C-2}}$$

spring index

Problem: A concentric spring is used as a valve spring in a heavy duty diesel engine. It consists of two helical compression springs having the same free length and same solid length. The composite spring is subjected to a maximum force of 6000 N and the corresponding deflection is 50 mm. The maximum torsional shear stress induced in each spring is 800 N/mm2. The spring index of each spring is 6. Assume same material for two springs and the modulus of rigidity of spring material is 81370 N/mm2. The diametral clearance between the coils is equal to the difference between their wire diameters.

Calculate:

(i) the axial force transmitted by each spring;

(ii) wire and mean coil diameters of each spring; and

(iii) number of active coils in each spring.

$2c \rightarrow$ diametral clearance

$c \rightarrow$ radial clearance

Given data: $P = 6000\ N$; $\delta = 50\ mm$, $\tau = 800\ N/mm^2$

$C = 6 \qquad 2c = d_1 - d_2$

$$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \qquad d_1 = 1.5 d_2 \quad -(1)$$

$$\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2} \Rightarrow \frac{P_1}{P_2} = (1.5)^2 = 2.25 \quad -(2)$$

$$P_1 = 2.25 P_2$$

$$P_1 + P_2 = P = 6000$$

$$2.25 P_2 + P_2 = 6000 \Rightarrow P_2 = 1846.15\ N$$

$$P_1 = 4153.85\ N$$

wire dia & Coil dia:

$$\tau = K_1 \left[\frac{8 P_1 C_1}{\pi d_1^2} \right]$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.2525$$

$$\tau_1 = K_1\left[\frac{8P_1C}{\pi d_1^2}\right] \Rightarrow 800 = 1.2525\left[\frac{8\times4183.85\times6}{\pi d_1^2}\right]$$

$$d_1 = 10\ mm \qquad \frac{P_1}{d_1^2} = \frac{P_2}{d_2^2}$$

$$D_1 = \underset{6}{C}\,\underset{\cdot 10}{d_1} = 60\ mm$$

$$\tau_2 = K_2\left[\frac{8P_2C}{\pi d_2^2}\right] \Rightarrow 800 = 1.2525\left[\frac{8\times1846.15\times6}{\pi d_2^2}\right]$$

$$d_2 = 7\ mm \qquad d_2^2 = \left(\frac{1.2525\times6\times1846.15\times8}{800\times\pi}\right)$$

$$D_2 = 42\ mm$$

No. of active coils!

$$\delta_1 = \frac{8P_1D_1^3N_1}{Gd_1^4} \Rightarrow 50 = \frac{8\times4183.15\times(60)^3\times N_1}{(81,370)\times(10)^4}$$

$$N_1 = 6\ coils \qquad (\text{Solid length})_1 = (\text{Solid length})_2$$

$$N_{t_1} = N_1 + 2 = 8\ coils \qquad (N_t\times d)$$

$$(L_t)_1 = (L_t)_2 \Rightarrow N_{t_1}\times d_1 = N_{t_2}\times d_2$$

$$N_{t_2} = 12\ coils$$

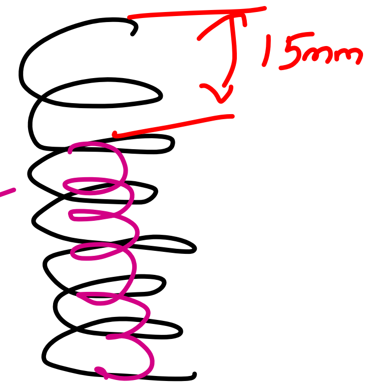
$$N_2 = N_{t_2} - 2 = 10\ coils$$

Problem: A concentric spring consists of two helical compression springs one inside the other. The free length of the outer spring is 15 mm greater than that of the inner spring. The wire diameter and mean coil diameter of the inner spring are 5 and 30 mm respectively. Also, the wire diameter and mean coil diameter of the outer spring are 6 and 36 mm respectively. The number of active coils in the inner and outer springs are 8 and 10 respectively. Assume same material for two springs and the modulus of rigidity of spring material is 81370 N/mm2. The composite spring is subjected to a maximum axial force of 1000 N. Calculate:

(i) the compression of each spring; δ_1, δ_2

(ii) the force transmitted by each spring; and P_1, P_2

(iii) the maximum torsional shear stress induced in each spring. τ_1, τ_2



Given data:

$P_t = 1000\,N$; $D_o = 36\,mm$

$D_i = 30\,mm$

$d_o = 6\,mm$ $\quad N_o = 10$

$d_i = 5\,mm$ $\quad N_i = 8$

$$K_i = \frac{G d_i^4}{8 D_i^3 N_i} = \frac{(81,370) \times 5^4}{8 \times (30)^3 \times 8} = 29.43\ N/mm$$

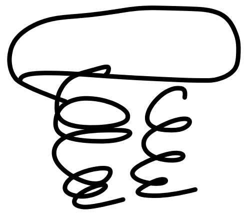
$$K_o = \frac{(81,370) \times 6^4}{8 \times (36)^3 \times 10} = 28.25\ N/mm$$

$P \rightarrow$ Force experienced by outer spring for first 15 mm.

$$P = K_o \times \underbrace{\delta}_{15\ mm} = 28.25 \times 15 = 423.75\ N$$

Remaining load shared by both springs $= 1000 - 423.75$

$= \underline{576.25\ N}$



In parallel conn of springs $K_{eq} = K_1 + K_2$

$$K_{eq} = K_o + K_i =$$
$$= 57.88 \ N/mm$$

Deflection due to remaining load

$$= \frac{\text{Remaining load}}{\text{Equivalent Stiffness}} = \frac{576.25}{57.68}$$

$x = 10mm$

"x" is the deflection of both inner & outer springs.

Total compression of outer spring $= 15 + x$

$= 25\ mm$

$\underline{\delta_o = \quad mm}$

Total comp of inner spring $= x$

$\underline{\delta_i = 10mm}$

Force transmitted by spring!

$P_o = K_o \times$ deflection of outer spring

$P_o = K_o \times \delta_o$

$= (28.25) \times (25) = 706.25\,N$

$P_i = K_i \times \delta_i$

$= 294.3\,N$

$(+)$ $\simeq 1000\,N$

max shear stress:

$$\tau_o = K\left[\frac{8 P_o D_o}{\pi d_o^3}\right]$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$C = \frac{D_o}{d_o} = \frac{36}{6} = 6$$

$K = 1.2525$

$$\tau_o = 1.2525\left[\frac{8 \times 706.25 \times 36}{\pi \times (6)^3}\right] = \quad N/mm^2$$

$$\tau_i = K\left[\frac{8 P_i D_i}{\pi d_i^3}\right]$$

$$C = \frac{D_i}{d_i} = \frac{30}{5} = 6$$

$\tau_i = \quad N/mm^2$

SURGE IN SPRING

When the natural frequency of vibrations of the spring coincides with the frequency of external periodic force, which acts on it, resonance occurs. In this state, the spring is subjected to a wave of successive compressions of coils that travels from one end to the other and back. This type of vibratory motion is called '*surge*' of spring. Surge is found in valve springs, which are subjected to periodic force.

The natural frequency of helical compression springs held between two parallel plates is given by,

$$\omega = \frac{1}{2}\sqrt{\frac{k}{m}}$$

The natural frequency of helical compression springs with one end on the flat plate and the other end free, supporting the external force is given by,

$$\omega = \frac{1}{4}\sqrt{\frac{k}{m}}$$

where,

k = stiffness of spring (N/m)

m = mass of spring (kg)

The mass of the spring is given by,

$$m = Al\rho \qquad \text{(c)}$$

where,

A = cross-sectional area of spring = $\left(\frac{\pi}{4} d^2\right)$

l = length of spring = $(\pi D N_t)$

ρ = mass density of spring material

Surge in springs is avoided by the following methods:

(i) The spring is designed in such a way that the natural frequency of the spring is 15 to 20 times the frequency of excitation of the external force. This prevents the resonance condition to occur.

(ii) The spring is provided with friction dampers on central coils. This prevents propagation of surge wave.

(iii) A spring made of stranded wire reduces the surge. In this case, the wire of the spring is made of three strands. The direction of winding of strands is opposite to the direction of winding of the coils while forming the spring. In case of compression of the coils, the spring tends to wind the individual wires closer together, which introduces friction. This frictional damping reduces the possibility of surge.

Surge is a serious problem in typical vibrations like valve springs and guns. It is not a problem in other applications where the external load is steady.

HELICAL TORSION SPRINGS

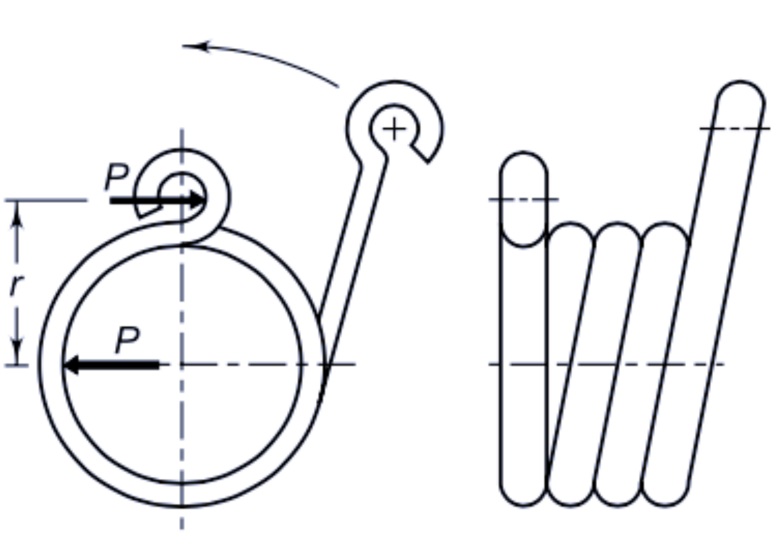


Fig. 10.27 *Helical Torsion Spring*

The primary stresses in this spring are flexural

Each individual section of the torsion spring is, in effect, a portion of a curved beam

$$\sigma_b = K\left(\frac{M_b\, y}{I}\right)$$

Stress conc factor due to curvature

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$y = \frac{d}{2} \;;\quad I = \frac{\pi d^4}{64}$$

$K \rightarrow$ wahl's Stress Conc factor

$$\sigma_b = K\left[\frac{32\, M_b}{\pi d^3}\right]$$

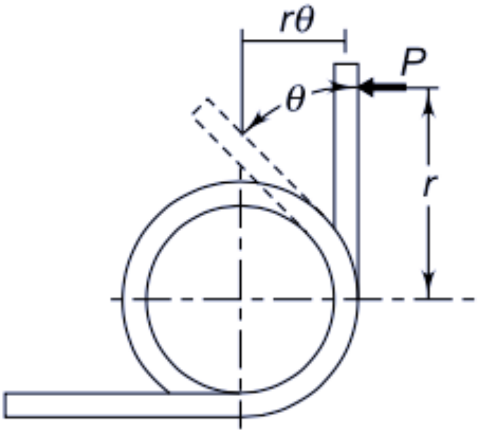
C - spring index

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} \qquad K_o = \frac{4C^2 + C - 1}{4C(C+1)}$$

Inner & outer fibres

$$\boxed{C = D/d}$$

$$M_b = P \times r$$



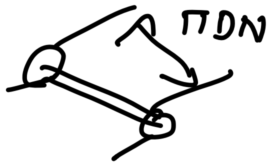


Fig. 10.28 *Angular Deflection of Spring*

Strain energy stored $\Rightarrow$ $U = \int_0^{\pi DN} \frac{(M_b)^2}{2EI} dx$

$$U = \frac{P^2 r^2}{2EI} \int_0^{\pi DN} dx = \frac{P^2 r^2 (\pi D N)}{2EI}$$

Castigliano's Theorem: 'When a body is elastically deflected by any combination of forces or moments, the deflection at any point and any direction is equal to the partial derivative of total strain energy of the body with respect to the force located at that point and acting in that direction'.

$$\delta = \frac{\partial U}{\partial P} \Rightarrow \frac{\partial}{\partial P}\left[\frac{P^2 r^2 (\pi D N)}{2EI}\right]$$

$$r\theta = \frac{r^2}{2EI}(\pi D N)\left(\frac{\partial P^2}{\partial P}\right) = \frac{P r^2 (\pi D N)}{EI}$$

$$\theta = \frac{P r (\cancel{\pi} D N)}{E\left(\frac{\cancel{\pi} d^4}{64}\right)} \Rightarrow \boxed{\theta = \frac{64 P r D N}{E d^4}}$$

Stiffness : $K = P/\delta$ $\frac{\text{Force}}{\text{displacement}}$

$K \rightarrow$ B.m req to produce unit angular disp

$$K = \frac{Pr}{\theta} \Rightarrow \boxed{K = \frac{Ed^4}{64DN}}$$

The design of the helical torsion spring is based on the torque-stress and the torque-deflection equations. The spring index is generally kept from 5 to 15. When it is less than 5, the strain on the coiling arbor of the torsion winder causes excessive tool breakage. When it is more than 15, the control over the spring pitch is lost.

Problem: It is required to design a helical torsion spring for a window shade. The spring is made of patented and cold-drawn steel wire of Grade-4. The yield strength of the material is 60% of the ultimate tensile strength and the factor of safety is 2. From space considerations, the mean coil diameter is kept as 18 mm. The maximum bending moment acting on the spring is 250 N-mm. The modulus of elasticity of the spring material is 207 000 N/mm^2. The stiffness of the spring should be 3 N-mm/rad. Determine the wire diameter and the number of active coils.

Given data:

$\sigma_{yt} = 0.6\sigma_{ut}$

$F_{OS} = 2$; $D = 18\,mm$

$(M_b)_{max} = 250$ N-mm ; $K = 3\,\frac{N\text{mm}}{rad}$

$E = 207{,}000\ N/mm^2$

From table we will choose

$d = 1.4\,mm$

Table 10.1 *Mechanical properties of patented and cold-drawn steel wires*

Wire diameter d (mm)	Minimum tensile strength (N/mm^2)			
	Gr.1	Gr.2	Gr.3	Gr.4
0.3	1720	2060	2460	2660
0.4	1700	2040	2430	2620
0.5	1670	2010	2390	2580
0.6	1650	1990	2360	2550
0.7	1630	1970	2320	2530
0.8	1610	1950	2280	2480
0.9	1590	1920	2250	2440
1.0	1570	1900	2240	2400
1.2	1540	1860	2170	2340
1.4	1500	1820	2090	2290
1.6	1470	1780	2080	2250
1.8	1440	1750	2030	2190
2.0	1420	1720	1990	2160
2.5	1370	1640	1890	2050
3.0	1320	1570	1830	1980
3.6	1270	1510	1750	1890
4.0	1250	1480	1700	1840
4.5	1230	1440	1660	1800
5.0	1190	1390	1600	1750
6.0	1130	1320	1530	1670
7.0	1090	1260	1460	1610
8.0	1050	1220	1400	1540

$\sigma_{ut} = 2290\ N/mm^2$

$\sigma_{yt} = 0.6 \times 2290$

$= 1374\ N/mm^2$

$\sigma_t = \frac{\sigma_{yt}}{F_{OS}} = 687\ N/mm^2$

$\sigma_b = K\left[\frac{32 M_b}{\pi d^3}\right]$

$K = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(12.857)^2 - 12.857 - 1}{4(12.857)(11.857)} = 1.061$

$C = D/d = \frac{18}{1.4} = 12.857$

$$\sigma_b = (1.061)\left[\frac{32 \times 250}{\pi \times (1.4)^3}\right] = 985 \ N/mm^2$$

$$(\sigma_b) > (\sigma_t)$$

$\rightarrow$ Design is not safe

Assume $d = 1.6$ mm ; $\sigma_{ut} = 2250 \ N/mm^2$

$$\sigma_t = \frac{0.6 \times \sigma_{ut}}{(FOS)} = 675 \ N/mm^2$$

$$C = D/d = \frac{18}{1.6} = 11.25$$

$$\sigma_b < \sigma_t$$

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = 1.071$$

$$\sigma_b = K_i\left[\frac{32 M_b}{\pi d^3}\right] = 665.83 \ N/mm^2$$

Design is safe

$d = 1.6$ mm

No. of active coils:

$$N = \frac{Ed^4}{64DK} = \frac{(207{,}000) \times (1.6)^4}{64 \times 18 \times 3}$$

$$N = 393 \text{ coils}$$

SPIRAL SPRINGS

A spiral spring consists of a thin strip of rectangular cross-section, which is wound in the form of spiral. It is also called 'power' spring or 'flat' spiral spring. The inner end of this strip is fixed in the arbor at the centre. The outer end is clamped to a drum called 'retaining' drum.

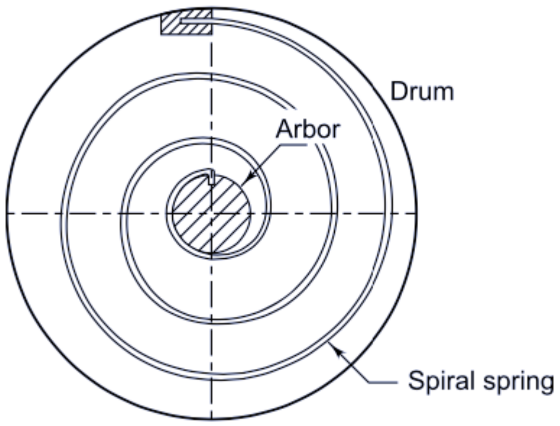
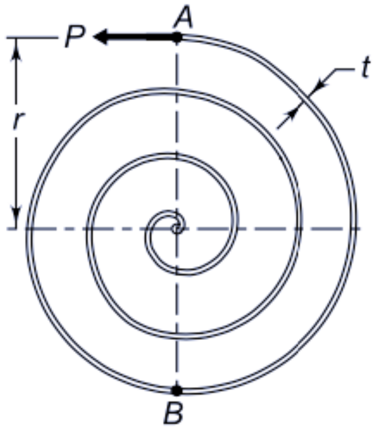


Fig. 10.29 *Spiral Spring*



Spiral springs are widely used in watches, cameras, instruments and automatic weapons. All types of toys are powered by this type of spring. They are also used as starters for small engines.

P = force induced at the outer end A due to winding of the arbor (N)

r = distance of centre of gravity of spiral from outer end (mm)

t = thickness of strip (mm)

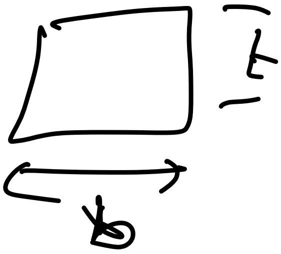
b = width of strip perpendicular to plane of paper (mm)

l = length of strip from outer end to inner end (mm)

Bending moment @ centre $= Pr = M$

Max Bending Moment occurs @ B.

$(M_b) = P\times(2r) = 2(Pr) = 2M$



$\sigma_b = \dfrac{M_b\, y}{I}$ $\qquad y = t/2, \quad I = \dfrac{bt^3}{12}$

$= \dfrac{(2M)(t/2)}{(bt^3/12)}$

$\sigma_b = \dfrac{12M}{bt^2}$

$\theta = \dfrac{Ml}{EI} = \dfrac{Ml}{E\left(\dfrac{bt^3}{12}\right)}$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

$$\frac{dy}{dx} = -\frac{M}{EI}\int_0^L dx$$

$\theta = \dfrac{12Ml}{Ebt^3}$

$\delta = r\theta \Rightarrow \delta = \dfrac{12Mlr}{Ebt^3}$

Strain Energy: $U = \frac{1}{2}\times M\times\theta = \frac{1}{2}\times M\times\dfrac{12Mlr}{Ebt^3}$

$$\boxed{U = \frac{6M^2 l}{Ebt^3}}$$

Problem: A flat spiral spring is required to provide a maximum torque of 1200 N-mm. The spring is made of a steel strip and the maximum bending stress should not exceed 800 N/mm^2. When the stress in the spring decreases from 800 to 0 N/mm^2, the arbor turns through three complete revolutions with respect to the retaining drum. The thickness of the steel strip is 1.25 mm and the modulus of elasticity is 207 000 N/mm^2. Calculate the width and length of the steel strip.

Given data: $M = 1200$ N-mm ; $\sigma_b = 800$ N/mm^2

$t = 1.25$ mm ; $E = 207{,}000$ N/mm^2

$$\sigma_b = \frac{12M}{bt^2} \Rightarrow 800 = \frac{12(1200)}{b(1.25)^2}$$

11.52

$$b = 12 \text{ mm}$$

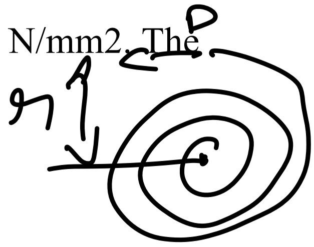
1 rev = 360°
= 2π

θ = 3 revolutions $= 6\pi$

$$\theta = \frac{12Ml}{Ebt^3} \Rightarrow 6\pi = \frac{12 \times 1200 \times l}{(207{,}000) \times 12 \times (1.25)^3}$$

$$l = 6350.68 \text{ mm} = 6.35 \text{ m}$$

Problem: A flat spiral spring, used in an electrical instrument, is required to exert a maximum force of 5 N at the free end against the retaining drum. The line of action of this force is 75 mm from the centre of gravity of the spiral. The spring is made of brass strip (E = 106 000 N/mm2) and the maximum bending stress should not exceed 100 N/mm2. The width and length of the strip are 12.5 and 750 mm. Calculate



(i) the thickness of strip; and

(ii) the number of degrees of rotation through which the arbor should be turned to produce the required force.

Given data: $P = 5\,N$; $r = 75\,mm$; $E = 106{,}000\ N/mm^2$

$\sigma_b = 100\ N/mm^2$; $b = 12.5\,mm$;

$l = 750\,mm$

$t = ?$

$\theta = ?$

$M = P \times r$

$$\sigma_b = \frac{12M}{bt^2}$$

$\pi\ rad = 180°$

$$t = 1.84\ mm \simeq 2\ mm$$

$$\theta = \frac{12Ml}{Ebt^3} = \frac{12 \times 5 \times 75 \times 750}{(106{,}000) \times 12.5 \times (2)^3} = 0.318\ rad$$

$$1\ degree = \frac{\pi}{180}\ rad$$

$$1\ rad = \frac{180}{\pi}\ degrees \Rightarrow$$

$$0.318\ rad = 0.318 \times \frac{180}{\pi} = 18.22°$$

MULTI LEAF SPRING:

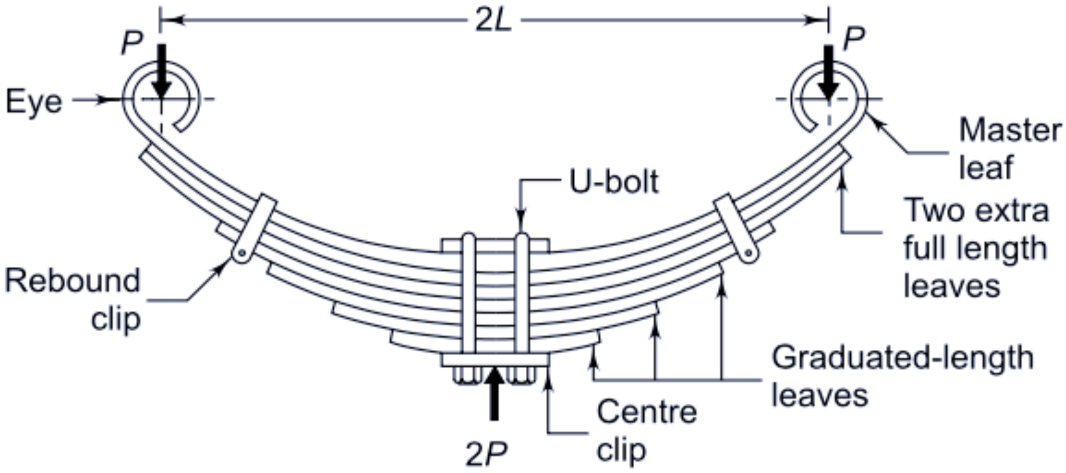


Fig. 10.31 *Semi-elliptic Leaf Spring*

$n = n_f + n_g$

n_f = number of extra full-length leaves

n_g = number of graduated-length leaves including master leaf

n = total number of leaves

b = width of each leaf (mm)

t = thickness of each leaf (mm)

L = length of the cantilever or half the length of semi-elliptic spring (mm)

P = force applied at the end of the spring (N)

P_f = portion of P taken by the extra full-length leaves (N)

P_g = portion of P taken by the graduated-length leaves (N)

The group of graduated-length leaves along with the master leaf can be treated as a triangular plate,

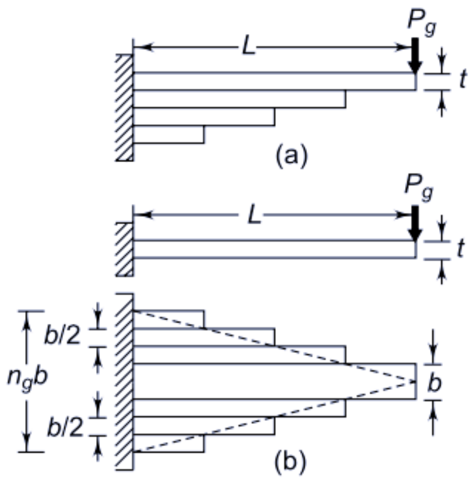


Fig. 10.32 *Graduated-length Leaves as Triangular Plate*

$$(\sigma_b)_g = \frac{M_b y}{I} = \frac{(P_g L)\,(t/2)}{\left[\frac{1}{12}\,(n_g b)\,(t^3)\right]}$$

$$(\sigma_b)_g = \frac{6P_g L}{n_g b t^2}$$

It can be proved that the deflection (d_g) at the load point of the triangular plate is given by,

$$\delta_g = \frac{P_g L^3}{2EI_{\text{max.}}} = \frac{P_g L^3}{2E\left[\frac{1}{12}(n_g b)\,(t^3)\right]}$$

$$\boxed{\delta = \frac{PL^3}{2EI}}$$

$$\delta_g = \frac{6P_g L^3}{En_g bt^3}$$

Similarly, the extra full-length leaves can be treated as a rectangular plate of thickness t and uniform width (nf b),

$$(\sigma_b)_f = \frac{M_b\, y}{I} = \frac{(P_f L)(t/2)}{\left[\frac{1}{12}\,(n_f b)(t^3)\right]} \Rightarrow (\sigma_b)_f = \frac{6P_f L}{n_f bt^2}$$

The deflection at the load point is given by,

$$\delta_f \quad \frac{P_f L^3}{3EI} = \frac{P_f L^3}{3E\left[\frac{1}{12}(n_f b)(t^3)\right]}$$

$$\boxed{\delta = \frac{PL^3}{3EI}}$$

$$\delta_f = \frac{4P_f L^3}{E n_f b t^3}$$

Since deflection of full-length leaves is equal to deflection of graduated leaves,

$$\delta_g = \delta_f$$

$$\frac{6P_g L^3}{E n_g b t^3} = \frac{4P_f L^3}{E n_f b t^3}$$

$$\frac{P_g}{P_f} = \frac{2n_g}{3n_f} \quad \text{— (1)}$$

$$P_g + P_f = P \quad \text{— (2)}$$

$$P_f = \frac{3n_f P}{(3n_f + 2n_g)}$$

$$P_g = \frac{2n_g P}{(3\, n_f + 2n_g)}$$

Substituting these values in stress equations:

$$(\sigma_b)_g = \frac{12PL}{(3n_f + 2n_g)\, bt^2}$$

$$(\sigma_b)_f = \frac{18PL}{(3n_f + 2n_g)\, bt^2}$$

$$(\sigma_b)_f = 1.5 \times (\sigma_b)_g$$

It is seen from the above equations that bending stresses in full-length leaves are 50% more than those in graduated-length leaves.

Deflection at the end of the spring is given by:

$$\delta = \frac{12PL^3}{Ebt^3(3n_f + 2n_g)}$$

NIPPING OF LEAF SPRINGS

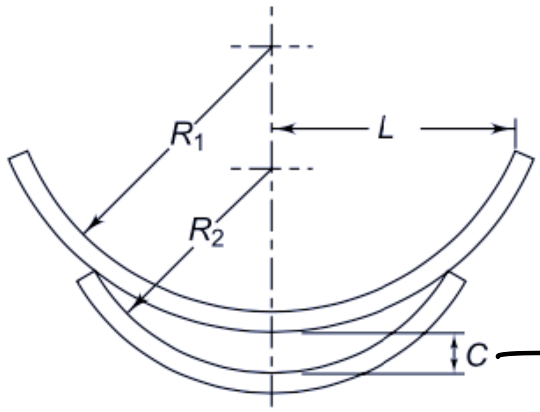
The stresses in extra full-length leaves are 50% more than the stresses in graduated-length leaves. One of the methods of equalising the stresses in different leaves is to **pre-stress** the spring. The pre-stressing is achieved by bending the leaves to different radii of curvature, before they are assembled with the centre clip.

The full-length leaf is given a greater radius of curvature than the adjacent leaf. The radius of curvature decreases with shorter leaves. The initial gap C between the extra full-length leaf and the graduated-length leaf before the assembly, is called a **'nip'.**

$R_1 \rightarrow$ radius of curvature is given small

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$R \rightarrow$ radius of curvature



$C \rightarrow$ nip

Fig. 10.34 *Nipping of Leaf Spring*

Nipping concept

$$(\sigma_b)_g = (\sigma_b)_f$$

$$\frac{P_g}{n_g} = \frac{P_f}{n_f} \quad / \quad P_g + P_f = P$$

$$P_g = \frac{n_g P}{n} \qquad n = n_g + n_f$$

$$P_f = \frac{n_f P}{n}$$

$$(\sigma_b)_g = \frac{6P_g L}{n_g b t^2}$$

$$(\sigma_b)_f = \frac{6P_f L}{n_f b t^2}$$

$$\delta_g = \frac{6P_g L^3}{E n_g b t^3}$$

$$\delta_f = \frac{4P_f L^3}{E n_f b t^3}$$

Under the maximum force P, the deflection of graduated-length leaves will exceed the deflection of extra full-length leaves by an amount equal to the initial nip C.

$$C = \frac{6P_g L^3}{En_g bt^3} - \frac{4P_f L^3}{En_f bt^3}$$

n -> total no. of leaves

P -> Total force applied

$$\boxed{C = \frac{2PL^3}{Enbt^3}}$$

The initial pre-load P_i required to close the gap C between the extra full-length leaves and graduated-length leaves is determined by considering the initial deflection of leaves.

Under the action of pre-load P_i,

$$C = (\delta_g)_i + (\delta_f)_i$$

$$\frac{2PL^3}{Enbt^3} = \frac{6(P_i/2)L^3}{En_g bt^3} + \frac{4(P_i/2)L^3}{En_f bt^3}$$

$$P_i = \frac{2n_g n_f P}{n(3n_f + 2n_g)}$$

$$(\sigma_b)_f = \frac{6PL}{nbt^2}$$

$$(\sigma_b) = \frac{6PL}{nbt^2}$$

Problem: A semi-elliptic leaf spring used for automobile suspension consists of three extra full-length leaves and 15 graduated-length leaves, including the master leaf. The centre-to-centre distance between two eyes of the spring is 1 m. The maximum force that can act on the spring is 75 kN. For each leaf, the ratio of width to thickness is 9:1. The modulus of elasticity of the leaf material is 207 000 N/mm^2. The leaves are pre-stressed in such a way that when the force is maximum, the stresses induced in all leaves are same and equal to 450 N/mm^2. Determine

(i) the width and thickness of the leaves;

(ii) the initial nip; and

(iii) the initial pre-load required to close the gap C between extra full-length leaves and graduated-length leaves.

Given data: $n_f = 3$; $n_g = 15$; $2L = 1$ m.

$2P = 75$ KN ; $E = 207{,}000$ N/mm^2 ; $n = n_f + n_g = 18$

$\sigma_b = 450$ N/mm^2

$b = ?$

$t = ?$

$C = ?$

$P_i = ?$

$$\sigma_b = \frac{6PL}{nbt^2} = \frac{6 \times 37{,}500 \times 500}{18 \times 9t \times t^2}$$

$$\frac{b}{t} = 9 \Rightarrow b = 9t$$

$$t = 11.55 \simeq 12 \text{ mm}$$

$$b = 108 \text{ mm}$$

Initial nip:

$$C = \frac{2PL^3}{Enbt^3} \Rightarrow C = \frac{2 \times 37{,}500 \times (500)^3}{(207{,}000) \times 18 \times 108 \times (12)^3}$$

$$C = 13.48 \text{ mm}$$

Initial pre-load :

$$P_i = \frac{2 n_g n_f P}{n(3 n_f + 2 n_g)} = \frac{2(15)(3)(37{,}500)}{18((3\times3)+(2\times15))}$$

$$\boxed{P_i = 4807 \text{ N}}$$

FLYWHEELS

A flywheel is a heavy rotating body that acts as a reservoir of energy. The energy is stored in the flywheel in the form of kinetic energy. The flywheel acts as an energy bank between the source of power and the driven machinery.

- In certain cases, the power is supplied at uniform rate, while the demand for power from the driven machinery is variable, e.g., a punch press driven by an electric motor.
- In punching and shearing machines, maximum power is required only during a small part of the cycle, when actual punching or shearing takes place. During the remaining part of the cycle, negligible power is required to overcome friction.
- Thus, the flywheel stores the kinetic energy during the idle portion of the work cycle by increasing its speed and delivers this kinetic energy during the peak-load period of punching or shearing. Therefore, when a flywheel is used between the motor and these machines, a smaller capacity motor is sufficient.

- In other applications, the power is supplied at variable rate, while the requirement of the driven machinery is at a uniform rate, e.g., machinery driven by an internal combustion engine.
- In IC engines, the power is generated at a variable rate. The flywheel absorbs the excess energy during the expansion stroke, when power developed in the cylinder exceeds the demand.
- This energy is delivered during suction, compression and exhaust strokes. The flywheel, therefore, enables the engine to supply the power at a practically uniform rate.

The functions of the flywheel are as follows:

1. To store and release energy when needed during the work cycle
2. To reduce the power capacity of the electric motor or engine
3. To reduce the amplitude of speed fluctuations

Casting

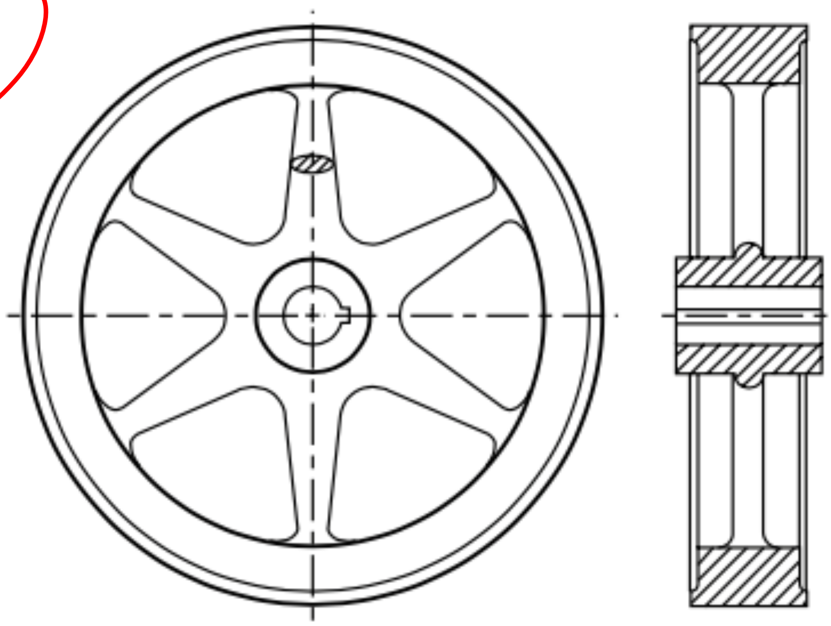


Fig. 21.1 *Solid One Piece Flywheel*

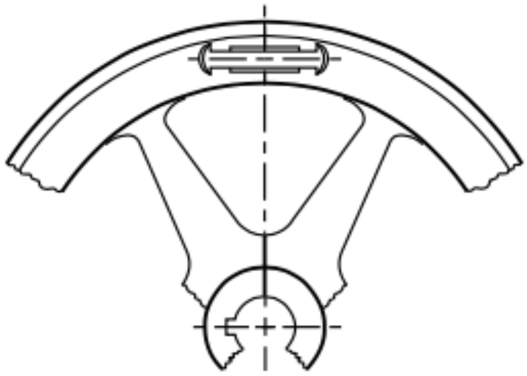


Fig. 21.2 *Split Flywheel*

FLYWHEEL AND GOVERNOR

Both flywheel and governor, are used in internal combustion engines to control the speed.

The function of the engine governor is to control the mean speed of the engine.

There is a basic difference between the functions of flywheel and governor. It is as follows:

1. The flywheel limits the inevitable fluctuations of speed during '*each cycle*', which arise from fluctuations of turning moment on the crankshaft. The governor controls the '*mean*' *speed* of the engine by varying the fuel supply to the engine.

2. The flywheel has no influence on the '*mean*' speed of the engine. It does not maintain a constant speed. The governor has no influence on '*cyclic*' speed fluctuations.
3. If the load on the engine is constant, the mean speed will be constant from cycle to cycle and the governor will not operate. On the other hand, the flywheel will be always acting. The operation of flywheel is continuous while that of governor is more or less intermittent.
4. A flywheel may not be used if the cyclic fluctuations of energy output are small or negligible. A governor is essential for all types of engines to adjust the fuel supply as per the demand.
5. The kind of energy stored in flywheel is kinetic energy. The kinetic energy is all available, 100% convertible into work without friction. The governor mechanism involves frictional losses.

FLYWHEEL MATERIALS

- Traditionally, flywheels are made of cast iron.

  1. Cast iron flywheels are the cheapest.
  2. Cast iron flywheel can be given any complex shape without involving machining operations.
  3. Cast iron flywheel has excellent ability to damp vibrations.

- However, cast iron has poor tensile strength compared to steel. The failure of cast iron flywheel is sudden and total. The machinability of cast iron flywheel is poor compared to steel flywheel.

Table 21.1 *Mass density of flywheel materials*

Material	*Mass density (kg/m^3) (ρ)*
Grey cast iron	
FG 150	7050
FG 200	7100
FG 220	7150
FG 260	7200
FG 300	7250
Steels	
Carbon steels	7800

$T_i \to$ Input torque

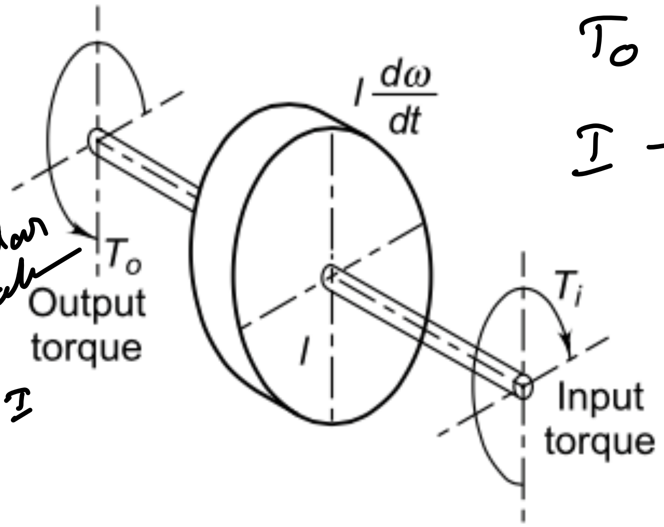
$T_o \to$ O/P "

$I \to$ mass M.O.I flywheel

$F = ma$

$T = I \times \alpha$ → angular accel

mass m.o.I

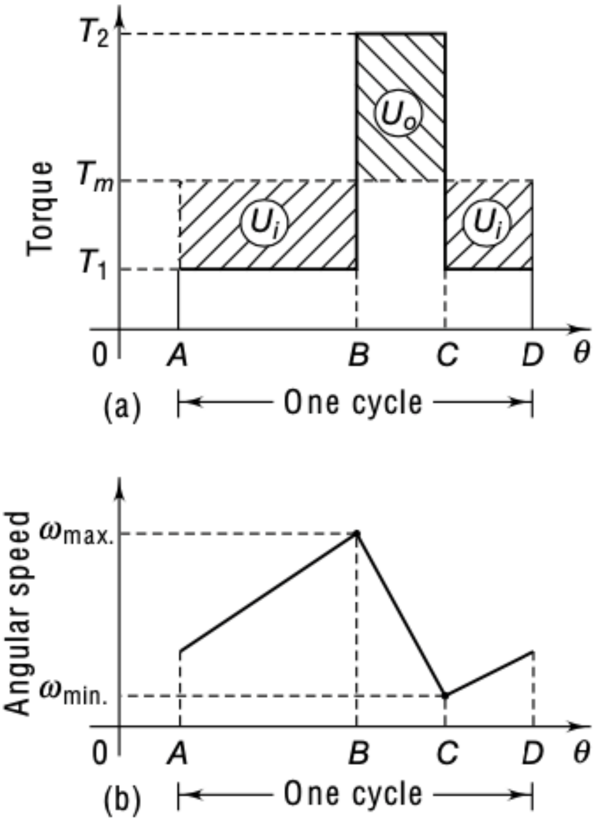


$\omega \to$ angular vel of shaft on flywheel

Fig. 21.3 *Torque on Flywheel Shaft*

$T = I\alpha \Rightarrow T_i - T_o = I\left(\frac{d\omega}{dt}\right)$

$a = \frac{dv}{dt}$



AB & CD → T_1

BC → T_2

$T_1 < T_m$

$T_2 > T_m$

Fig. 21.4 *(a) Torque Diagram (b) Speed Diagram*

During AB & CD, energy is supplied to flywheel:

$$U_i = \int_A^B (T_m - T_1)\, d\theta + \int_C^D (T_m - T_1)\, d\theta$$

BC → energy is taken from flywheel

$$U_o = \int_B^C (T_2 - T_m)\, d\theta$$

From point B → C

Change in K.E $= \frac{1}{2} I \left(\omega_{max}^2 - \omega_{min}^2\right)$

$$U_o = \frac{1}{2} I (\omega_{max} + \omega_{min})(\omega_{max} - \omega_{min}) \quad \text{— (1)}$$

$(\omega_{max} - \omega_{min})$ → maximum fluctuation of speed

$\frac{(\omega_{max} - \omega_{min})}{\text{Mean speed}}$ → Co-eff of fluctuation of speed

$$C_s = \frac{\omega_{max} - \omega_{min}}{\omega}$$

→ Mean speed

$$\omega = \left(\frac{\omega_{max} + \omega_{min}}{2}\right)$$

→ Substituting these in eqn (1)

$$U_0 = \frac{1}{2} I (\omega_{max} + \omega_{min}) \times \omega \times C_s$$

$$= \frac{1}{\cancel{2}} I (\cancel{2}\omega)(\omega \times C_s)$$

$$\boxed{U_0 = I\omega^2 C_s}$$

$$C_s = \frac{\omega_{max} - \omega_{min}}{\omega} = \frac{\omega_{max} - \omega_{min}}{\frac{(\omega_{max} + \omega_{min})}{2}}$$

$$= \frac{2[\omega_{max} - \omega_{min}]}{(\omega_{max} + \omega_{min})}$$

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2[N_{max} - N_{min}]}{(N_{max} + N_{min})}$$

Table 21.2 *Coefficients of fluctuations of speed*

Type of Equipment	C_s
Punching, shearing and forming presses	0.200
Compressor (belt driven)	0.120
Compressor (gear driven)	0.020
Machine tools	0.025
Reciprocating pumps	0.040
Geared drives	0.020
Internal combustion engines	0.030
D.C. generators (direct drive)	0.010
A.C. generator (direct drive)	0.005

COEFFICIENT OF FLUCTUATION OF ENERGY

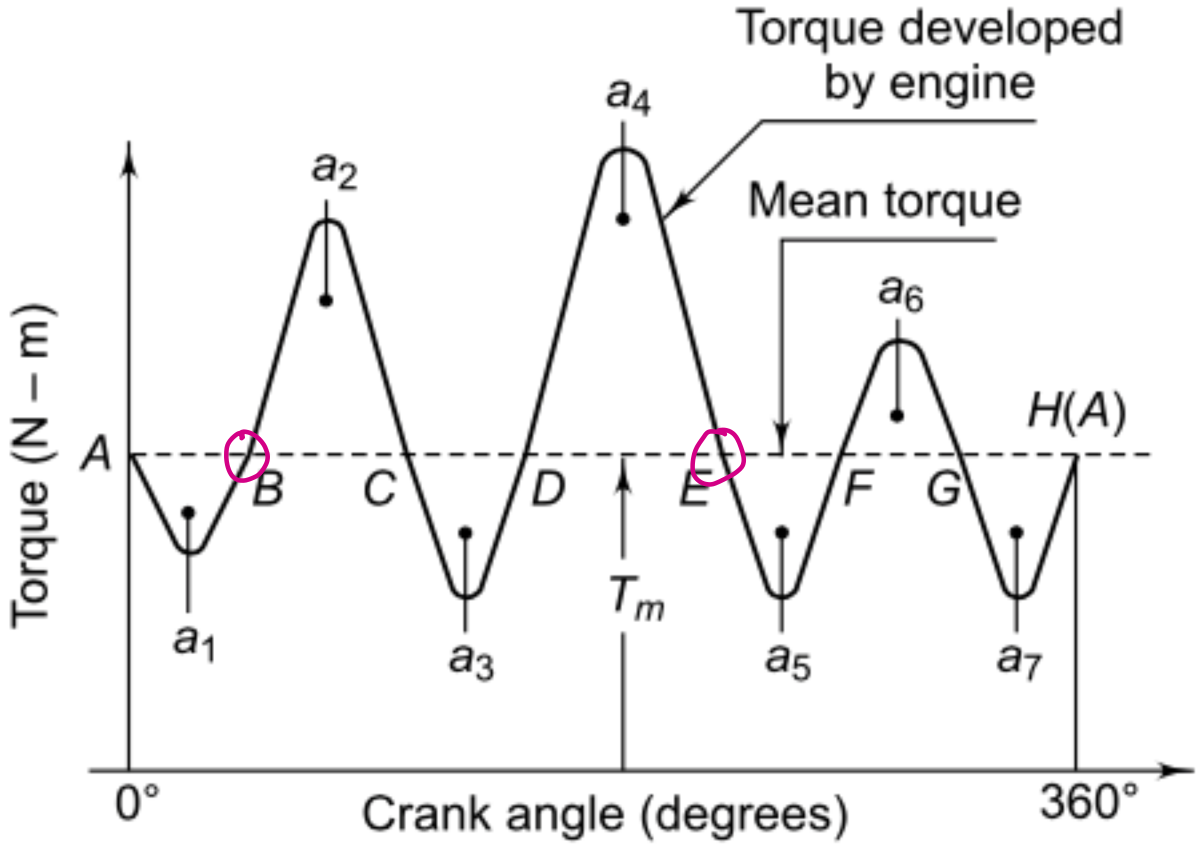


Fig. 21.5 *Turning Moment Diagram*

Energy @ A = U

Energy @ B = $U - a_1$

@ C = $U - a_1 + a_2$

@ D = $U - a_1 + a_2 - a_3$

@ E = $U - a_1 + a_2 - a_3 + a_4$

@ F = $U - a_1 + a_2 - a_3 + a_4 - a_5$

@ G = $U - a_1 + a_2 - a_3 + a_4 - a_5 + a_6$

@ H = $U - a_1 + a_2 - a_3 + a_4 - a_5 + a_6 - a_7 = U$

Max Energy = Energy @ Point E.

$$= \underline{U - a_1 + a_2 - a_3 + a_4}$$

Min Energy = Energy @ B

$$= \underline{U - a_1}$$

(Max − Min) energy = Max Fluctuation of energy

$$\frac{(\text{Max} - \text{Min})\ \text{Energy}}{\text{Work done / cycle}} = \text{Co-eff of fluctuation of Energy}$$

U_0 = Change in Energy $= a_2 - a_3 + a_4$

Table 21.3 *Coefficient of fluctuations of energy*

Type of engine	C_e
Single-cylinder, double-acting steam engine	0.21
Cross-compound steam engine	0.096
Single-cylinder, four-stroke petrol engine	1.93
Four-cylinder, four-stroke petrol engine	0.066
Six-cylinder, four-stroke petrol engine	0.031

work done per cycle := area enclosed by curves below mean torque line

$$W.D/\text{cycle} = (2\pi)T_m \rightarrow \text{2-stroke engine}$$

$$= (4\pi)T_m \rightarrow \text{4-stroke engine}$$

SOLID DISK FLYWHEEL

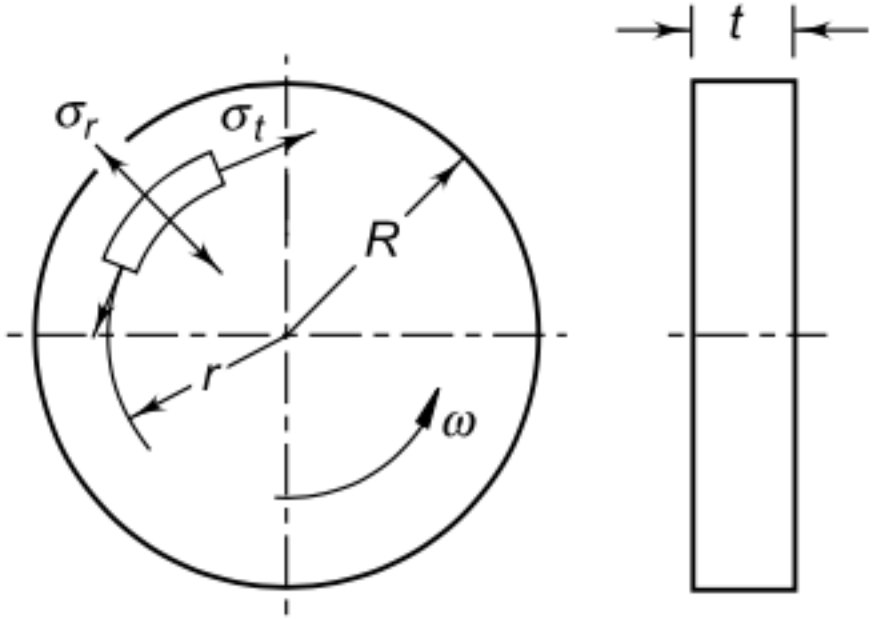


Fig. 21.6 *Solid Disk Flywheel*

$I = \frac{mR^2}{2}$

I – m.o.I

$R \to$ outer radius of the disk.

$m = F/a$

$m = W/g$

mass = density × volume

= ρ × area × thickness

$m = \rho \times \pi R^2 \times t$

$$\boxed{I = \frac{\pi}{2}\rho t R^4}$$

There are two principal stresses in the rotating disk—the tangential stress σ_t, and radial stress σ_r. The general equations for these stresses at a radius r are as follows:

$$\sigma_t = \frac{\rho v^2}{10^6}\left(\frac{\mu+3}{8}\right)\left[1-\left(\frac{3\mu+1}{\mu+3}\right)\left(\frac{r}{R}\right)^2\right]$$

$$\sigma_r = \frac{\rho v^2}{10^6}\left(\frac{\mu+3}{8}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$$

where,

σ_t = tangential stress at radius r (N/mm^2)

σ_r = radial stress at radius r (N/mm^2)

μ = Poisson's ratio (0.3 for steel and 0.27 for cast iron) $= 1/m$

v = peripheral velocity (m/s) ($R\omega$)

The maximum tangential stress and maximum radial stress are equal and both occur at ($r = 0$).

Therefore,

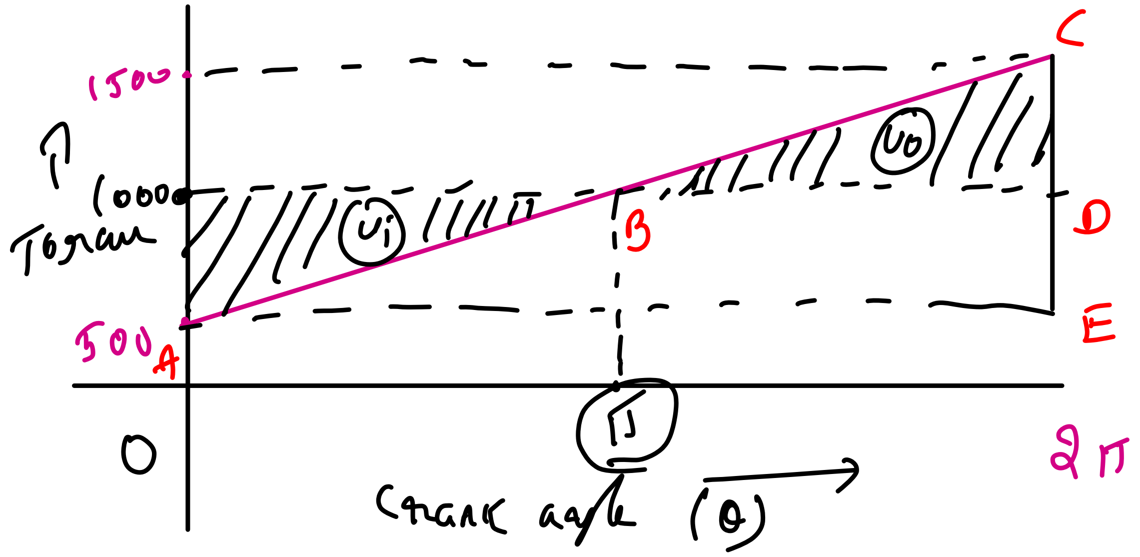
$$(\sigma_t)_{\text{max.}} = (\sigma_r)_{\text{max.}} = \frac{\rho v^2}{10^6}\left(\frac{\mu+3}{8}\right) \qquad (21.13)$$

Problem: A machine is driven by a motor, which exerts a constant torque. The resisting torque of the machine increases uniformly from 500 N-m to 1500 N-m through a 360° rotation of the driving shaft and drops suddenly to 500 N-m again at the beginning of the next revolution. The mean angular velocity of the machine is 30 rad/s and the coefficient of speed fluctuations is 0.2. A solid circular steel disk, 25 mm thick, is used as flywheel. The mass density of steel is 7800 kg/m^3 while Poisson's ratio is 0.3. Calculate the outer radius of the flywheel disk and the maximum stresses induced in it.

Given data: $\omega = 30$ rad/s ; $C_s = 0.2$; $T_{min} = 500$

$T_{max} = 1500$ N-m

$\rho = 7800$ kg/m^3 ; $\mu = 0.3$; $t = 25$ mm



U_0 = Area of BCD

$= \frac{1}{2} \times BD \times CD = \frac{1}{2} \times (2\pi - \pi) \times (1500 - 1000)$

$= 250\pi$ N-m (or) J

$U_0 = I\omega^2 C_s \Rightarrow I = \frac{U_0}{\omega^2 C_s}$

$= \frac{250\pi}{(30)^2 \times (0.2)} = 4.36$ kg-m^2

$I = \frac{\pi}{2} \rho t R^4$

$R^4 = \frac{2I}{\pi \rho t} = \frac{2 \times 4.36}{\pi \times 7800 \times 25 \times 10^{-3}}$

$R = 0.345\,m \simeq 0.35\,m$

Max stress → tangential (or) radial.

$(\sigma_t)_{max} = (\sigma_r)_{max} = \frac{\rho v^2}{10^6}\left(\frac{\mu + 3}{8}\right)$ MPa

$= \frac{7800 \times (0.35 \times 30)^2}{10^6}\left(\frac{0.3 + 3}{8}\right)$

$= 0.35$ MPa (or) 0.35 N/mm^2.

Problem: The torque developed by an engine is given by the following equation:

$$T = 14\,250 + 2200 \sin 2\theta - 1800 \cos 2\theta$$

where T is the torque in N-m and θ is the crank angle from the inner dead centre position. The resisting torque of the machine is constant throughout the work cycle. The coefficient of speed fluctuations is 0.01. The engine speed is 150 rpm. A solid circular steel disk, 50 mm thick, is used as a flywheel. The mass density of steel is 7800 kg/m^3. Calculate the radius of the flywheel disk.

Given data: $N = 150\,rpm$, $C_s = 0.01$; $t = 50\,mm$;

$$\rho = 7800\ kg/m^3$$

Mean torque $T_m = 14250$ N-m

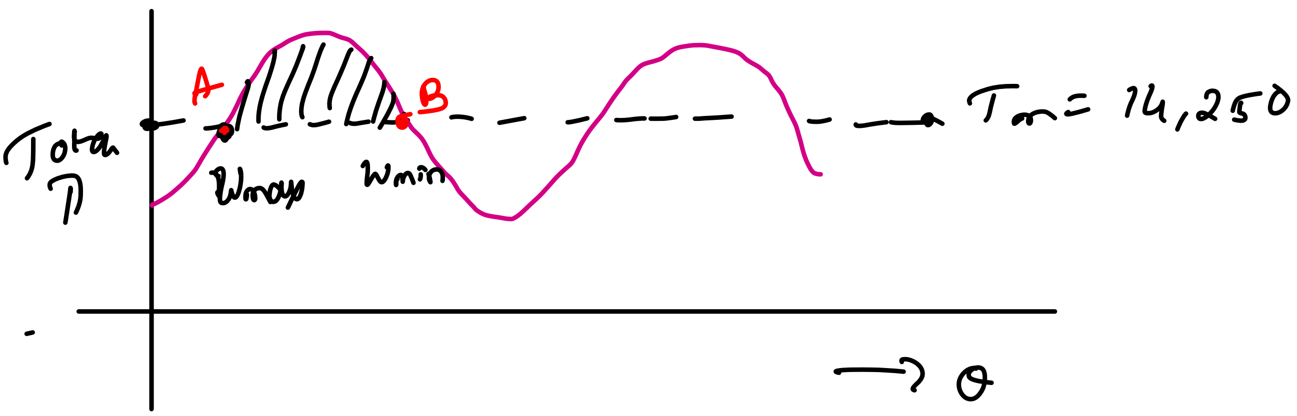
when $T = T_m$

$$\cancel{14,250} + 2200 \sin 2\theta - 1800 \cos 2\theta = \cancel{14,250}$$
$$= 0$$

$$\tan 2\theta = \frac{1800}{2200}$$

$2\theta = 39.29^\circ$ (or) $2\theta = 180 + 39.29^\circ$

$\theta = 19.645^\circ$ (or) $\theta = 109.645^\circ$



$$U_0 = \int_{\theta_A}^{\theta_B} (T - T_m)\, d\theta$$

$$= \int_{19.645}^{109.645} (2200 \sin 2\theta - 1800 \cos 2\theta)\, d\theta$$

$$= 2842.2\ \text{J}$$

$$U_0 = I \omega^2 C_s \Rightarrow I = \frac{U_0}{\omega^2 C_s}$$

$$= \frac{2842.2}{\left(\frac{2\pi \times 150}{60}\right)^2 \times 0.01}$$

$$I = 1151.9\ \text{kg-m}^2$$

$$R^4 = \frac{2I}{\pi \rho t} = \frac{\times 1151.9}{\pi \times 7800\,(50 \times 10^{-3})}$$

$$\boxed{R = 1175\ \text{mm}}$$

//

RIMMED FLYWHEEL

$V_0 = I\omega^2 C_s$

Due to the complicated geometric shapes of its component parts, it is difficult to determine the exact moment of inertia of this flywheel. Therefore, the analysis of such a flywheel is done by using anyone of the following two assumptions:

i. The spokes, the hub and the shaft do not contribute any moment of inertia, and the entire moment of inertia is due to the rim alone.

ii. The effect of spokes, the hub and the shaft is to contribute 10% of the required moment of inertia, while the rim contributes 90%.

$I = \frac{mR^2}{2}$

$I_r = KI$

I_r – m.I of rim

I → req m.I

The factor K is equal to 1 when the entire moment of inertia is due to the rim alone. The factor K is taken as 0.9, when it is assumed that the rim contributes 90% of the required moment of inertia.

$V_0 = I\omega^2 C_s \Rightarrow I = \frac{V_0}{\omega^2 C_s}$

$$\boxed{I_r = \frac{K V_0}{\omega^2 C_s}}$$

The thickness of the rim is usually very small compared with the mean radius. Therefore, it is assumed that the radius of gyration of the rim is equal to the mean radius

$I_r = m_r R^2$

m_r → mass of the rim (kg)

R → mean radius of rim

$I_r \propto m$

$I_r \propto R^2$

mass of the flywheel can be considerably reduced by increasing the mean radius for a required amount of moment of inertia. The aim should be to use the largest possible radius because it reduces the weight.

There are two limiting factors—speed and availability of space.

Flywheels are usually made of grey cast iron, for which the limiting mean rim velocity is 30 m/s. When the velocity exceeds this limit, there is a possibility of bursting due to centrifugal force,

$$v = \omega R \le 30$$

$$\boxed{R < \frac{30}{\omega}}$$

For high-speed applications, cast steel is used as flywheel material.

STRESSES IN RIMMED FLYWHEEL

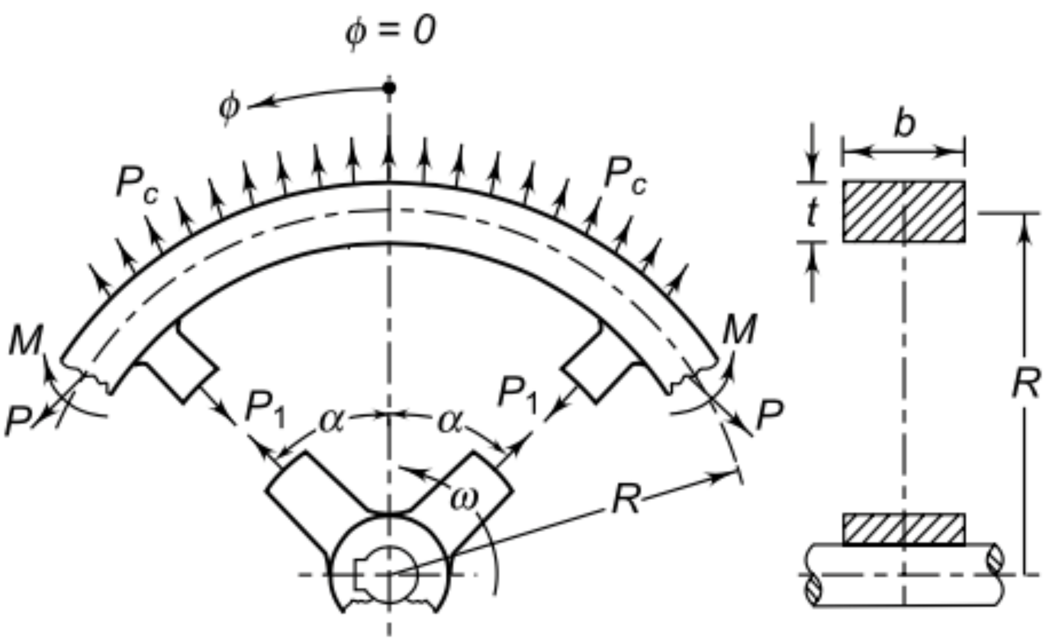


Fig. 21.9 *Forces and Moments in Rimmed Flywheel*

Tensile force = P_1 in spokes

$A_1 \rightarrow$ Area of spokes

$$\sigma_t = P_1 / A_1$$

Final total stress in rim

$$\sigma_t = \frac{P}{A} \pm \frac{My}{I}$$

$$= \frac{P}{bt} + \frac{6M}{bt^2}$$

$$I = \frac{bt^3}{12}$$

$$y = t/2$$

Problem: The turning moment diagram of a multi-cylinder engine is drawn with a scale of (1 mm = 1°) on the abscissa and (1 mm = 250 N-m) on the ordinate. The intercepted areas between the torque developed by the engine and the mean resisting torque of the machine, taken in order from one end are–350, +800, –600, +900, –550, +450 and –650 mm^2. The engine is running at a mean speed of 750 rpm and the coefficient of speed fluctuations is limited to 0.02. A rimmed flywheel made of grey cast iron FG 200 (ρ = 7100 kg/m^3) is provided. The spokes, hub and shaft are assumed to contribute 10% of the required moment of inertia. The rim has rectangular cross-section and the ratio of width to thickness is 1.5.

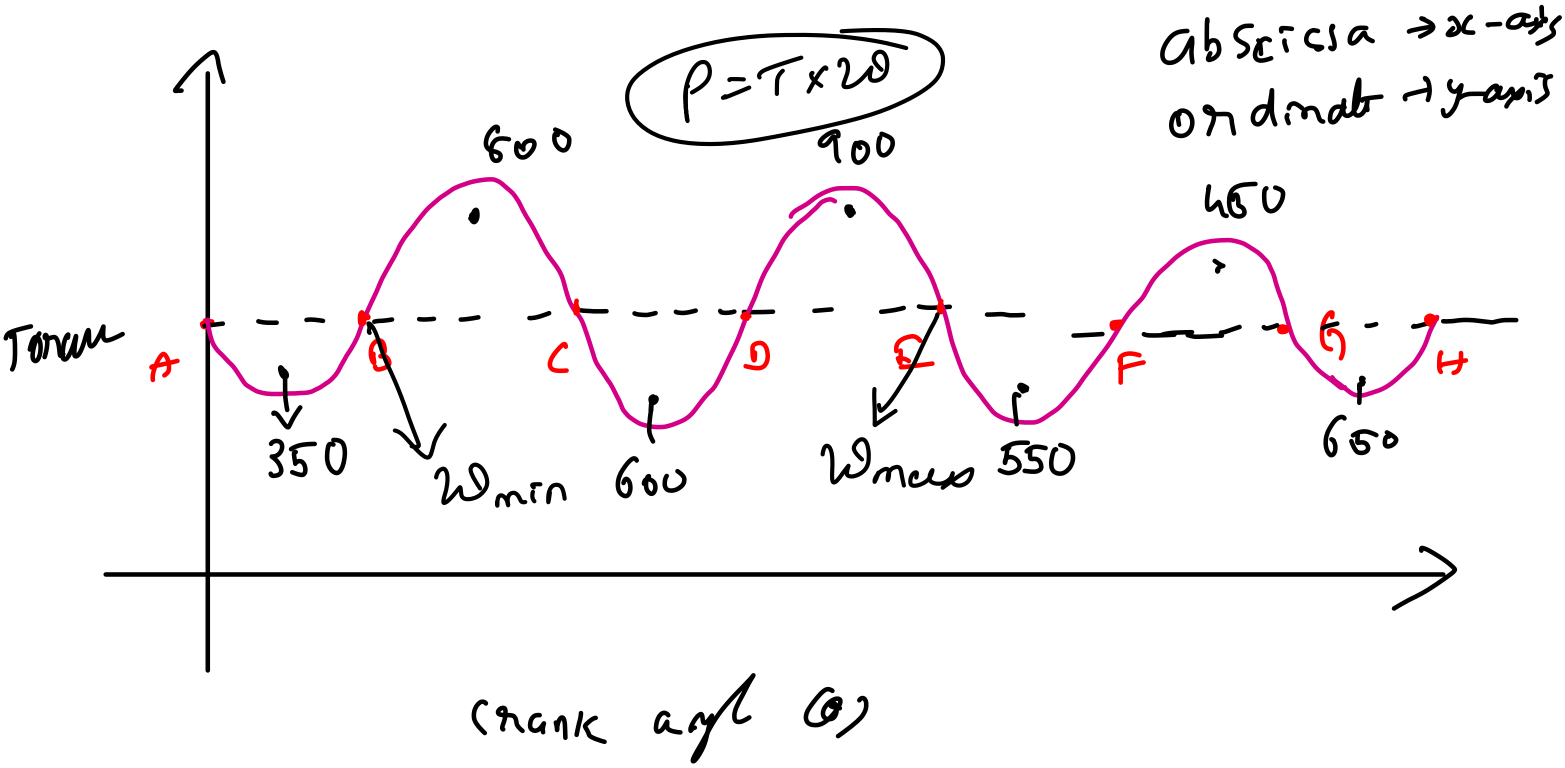
Determine the dimensions of the rim.

w, t, R

Given data:

$n = 750$ RPM ; $C_s = 0.02$; $b/t = 1.5$

$k = 0.9$; $\rho = 7100$ kg/m^3



Energy @ A = U

@ B = U − 350

@ C = U − 350 + 800 = U + 450

@ D = U + 450 − 600 = U − 150

@ E = U − 150 + 900 = U + 750

@ F = U + 750 − 550 = U + 200

@ G = U + 200 + 450 = U + 650

@ H = U + 650 − 650 = U

Max energy @ E → max Angular vel

min " @ B → Min angular vel

$U_E - U_B = U_0$ $\qquad 1^\circ = \frac{\pi}{180}$

$U_0 = (U + 750) - (U - 350)$

$U_0 = 1100 \text{ mm}^2$ $\qquad$ 1mm×1mm

$= 1100 \times 250 \times 1^\circ = 1100 \times 250 \times \frac{\pi}{180}$

$= 4799.65$ N-m $\qquad U_0 = I\omega^2 C_s$

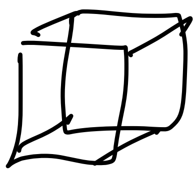
$\omega = \frac{2\pi N}{60} = \frac{2\pi(750)}{60} = (25\pi)$ rad/s

$I_R = KI = \frac{K \times U_0}{\omega^2 C_s} = \frac{0.9 \times 4799.65}{(25\pi)^2 \times 0.02}$ $\qquad U_0 = I\omega^2 C_s$

$I_R = 35 \text{ kg-m}^2$

$R < \frac{30}{\omega} \Rightarrow R < \frac{30}{25\pi}$ $\qquad v < 30 \text{ ms}^{-1}$

$R < 0.381$ m



$R = 0.35$ m

$I_R = m_R R^2 \Rightarrow m_R = I_R / R^2 = \frac{35}{(0.35)^2}$

$m_{r} = 285.71$ Kg

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\times \rho \qquad m = \rho \times V$$

V = area × l

$$285.71 = 2\pi(0.35)\left(\frac{1.5t}{10^6}\right)(t)\times 7100$$

$b = 1.5t$

$t = 110$ mm

$b = 1.5 \times t = 170$ mm

Dimensions of the Rim are:

$R = 0.35m = 350$ mm

$b = 170$ mm

$t = 110$ mm

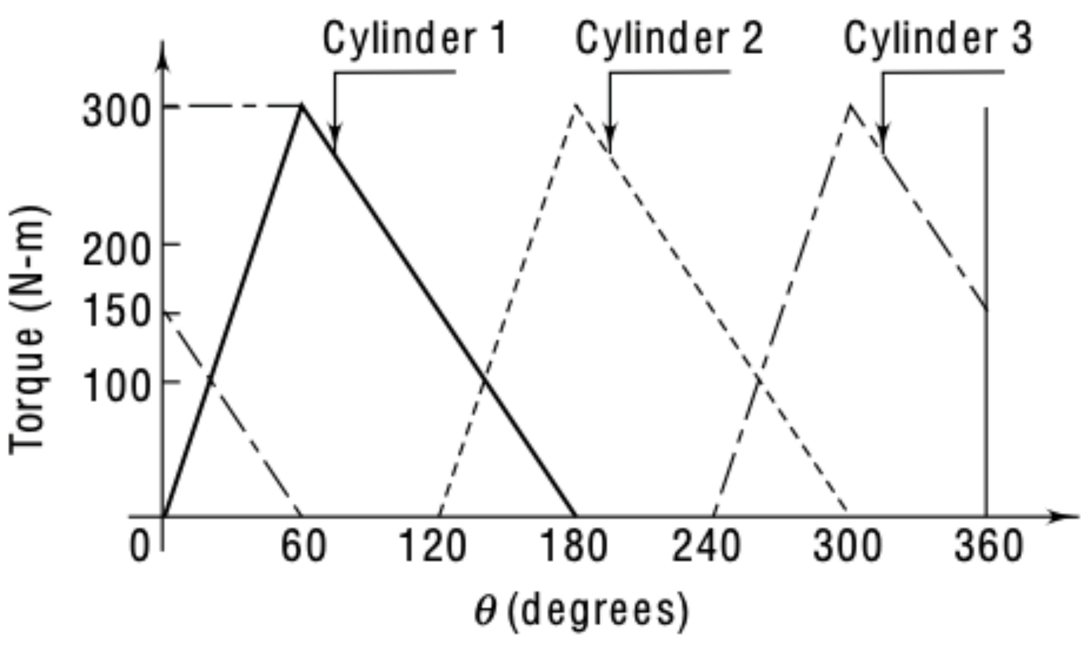
Problem: It is required to design a rimmed flywheel for an engine consisting of three single acting cylinders with their cranks set equally at 120° to each other. The torque-crank angle diagram for each cylinder consists of a triangle with the following values:

Crank angle (deg.)	*0*	*60*	*180*	*180 to 360*
Torque (N-m)	*0*	*300*	*0*	*0*

The engine runs at a mean speed of 240 rpm and the coefficient of speed fluctuations is limited to 0.03. The resisting torque of the machine is constant throughout the work cycle. From considerations of space, the mean radius of the rim should not exceed 0.25 m. It can be assumed that the rim contributes 90% of the required moment of inertia. The flywheel is made of grey cast iron FG(ρ = 7150 kg/m^3). The cross-section of the rim is square. Determine the dimensions of the cross-section of the rim.

Given data: $n = 240$ rpm; $C_s = 0.03$; $b/t = 1$

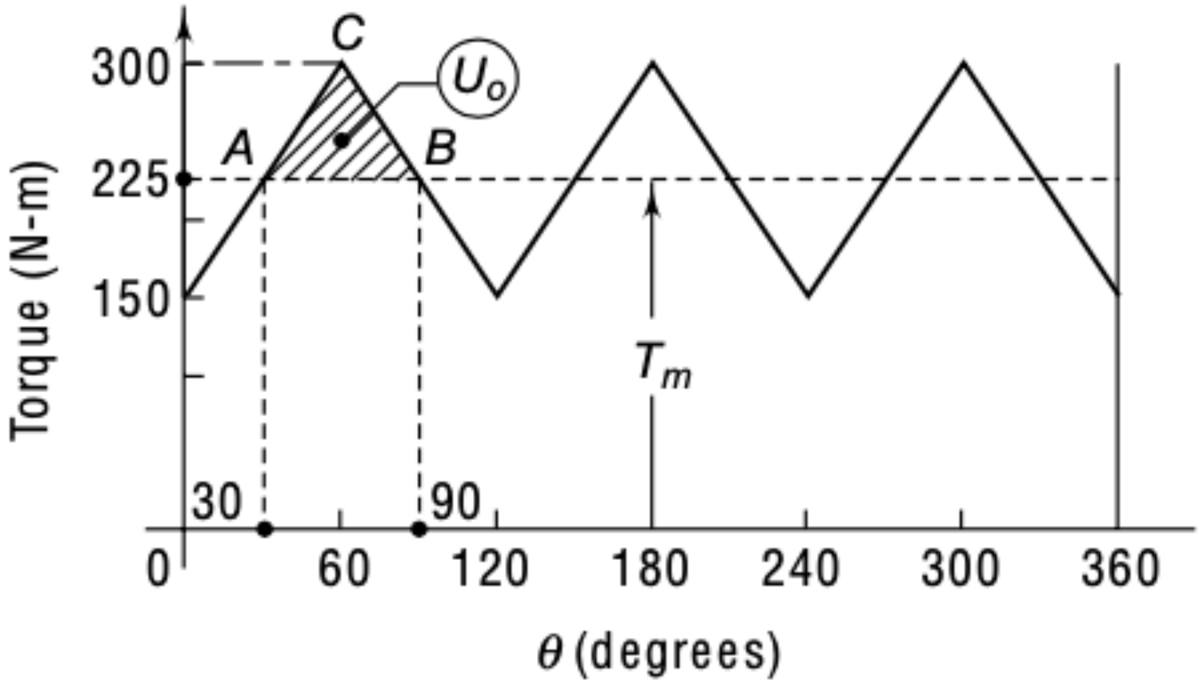
$K = 0.9$; $\rho = 7150$ kg/m^3 $R \leq 0.25$ m



Work done/cycle = area of 3 $\triangle^{le}$ = 3× area of each $\triangle^{le}$

$$= 3 \times \left[\frac{1}{\cancel{2}} \times \pi \times \overset{150}{\cancel{300}}\right]$$

$$= 450\pi \text{ J}$$



$$\text{Mean torque} = \frac{\text{W.D per revolution}}{\text{crank angle per revolution}}$$

$$\text{Mean torque} = \frac{450\cancel{\pi}}{2\cancel{\pi}} = 225 \text{ N-m}$$

$$1^\circ = \frac{\pi}{180}$$

$$U_0 = \text{Area of } \triangle^{le} ABC = \frac{1}{2} \times \frac{(90-30)}{180} \times \pi \times (300-225)$$

$$U_0 = 39.26 \text{ N-m}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 8\pi \text{ (rad/s)}$$

$$I_r = K \times I = \frac{0.9 \times U_0}{\omega^2 C_s} = \frac{0.9 \times 39.26}{(8\pi)^2 \times 0.03}$$

$$I_r = \underline{1.864} \; kg\text{-}m^2$$

$R = 0.25$

$$I_r = m_r R^2 \Rightarrow 1.864 = m_r \times (0.25)^2$$

$$m_r = 29.824 \; kg = \rho \times v$$

$$m_r = \underline{\rho} \times 2\pi R \left(\frac{t}{1000} \times \frac{t}{1000} \right) \qquad [b = \underline{t}]$$

$$29.84 = 7150 \times 2\pi \times 0.25 \times t^2 \times 10^{-6}$$

$$\boxed{t = b = 55 \; mm}$$

UNIT-4

Bearings

Bearing is a mechanical element that permits relative motion between two parts, such as the shaft and the housing, with minimum friction. The functions of the bearing are as follows:

1. The bearing ensures free rotation of the shaft or the axle with minimum friction.
2. The bearing supports the shaft or the axle and holds it in the correct position.
3. The bearing takes up the forces that act on the shaft or the axle and transmits them to the frame or the foundation.

Bearings are classified in different ways. Depending upon the direction of force that acts on them, bearings are classified into two categories—**radial and thrust bearings**

A radial bearing supports the load, which is perpendicular to the axis of the shaft.

A thrust bearing supports the load, which acts along the axis of the shaft.

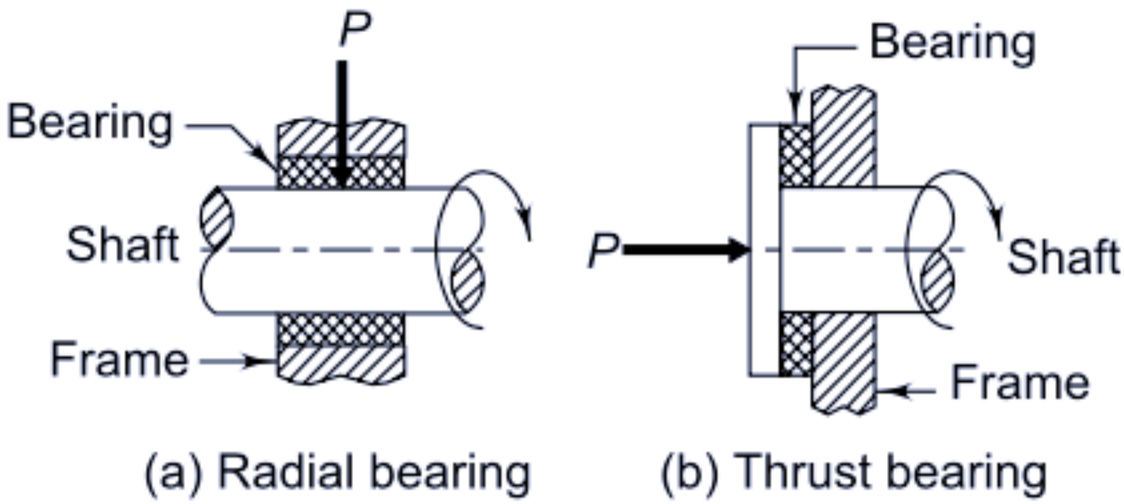


Fig. 15.1 *Radial and Thrust Bearings*

The most important criterion to classify the bearings is the type of friction between the shaft and the bearing surface.

Sliding contact bearings and **Rolling contact** bearings

Sliding contact bearings are also called **plain bearings, journal bearings or sleeve bearings**.

In this case, the surface of the shaft slides over the surface of the bush resulting in friction and wear. In order to reduce the friction, these two surfaces are separated by a film of lubricating oil.

Rolling contact bearings are also called **antifriction bearings or simply ball bearings**.

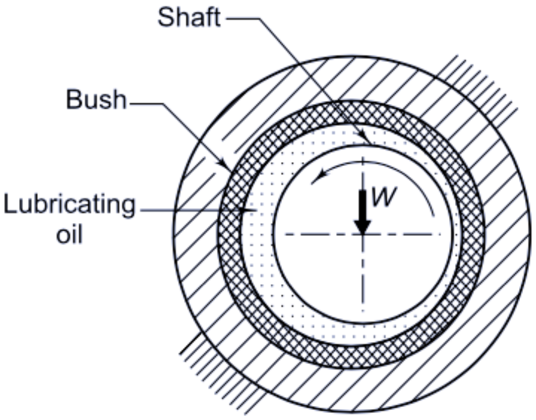
Rolling elements, such as balls or rollers, are introduced between the surfaces that are in relative motion. In this type of bearing, sliding friction is replaced by rolling friction.

Sliding contact bearings are used in the following applications:

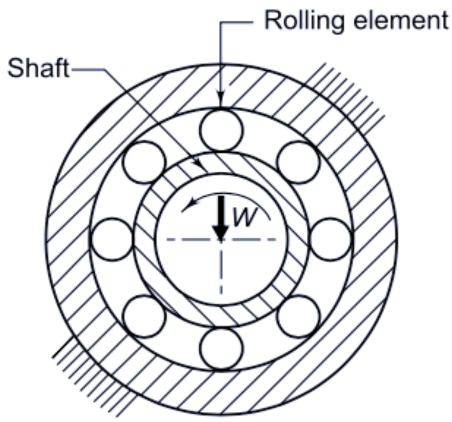
a) crankshaft bearings in petrol and diesel engines;
b) centrifugal pumps;
c) large size electric motors;
d) steam and gas turbines; and
e) concrete mixers, rope conveyors and marine installations.

Rolling contact bearings are used in the following applications:

1. machine tool spindles;
2. automobile front and rear axles;
3. gear boxes;
4. small size electric motors; and
5. rope sheaves, crane hooks and hoisting drums.



(a) Sliding contact bearing



(b) Rolling contact bearing

TYPES OF ROLLING CONTACT BEARINGS

For starting conditions and at moderate speeds, the frictional losses in rolling contact bearing are lower than that of equivalent hydrodynamic journal bearing.

> This is because the sliding contact is replaced by rolling contact resulting in low coefficient of friction. Therefore, rolling contact bearings are called 'antifriction' bearings.

A rolling contact bearing consists of four parts— **inner and outer races**, a **rolling element** like ball, roller or needle and a **cage** which holds the rolling elements together and spaces them evenly around the periphery of the shaft.

Depending upon the type of rolling element, the bearings are classified as

i. ball bearing,
ii. cylindrical roller bearing,
iii. taper roller bearing and
iv. needle bearing.

Depending upon the direction of load, the bearings are also classified as **radial bearing and thrust bearing.**

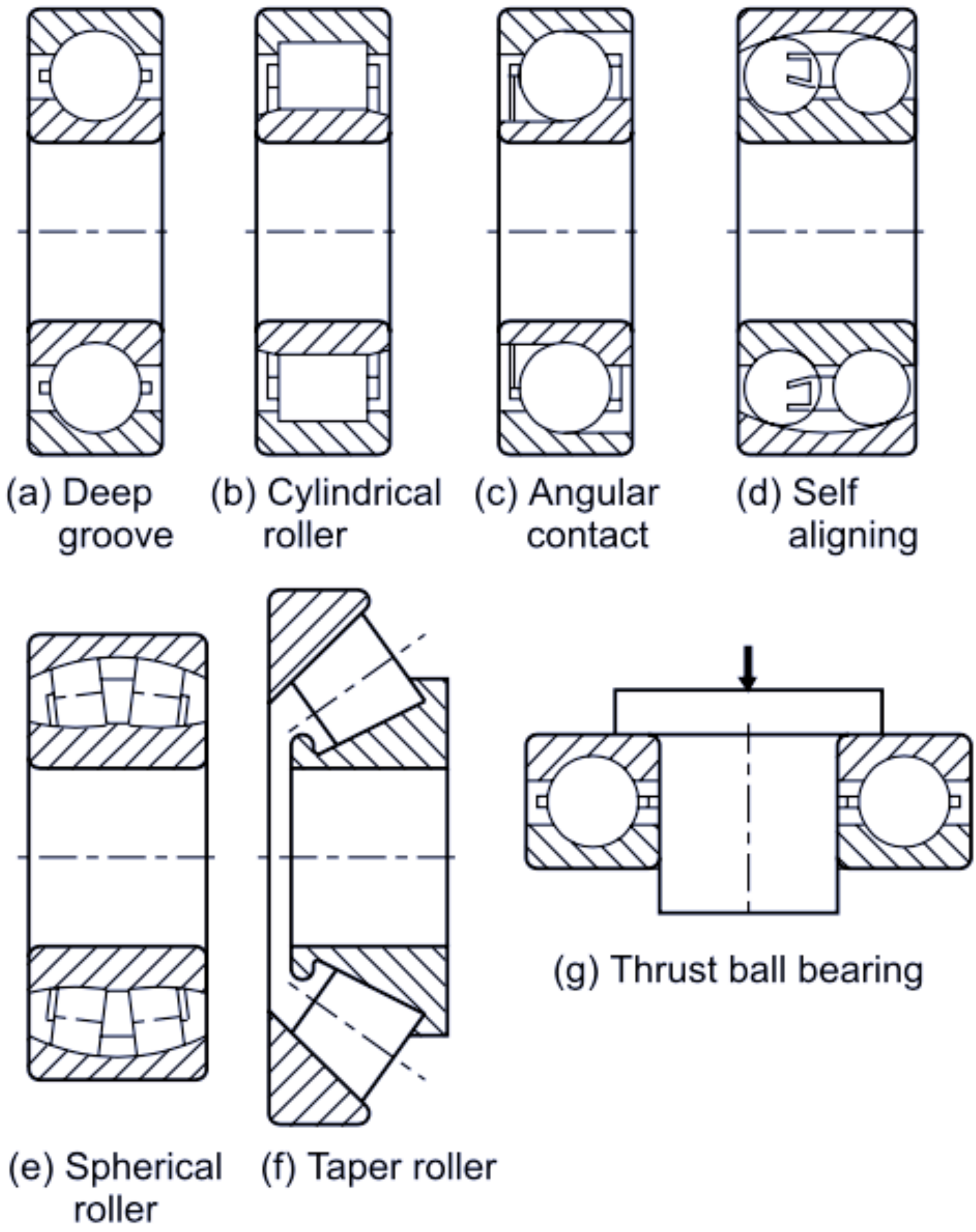


Fig. 15.3 *Types of Rolling Contact Bearing*

Deep Groove Ball Bearing The most frequently used bearing is the deep groove ball bearing.

The radius of the ball is slightly less than the radii of curvature of the grooves in the races.

Deep groove ball bearing has the following advantages:

1. Due to relatively large size of the balls, deep groove ball bearing has high load carrying capacity.
2. Deep groove ball bearing takes loads in the radial as well as axial direction.
3. Due to point contact between the balls and races, frictional loss and the resultant temperature rise is less in this bearing. The maximum permissible speed of the shaft depends upon the temperature rise of the bearing. Therefore, deep groove ball bearing gives excellent performance, especially in high speed applications.
4. Deep groove ball bearing generates less noise due to point contact.
5. Deep groove ball bearings are available with bore diameters from a few millimetres to 400 millimetres.

The disadvantages of deep groove ball bearings are as follows:

6. Deep groove ball bearing is not self- aligning. Accurate alignment between axes of the shaft and the housing bore is required.
7. Deep groove ball bearing has poor rigidity compared with roller bearing. This is due to the point contact compared with the line contact in case of roller bearing. It is unsuitable for machine tool spindles where rigidity is important consideration.

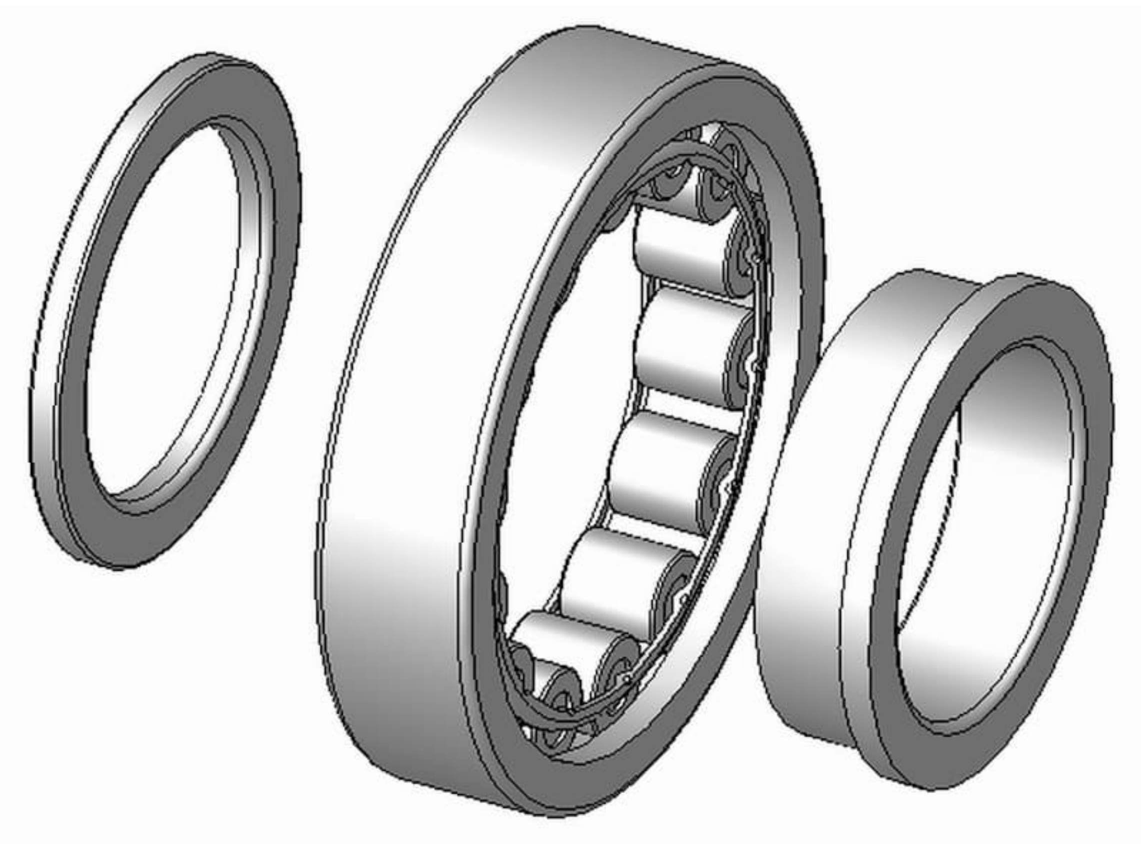
Cylindrical Roller Bearing When maximum load carrying capacity is required in a given space, the point contact in ball bearing is replaced by the line contact of roller bearing. A cylindrical roller bearing consists of relatively short rollers that are positioned and guided by the cage.

Cylindrical roller bearing offers the following advantages:

a. Due to line contact between rollers and races, the radial load carrying capacity of the cylindrical roller bearing is very high.
b. Cylindrical roller bearing is more rigid than ball bearing.
c. The coefficient of friction is low and frictional loss is less in high-speed applications.

The disadvantages of cylindrical roller bearing are as follows:

a. In general, cylindrical roller bearing cannot take thrust load.
b. Cylindrical roller bearing is not self-aligning. It cannot tolerate misalignment. It needs precise alignment between axes of the shaft and the bore of the housing.
c. Cylindrical roller bearing generates more noise.



Angular Contact Bearing In angular contact bearing, the grooves in inner and outer races are so shaped that the line of reaction at the contact between balls and races makes an angle with the axis of the bearing. This reaction has two components— radial and axial. Therefore, angular contact bearing can take radial and thrust loads. Angular contact bearings are often used in pairs, either side by side or at the opposite ends of the shaft, in order to take the thrust load in both directions.

Angular contact bearings offer the following advantages:

a. Angular contact bearing can take both radial and thrust loads.
b. In angular contact bearing, one side of the groove in the outer race is cut away to permit the insertion of larger number of balls than that of deep groove ball bearing. This permits the bearing to carry relatively large axial and radial loads. Therefore, the load carrying capacity of angular contact bearing is more than that of deep groove ball bearing.

The disadvantages of angular contact bearings are as follows:

a. Two bearings are required to take thrust load in both directions.
b. The angular contact bearing must be mounted without axial play.
c. The angular contact bearing requires initial pre-loading.

Self-aligning Bearings There are two types of self-aligning rolling contact bearings, viz., self- aligning ball bearing and spherical roller bearing.

The self-aligning ball bearing consists of two rows of balls, which roll on a common spherical surface in the outer race. In this case, the assembly of the shaft, the inner race and the balls with cage can freely roll and adjust itself to the angular misalignment of the shaft.

There is similar arrangement in the spherical roller bearing, where balls are replaced by two rows of spherical rollers, which run on a common spherical surface in the outer race. Compared with the self- aligning ball bearing, the spherical roller bearing can carry relatively high radial and thrust loads.

They are therefore particularly suitable for applications where misalignment can arise due to errors in mounting or due to deflection of the shaft. They are used in agricultural machinery, ventilators, and railway axle-boxes.

Taper roller bearing: The taper roller bearing consists of rolling elements in the form of a frustum of cone. They are arranged in such a way that the axes of individual rolling elements intersect in a common apex point on the axis of the bearing. In kinematics' analysis, this is the essential requirement for pure rolling motion between conical surfaces. In taper roller bearing, the line of resultant reaction through the rolling elements makes an angle with the axis of the bearing. Therefore, taper roller bearing can carry both radial and axial loads.

a. Taper roller bearing can take heavy radial and thrust loads.
b. Taper roller bearing has more rigidity.
c. Taper roller bearing can be easily assembled and disassembled due to separable parts.

The disadvantages of taper roller bearing are as follows:

a) It is necessary to use two taper roller bearings on the shaft to balance the axial force.
b) It is necessary to adjust the axial position of the bearing with pre-load. This is essential to coincide the apex of the cone with the common apex of the rolling elements.
c) Taper roller bearing cannot tolerate misalignment between the axes of the shaft and the housing bore.
d) Taper roller bearings are costly.

Thrust Ball Bearing A thrust ball bearing consists of a row of balls running between two rings—the shaft ring and the housing ring. Thrust ball bearing carries thrust load in only one direction and cannot carry any radial load. The use of a large number of balls results in high thrust load carrying capacity in smaller space.

The <u>disadvantages</u> of thrust bearings are as follows:

a) Thrust ball bearing cannot take radial load.
b) It is not self-aligning and cannot tolerate misalignment.
c) Their performance is satisfactory at low and medium speeds. At high speeds, such bearings give poor service because the balls are subjected to centrifugal forces and gyroscopic couple.
d) Thrust ball bearings do not operate as well on horizontal shafts as they do on vertical shafts.
e) Thrust ball bearing requires continuous pressure applied by springs to hold the rings together.

PRINCIPLE OF SELF-ALIGNING BEARING

In many applications, the bearing is required to tolerate a small amount of misalignment between the axes of the shaft and the bearing. The misalignment may be due to deflection of the shaft under load or due to tolerances of individual components.

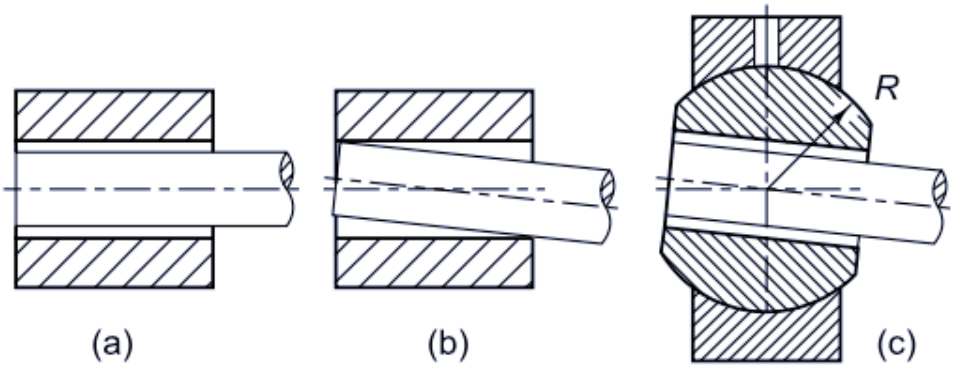


Fig. 15.4 *Self aligning Bearing: (a) Shaft aligned with Bearing (b) Shaft misaligned with Bearing (c) Self aligning Bearing*

In self-aligning bearing, the external surface of the bearing bush is made spherical as shown in Fig. 15.4(c). The centre of this spherical surface is at the centre of the bearing. Therefore, the bush is free to roll in its seat and align itself with the journal.

SELECTION OF BEARING-TYPE

The guidelines for selecting a proper type of bearing are as follows:

1. For low and medium radial loads, ball bearings are used, whereas for heavy loads and large shaft diameters, roller bearings are selected.
2. Self-aligning ball bearings and spherical roller bearings are used in applications where a misalignment between the axes of the shaft and housing is likely to exist.
3. Thrust ball bearings are used for medium thrust loads whereas for heavy thrust loads, cylindrical roller thrust bearings are recommended. Double acting thrust bearings can carry the thrust load in either direction.
4. Deep groove ball bearings, angular contact bearings and spherical roller bearings are suitable in applications where the load acting on the bearing consists of two components— radial and thrust.

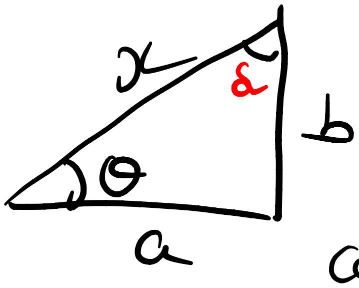
STATIC LOAD CARRYING CAPACITY

Static load is defined as the load acting on the bearing when the shaft is stationary. It produces permanent deformation in balls and races, which increases with increasing load. The permissible static load, therefore, depends upon the permissible magnitude of permanent deformation.

The static load carrying capacity of a bearing is defined as the static load which corresponds to a total permanent deformation of balls and races, at the most heavily stressed point of contact, equal to 0.0001 of the ball diameter.

STRIBECK'S EQUATION

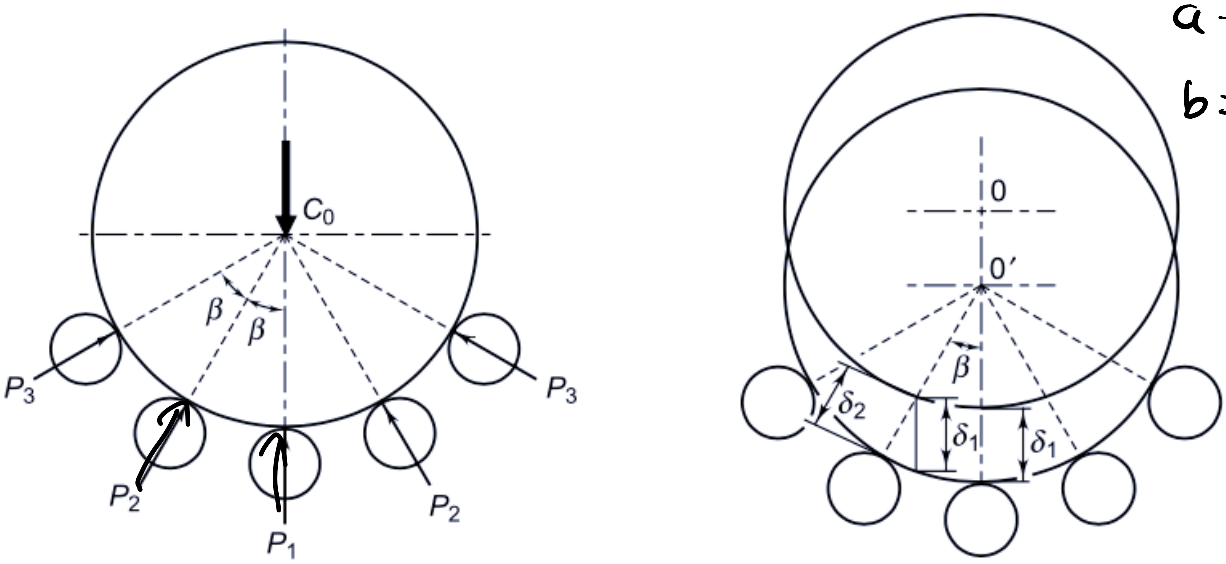
A. The races are rigid and retain their circular shape.
B. The balls are equally spaced.
C. The balls in the upper half do not support any load.



$\cos\theta = \frac{a}{x}$

$a = x\cos\theta$

$b = x\sin\theta$



(a) (b)

$$C_0 = P_1 + P_2\cos\beta + P_3\cos 2\beta + P_2\cos\beta + P_3\cos 2\beta$$

$$C_0 = P_1 + 2P_2\cos\beta + 2P_3\cos 2\beta \quad \text{— (1)}$$

$\delta_1, \delta_2, \delta_3 - - -$ → radial deflection of the balls

$$\delta_2 = \delta_1 \cos\beta \Rightarrow \frac{\delta_2}{\delta_1} = \cos\beta \quad — (2)$$

By Hertz's equation

$$\delta \propto (P)^{2/3}$$

$$\delta_1 = C_1 (P_1)^{2/3} \quad ; \quad \delta_2 = C_1 (P_2)^{2/3}$$

$$\frac{\delta_2}{\delta_1} = \left(\frac{P_2}{P_1}\right)^{2/3} \quad — (3)$$

Subs, eqn (2) in (3)

$$\left(\frac{P_2}{P_1}\right)^{2/3} = \cos\beta$$

$$P_2 = P_1 (\cos\beta)^{3/2} \quad — (3a)$$

$$\frac{P_3}{P_2} = (\cos\beta)^{3/2} \Rightarrow P_3 = P_2 (\cos\beta)^{3/2} \quad ; \quad P_3 = P_1 (\cos 2\beta)^{3/2} \quad — (3b)$$

Subs eqn 3(a) & 3 (b) in (1)

Z	8	10	12	15
M	1.84	2.28	2.75	3.47
(z/M)	4.35	4.38	4.36	4.37

$$C_0 = P_1 + 2\left[P_1 (\cos\beta)^{3/2}\right]\cos\beta + 2\left[P_1 (\cos(2\beta))^{3/2} \times \cos 2\beta\right]$$

$$C_0 = P_1 \left[1 + 2(\cos\beta)^{5/2} + 2(\cos 2\beta)^{5/2} + \cdots\right]$$

$C_0 = P_1 M$ — (4) $\quad M = 1 + 2(\cos\beta)^{5/2} + 2(\cos 2\beta)^{5/2} + \ldots$

$\beta = \frac{360}{Z}$ $\quad Z \rightarrow$ no. of balls.

$Z/m = 5 \Rightarrow m = \frac{Z}{5}$ — (4a)

(4a) in (4)

$\Rightarrow \boxed{C_0 = \frac{P_1 Z}{5}} \rightarrow$ (5)

$P_1 = K d^2$ $\quad K \rightarrow$ radii of curvature factor

$d \rightarrow$ dia of the balls

$\boxed{C_0 = \frac{K d^2 Z}{5}} \rightarrow$ Stribeck's equation.

DYNAMIC LOAD CARRYING CAPACITY

The dynamic load carrying capacity of the bearing is based on the fatigue life of the bearing.

- *The life of an individual ball bearing is defined as the number of revolutions (or hours of service at some given constant speed), which the bearing runs before the first evidence of fatigue crack in balls or races.*
- Since the life of a single bearing is difficult to predict, it is necessary to define the life in terms of the statistical average performance of a group of bearings.

Bearings are rated on one of the two criteria—the average life of a group of bearings or the life, which 90% of the bearings will reach or exceed.

The second criterion is widely used in bearing industry.

The rating life of a group of apparently identical ball bearings is defined as the number of revolutions that 90% of the bearings will complete or exceed before the first evidence of fatigue crack. → L_{10}

EQUIVALENT BEARING LOAD

The equivalent dynamic load is defined as the constant radial load in radial bearings (or thrust load in thrust bearings), which if applied to the bearing would give same life as that which the bearing will attain under actual condition of forces.

$$P = XVF_r + YF_a$$

F_r → radial load

F_a → axial load

P → equivalent dynamic load

V → race rotation factor

X → factor of radial load

Y → thrust factor

when inner race is rotating & outer race is stationary ⇒ $V = 1$

[redacted]

when outer race is rotating & inner race is stationary $\Rightarrow$ $V = 1.2$

$$P = XF_r + YF_a$$

For pure radial load $\Rightarrow$ $X = 1$, $Y = 0$

$$P = F_r$$

For pure axial load $\Rightarrow$ $X = 0$, $Y = 1$

$$P = F_a$$

Load-Life Relationship:

$$L_{10} = \left(\frac{C}{P}\right)^{p}$$

P → dynamic load

L_{10} → rated bearing life in million revs

C → dynamic load capacity

$p = 3$ → ball bearing

$p = 10/3$ → roller bearing

$$C = P(L_{10})^{1/p}$$

$$C = P(L_{10})^{1/3} \rightarrow \text{Ball bearing}$$

$$C = P(L_{10})^{0.3} \rightarrow \text{roller bearing}$$

$$L_{10} = \frac{60 n L_{10h}}{10^6}$$

$n \rightarrow rpm$

$L_{10h} \rightarrow$ rated life in hours.

Problem: In a particular application, the radial load acting on a ball bearing is 5 kN and the expected life for 90% of the bearings is 8000 h. Calculate the dynamic load carrying capacity of the bearing, when the shaft rotates at 1450 rpm.

$F_a \rightarrow$ thrust load

Given data: $F_r = 5\,kN$; $L_{10h} = 8000\,h$

$N = 1450$ RPM

$$L_{10} = \frac{60 \times n \times L_{10h}}{10^6} = \frac{60(1450)(8000)}{10^6}$$

$$L_{10} = 696 \text{ million rev} \qquad C = P(L_{10})^{1/p}$$

The load is purely radial,

$$\underline{P = F_r = 5000\,N}$$

$$L_{10} = \left(\frac{C}{P}\right)^p \qquad p = 3 \text{ (ball bearing)}$$

$$696 = \left(\frac{C}{5000}\right)^3$$

$$C = (696)^{1/3} \times 5000 = 44{,}310.5\,N$$

Problem: A taper roller bearing has a dynamic load capacity of 26 kN. The desired life for 90% of the bearings is 8000 h and the speed is 300 rpm. Calculate the equivalent radial load that the bearing can carry.

C $\qquad$ P

Given data:

$C = 26\,kN$; $L_{10h} = 8000\,h$; $N = 300$ Rpm

$F_r = ?$

$C = P \times (L_{10})^{1/p}$

$$L_{10} = \frac{60 \times n \times L_{10h}}{10^6} = \frac{60 \times 300 \times 8000}{10^6}$$

$L_{10} = 144$ million revs

$$L_{10} = \left(\frac{C}{P}\right)^p$$

$p = 10/3$ (roller bearing)

$$P = \frac{C}{(L_{10})^{1/p}} = \frac{26 \times 10^3}{(144)^{0.3}}$$

$P = 5854\,N$

Since bearing is subjected to only radial load

$P = F_r = 5854\,N$

SELECTION OF BEARING LIFE

Table 15.1 *Bearing life for wheel applications*

Wheel application	*Life (million rev.)*
Automobile cars	50
Trucks	100
Trolley cars	500
Rail-road cars	1000

Table 15.2 *Bearing life for industrial applications*

(i)	Machines used intermittently such as lifting tackle, hand tools and household appliances	4000–8000 h
(ii)	Machines used for eight hours of service per day, such as electric motors and gear drives	12 000–20 000 h
(iii)	Machines used for continuous operation (24 h per day) such as pumps, compressors and conveyors	40 000–60 000 h

LOAD FACTOR:

The forces acting on the bearing are calculated by considering the equilibrium of forces in vertical and horizontal planes. These elementary equations do not take into consideration the effect of dynamic load. The forces determined by these equations are multiplied by a *load factor* to determine the dynamic load carrying capacity of the bearing. Load factors are used in applications involving gear, chain and belt drives.

Table 15.3 *Values of load factor*

Types of drive	*Load factor*
(A) *Gear drives*	
(i) Rotating machines free from impact like electric motors and turbo-compressors	1.2–1.4
(ii) Reciprocating machines like internal combustion engines and compressors	1.4–1.7
(iii) Impact machines like hammer mills	2.5–3.5
(B) *Belt drives*	
(i) V-belts	2.0
(ii) Single-ply leather belt	3.0
(iii) Double-ply leather belt	3.5
(C) *Chain drives*	1.5

Problem: A single-row deep groove ball bearing is subjected to a pure radial force of 3 kN from a shaft that rotates at 600 rpm. The expected life L10h of the bearing is 30 000 h. The minimum acceptable diameter of the shaft is 40 mm. Select a suitable ball bearing for this application.

Given data:

$F_r = 3000\ N = P$; $N = 600$ RPM

$L_{10h} = 30{,}000\ h$; $d = 40\ mm$ $\quad C = P\left[L_{10}\right]^{1/K}$

Dynamic load capacity:

$$L_{10} = \frac{L_{10h} \times n \times 60}{10^6} = \frac{30{,}000 \times 600 \times 60}{10^6}$$

$L_{10} = 1080$ million rev

Ball bearings

$$L_{10} = \left(\frac{C}{P}\right)^{k} \qquad k = 3$$

$$C = (L_{10})^{1/k} \times P = (1080)^{1/3} \times 3000$$

$$C = 30,779.56 \text{ N}$$

$1 \text{ kgf} = 9.805 \text{ N}$

$$C = \frac{30,779.56}{9.805} = 3139.16 \text{ kgf}$$

From Data book; For $d = 40$ mm, & $C = 3139$ kgf,

Bearing 6308 is most suitable.

Problem: A single-row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The shaft rotates at 1200 rpm. The expected life L_{10h} of the bearing is 20 000 h. The minimum acceptable diameter of the shaft is 75 mm. Select a suitable ball bearing for this application.

Given data: $F_r = 8\,kN$; $F_a = 3\,kN$; $N = 1200\,RPM$

$L_{10h} = 20{,}000\,h$; $d = 75\,mm$

$$L_{10} = \frac{60\,n\,L_{10h}}{10^6} = \frac{60 \times 1200 \times 20{,}000}{10^6}$$

$$L_{10} = 1440 \text{ million rev}$$

$P \rightarrow$ dynamic Load

$$P = X F_r + Y F_a$$

From data book (Equivalent bearing Load)

$X = 0.56$; $Y = 1.5$; $F_r = 8000\,N$; $F_a = 3000\,N$

$$P = X F_r + Y F_a = 0.56(8000) + 1.5(3000)$$

$$\underline{P = 8980\,N}$$

$$C = P(L_{10})^{1/3} = 8980(1440)^{1/3} = 101{,}406.04\,N$$

$$C = 10{,}343\ Kgf$$

From data book, for $C = 10{,}343$ kgf & $d = 75$ mm,

Bearing 6415 is suitable.

For 6415 → $C_0 = 10{,}160$ kgf

$= 99{,}608.64$ N

$$\frac{F_a}{F_r} = \frac{3000}{8000} = 0.375$$

$\frac{F_a}{C_0} = 0.04 \rightarrow e \rightarrow 0.24 \rightarrow y = 1.8$

$$\frac{F_a}{C_0} = \frac{3000}{99{,}608} = 0.03$$

$\frac{F_a}{C_0} = 0.03$

0.025 → 2

0.04 → 1.8

(0.03) x

From data book, for $\frac{F_a}{C_0} = 0.03$,

$e = 0.24$ (app)

$$\frac{F_a}{F_r} > e$$

$$\frac{|y_2 - y_1|}{|x_2 - x_1|} \times (x_2 - x)$$

$$y = 2 - \left[\frac{(2 - 1.8)}{(0.04 - 0.025)} \times (0.04 - 0.03)\right]$$

$y = 1.866$ & $X = 0.56$

$P = XF_r + YF_a = (0.56)8000 + (1.866)3000$

$P = 10{,}078\ N$

$C = P\,(L_{10})^{1/3} = (10{,}078)\,(1440)^{1/3}$

$C = 113{,}805.133\ N$

$d = 75\ mm$

$C = 11{,}608.03\ Kgf$

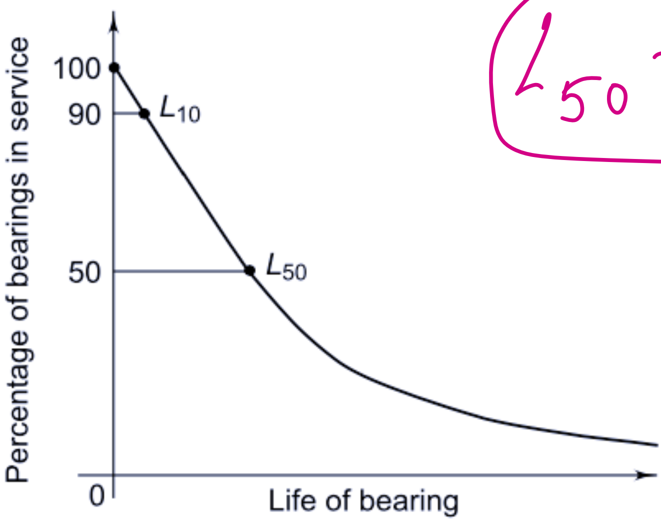
Our bearing <u>6415 is safe</u> ✓

BEARING WITH A PROBABILITY OF SURVIVAL OTHER THAN 90 PER CENT

In the definition of rating life, it is mentioned that the rating life is the life that 90% of a group of identical bearings will complete or exceed before fatigue failure.

$$R = \frac{\text{No. of bearings which have successfully completed } L \text{ million revolutions}}{\text{Total number of bearings under test}}$$

In certain applications, where there is risk to human life, it becomes necessary to select a bearing having a reliability of more than 90%



$L_{50} = 5 \times L_{10}$

The relationship between bearing life and reliability is given by a statistical curve known as Wiebull distribution.

$$R = e^{-(L/a)^b}$$

$R \to$ Reliability (fraction)

$L \to$ life of bearing

$a, b \to$ constants

$$\frac{1}{R} = e^{(L/a)b}$$

$$\boxed{e^{-x} = \frac{1}{e^{x}}}$$

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b \quad -(1)$$

If L_{10} is the life of 90% Successful bearing

$\downarrow$

R_{90}

$$\log_e\left(\frac{1}{R_{90}}\right) = \left(\frac{L_{10}}{a}\right)^b \quad -(2)$$

$$(1)/(2) \Rightarrow \left(\frac{L}{L_{10}}\right)^b = \frac{\log\left(\frac{1}{R}\right)}{\log\left(\frac{1}{R_{90}}\right)}$$

$$\frac{L}{L_{10}} = \left[\frac{\log\left(\frac{1}{R}\right)}{\log\left(\frac{1}{R_{90}}\right)}\right]^{1/b} \quad -(3)$$

$$L_{50} = 5 \times L_{10} \longrightarrow R_{50} = 0.5$$

$$\frac{L_{50}}{L_{10}} = \left[\frac{\log\left(\frac{1}{R_{50}}\right)}{\log\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

Solving we get

$$\boxed{b = 1.17;\ a = 6.84}$$

Total reliability in a system of "N" bearings

$$\boxed{R_s = (R)^N}$$

$$R_s = (0.9)^{10}$$

$$= 0.348$$

Problem: A single-row deep groove ball bearing is subjected to a radial force of 8 kN and a thrust force of 3 kN. The values of X and Y factors are 0.56 and 1.5 respectively. The shaft rotates at 1200 rpm. The diameter of the shaft is 75 mm and Bearing No. 6315 (C =112 000 N) is selected for this application.

1. Estimate the life of this bearing, with 90% reliability.
2. Estimate the reliability for 20,000 h life.

Given data:

$F_r = 8000\,N$; $F_a = 3000\,N$; $x = 0.56$, $Y = 1.5$

$n = 1200\,rpm$; $d = 75\,mm$; $C = 112{,}000\,N$

$R_{90} \rightarrow L_{10}$

$k = 3$

$$L_{10} = \left(\frac{C}{P}\right)^k$$

$$P = XF_r + YF_a = (0.56 \times 8000) + (1.5 \times 3000)$$
$$= 8980\,N$$

$$L_{10} = \left(\frac{112{,}000}{8980}\right)^3 = 1940.10 \text{ million rev}$$

$$L_{10} = \frac{60 n L_{10h}}{10^6} \Rightarrow L_{10h} = \frac{L_{10} \times 10^6}{60 \times n}$$

" R_{90}

$n \rightarrow 1200\,rpm$

$$L_{10h} = 26945.83 \text{ hours}$$

Reliability for 20,000 h :

$$\frac{L}{L_{10}} = \left[\frac{\log\left(\frac{1}{R}\right)}{\log\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

$$\log\left(\frac{1}{R}\right) = \log\left(\frac{1}{R_{90}}\right)\left[\frac{L}{L_{10}}\right]^{b}$$

$$= \log_e\left(\frac{1}{0.9}\right)\left[\frac{20,000}{26,945.83}\right]^{1.17}$$

$R_{90} = 0.9;$
$b = 1.17$
$L = 20,000\ h$

$$\log\left(\frac{1}{R}\right) = 0.04575574 \times 0.7055535$$

$$\log_e\left(\frac{1}{R}\right) = 0.0322842$$

$$\frac{1}{R} = 1.03281$$

$$R = 0.9682$$

$$R = 96.82\%$$

LUBRICATION OF ROLLING CONTACT BEARINGS

a) The purpose of lubrication in antifriction bearings is to reduce the friction between balls and races.
b) The other objectives are dissipation of frictional heat, prevention of corrosion and protection of the bearing from dirt and other foreign particles.

There are two types of lubricants—oil and grease.

Compared with grease, oil offers the following advantages:

i. It is more effective in carrying frictional heat.
ii. It feeds more easily into contact areas of the bearing under load.
iii. It is more effective in flushing out dirt, corrosion and foreign particles from the bearing.

The advantages offered by grease lubricated bearings are

a) simple housing design
b) less maintenance cost
c) better sealing against rust and
d) less possibility of leakage

The guidelines for selecting the lubricant are as follows:

1. When the temperature is less than 100°C, grease is suitable, while lubricating oils are preferred for applications where the temperature exceeds 100°C.
2. When the product of [bore (in mm) X speed (in rpm)] is below 200 000, grease is suitable. For higher values, lubricating oils are recommended.
3. Grease is suitable for low and moderate loads, while lubricating oils are used for heavy duty applications.
4. If there is a central lubricating system, which is required for the lubrication of other parts, the same lubricating oil is used for bearings, e.g., gearboxes.

SLIDING CONTACT BEARINGS

BASIC MODES OF LUBRICATION

Lubrication is the science of reducing friction by application of a suitable substance called lubricant, between the rubbing surfaces of bodies having relative motion. The lubricants are classified into following three groups:

(i) Liquid lubricants like mineral or vegetable oils
(ii) Semi-solid lubricants like grease
(iii) Solid lubricants like graphite or molybdenum disulphide

The objectives of lubrication are as follows:

(i) to reduce friction
(ii) to reduce or prevent wear
(iii) to carry away heat generated due to friction
(iv) to protect the journal and the bearing from corrosion

The basic modes of lubrication are **thick-and thin film** lubrication. In addition, sometimes a term 'zero film' bearing is used. Zero film bearing is a bearing which operates without any lubricant, i.e., without any film of lubricating oil.

Thick film lubrication describes a condition of lubrication, where *two surfaces of the bearing in relative motion are completely separated by a film of fluid*. Since there is no contact between the surfaces, the properties of surface, like surface finish, have little or no influence on the performance of the bearing. The resistance to relative motion arises from the viscous resistance of the fluid. Therefore, the viscosity of the lubricant affects the performance of the bearing.

Thick film lubrication is further divided into two groups:

hydrodynamic and hydrostatic lubrication.

Hydrodynamic lubrication is defined as a system of lubrication in which the load-supporting fluid film is created by the shape and relative motion of the sliding surfaces.

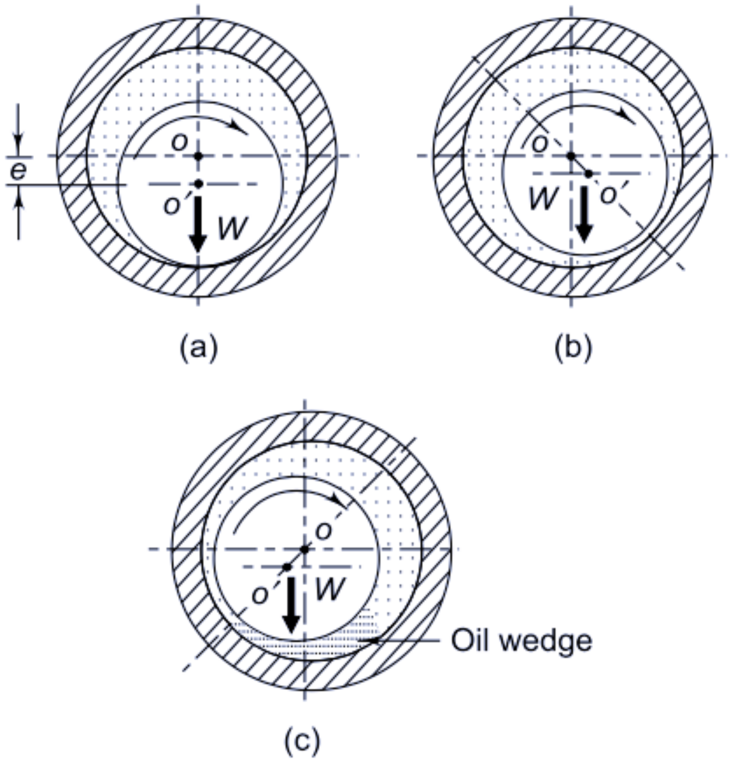


Fig. 16.1 *Hydrodynamic Lubrication (a) Journal at Rest (b) Journal Starts to Rotate (c) Journal at Full Speed*

Since more and more fluid is forced into the wedge-shaped clearance space, pressure is generated within the system. Since the pressure is created within the system due to rotation of the shaft, this type of bearing is known as *self-acting bearing*.

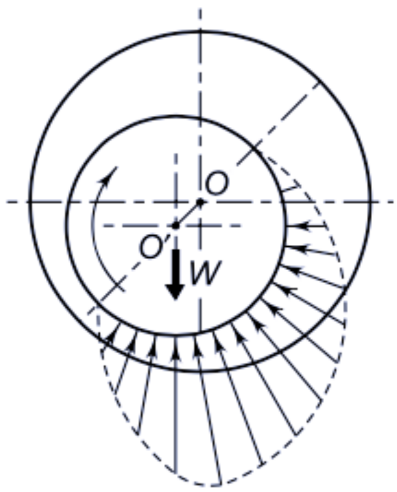


Fig. 16.2 *Pressure Distribution in Hydrodynamic Bearing*

This mode of lubrication is seen in bearings mounted on engines and centrifugal pumps

A journal bearing is a sliding contact bearing working on hydrodynamic lubrication and which supports the load in radial direction. The portion of the shaft inside the bearing is called journal and hence the name 'journal' bearing.

There are two types of hydrodynamic journal bearings, namely, full journal bearing and partial bearing.

- In *full journal bearing*, the angle of contact of the bushing with the journal is 360°. Full journal bearing can take loads in any radial direction.
- In *partial bearings*, the angle of contact between the bush and the journal is always less than 180°. Most of the partial bearings in practice have 120° angle of contact. Partial bearing can take loads in only one radial direction. Partial bearings are used in railroad-cars.

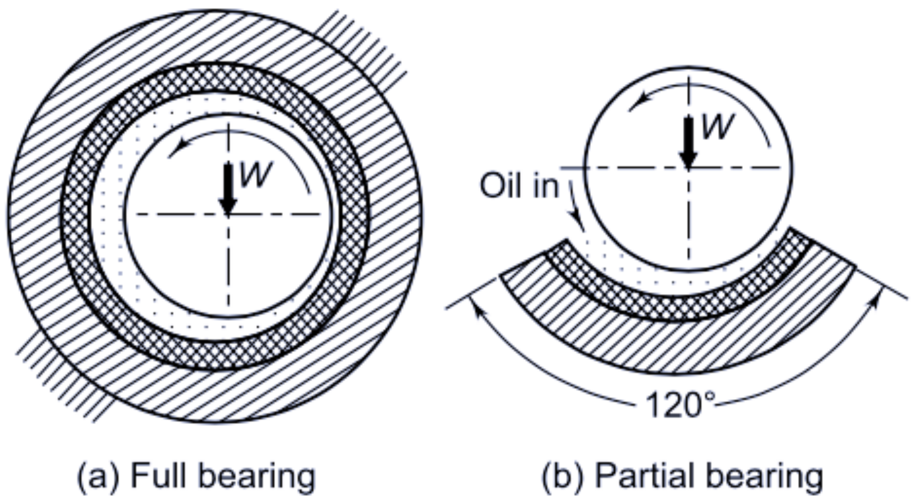


Fig. 16.3 *Full and Partial Bearings*

The advantages of partial bearings compared to full journal bearing are as follows:

(i) Partial bearing is simple in construction.

(ii) It is easy to supply lubricating oil to the partial bearing.

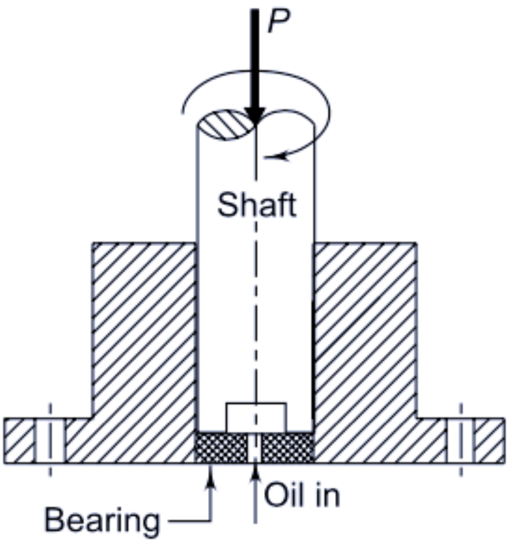
(iii) The frictional loss in partial bearing is less. Therefore, temperature rise is low.

There are two terms with reference to full and partial bearings, namely, 'clearance' bearing and 'fitted' bearing.

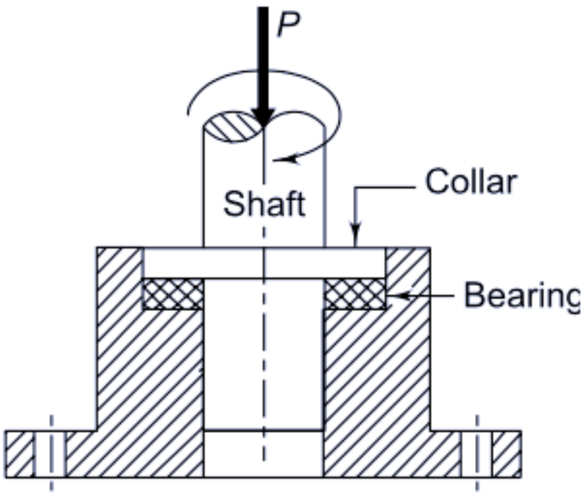
1. A clearance bearing is a bearing in which the radius of the journal is less than the radius of the bearing. Therefore, there is a clearance space between the journal and the bearing. Most of the journal bearings are of this type.
2. A fitted bearing is a bearing in which the radius of the journal and the bearing are equal. Obviously, fitted bearing must be partial bearing and the journal must run eccentric with respect to the bearing in order to provide space for lubricating oil.

There are two types of thrust bearings which take axial load, namely 'footstep' bearing and 'collar' bearing

1. The footstep bearing or simply 'step' bearing is a thrust bearing in which the end of the shaft is in contact with the bearing surface.
2. The collar bearing is a thrust bearing in which a collar integral with the shaft is in contact with the bearing surface. In this case, the shaft continues through the bearing. The shaft can be with single collar or can be with multiple collars.



(a) Footstep bearing



(b) Collar bearing

Hydrostatic lubrication is defined as a system of lubrication in which the load supporting fluid film, separating the two surfaces is created by an external source, like a pump, supplying sufficient fluid under pressure. Since the lubricant is supplied under pressure, this type of bearing is called externally pressurised bearing.

Hydrostatic bearings are used on vertical turbo generators, centrifuges and ball mills.

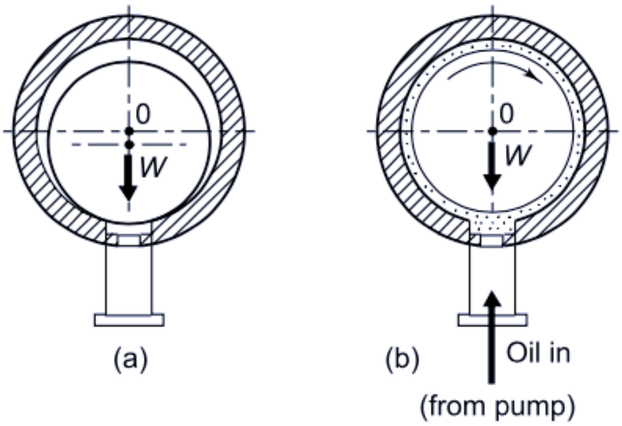


Fig. 16.5 *Hydrostatic Lubrication: (a) Journal at Rest (b) Journal at Full Speed*

Compared with hydrostatic bearings, hydro- dynamic bearings are simple in construction, easy to maintain and lower in initial as well as maintenance cost. Hydrostatic bearings, although costly, offer the following advantages:

(i) high load carrying capacity even at low speeds;

(ii) no starting friction; and

(iii) no rubbing action at any operating speed or load.

Thin film lubrication, which is also called boundary lubrication, is defined as a condition of lubrication where the lubricant film is relatively thin and there is **partial metal to metal contact**. This mode of lubrication is seen in door hinges and machine tool slides. The conditions resulting in boundary lubrication are excessive load, insufficient surface area or oil supply, low speed and misalignment.

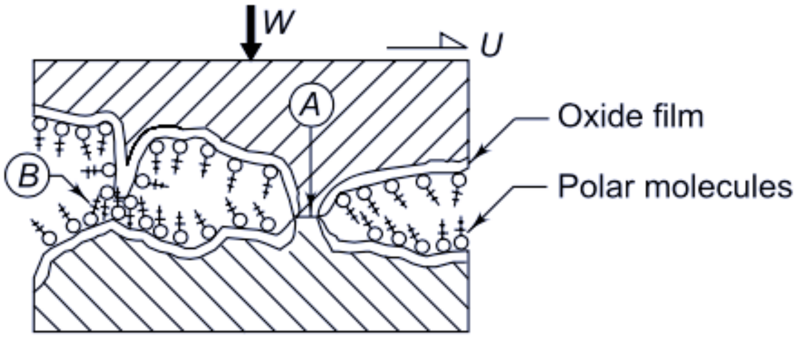


Fig. 16.6 *Boundary Lubrication: (a) Metal to Metal Contact (b) Cluster of Molecules*

There are certain fatty acids which contain polar molecules. Molecules in which there is a permanent separation of positive and negative charges are called polar molecules. Their polarity has a tendency to orient and stick to the surface in a particular fashion.

- The clusters of polar molecules, cohering to one another and adhering to the surface, form a compact film which prevents metal to metal contact as is seen in the region B.
- This results in partial lubrication. There is also a zone (region A) where metal to metal contact takes place, junctions are formed at high spots and shearing takes place due to relative motion.

The performance of bearing under boundary lubrication depends upon two factors, namely, the chemical composition of the lubricating oil, such as polar molecules (at the region B), and surface roughness (at region A). The hydrodynamic bearing also operates under the boundary lubrication when the speed is very low or when the load is excessive.

BEARING MATERIALS

The desirable properties of a good bearing material are as follows:

(i) When metal to metal contact occurs, the bearing material should not damage the surface of the journal. It should not stick or weld to the journal surface.

(ii) It should have high compressive strength to withstand high pressures without distortion.

(iii) In certain applications like connecting rods or crankshafts, bearings are subjected to fluctuating stresses. The bearing material, in these applications, should have sufficient endurance strength to avoid failure due to pitting

(iv) The bearing material should have the ability to yield and adopt its shape to that of the journal. This property is called conformability. When the load is applied, the journal is deflected resulting in contact at the edges. A conformable material adjusts its shape under these circumstances.

(v) The dirt particles in lubricating oil tend to jam in the clearance space and, if hard, may cut scratches on the surfaces of the journal and bearing. The bearing material should be soft to allow these particles to get embedded in the lining and avoid further trouble. This property of the bearing material is called embeddability.

(vi) In applications like engine bearings, the excessive temperature causes oxidation of lubricating oils and forms corrosive acids. The bearing material should have sufficient corrosion resistance under these conditions.

(vii) The bearing material should have reasonable cost and should be easily available in the market.

- The most popular bearing material is **babbitt**. Due to its silvery appearance, babbitt is called 'white' metal. There are two varieties of babbitts— lead-base and tin-base, depending upon the major alloying element.
- Babbitts have excellent conformability and embeddability. Tin-base babbitts have better corrosion resistance and can be easily bonded to steel shells. High cost and shortage of tin are their main limitations.

- The original formula for this bearing metal was 89.3% tin, 7.1% antimony and 3.6% copper

There are ten grades of white metals4 and some among them and their typical applications are as follows:

(i) Grade 90 and Grade 84 are used for bearings in petrol and diesel engines and in crossheads of steam engines.

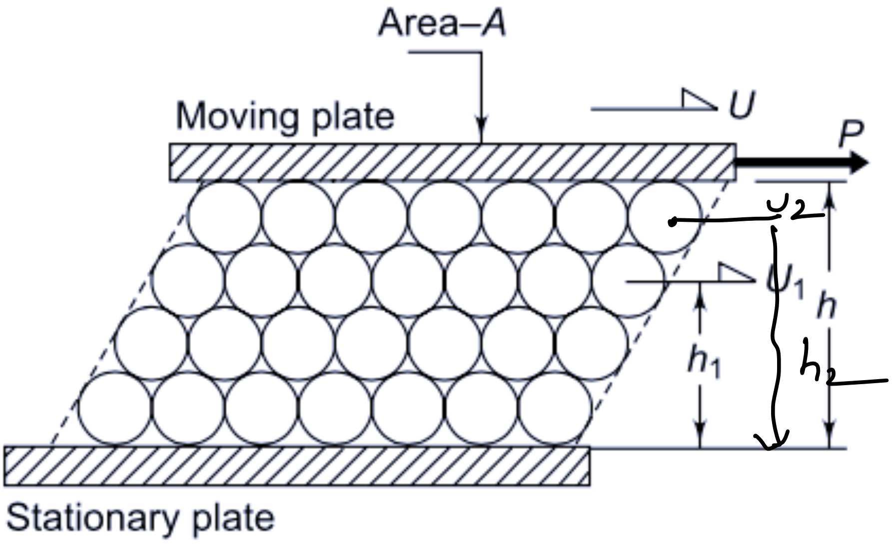
(ii) Grade 75 is used for repair jobs in mills and marine installations.

(iii) Grade 69 is used in underwater applications and gland packing.

(iv) Grade 60 is used in dynamos, electric motors, centrifugal pumps and other medium speed applications.

The other bearing materials are bronze, copper, lead, aluminium alloys and plastics.

VISCOSITY

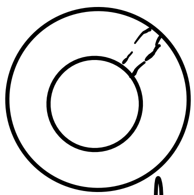


$$\frac{U}{h} = \frac{U_1}{h_1} = \frac{U_2}{h_2}$$

P → tangential force   A → Area

$$\frac{\text{Force}}{\text{unit Area}} \propto \text{rate of shear}$$

Newton's law of viscosity



$$\frac{\text{velocity}}{\text{distance}}$$

$$\frac{P}{A} \propto \left(\frac{U}{h}\right)$$

$$P \propto A\left(\frac{U}{h}\right) \Rightarrow P = \mu A\left(\frac{U}{h}\right)$$

co-eff of viscosity

$$\mu = \frac{Ph}{AU}$$ $N\text{-}s/mm^2$ (or) $N\text{-}s/m^2$

$\Rightarrow 10^6\ N\text{-}s/m^2$

MPa-s

N/m^2 - Pa

Common unit is poise

$$1\,\text{poise} = 1\ \text{dyne-s}/\text{cm}^2 = 0.1\ \text{N-s}/\text{m}^2$$

$$= 10^{-7}\ \text{N-s}/\text{mm}^2$$

$1\,\text{m} = 10^3\,\text{mm}$

$1\,\text{m}^2 = 10^6\,\text{mm}^2$

$$1\,\text{centipoise} = \frac{1}{100}\ \text{poise}$$

$$= \frac{1}{100}\left(\frac{\text{dyne-s}}{\text{cm}^2}\right)$$

$$= \frac{1}{100}\left(0.1\ \frac{\text{N-s}}{\text{m}^2}\right)$$

$$= \frac{1}{100} \times \frac{1}{10}\ \frac{1}{10^6}\ \frac{\text{N-s}}{\text{mm}^2}$$

$$1\,\text{CP} = 10^{-9}\ \text{N-s}/\text{mm}^2$$

$\mu \rightarrow$ viscosity in $\text{N-s}/\text{mm}^2$

$Z \rightarrow$ " " centipoise

$$\boxed{\mu = \frac{Z}{10^9}}$$

MEASUREMENT OF VISCOSITY

The popular method of determining viscosity is to measure the time required for a given volume of oil to pass through a capillary tube of standard dimensions. The oil is kept in a reservoir, which is immersed in the constant temperature bath.

Based on this principle, there are three commercial viscometers

A. Saybolt viscometer → USA
B. Redwood viscometer and → UK
C. Engler viscometer → India

In the Saybolt universal viscometer, 60 cm^3 of lubricating oil is passed through a capillary tube of standard dimensions and the time is measured in seconds. The unit of viscosity is called Saybolt Universal Seconds (SUS)

$$Z_k = \left[0.22t - \frac{180}{t}\right]$$

Z_k → kinematic viscosity

$\nu = \mu/\rho$ centistokes

In Engler viscometer calculating the ratio of time of outflow of 50 ml of liquid at 20 ° C to the time of outflow of the same volume of distilled water at 25 °C, kinematic viscosity is found out.

VISCOSITY INDEX

The viscous resistance of lubricating oil is due to intermolecular forces. As the temperature increases, the oil expands and the molecules move further apart, decreasing the intermolecular force in consequence. Therefore, the viscosity of the lubricating oil decreases with increasing temperature.

$\mu \rightarrow$ Co-eff of viscosity

$$\log \mu = A + \frac{B}{T}$$

$T \rightarrow$ Temp

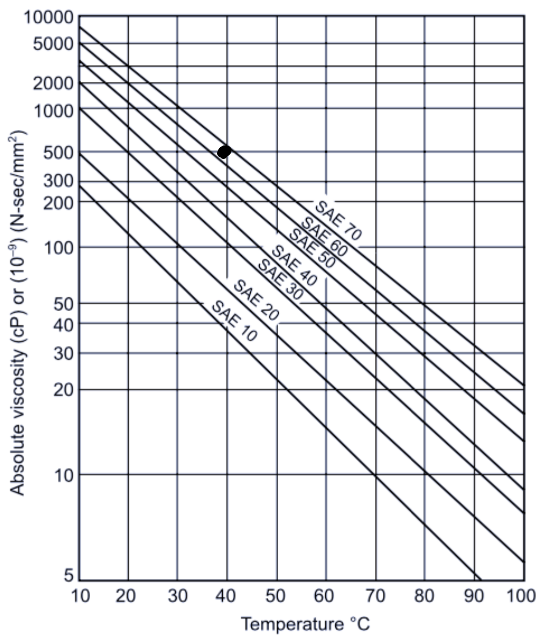


Fig. 16.8 *Viscosity–Temperature Relationship*

The rate of change of viscosity with respect to temperature is indicated by a number called Viscosity Index (VI). *The viscosity index is defined as an arbitrary number used to characterize the variation of the kinematic viscosity of lubricating oil with temperature.*

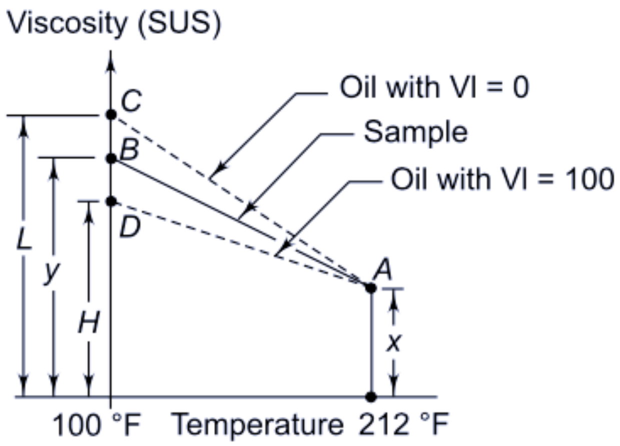


Fig. 16.9 *Viscosity Index*

$$VI = \left(\frac{L-y}{L-H}\right) \times 100\%$$

$VI = 70$

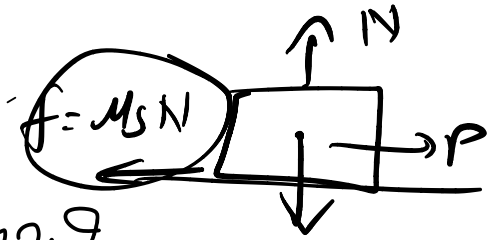
$VI = 50$

PETROFF'S EQUATION

Petroff's equation is used to determine the coefficient of friction in journal bearings. It is based on the following assumptions:

(i) The shaft is concentric with the bearing.

(ii) The bearing is subjected to light load.



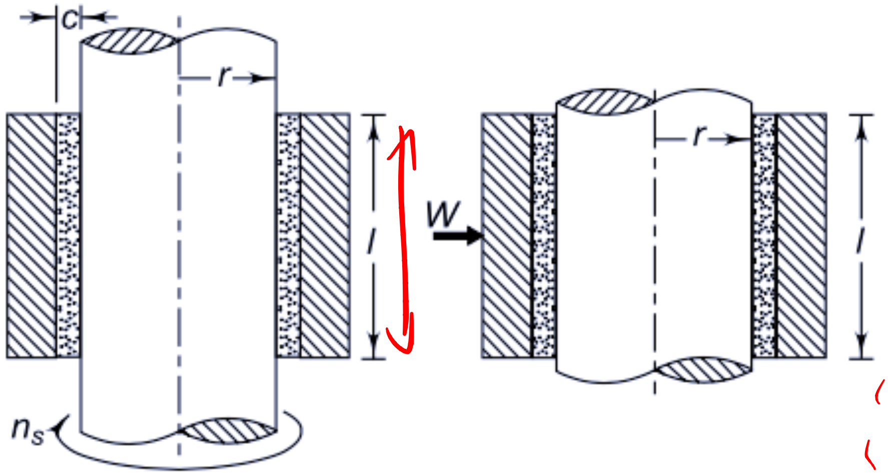
$U = r\omega$

$\omega = \dfrac{2\pi N}{60}$ → rpm

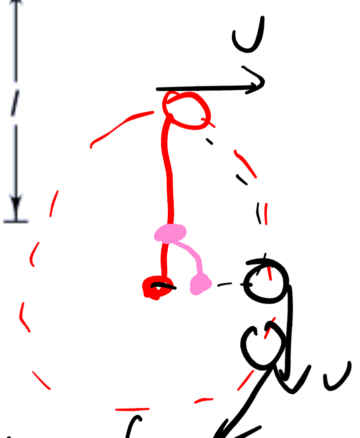
$= 2\pi n_s$

$U = (2\pi r) n_s$ — (1)

r = radius of the journal (mm)
l = length of the bearing (mm)
c = radial clearance (mm)
n_s = journal speed (rev/sec)



(a) (b)



$P = \mu A \left(\dfrac{U}{h}\right)$

P → tangential friction force

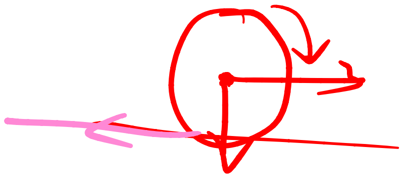
A → Area of Journal surf $= (2\pi r) l$

$P = \mu (2\pi r l)(2\pi r n_s) \times \dfrac{1}{C}$ $h = C$

$P = \dfrac{4\pi^2 r^2 l \mu n_s}{C}$ — (2)

$U = r\omega$
$= r \times 2\pi n_s$

Frictional torque $(M_t)_f = P \times r = \dfrac{4\pi^2 r^3 l \mu n_s}{c}$



Unit bearing pressure:

$$p = \frac{\text{Force}}{\text{area}} = \frac{W}{(2rl)}$$

$\mu = \dfrac{f}{N}$

$f = \mu N$

Load ← $W = 2prl$

Frictional force $= fW$ → Co-eff of friction

" torque $= (fW) r$

$$(M_t)_f = f(2prl) \times r$$

$$= f(2pr^2 l) \quad \text{— (3)}$$

Equating (3) & (2)

$$\frac{4\pi^2 r^3 l \mu n_s}{c} = f(2pr^2 l)$$

$$f = \frac{2\pi^2 r \mu n_s}{cp} \Rightarrow \boxed{f = (2\pi^2)\left(\frac{r}{c}\right)\left(\frac{\mu n_s}{p}\right)}$$

Petroff's equation.

MCKEE'S INVESTIGATION – BEARING CHARACTERISTIC NUMBER

In hydrodynamic bearings, initially the journal is at rest. There is no relative motion and no hydrodynamic film.

- Therefore, there is metal to metal contact between the surfaces of the journal and the bearing. As the journal starts to rotate, it takes some time for the hydrodynamic film to build sufficient pressure in the clearance space.
- During this period, there is partial metal to metal contact and a partial lubricant film. This is thin film lubrication.
- As the speed is increased, more and more lubricant is forced into the wedge-shaped clearance space and sufficient pressure is built up, separating the surfaces of the journal and the bearing. This is thick film lubrication.
- Therefore, there is a transition from thin film lubrication to thick film lubrication as the speed increases**.**

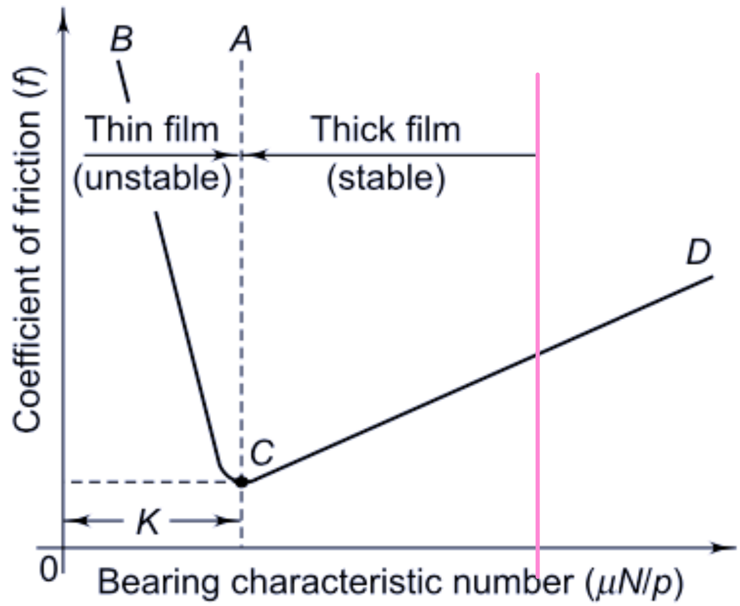


Fig. 16.11 *μN/p Curve*

There are two distinct parts of the curve—*BC* and *CD*.

1. In the region *BC*, there is partial metal to metal contact and partial patches of lubricant. This is the condition of thin film or boundary lubrication.
2. In the region *CD*, there is relatively thick film of lubricant and hydrodynamic lubrication takes place.

3. *AC* is the dividing line between these two modes of lubrication. The region to the left of the line *AC* is the thin film zone while the region to the right of the line *AC* is the thick film zone.
4. It is observed that the coefficient of friction is minimum at *C* or at the transition between these two modes. The value of the bearing characteristic number corresponding to this minimum coefficient is called the *bearing modulus*. It is denoted by *K* in the figure.

The bearing should **not be operated** near the critical value *K* at the point *C*. A slight drop in the speed (*N*) or a slight increase in the load (*p*) will reduce the value of ($\mu N/p$) resulting in boundary lubrication.

a. In order to avoid seizure, the operating value of the bearing characteristic number (μN/p) should be at least 5 to 6 times that when the coefficient of friction is minimum. (5 K to 6 K or 5 to 6 times the bearing modulus).
b. If the bearing is subjected to fluctuating loads or impact conditions, the operating value of the bearing characteristic number (μN/p) should be at least 15 times that when the coefficient of friction is minimum. (15 K or 15 times the bearing modulus).

It is observed from the (μN/p) curve that when viscosity of the lubricant is very low, the value of (μN/p) parameter will be low and boundary lubrication will result. Therefore, if the viscosity of the lubricant is very low then the lubricant will not separate the surfaces of the journal and the bearing and metal to metal contact will occur resulting in excessive wear at the contacting surfaces.

The (μN/p) curve is important because it defines the stability of hydrodynamic journal bearings and helps to visualize the transition from boundary lubrication to thick film lubrication.

HYDROSTATIC STEP BEARING

W = thrust load (N)

Ro = outer radius of the shaft (mm)

Ri = radius of the recess or the pocket (mm)

Pi = supply of inlet pressure (N/mm^2) or (MPa)

Po = outlet or atmospheric pressure (N/mm^2) or (MPa)

ho = fluid film thickness (mm)

Q = flow of the lubricant (mm^3 /s)

μ = viscosity of the lubricant (N-s/mm^2) or (MPa-s)

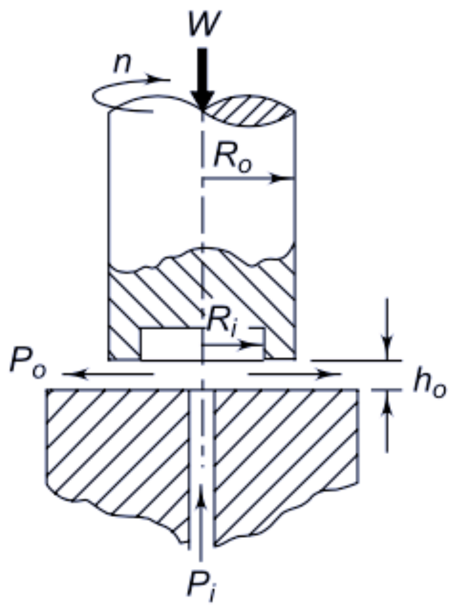


Fig. 16.13 *Hydrostatic Step Bearing*

$$Q = \frac{\Delta p b h^3}{12 \mu l}$$

$b \rightarrow$ width

$l \rightarrow$ length of flow

$h = h_o$

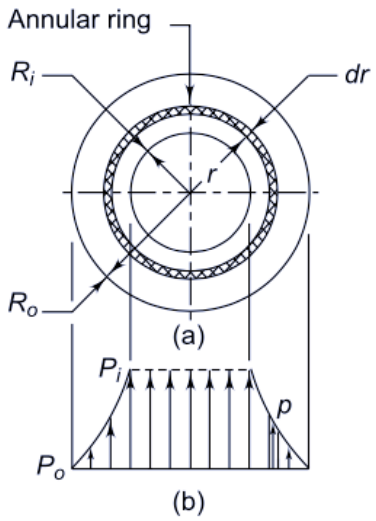
$\Delta p = dp$

$l = dr$

$b = 2\pi r$

$$Q = \frac{dp \times \cancel{2}\pi r \times (h_0)^3}{\cancel{12}\,6\, \mu \times dr}$$

$$Q = -\left(\frac{\pi r h_0^3}{6\mu}\right)\frac{dp}{dr}$$



Pressure Distribution in Hydrostatic Bearing

$$dp = -\left(\frac{6\mu Q}{\pi h_0^3}\right)\frac{dr}{r} \quad -(1)$$

$$\int \frac{1}{x} = \log x$$

Integrating

$$p = -\left(\frac{6\mu Q}{\pi h_0^3}\right)\log_e r + C \quad -(2)$$

$$p = 0 \ @ \ r = R_o$$

$$C = \left(\frac{6\mu Q}{\pi h_0^3}\right)\log_e R_o \quad -(3)$$

using (3) in (2)

$$p = \left(\frac{6\mu Q}{\pi h_0^3}\right)\left[\log_e R_o - \log_e r\right]$$

$$p = \left(\frac{6\mu Q}{\pi h_0^3}\right)\log_e\left(\frac{R_o}{r}\right) \quad -(a)$$

$$@ \ r = R_i \Rightarrow P = P_i$$

$$P_i = \left(\frac{6\mu Q}{\pi h_0^3}\right)\log\left(\frac{R_o}{R_i}\right) \quad -(4)$$

$$\boxed{Q = \frac{\pi P_i h_0^3}{6\mu \log_e\left(\frac{R_o}{R_i}\right)}} \quad -(5)$$

1) Constant press area till R_i

2) variable " " from $R_i \to R_o$

Load = (press) × Area

$$W = P_i \times \pi R_i^2 + \int_{R_i}^{R_o} P(2\pi r)\,dr$$

$$W = \pi P_i R_i^2 + \frac{12\mu Q}{h_0^3}\int_{R_i}^{R_o} \log_e\left(\frac{R_o}{r}\right) r\,dr \quad -(6)$$

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

$\int uv = u\int v - \int du \int v$

lets say $u = \log_e\left(\frac{R_o}{r}\right)$ & $dv = r\,dr$

$$du = \left(\frac{r}{R_o}\right)(R_o)\left(\frac{-1}{r^2}\right)dr = \left(\frac{-1}{r}\right)dr$$

$\int dv = v$

$$\int dv = \int r\,dr = \frac{r^2}{2} = v$$

$$\int uv = \int u\,dv = uv - \int v\,du$$

$$\int \log_e\left(\frac{R_o}{r}\right) r\,dr = \log_e\left(\frac{R_o}{r}\right) \times \frac{r^2}{2} + \int \frac{r^{\cancel{2}}}{2} \times \frac{+1}{\cancel{r}}\,dr$$

$$= \frac{r^2}{2}\log\left(\frac{R_o}{r}\right) + \frac{r^2}{4} \quad -(7)$$

using these values in (6)

$$\int_{R_i}^{R_o} \log_e\left(\frac{R_o}{r}\right) r\,dr = \left[\frac{r^2}{2}\log\left(\frac{R_o}{r}\right) + \frac{r^2}{4}\right]_{R_i}^{R_o}$$

$$= \left(\frac{R_o^2 - R_i^2}{4}\right) - \frac{R_i^2}{2}\log\left(\frac{R_o}{R_i}\right)$$

using this in (6)

$$\boxed{W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log_e\left(\frac{R_o}{R_i}\right)}\right]}$$

ENERGY LOSSES IN HYDROSTATIC BEARING

The total energy loss in a hydrostatic step bearing consists of two factors—the energy required to pump the lubricating oil and energy loss due to viscous friction.

1) Energy supplied to pump the oil

2) Frictional losses due to viscosity

$m = \rho \times V$

$= \rho \times Q$

$E = mgh$

$= \rho Q g h$

$\frac{N\text{-}m}{s} = \frac{J}{s} = W$

$E_p = Q \Delta P$

$= Q(P_i - P_o) \left(\frac{mm^3}{s}\right) \times \frac{N}{mm^2}$

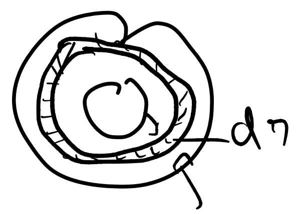
$\boxed{E = Q \times (\rho g h)}$

$\boxed{E = Q \times \Delta P}$

$= Q(P_i - P_o)(N\text{-}mm/s)$

$= Q(P_i - P_o) \times 10^{-3} (N\text{-}m/s)$ (or) watt

$(E_p)_{kW} = Q(P_i - P_o) \times 10^{-6}$ kW



$(E)_f$

Energy = Torque × ω

Torque = Force × dist

$\tau = \mu \frac{du}{dy}$

$\left(\mu \frac{u}{y}\right)$

$dF = \mu A \left(\frac{u}{h}\right)$

$A = (2\pi r)\,dr$ $\quad U = \omega r = \left(\frac{2\pi n}{60}\right) r$ $\quad h = h_0$

$$dF = \mu \times 2\pi r\,dr \times \left(\frac{2\pi n}{60}\right) r \times h_0$$

$$dF = \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_0}\right) r^2\,dr$$

Elementary frictional torque

$$d(M_t)_f = dF \times dist$$

$$d(M_t)_f = dF \times r$$

$$(M_t)_f = \int d(M_t)_f = \int dF \times r$$

$$= \left(\frac{4\pi^2}{60}\right)\left(\frac{\mu n}{h_0}\right)\int_{R_i}^{R_o} r^3\,dr$$

$$= \left(\frac{\pi^2}{60}\right)\left(\frac{\mu n}{h_0}\right)\left(r^4\right)_{R_i}^{R_o}$$

$$= \frac{\pi^2}{60}\,\frac{\mu n\left(R_o^4 - R_i^4\right)}{h_0} \quad \text{N-mm}$$

$$(E)_f = \frac{\pi^2}{60}\left(\frac{\mu n\left(R_o^4 - R_i^4\right)}{h_0}\right) \times \frac{2\pi n}{60} \times 10^{-3} \times 10^{-3}$$

$$(E)_f = \left(\frac{2\pi^3}{3600 \times 10^6}\right)\left(\frac{\mu n^2\left(R_o^4 - R_i^4\right)}{h_0}\right)$$

$$(E)_f = \left(\frac{1}{58.05 \times 10^6}\right)\left(\frac{\mu n^2\left(R_o^4 - R_i^4\right)}{h_0}\right)$$

Total Energy lost

$$= \underline{E_p + E_f}$$

where E_p = Pump loss = $Q(P_i - P_o) \times 10^{-6}$ kW

E_f = friction loss due to viscosity $= \left(\frac{1}{58.05}\right) \frac{\mu n^2 (R_o^4 - R_i^4)}{h_0} \times 10^{-6}$ kW

Problem: The following data is given for a hydrostatic thrust bearing:

thrust load = 500 kN | shaft speed = 720 rpm

shaft diameter = 500 mm | recess diameter = 300 mm

film thickness = 0.15 mm | viscosity of lubricant = 160 SUS

specific gravity = 0.86

Calculate

i. supply pressure;
ii. flow requirement in litres/min;
iii. power loss in pumping; and
iv. frictional power loss.

Given data:

$W = 500\,kN$

$n = 720$ Rpm

$D_o = 500\,mm = 0.5\,m$

$\rho = 0.86 \times 1000$ $\quad D_i = 300\,mm = 0.3\,m$

$h_o = 0.15\,mm$

viscosity = 160 SUS

1) Supply press:

$$W = \frac{\pi P_i}{2}\left[\frac{R_o^2 - R_i^2}{\log\left(\frac{R_o}{R_i}\right)}\right]$$

$$P_i = \frac{2W \log_e\left(\frac{R_o}{R_i}\right)}{\pi\left(R_o^2 - R_i^2\right)}$$

$$= \frac{2 \times 500 \times 10^3 \log_e\left(\frac{250}{150}\right)}{\pi\left(250^2 - 150^2\right)}$$

$$= 4.065\ N/mm^2$$

(or) MPa

$$\frac{2 \times 500 \times 10^3\left[\log\left(\frac{0.25}{0.15}\right)\right]}{\pi\left[(0.25)^2 - (0.15)^2\right]}$$

ln

2.303 × log

$$= \frac{10^3 \times 10^3 \times 0.510}{\pi\left(250^2 - 150^2\right)}$$

2) Flow rate:

$$Q = \frac{\pi P_i h_0^3}{6\mu \ln\left(\frac{R_0}{R_i}\right)}$$

160 SUS

$$Z_k = \left[0.22t - \frac{180}{t}\right] \text{ Centistokes}$$

$= 34.075$ CSt → kinematic viscosity (ν)

$$Z = \rho \times Z_k = 0.86 \times 34.075$$

$= 29.3045$ Cpoise

$$\mu = \frac{Z}{10^9} = 2.93 \times 10^{-8} \text{ NS/mm}^2$$

$$Q = \frac{\pi \times 4.065 \times (0.15)^3}{6 \times 2.93 \times 10^{-8} \times \ln\left(\frac{250}{150}\right)}$$

1 m = 1000 mm

$$Q = 479.946 \times 10^3 = 0.479$$

$\simeq 0.48 \times 10^6$ mm³/s

1 m³/s = 1000 lit/s

$= 0.48 \times 10^6 (10^{-3})^3$ m³/s

$= 0.48 \times 10^{-3}$ m³/s

$= 0.48 \times 10^{-3} \times 10^3$ lit/s

$= 0.48 \times 60 \frac{\text{lit}}{\text{min}}$

$$\boxed{Q = 28.8 \text{ l/min}}$$

3) power loss in pumping:

$$(E_p) = Q(P_i - P_0) \times 10^{-6} \text{ KW}$$

$$= (0.48 \times 10^6)(4.065 - 0) \times 10^{-6}$$

$$= 1.95 \text{ KW}$$

4) Frictional power loss:

$$E_f = \frac{1}{58.05 \times 10^6}\left[\frac{\mu n^2 (R_o^4 - R_i^4)}{h_o}\right]$$

$$= \frac{1}{(58.05 \times 10^6)}\left[\frac{2.93 \times 10^{-8} \times (720)^2 (250^4 - 150^4)}{(0.15)}\right]$$

$$= 5.93 \text{ KW}$$

$$\text{Total Energy loss} = E_p + E_f$$

$$= 1.95 + 5.93$$

$$= 7.88 \text{ KW}$$

Problem: The following data is given for the hydrostatic step bearing of a vertical turbo generator:

thrust load = 450 kN

shaft diameter = 400 mm

recess diameter = 250 mm

shaft speed = 750 rpm

viscosity of lubricant = 30 cP

Draw a neat sketch showing the effect of film thickness on energy losses. Calculate the optimum film thickness for minimum power loss.

Given datas

$W = 450$ KN ; $D_o = 400$ mm $n = 750$ RPM

$D_i = 250$ mm

$Z = 30$ cP $h_o = ?$

1) Supply press;

$$P_i = \frac{2W \ln\left(\frac{R_o}{R_i}\right)}{\pi (R_o^2 - R_i^2)} = \frac{2 \times 450 \times 10^3 \ln\left(\frac{200}{125}\right)}{\pi (200^2 - 125^2)}$$

$$P_i = 5.53 \text{ N/mm}^2$$

2) Flow:

$$Q = \frac{\pi P_i h_o^3}{6 \mu \ln\left(\frac{R_o}{R_i}\right)} = \frac{\pi \times 5.53 \times h_o^3}{6 \times 3 \times 10^{-8} \ln\left(\frac{200}{125}\right)}$$

$$\mu = \frac{Z}{10^9}$$

$$Q = \frac{17.34 h_o^3}{18 \times 10^{-8} \times 0.47} = 205 \times 10^6 h_o^3 \text{ mm}^3/\text{s}$$

3) Energy losses:

a) Pumping loss:

$$E_p = Q(P_i - P_o) \times 10^{-6} \text{ kW}$$

$$= (205\times10^{6})\, h_o^3 \times (5.52 - 0) \times 10^{-6} \text{ kW}$$

$$= 1131.60\, h_o^3 \text{ kW}$$

b) Frictional losses:

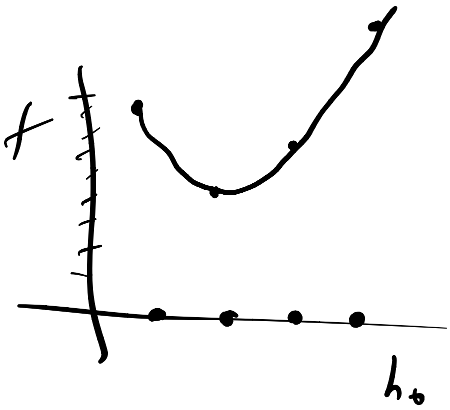
$$E_f = \frac{1}{58.05\times10^6}\left(\frac{\mu n^2 (R_o^4 - R_i^4)}{h_o}\right)$$

$\frac{dE}{dh_o} = 0 \to$ func will be min

$$= \frac{1}{h_o}\left[\frac{3\times10^{-8}\times(200^4 - 125^4)\times 750^2}{58.05\times10^6}\right]$$

$E = f(h_o)$

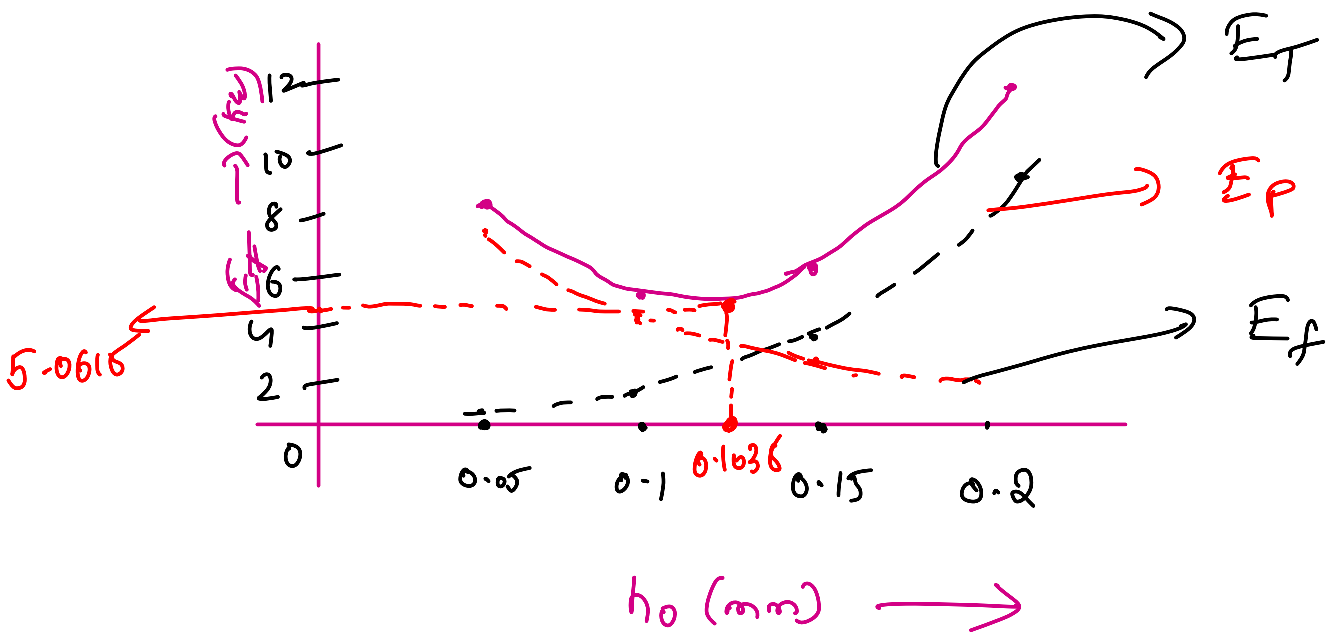
$$E_f = \frac{0.3941}{h_o} \text{ kW}$$



Total loss = $E_p + E_f$

$$E_T = 1131.6\, h_o^3 + \frac{0.3941}{h_o}$$

h_o	E_p	E_f	E_T
0.05	0.141	7.882	8.023
0.10	1.131	3.941	5.072
0.15	3.819	2.627	6.446
0.20	9.052	1.970	11.022
0.1038	1.2655	3.796	5.061



Optimum film thickness:

For Energy loss (total) to be minimum, the derivative of E_T should be equal to zero.

$$\frac{d}{dh_0}(E_T) = 0$$

$$\frac{d}{dh_0}\left[1131.6\, h_0^3 + \frac{0.3941}{h_0}\right] = 0$$

$$1131.6\,(3h_0^2) - \frac{0.3941}{h_0^2} = 0$$

$$h_0^4 = \frac{0.3941}{1131.6 \times 3}$$

$$\boxed{h_0 = 0.1038\,mm}$$

$\int \frac{1}{x} = \log x$

$\frac{1}{x}$

$\frac{d}{dx}(x^n) = n x^{n-1}$

$\frac{d}{dx}(x^{-1}) = -1 \times x^{-2}$

$= -1/x^2$

REYNOLD'S EQUATION

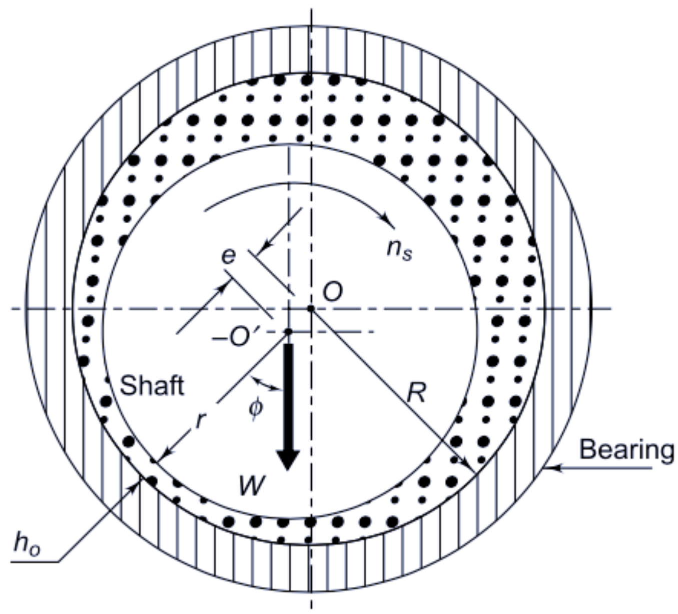
The **theory of hydrodynamic lubrication** is based on a differential equation derived by Osborne Reynold. This equation is based on the following assumptions:

i. The lubricant obeys Newton's law of viscosity.
ii. The lubricant is incompressible.
iii. The inertia forces in the oil film are negligible.
iv. The viscosity of the lubricant is constant.
v. The effect of curvature of the film with respect to film thickness is neglected. It is assumed that the film is so thin that the pressure is constant across the film thickness.
vi. The shaft and the bearing are rigid.
vii. There is a continuous supply of lubricant.

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right]=6\mu U\left(\frac{\partial h}{\partial x}\right)$$

There is no exact soln ~~for~~ for this eqn:

RAIMONDI AND BOYD METHOD → Approximate soln for Reynold's eqn:



$$C = R - r \quad -(1)$$

C → Radial clearance
R → Radius of bearing
r → " " Journal

OO' → Radial dist b/n bearing center & shaft centre
→ eccentricity = e

Eccentricity ratio $\varepsilon = \frac{e}{c}$ — (2) $e = \varepsilon \times c$

Radius of bearing $h_0 \rightarrow$ minimum film thickness

$$R = e + r + h_0 \quad -(3)$$

$$c = R - r = e + r + h_0 - r = e + h_0$$

$$c = c\varepsilon + h_0 \Rightarrow c(1-\varepsilon) = h_0$$

$$\boxed{\varepsilon = 1 - \frac{h_0}{c}}$$

The Sommerfeld number contains all variables, which are controlled by the designer.

The angle ϕ shown in Fig is called the angle of eccentricity or attitude angle. It locates the position of minimum film thickness with respect to the direction of load. The coefficient of friction variable (CFV) is given by,

$\frac{h_0}{c} \rightarrow$ min film thickness variable.

$$\boxed{S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{P}}$$

$n_s \rightarrow$ rps

$P \rightarrow$ unit bearing press

Co-eff of friction variable:

$$(CFV) = \left(\frac{r}{c}\right) f$$

$f \rightarrow$ co-eff of friction

Frictional torque

$$(M_t)_f = f W r$$

Frictional power:

$$P = \frac{2\pi n t}{60}$$

$$= (2\pi n_s)(f w r) \quad N\text{-}mm/s$$

$$(P)_f = (2\pi n_s)(f w r) \times 10^{-6} \quad kW$$

Flow variable:

$$(FV) = \frac{Q}{r c n_s l}$$

$l \rightarrow$ length of bearing (mm)

$Q \rightarrow$ flow of lubricant (mm^3/s)

TEMPERATURE RISE

Heat is generated in the bearing due to viscosity of the lubricating oil. The frictional work is converted into heat, which increases the temperature of the lubricant.

$$P_f = (2\pi n_s) f w r \times 10^{-6} \text{ KW (or) kJ/s}$$

↓

Heat generates

$$H_g = (2\pi n_s) f w r \times 10^{-6} \text{ kJ/s} \quad - (a)$$

$$f = \left(\frac{c}{r}\right) CFV \qquad w = (P) \times (2r \times l)$$

$$w = 2prl$$

using in (a)

$$H_g = (4\pi)(r c n_s l P) CFV \times 10^{-6} \quad - (1)$$

Heat carried away by oil:

$$H_c = m C_p \Delta t \quad - (b)$$

$m \to$ mass of lubricant oil (kg/s)

$C_p \to$ Specific heat of oil (kJ/kg°C)

$\Delta t \to$ Change in temp

$$m = \rho \times V/s \qquad V/s \to Q$$

$$m = \rho Q \times 10^{-6} \text{ kg/s}$$

$$Q = r c n_s l (FV)$$

$$m = \rho (r c n_s l)(FV) \times 10^{-6} \text{ kg/s} \quad -(c)$$

using (c) in (b)

$$H_c = C_p \Delta t \left[\rho r c n_s l (FV) \right] \times 10^{-6} \quad - (2)$$

Heat gen = Heat carried away.

$$H_g = H_c$$

$$4\pi (r c n_s l \, p)(CFV) \times 10^{-6} = C_p \Delta t \left[\rho r c n_s l (FV) \right] \times 10^{-6}$$

$$\Delta t = \frac{(4\pi p)}{(\rho C_p)} \left[\frac{CFV}{FV} \right]$$

For most lubricating oils:

$$\rho = 0.86 \quad \& \quad C_p = 1.76 \text{ kJ/kg}\,°C$$

$$\boxed{\Delta t = \frac{8.3 p (CFV)}{(FV)}}$$

$T_i \rightarrow$ inlet temp.

$$\boxed{T_{avg} = T_i + \frac{(\Delta t)}{2}} \rightarrow \text{Average temp of lubricating oil}$$

Table 16.1 *Dimensionless performance parameters for full journal bearing with side flow*

$\left(\frac{l}{d}\right)$	ε	$\left(\frac{h_o}{c}\right)$	S	ϕ	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{rcn_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{max.}}\right)$
∞	0	1.0	∞	(70.92)	∞	π	0	–
	0.1	0.9	0.240	69.10	4.80	3.03	0	0.826
	0.2	0.8	0.123	67.26	2.57	2.83	0	0.814
	0.4	0.6	0.0626	61.94	1.52	2.26	0	0.764
	0.6	0.4	0.0389	54.31	1.20	1.56	0	0.667
	0.8	0.2	0.021	42.22	0.961	0.760	0	0.495
	0.9	0.1	0.0115	31.62	0.756	0.411	0	0.358
	0.97	0.03	–	–	–	–	0	–
	1.0	0	0	0	0	0	0	0
1	0	1.0	∞	(85)	∞	π	0	–
	0.1	0.9	1.33	79.5	26.4	3.37	0.150	0.540
	0.2	0.8	0.631	74.02	12.8	3.59	0.280	0.529
	0.4	0.6	0.264	63.10	5.79	3.99	0.497	0.484
	0.6	0.4	0.121	50.58	3.22	4.33	0.680	0.415
	0.8	0.2	0.0446	36.24	1.70	4.62	0.842	0.313
	0.9	0.1	0.0188	26.45	1.05	4.74	0.919	0.247
	0.97	0.03	0.00474	15.47	0.514	4.82	0.973	0.152
	1.0	0	0	0	0	0	1.0	0
$\left(\frac{1}{2}\right)$	0	1.0	∞	(88.5)	∞	π	0	–
	0.1	0.9	4.31	81.62	85.6	3.43	0.173	0.523
	0.2	0.8	2.03	74.94	40.9	3.72	0.318	0.506
	0.4	0.6	0.779	61.45	17.0	4.29	0.552	0.441
	0.6	0.4	0.319	48.14	8.10	4.85	0.730	0.365
	0.8	0.2	0.0923	33.31	3.26	5.41	0.874	0.267
	0.9	0.1	0.0313	23.66	1.60	5.69	0.939	0.206
	0.97	0.03	0.00609	13.75	0.610	5.88	0.980	0.126
	1.0	0	0	0	0	–	1.0	0
$\left(\frac{1}{4}\right)$	0	1.0	∞	(89.5)	∞	π	0	–
	0.1	0.9	16.2	82.31	322.0	3.45	0.180	0.515
	0.2	0.8	7.57	75.18	153.0	3.76	0.330	0.489
	0.4	0.6	2.83	60.86	61.1	4.37	0.567	0.415
	0.6	0.4	1.07	46.72	26.7	4.99	0.746	0.334
	0.8	0.2	0.261	31.04	8.8	5.60	0.884	0.240
	0.9	0.1	0.0736	21.85	3.50	5.91	0.945	0.180
	0.97	0.03	0.0101	12.22	0.922	6.12	0.984	0.108
	1.0	0	0	0	0	–	1.0	0

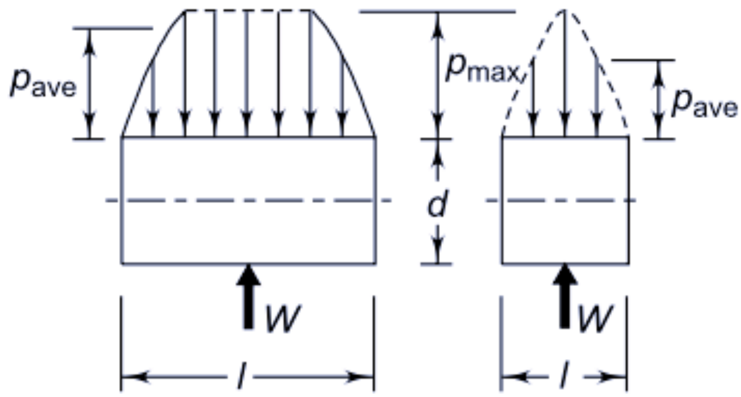
BEARING DESIGN—SELECTION OF PARAMETERS

(i) length-to-diameter ratio;
(ii) unit bearing pressure;
(iii) start-upload;
(iv) radial clearance;
(v) minimum oil film thickness; and
(vi) maximum oil film temperature.

Length to Diameter Ratio

Diameter of the shaft is determined by strength or rigidity considerations and not on the basis of bearing capacity. The shaft diameter is usually determined by using the criteria such as permissible stress, permissible lateral deflection or permissible angle of twist. Therefore, it is the bearing length that the designer has to decide to obtain a given bearing capacity.

The **length to diameter ratio (*l*/*d*) affects the performance of the bearing**. As the ratio increases, the resulting film pressure increases as shown in Fig. A **long bearing, therefore, has more load carrying capacity compared with a short bearing**. A short bearing, on the other hand, has greater side flow, which improves heat dissipation.



(a) $l/d = 2$ (b) $l/d = 1$

The long bearings are more susceptible to metal to metal contact at the two edges, when the shaft is deflected under load. The longer the bearing, the more difficult it is to get sufficient oil flow through the passage between the journal and the bearing.

Therefore, the design trend is to use (*l*/*d*) ratio as 1 or less than 1. In practice, the (*l*/*d*) ratio varies from 0.5 to 2.0, but in the majority of applications, it is taken as 1 or less than 1.

1. (a) When (*l*/*d*) ratio is more than 1, the bearing is called '*long*' bearing.
2. (b) When (*l*/*d*) ratio is less than 1, the bearing is called '*short*' bearing.
3. (c) When (*l*/*d*) ratio is equal to 1, the bearing is called '*square*' bearing.

Unit Bearing Pressure

The unit bearing pressure is the load per unit of projected area of the bearing in running condition. It depends upon a number of factors, such as bearing material, operating temperature, the nature and frequency of load and service conditions.

Table 16.2 *Permissible bearing pressures*

Application		*Unit bearing pressure (p) (N/mm²)*
(i)	*Diesel engines*	
	Main bearing	5–10
	Crank pin	7–14
	Gudgeon pin	13–14
(ii)	*Automotive engines*	
	Main bearing	3–4
	Crank pin	10–14
(iii)	*Air compressors*	
	Main bearing	1–1.5
	Crank pin	1.5–3.0
(iv)	*Centrifugal pumps*	
	Main bearing	0.5–0.7
(v)	*Electric motors*	
	Main bearing	0.7–1.5
(vi)	*Transmission shafting*	
	Light duty	0.15
	Heavy duty	1.00
(vii)	*Machine tools*	
	Main bearing	2

Start-up Load The unit bearing pressure for starting conditions should not exceed 2 N/mm^2. The start-up load is the static load when the shaft is stationary. It mainly consists of the dead weight of the shaft and its attachments. The start- up load can be used to determine the minimum length of the bearing on the basis of starting conditions.

Radial Clearance The radial clearance should be small to provide the necessary velocity gradient. However, this requires costly finishing operations, rigid mountings of the bearing assembly and clean lubricating oil without any foreign particles. This increases the initial and maintenance costs. The practical value of radial clearance is 0.001 mm per mm of the journal radius.

Material	*Radial clearance*
Babbitts	(0.001) r to (0.00167) r
Copper–lead	(0.001) r to (0.01) r
Aluminium–alloy	(0.002) r to (0.0025) r

Minimum oil film thickness

The surface finish of the journal and the bearing is governed by the value of the minimum oil film thickness selected by the designer and vice versa. There is a lower limit for the minimum oil film thickness, below which metal to metal contact occurs and the hydrodynamic film breaks. This lower limit is given by,

$$h_0 = (0.0002)r$$

Maximum Oil Film Temperature

The lubricating oil tends to oxidise when the operating temperature exceeds 120°. Also, the surface of babbitt bearing tends to soften at 125°C (for bearing pressure of 7 N/mm^2) and at 190°C (for bearing pressure of 1.4 N/mm^2). Therefore, the operating temperature should be kept within these limits. In general, the limiting temperature is 90°C for bearings made of babbitts.

Problem: The following data is given for a 360° hydrodynamic bearing:

radial load = 3.2 kN	journal speed = 1490 rpm
journal diameter = 50 mm	bearing length = 50 mm
radial clearance = 0.05 mm	viscosity of lubricant = 25 cP

Assuming that the total heat generated in the bearing is carried by the total oil flow in the bearing, calculate

(i) coefficient of friction;
(ii) power lost in friction;
(iii) minimum oil film thickness;
(iv) flow requirement in 1itres/min; and
(v) temperature rise.

Given data:

$W = 3.2$ kN ; $n = 1490$ rpm

$d = 50$ mm ; $l = 50$ mm

$c = 0.05$ mm ; $Z = 25$ cP

$H_s = H_c$

1) Co-eff of friction:

$$f = \left(\frac{r}{c}\right) CFV \qquad CFV = \left(\frac{c}{r}\right) f$$

$Z = 25$ cP

$$\mu = \frac{Z}{10^9} \Rightarrow \mu = 25 \times 10^{-9}\ \frac{N\text{-}s}{mm^2}$$

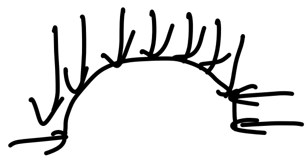
$$\tau = \mu\left(\frac{v}{y}\right)$$

$$\frac{N}{mm^2} = (\mu)\left[\frac{mm/s}{mm}\right]$$

$$\mu = N\text{-}s/mm^2$$

unit bearing press

$$p = \frac{W}{\text{proj. area}} = \frac{W}{(2r)\times l}$$



$$= \frac{3200}{(50 \times 50)}$$

$$p = 1.28\ N/mm^2$$

Sommerfeld number:

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{P} = \left(\frac{25}{0.05}\right)^2 \times \frac{25\times10^{-9} \times 1490}{1.28 \times 60}$$

$$S = 0.121$$

$$l/d = \frac{50}{50} = 1$$

From table, for $l/d = 1$ & $S = 0.121$

$$\frac{h_0}{c} = 0.4 \;;\; \left(\frac{r}{c}\right)f = 3.22 \;;\; \frac{Q}{r c n_s l} = 4.33$$

CFV FV

1) Co-eff of friction: (f)

$$\left(\frac{r}{c}\right)f = 3.22 \Rightarrow f = \frac{3.22 \times 0.05}{25}$$

$$f = 6.44 \times 10^{-3}$$

2) Friction Power:

$W \rightarrow fW \rightarrow$ friction load

$fWr \rightarrow$ " torque

$fWr \times \omega \rightarrow$ " power

$$(P_f) = \left(\frac{2\pi n}{60}\right)(fWr) \times 10^{-6} \text{ KW}$$

$$= \left(\frac{2\pi \times 1490}{60}\right) \times \left(6.44\times10^{-3} \times 3200 \times 25\right) \times 10^{-6}$$

$$\boxed{P_f = 0.08 \text{ kW}}$$

3) min oil film thickness (h_0):

$$\frac{h_0}{c} = 0.4 \Rightarrow h_0 = 0.05 \times 0.4$$

$$h_0 = 0.02 \text{ mm}$$

4) Flow rate in litres/min:

$$\frac{Q}{r c n_s l} = 4.33 \Rightarrow Q = 25 \times 0.05 \times \frac{1490}{60} \times 50 \times 4.33$$

m^3/s

$$Q = 6720.52 \text{ mm}^3/s$$

$1 m^3/s = 1000 \text{ l}/s$

$$Q = 6720.52 (10^{-3} m)^3/s$$

$1 mm = 10^{-3} m$

$$Q = 6720.52 \times 10^{-9} m^3/s$$

$$= 6720.52 \times 10^{-9} \times 10^3 \text{ lit}/s$$

$1 \text{ l}/s = 60 \text{ l}/m$

$$= 6720.52 \times 10^{-6} \times 60 \text{ l}/m$$

$$\boxed{Q = 0.4032 \text{ litres/min}}$$

$$CFV = \left(\frac{r}{c}\right) f$$

$$(FV) = \frac{Q}{r c n_s l}$$

5) Temp rise: $\Delta t = \frac{8.3 p (CFV)}{(FV)}$

$$\Delta t = \frac{8.3 \times 1.28 \times 3.22}{4.33}$$

$$\boxed{\Delta t = 7.9^\circ C}$$

Problem: An oil ring bearing of a transmission shaft is shown in Fig. 16.28. There is no hydrodynamic action over the width of 4 mm of the oil ring. The total radial load acting on the journal is 20 kN and the journal rotates at 1450 rpm. The radial clearance and minimum film thickness are 20 to 5 microns respectively. Calculate

(i) viscosity of the lubricant; and

(ii) required quantity of oil.

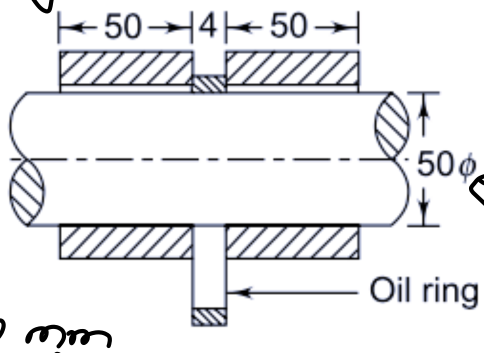
$\frac{h_0}{c} = \frac{5\times10^{-6}}{20\times10^{-6}}$

$1 \text{ micron} = 10^{-6}$ m

R. Ø

Given data: $W_T = 20$ kN $l = 50$ mm

$n = 1450$ RPM



$c = 20$ microns $= 20\times10^{-3}$ mm

$h_0 = 5$ micron $= 5\times10^{-3}$ mm

$W_1 = W_2 = 10$ kN $d = 50$ mm

$$p = \frac{W}{(2r\times l)} = \frac{10\times10^{3}}{(50\times50)} = 4 \text{ N/mm}^2$$

$h_0/c\,(\frac{r}{c})$ +

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$$

$$\frac{h_0}{c} = \frac{5\times10^{-3}}{20\times10^{-3}} = 0.25$$

$$\frac{l}{d} = \frac{50}{50} = 1$$

From the table for $l/d = 1$ & $h_0/c = 0.25$

$$S = 0.0446 + (0.121 - 0.0446)\left(\frac{0.25-0.2}{0.4-0.2}\right)$$

$$S = 0.0637$$

$$\frac{Q}{rcn_s l} = 4.62 - (4.62 - 4.33)\left[\frac{0.25-0.2}{0.4-0.2}\right]$$

$$\frac{Q}{rcn_s l} = \cancel{4.5475} \qquad \frac{Q}{25\times 20\times 10^{-3}\times \frac{1450}{60}\times 50} = 4.547$$

1) <u>viscosity of lubricant</u> :

$$\mu = \left(\frac{c}{r}\right)^2 \times \frac{S\times P}{n_s}$$

$$= \left(\frac{20\times 10^{-3}}{25}\right)^2 \times \frac{0.0637\times 4}{1450}\times 60$$

$$\mu = 6.74\times 10^{-9}\ N\text{-}s/mm^2$$

$$\boxed{\mu = 6.74\ cP}$$

2) <u>Flow of oil</u> :

$$\frac{Q}{rcn_s l} = 4.5475 \Rightarrow Q = 25\times 20\times 10^{-3}\times \frac{1450}{60}\times 50\times 4.5475$$

$$Q_1 = 2747.44\ mm^3/s$$

$$Q_1 = Q_2$$

$$Q_T = Q_1 + Q_2$$

$$= 2Q_1 = 5494.88\ mm^3/s$$

$$Q_T = 5494.88\times 60\times 10^{-6}\ l/m$$

$$\boxed{Q_T = 0.33\ l/m}$$

LUBRICATING OILS

$1\,mm^3/s = 10^{-9}\,m^3/s$ $\quad$ $1\,m^3/s = 1000\,l/s$

The desirable properties of lubricating oil are as follows:

$1\,mm^3/s = (1\times10^{-3})^3\,m^3/s$

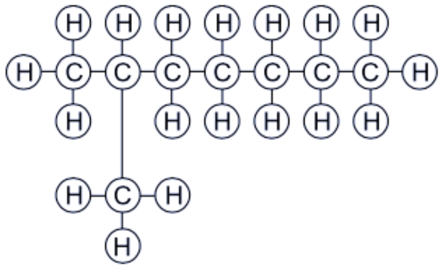
1. It should be available in a wide range of viscosities. $1\,mm^3/s = 10^{-9}\times10^{3}\,l/s$
2. There should be little change in viscosity of the oil with change in temperature.
3. The oil should be chemically stable with the bearing material and atmosphere at all temperatures encountered in the application. $1\,mm^3/s = \underline{10^{-6}\times 60}\,l/m$
4. The oil should have sufficient specific heat to carry away frictional heat, without abnormal rise in temperature.
5. It should be commercially available at reasonable cost.

Lubricating oils are divided into two groups— mineral oils and vegetable or animal oils.

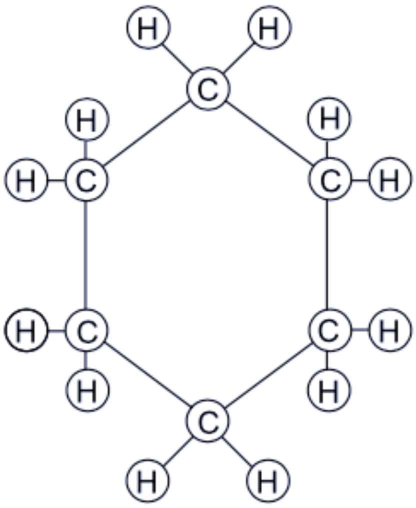
Mineral oils consist of hydrocarbons, which are obtained by the distillation of crude oil.

There are two different classes of mineral oils—those with a paraffinic series and those with a naphthenic series. A *paraffinic* oil is composed of straight and branched chains of hydrocarbons defined by the general formula $C_nH_{(2n+2)}$. A *naphthenic oil* is composed of a saturated single-ring formation of hydrocarbons defined by the general formula C_nH_{2n}.

SAE number	*Saybolt universal viscosity at 210°F*	
	Minimum	*Maximum*
20	45	less than 58
30	58	less than 70
40	70	less than 85
50	85	less than 110



(a) Paraffinic oil (n = 8) (C_8H_{18})



(b) Naphthenic oil (n = 6) (C_6H_{12})

Compared with vegetable or animal oils, mineral oils offer the following advantages:

I. Mineral oils are chemically inert.
II. They have a wide range of viscosities, corresponding to different values of n in the general formula.
III. They have little tendency to oxidise or form corrosive acids.
IV. After periodic filtration, they can be reused without any loss or change of their properties.
V. At normal temperature, they are not liable to spontaneous ignition.

Vegetable oils used for lubrication are castor oil, rapeseed oil, palm oil and olive oil. Lubricating oils of animal origin are lard oil, tallow oil and certain oils obtained from marine species, such as whales, sperms or dolphin jaws. The **advantages of vegetable and animal oils** are as follows:

1. These oils are sometimes referred to as *fixed oils* because they are non-volatile, unless there is chemical decomposition. This property prevents them from being expelled from intimate contacts of solid surfaces by frictional heat.
2. They retain their viscosities at high temperature much better than mineral oils.
3. These oils are called '*polar*' compounds. They have a long chain of molecules with positive and negative charges at the two ends. One end of the polar molecule adheres to the surface of the journal or bearing and the long chain of molecules extends into clearance space. They form 'clusters' which prevents metal to metal contact in boundary lubrication.

The main drawbacks of vegetable or animal oils are as follows:

i. At low temperature, these oils solidify and become '*fats*'. The fat is melted at about 65°C and becomes oil.
ii. These oils react with oxygen in the atmosphere and become acidic. In some cases they change from the liquid state to an elastic solid form. Due to this reason, they are termed '*drying oils*'.

iii. They are subjected to saponification either by contact with base metals or with hot water. They produce glycerol and some organic acids, which attack metallic surfaces and form metallic soaps.

Castor oil was used in the past as a lubricant in racing cars and aero-engines. Rapeseed oil is added to mineral oil to increase viscosity. Cottonseed oil is mainly used as a thickener in mineral oils. Lard oil is used as cutting oil, while tallow oil is used as cylinder oil. In light machine tools, sperm oil is used for spindle lubrication.

Gears

Spur Gears

MECHANICAL DRIVES

Belt, chain and gear drives are often called '*mechanical*' drives.

- *A mechanical drive is defined as a mechanism, which is intended to transmit mechanical power over a certain distance, usually involving a change in speed and torque*

A mechanical drive is used on account of the following reasons:

i. The torque and speed of the machine are always different than that of electric motor or engine. Machines usually run at low speed and require high torque. For example, in case of overhead travelling crane, the motor runs at 1440 rpm while the speed of the rope drum is as low as 20 rpm.
ii. In certain machines, variable speeds are required for the operation, whereas the prime mover runs at constant speed. For example, in case of lathe, the motor runs at constant speed, while different speeds are required for the spindle of the chuck to turn the jobs of different materials and with different feeds and depth of cut.
iii. Standard electric motors are designed for uniform rotary motion. However, in some machines like shaper or planer, linear motions with varying velocities are required.

Mechanical drives are classified into two groups according to their principle of operation. The two broad groups are as follows:

(i) Mechanical drives that transmit power by means of friction, e.g., belt drive and rope drive

(ii) Mechanical drives that transmit power by means of engagement, e.g., chain drives and gear drives

GEAR DRIVES

Gears are defined as toothed wheels or multilobed cams, which transmit power and motion from one shaft to another by means of successive engagement of teeth. Gear drives offer the following advantages compared with chain or belt drives:

1. It is a positive drive and the velocity ratio remains constant.
2. The centre distance between the shafts is relatively small, which results in compact construction.
3. It can transmit very large power, which is beyond the range of belt or chain drives.
4. It can transmit motion at very low velocity, which is not possible with the belt drives.
5. The efficiency of gear drives is very high, even up to 99 per cent in case of spur gears.
6. A provision can be made in the gearbox for gear shifting, thus changing the velocity ratio over a wide range.

Gear drives are, however, costly and their maintenance cost is also higher.

⇒ The manufacturing processes for gears are complicated and highly specialized.

⇒ Gear drives require careful attention for lubrication and cleanliness.

⇒ They also require precise alignment of the shafts.

CLASSIFICATION OF GEARS

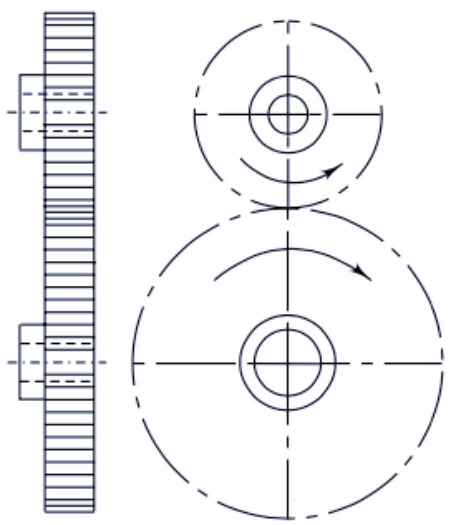


Fig. 17.1 *Spur Gears*

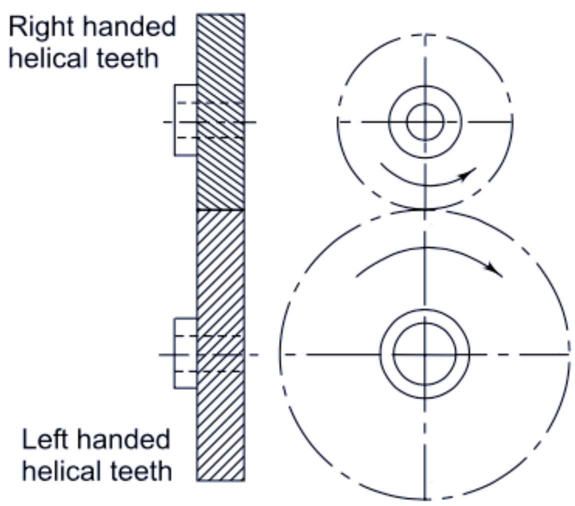


Fig. 17.2 *Helical Gears*

Gears are broadly classified into four groups, viz., spur, helical, bevel and worm gears.

In case of spur gears, the teeth are cut parallel to the axis of the shaft. As the teeth are parallel to the axis of the shaft, spur gears are used only when the shafts are parallel. The profile of the gear tooth is in the shape of an involute curve and it remains identical along the entire width of the gear wheel. Spur gears impose radial loads on the shafts.

In Helical gears the teeth of these gears are cut at an angle with the axis of the shaft. Helical gears have an involute profile similar to that of spur gears. However, this involute profile is in a plane, which is perpendicular to the tooth element. The magnitude of the helix angle of pinion and gear is same; however, the hand of the helix is opposite. A right-hand pinion meshes with a left-hand gear and vice versa. Helical gears impose radial and thrust loads on shafts.

There is a special type of helical gear, consisting of two helical gears with the opposite hand of helix, as shown in Fig below. It is called **herringbone gear**. The construction results in equal and opposite thrust reactions, balancing each other and imposing no thrust load on the shaft. Herringbone gears are used only for parallel shafts.

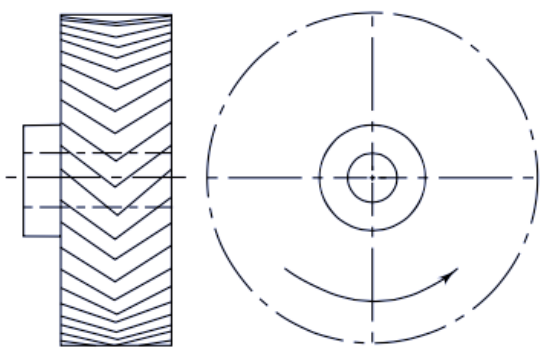


Fig. 17.3 *Herringbone Gear*

Bevel gears, as shown, have the shape of a truncated cone. The size of the gear tooth, including the thickness and height, decreases towards the apex of the cone. Bevel gears are normally used for shafts, which are at right angles to each other. This, however, is not a rigid condition and the angle can be slightly more or less than 90 degrees. The tooth of the bevel gears can be cut straight or spiral. Bevel gears impose radial and thrust loads on the shafts.

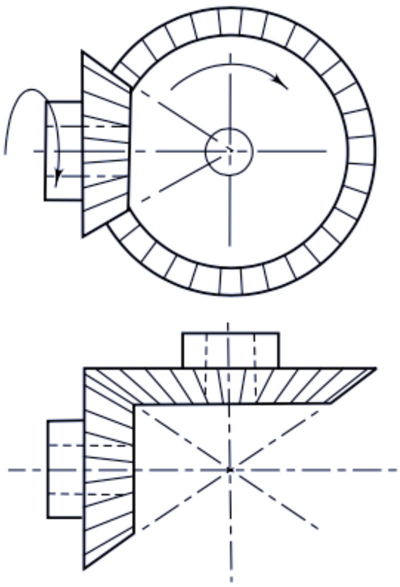
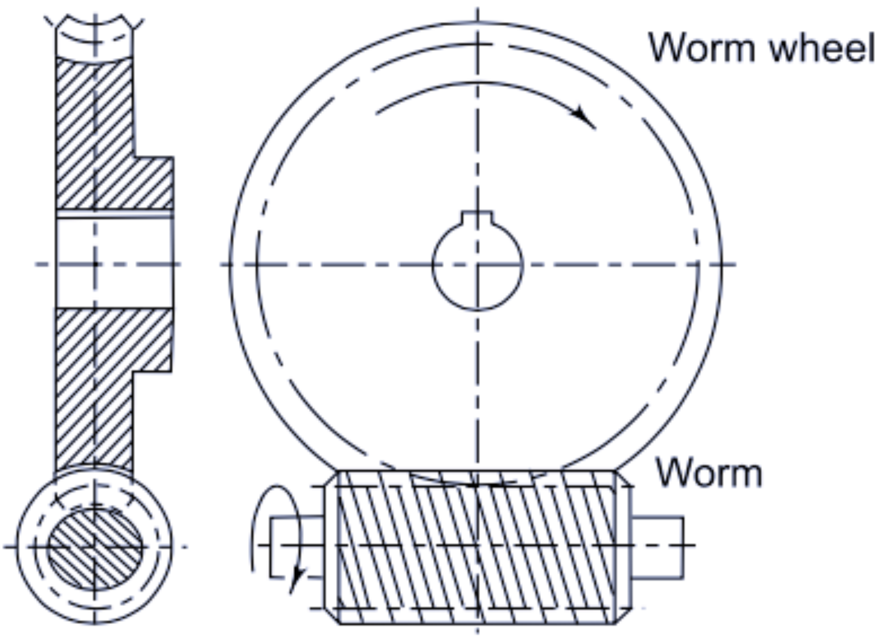


Fig. 17.4 *Bevel Gears*

The *worm gears*, as shown, consist of a worm and a worm wheel. The worm is in the form of a threaded screw, which meshes with the matching wheel. The threads on the worm can be single or multi-start and usually have a small lead. Worm gear drives are used for shafts, the axes of which do not intersect and are perpendicular to each other. The worm imposes high thrust load, while the worm wheel imposes high radial load on the shafts. Worm gear drives are characterized by high speed reduction ratio.



LAW OF GEARING

The fundamental law of gearing states *'The common normal to the tooth profile at the point of contact should always pass through a fixed point, called the pitch point, in order to obtain a constant velocity ratio'*.

$\omega_1 \rightarrow$ ang of I^{st} gear

$\omega_2 \rightarrow II^{nd}$ gear

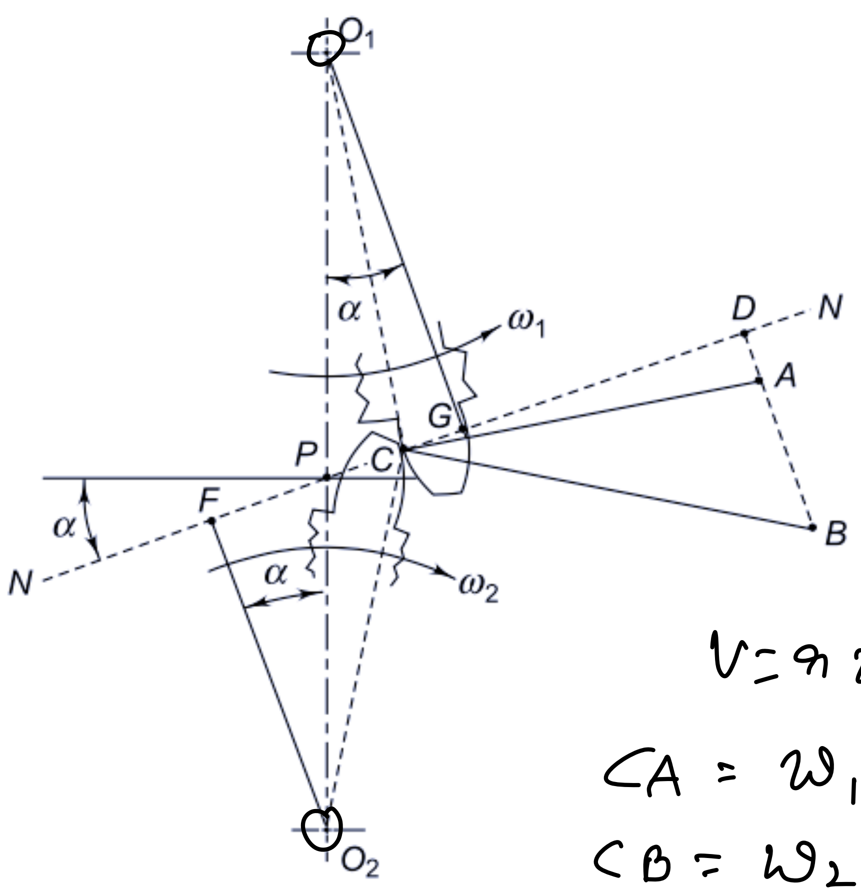


Fig. 17.6 *Law of Gearing*

$\vec{CA} \rightarrow$ vel vector of I^{st} gear

$\vec{CB} \rightarrow$ vel vector of II^{nd} gear

$$v = r\omega$$

$$CA = \omega_1 \times O_1C$$

$$CB = \omega_2 \times O_2C$$

$$\frac{\omega_1}{\omega_2} = \left(\frac{CA}{CB}\right) \times \left(\frac{O_2C}{O_1C}\right) \quad \text{——— (1)}$$

ΔO_1CG & ΔCAD are similar

$$\frac{O_1C}{O_1G} = \frac{CA}{CD} \Rightarrow \frac{O_1C}{CA} = \frac{O_1G}{CD} \quad - (2)$$

ΔO_2FC & ΔCDB are similar

$$\frac{O_2C}{CB} = \frac{O_2F}{CD} \quad — (3)$$

From (2) & (3)

$$\frac{CA}{CB} = \frac{O_1C}{O_2C} \times \frac{O_2F}{O_1G} \quad -(4)$$

using (4) in (1)

$$\frac{w_1}{w_2} = \frac{\cancel{O_1C}}{\cancel{O_2C}} \times \frac{O_2F}{O_1G} \times \frac{\cancel{O_2C}}{\cancel{O_1C}}$$

$$\frac{w_1}{w_2} = \frac{O_2F}{O_1G} \quad -(5)$$

ΔO_2FP & ΔO_1GP are similar

$$\frac{O_2F}{O_1G} = \frac{O_2P}{O_1P}$$

$$\frac{w_1}{w_2} = \frac{O_2P}{O_1P} \quad -(6)$$

From Δle, $O_1P + O_2P = O_1O_2$ = centre distance
constant

For $\frac{w_1}{w_2}$ to be constant, O_1P & O_2P should not change.

which means, the point "P" is a fixed point.

Only involute and cycloidal curves satisfy the fundamental law of gearing. The meaning of these curves is as follows:

i. An involute is a curve traced by a point on a line as the line rolls without slipping on a circle.
ii. A cycloid is a curve traced by a point on the circumference of a generating circle as it rolls without slipping along the inside and outside of another circle.
    1. The cycloid profile consists of two curves, namely, epicycloid and hypocycloid.
    2. An *epicycloid* is a curve traced by a point on the circumference of a generating circle as it rolls without slipping on the outside of the pitch circle.
    3. A *hypocycloid* is a curve traced by a point on the circumference of a generating circle as it rolls without slipping on the inside of the pitch circle.

Cycloidal tooth offers the following advantages compared with involute tooth:

a) In case of cycloidal gears, a convex flank on one tooth comes in contact with the concave flank of the mating tooth. This increases the contact area and also the wear strength. In involute gears, the contact is between two convex surfaces on mating teeth, resulting in smaller contact area and lower wear strength.
b) The phenomenon of interference does not occur at all in cycloidal gears.

However, cycloidal teeth are rarely used in practice due to the following disadvantages:

(i) Cycloidal tooth is made of two curves— hypocycloid curve below the pitch circle and epicycloid curve above the pitch circle. It is very difficult to manufacture an accurate profile consisting of two curves. The profile of an involute tooth is made of a single curve and only one cutter is necessary to manufacture one complete set of pinion and gear. This results in reduction in manufacturing cost.
(ii) In case of an involute profile, the common normal at the point of contact always passes through the pitch point P and maintains a constant inclination a with the common tangent to the two pitch circles. The angle a is called the pressure angle.

Therefore, the pressure angle remains constant in involute tooth. In case of cycloidal tooth, the pressure angle varies. The pressure angle has maximum value at the beginning of engagement and reduces to zero when the point of contact coincides with the pitch point. It again increases to maximum value in the reverse direction.

- It is due to these reasons that cycloidal curves have become obsolete.
- However, they are still in use in some of applications, such as spring driven watches and clocks, and some instruments.
- In these applications, their ability to provide satisfactory operation with very small number of teeth is used to advantage.

TERMINOLOGY OF SPUR GEARS

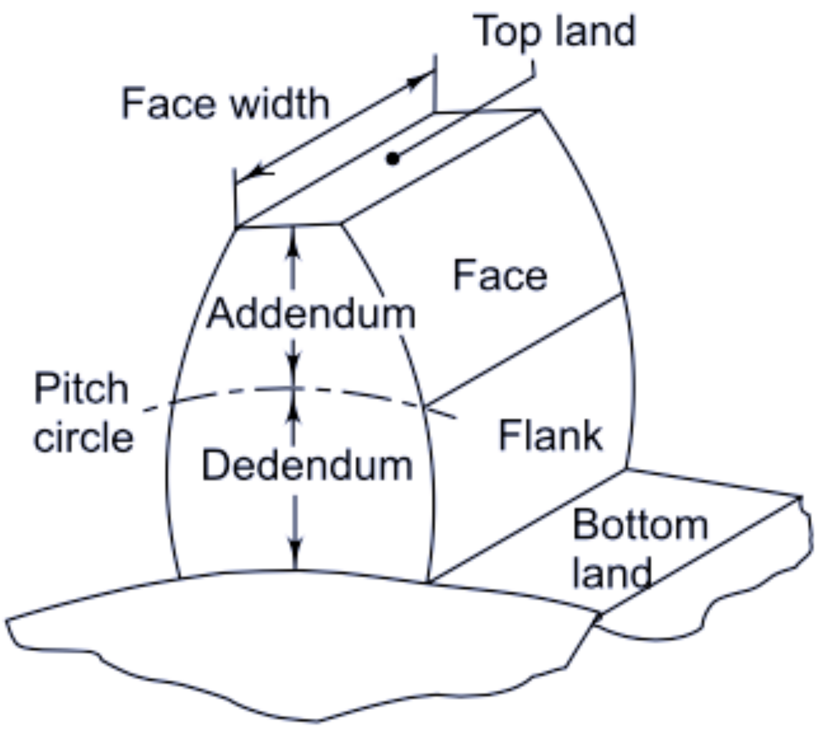


Fig. 17.7 *Gear Nomenclature*

(i) Pinion A pinion is the smaller of the two mating gears.

(ii) Gear A gear is the larger of the two mating gears.

(iii) Velocity Ratio (i) Velocity ratio is the ratio of angular velocity of the driving gear to the angular velocity of the driven gear. It is also called the speed ratio.

(iv) Transmission Ratio (i') The transmission ratio (i') is the ratio of the angular speed of the first driving gear to the angular speed of the last driven gear in a gear train.

(v) Pitch Surface The pitch surfaces of the gears are imaginary planes, cylinders or cones that roll together without slipping.

(vi) Pitch Circle The pitch circle is the curve of intersection of the pitch surface of revolution and the plane of rotation. It is an imaginary circle that rolls without slipping with the pitch circle of a mating gear. The pitch circles of a pair of mating gears are tangent to each other.

(vii) Pitch Circle Diameter The pitch circle diameter is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called *pitch diameter*. The pitch circle diameter is denoted by d ¢.

(viii) Pitch Point The pitch point is a point on the line of centres of two gears at which two pitch circles of mating gears are tangent to each other.

(ix) Top Land The top land is the surface of the top of the gear tooth.

(x) Bottom Land The bottom land is the surface of the gear between the flanks of adjacent teeth.

(xi) Involute An involute is a curve traced by a point on a line as the line rolls without slipping on a circle.

(xii) Base Circle The base circle is an imaginary circle from which the involute curve of the tooth profile is generated. The base circles of two mating gears are tangent to the pressure line.

(xiii) Addendum Circle The addendum circle is an imaginary circle that borders the tops of gear teeth in the cross section.

(xiv) Addendum (h_a) The addendum (h_a) is the radial distance between the pitch and the addendum circles. Addendum indicates the height of the tooth above the pitch circle.

(xv) Dedendum Circle The dedendum circle is an imaginary circle that borders the bottom of spaces between teeth in the cross section. It is also called *root* circle.

(xvi) Dedendum (h_f) The dedendum (h_f) is the radial distance between pitch and the dedendum circles. The dedendum indicates the depth of the tooth below the pitch circle.

(xvii) Clearance (c) The clearance is the amount by which the dedendum of a given gear exceeds the addendum of its mating tooth.

(xviii) Face of Tooth The surface of the gear tooth between the pitch cylinder and the addendum cylinder is called the face of tooth.

(xix) Flank of Tooth The surface of the gear tooth between the pitch cylinder and the root cylinder is called flank of the tooth.

(xx) Face Width (b) Face width is the width of the tooth measured parallel to the axis.

(xxi) Fillet Radius The radius that connects the root circle to the profile of the tooth is called fillet radius.

(xxii) Circular Tooth Thickness The length of the arc on the pitch circle subtending a single gear tooth is called circular tooth thickness. Theoretically, circular tooth thickness is half of the circular pitch.

(xxiii) Tooth Space The width of the space between two adjacent teeth measured along the pitch circle is called the tooth space. Theoretically, tooth space is equal to circular tooth thickness or half the circular pitch.

(xxiv) Working Depth (h_k) The working depth is the depth of engagement of two gear teeth, that is, the sum of their addendums.

(xxv) Whole Depth (h) The whole depth is the total depth of the tooth space, that is, the sum of the addendum and dedendum. Whole depth is also equal to working depth plus clearance.

(xxvi) Centre Distance The centre distance is the distance between centres of pitch circles of mating gears. It is also the distance between centres of base circles of mating gears.

(xxvii) Pressure Angle The pressure angle is the angle which the line of action makes with the common tangent to the pitch circles. The pressure angle is also called the angle of obliquity. It is denoted by α.

(xxviii) Line of Action The line of action is the common tangent to the base circles of mating gears. The contact between the involute surfaces of mating teeth must be on this line to give a smooth operation. The force is transmitted from the driving gear to the driven gear on this line.

(xxix) Arc of Contact The arc of contact is the arc of the pitch circle through which a tooth moves from the beginning to the end of contact with mating tooth.

(xxx) Arc of Approach The arc of approach is the arc of the pitch circle through which a tooth moves from its beginning of contact until the point of contact arrives at the pitch point.

(xxxi) Arc of Recess The arc of recess is the arc of the pitch circle through which a tooth moves from the contact at the pitch point until the contact ends.

(xxxii) Contact Ratio (m_p) The number of pairs of teeth that are simultaneously engaged is called contact ratio. If there are two pairs of teeth in contact all the time, the contact ratio is 2. As the two gears rotate, smooth and continuous transfer of power from one pair of meshing teeth to the following pair is achieved when the contact of the first pair continues until the following pair has established contact. Some overlapping is essential for this purpose. Therefore, the contact ratio is always more than 1. Other things being, the greater the contact ratio, the smoother the action of gears. The contact ratio for smooth transfer of motion is usually taken as 1.2. In industrial gearboxes for power transmission, the contact ratio is usually more than 1.4 (1.6 to 1.7).

(xxxiii) Circular Pitch The circular pitch (p) is the distance measured along the pitch circle between two similar points on adjacent teeth. Therefore,

$$p = \frac{\pi d'}{z}$$

where z is the number of teeth.

(xxxiv) Diametral Pitch The diametral pitch (P) is the ratio of the number of teeth to the pitch circle diameter. Therefore,

$$P = \frac{z}{d'}$$

The module (m) is defined as the inverse of the diametral pitch. Therefore,

$$m = \frac{1}{P} = \frac{d'}{z}$$

$\frac{D}{T}$

$$d' = mz$$

The centre to centre distance between two gears having zp and zg teeth is given by

$$a = \frac{1}{2}(d'_p + d'_g) = \frac{1}{2}(mz_p + mz_g)$$

$$a = \frac{m(z_p + z_g)}{2}$$

where,
a = centre to centre distance (mm)

z_p = number of teeth on pinion

z^g = number of teeth on gear

The gear ratio (i) that is, the ratio of the number of teeth on gear to that on pinion is given by,

$$i = \frac{n_p}{n_g} = \frac{z_g}{z_p}$$

$$i = \frac{T_g}{T_p} = \frac{N_p}{N_g}$$

where n_p = speed of pinion (rpm)
n_g = speed of gear (rpm)

There are a number of methods to manufacture gears. They include casting, blanking and machining. However, power transmitting gears are made of steel and made by the following methods:

a. Milling
b. Rack generation
c. Hobbing
d. Fellow gear shaper method

The **hobbing process** accounts for the manufacture of a major quantity of gears that are used for power transmission.

STANDARD SYSTEMS OF GEAR TOOTH

All standard systems prescribe the involute profile for gear tooth. The reasons are as follows:

1. The involute profile satisfies the fundamental law of gearing at any centre distance.
2. All involute gears of a given module and pressure angle are completely interchangeable.
3. All involute gears of a given module and pressure angle can be machined from one single tool.
4. The basic rack of an involute profile has straight sides. It is comparatively easy to machine straight sides. Further, straight sides can be more accurately machined compared with a curved surface.
5. As light change in the centre distance, which might be caused by incorrect mounting, has no effect upon the shape of the involute. In addition, the pitch point is still fixed and the law of gearing is satisfied. Therefore, the velocity ratio remains constant.

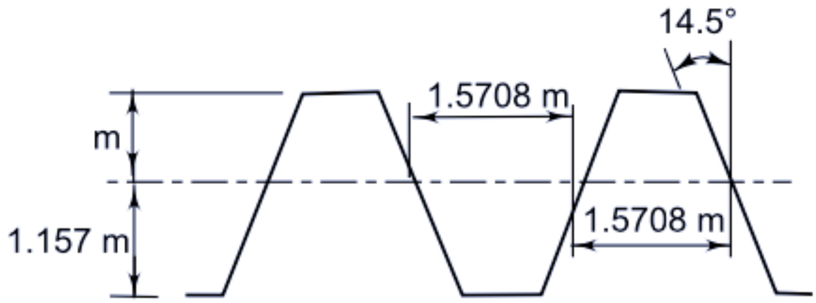
There are three standard systems for the shape of gear teeth. They are as follows:

(i) 14.5° full depth involute system

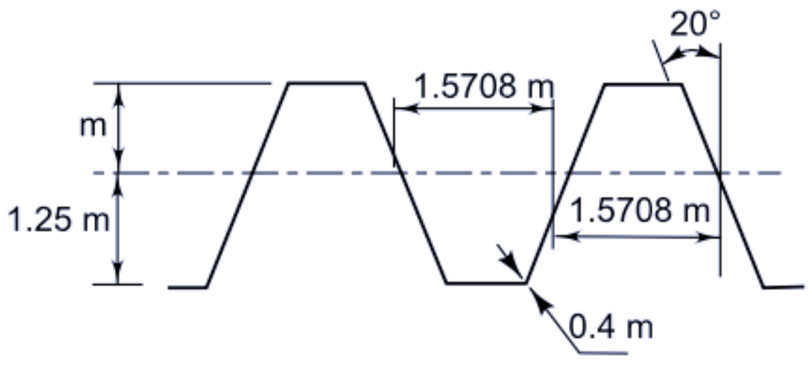
(ii) 20° full depth involute systems

(iii) 20°stubinvolutesystem

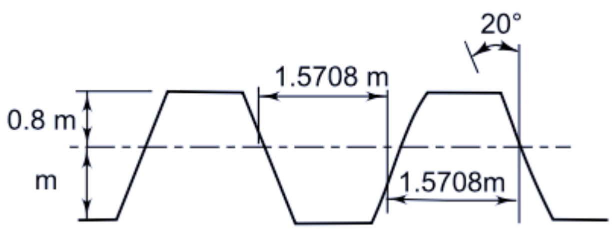
As the number of teeth on the gear is increased, the involute outline becomes straighter and straighter. When the number of teeth is infinity or when the pitch circle radius approaches infinity, the gear becomes a rack with straight-sided teeth. This rack is called the 'basic' rack, which is standardized in each system of gearing.



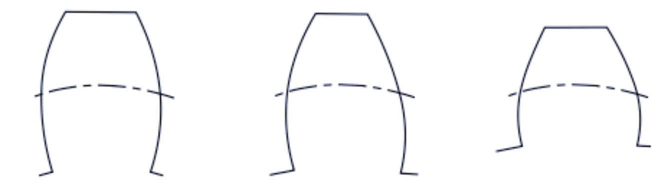
(a) 14.5° full depth involute system



(b) 20° full depth involute system



(c) 20° stub tooth involute system



(a) 14.5° full depth (b) 20° full depth (c) 20° stub

GEAR TRAINS

A gear train consists of two or more gears transmitting power from the driving shaft to the driven shaft. The gear trains are classified into the following categories:

- Simple gear train
- Compound gear train
- Reverted gear train
- Epicyclic gear train

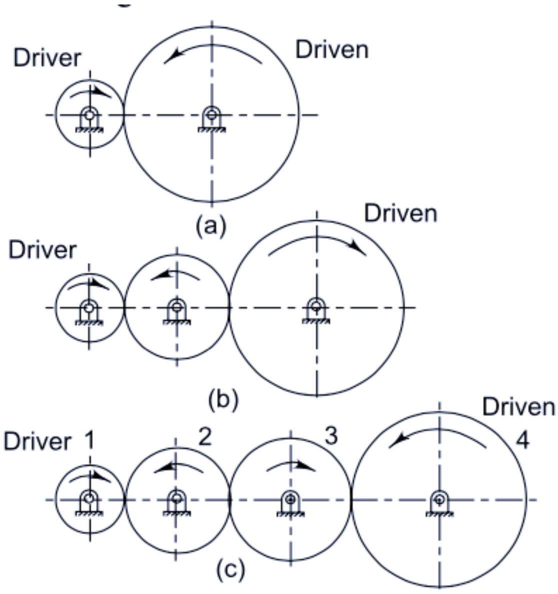


Fig. 17.15 *Simple Gear Trains*

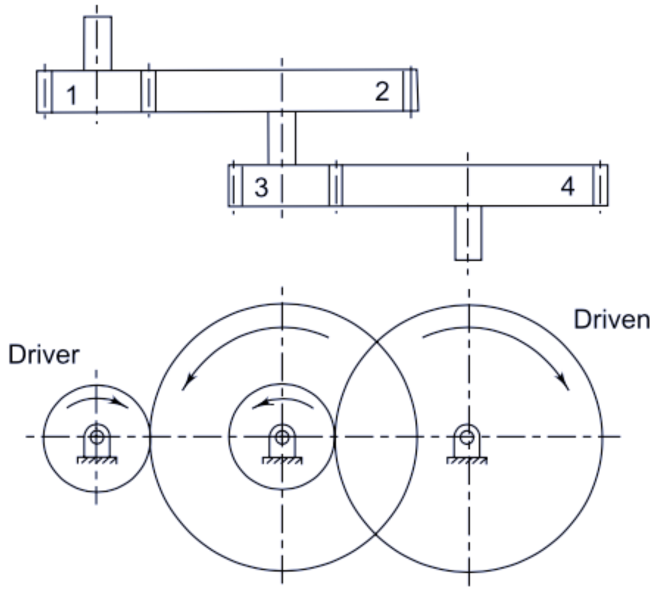


Fig. 17.16 *Compound Gear Train*

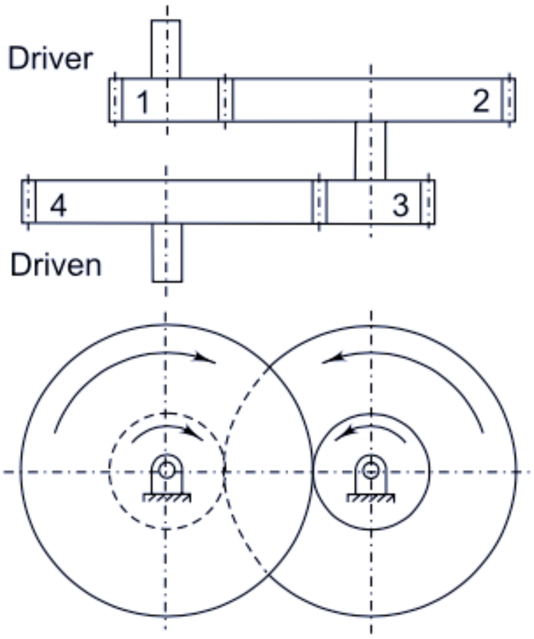


Fig. 17.17 *Reverted Gear Train*

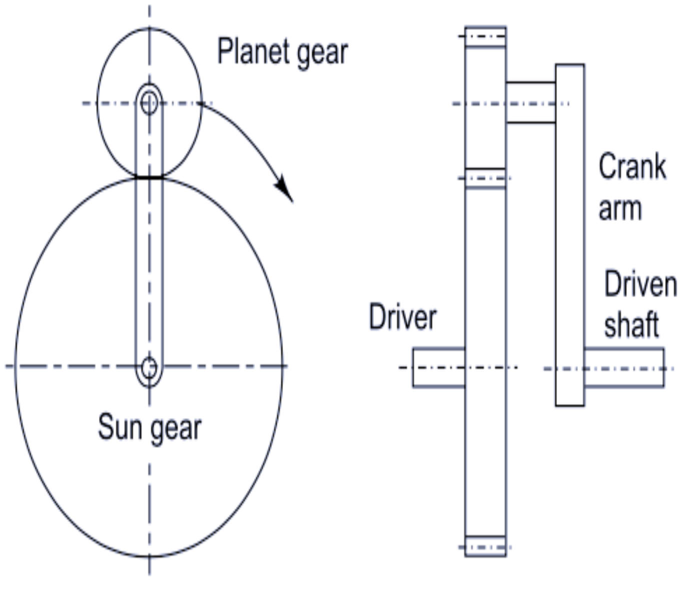


Fig. 17.18 *Epicyclic Gear Train*

INTERFERENCE AND UNDERCUTTING

The phenomenon of tooth profiles overlapping and cutting into each other is called '*interference*'. In this case, the tip of the tooth overlaps and digs into the root section of its mating gear. Interference is non-conjugate action and results in excessive wear, vibrations and jamming.

When the gears are generated by involute rack cutters, this interference is automatically eliminated because the cutting tool removes the interfering portion of the flank. This is called '*undercutting*'. Undercutting solves the problem of interference. However, an undercut tooth is considerably weaker. Undercutting not only weakens the tooth, but also removes a small involute portion adjacent to the base circle. This loss of involute profile may cause a serious reduction in the length of the contact.

The following methods can eliminate interference:

(i) Increase the Number of Teeth on the Pinion

Increasing the number of teeth increases the size of the gearbox and also increases the pitch line velocity. This is not desirable. The minimum number of teeth to avoid interference and undercutting is as follows:

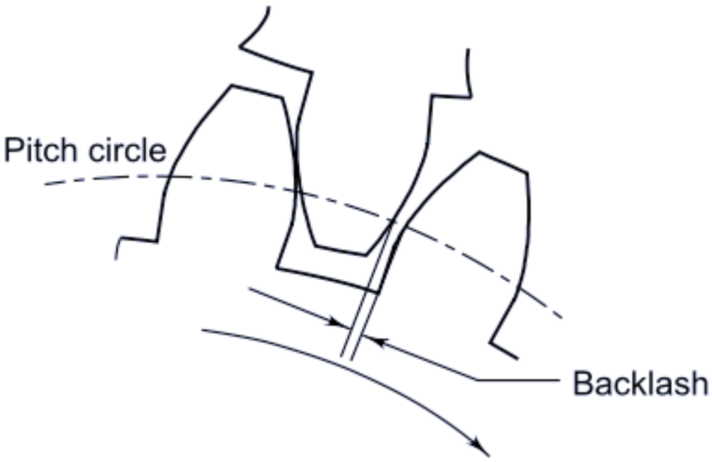
For 14.5° full depth system	-	32 Teeth
For 20° full depth system	-	17 Teeth
For 20° stub system	-	14 Teeth

(ii) Increase Pressure Angle This results in smaller base circle so that more portion of the tooth profile becomes involute.

(iii) Use Long and Short Addendum Gearing In this method, the addendum of the pinion is made longer than the standard addendum. Also, the addendum of the mating gear is made shorter than the standard addendum. However, this results in non-standard and non-interchangeable gears.

BACKLASH

Backlash is defined as the amount by which the width of tooth space exceeds the thickness of the engaging tooth measured along the pitch circle.



1. Backlash prevents the mating teeth from jamming together. The mating teeth do not make contact on both sides simultaneously. This makes the teeth roll together freely and smoothly.
2. Backlash compensates for machining errors.
3. Backlash compensates for thermal expansion of teeth.

FORCE ANALYSIS

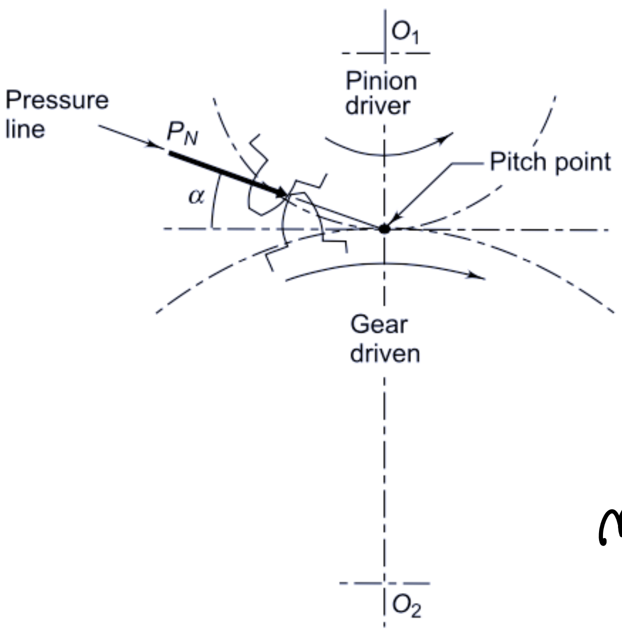


Fig. 17.20 *Gear Tooth Force*

P_N → Resultant force

Seperating

P_r → radial component force

P_t → tangential component

useful force

$$T = \frac{P \times 10^6}{\omega}$$

$$M_t = \frac{P}{\frac{2\pi N}{60}} \times 10^6 \text{ N-mm}$$

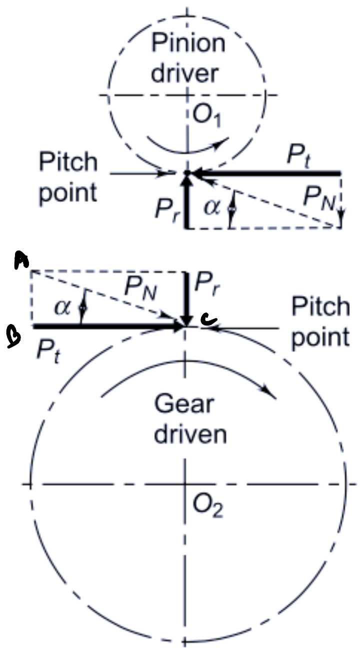
$$M_t = \frac{60 \times 10^6 P}{2\pi n} \text{ N-mm}$$

$P \rightarrow$ power in Kw

$n \rightarrow$ speed in rpm

Tangential force P_t acts @ pitch circle radius

$$\text{Torque } M_t = P_t \times r_p$$



$$P_t = \frac{M_t}{(d_p/2)}$$

$$\boxed{P_t = \frac{2M_t}{d_p}}$$

From figure $P_r = P_t \tan\alpha$

$$P_N = \frac{P_t}{\cos\alpha}$$

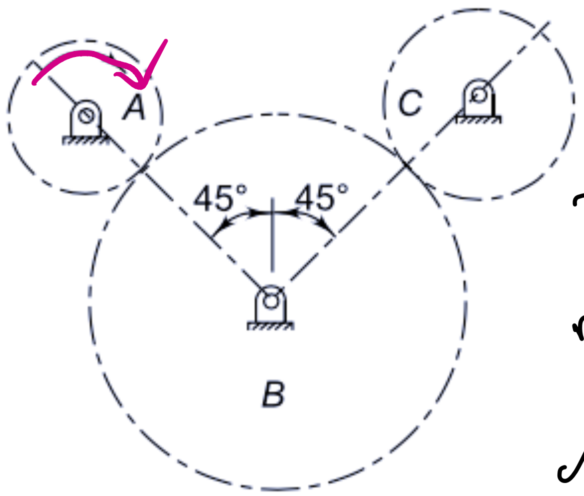
The above analysis of the gear tooth force is based on the following assumptions:

1. As the point of contact moves, the magnitude of the resultant force *PN* changes. This effect is neglected in the above analysis.
2. It is assumed that only one pair of teeth takes the entire load. At times there are two pairs, which are simultaneously in contact and share the load. This aspect is neglected in the analysis.
3. The analysis is valid under static conditions, i.e., when the gears are running at very low velocities. In practice, there is dynamic force in addition to force due to power transmission. The effect of this dynamic force is neglected in the analysis.

Problem: The pitch circles of a train of spur gears are shown in Fig.. Gear A receives 3.5 kW of power at 700 rpm through its shaft and rotates in the clockwise direction. Gear B is the idler gear while the gear C is the driven gear. The number of teeth on gears A, B and C are 30, 60 and 40 respectively, while the module is 5 mm. Calculate

(i) the torque on each gear shaft; and

(ii) the components of gear tooth forces.



Given data:

$P = 3.5\, kW$

$n = 700\ rpm$

$T_A = 30;\ T_B = 60;\ T_C = 40$

$m = 5\ mm\ ;\ \alpha = 20°$

Module of mating gears is same -

$$\text{module} = \frac{D}{T}$$

$$m_A = m_B = m_C$$

$$m_A = \frac{D_A}{T_A} \Rightarrow 5 = \frac{D_A}{30} \Rightarrow D_A = 150 mm$$

$$D_B = m \times T_B = 5 \times 60 = 300\ mm$$

$$m = \frac{D}{T}$$

$$D_C = 200\ mm$$

$$(M_t)_A = \frac{P}{\omega} = \frac{60 \times 10^6 \times P}{2 \pi n_A}\ N\text{-}mm$$

$$= \frac{60 \times 10^6 \times 3.5}{2 \pi \times 700}$$

$$(M_t)_A = 47746.48\ N\text{-}mm$$

Gear B is Idler gear. No torque is transmitted to it's shaft.

$$(M_t)_B = 0$$

$$P_A = P_C$$

$$(M_t)_A \times n_A = (M_t)_C \times n_C \Rightarrow$$

$$(M_t)_C = (M_t)_A \times \left(\frac{n_A}{n_C}\right)$$

$$\frac{n_A}{n_C} = \frac{T_C}{T_A}$$

$$\Rightarrow (M_t)_C = 47746.48 \times \left(\frac{40}{30}\right)$$

$$\boxed{(M_t)_C = 63661.97 \text{ N-mm}}$$

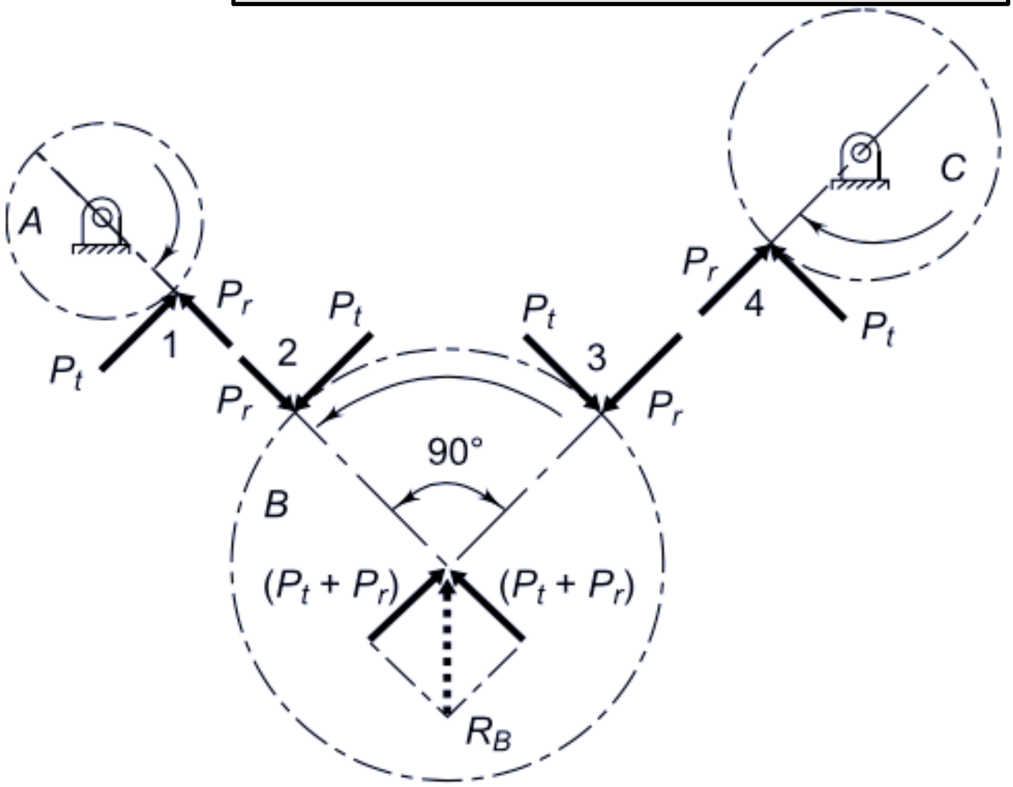


Fig. 17.24 *Free-body Diagram of Forces*

Force Components:

$$(P_t)_{AB} = \frac{2 \times (M_t)_A}{d_A} = \frac{2 \times (47746.48)}{150} = 636.61 \text{ N}$$

$$(P_r)_{AB} = (P_t)_{AB} \tan \phi \Rightarrow 636.61 \times \tan 20^\circ = (P_r)_{AB}$$

$(P_a)_{AB} = 231.70 \text{ N}$

Gear B → idler → It transfers all the torque.

$$(\text{Torque})_{AB} = (\text{Torque})_{BC}$$

$$(P_t)_{AB} \times \cancel{\frac{d_B}{2}} = (P_t)_{BC} \times \cancel{\frac{d_B}{2}}$$

$$(P_t)_{AB} = (P_t)_{BC} = P_t = 636.61 \text{ N}.$$

$$(P_a)_{BC} = (P_t)_{BC} \tan\psi = 231.7 \text{ N}$$

Problem: A planetary gear train is shown in Fig. The sun gear A rotates in a clockwise direction and transmits 5 kW of power at 1440 rpm to the gear train. The number of teeth on the sun gear A, the planet gear B and the fixed ring gear C are 30, 60 and 150 respectively.

The module is 4 mm and the pressure angle is 20°. Draw a free-body diagram of forces and calculate the torque that the arm D can deliver to its output shaft.

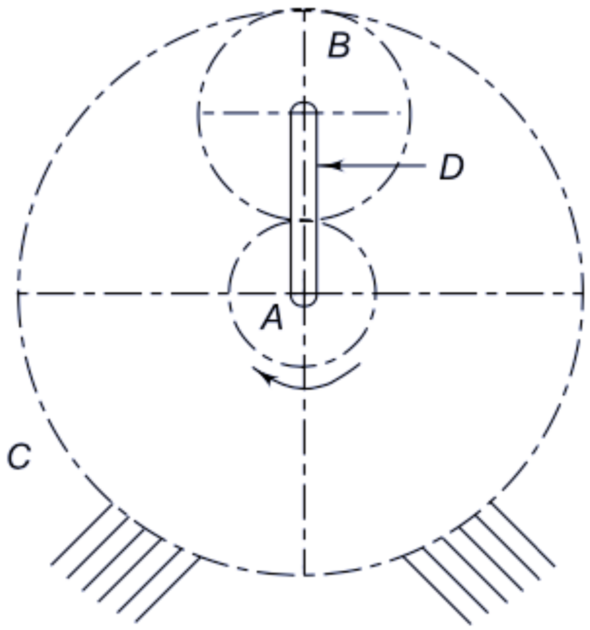


Fig. 17.25 *Planetary Gear Train*

Given data:

$P = 5 kW$

$n = 1440 rpm$

$T_A = 30;\ T_B = 60;\ T_C = 150$

$m = 4 mm \Rightarrow m_a = m_b = m_c$

$\delta = 20^\circ$

$m = \frac{D_B}{T_B}$

$m_A = 4mm = \frac{D_A}{T_A} \Rightarrow D_A = 120 mm$

$D_B = 240 mm$

$D_C = 600 mm$

Length of Arm 'D' $L_D = \frac{D_A + D_B}{2} = 180 mm$

$(M_t)_A = \frac{60 \times P \times 10^6}{2\pi n_a} = \frac{60 \times 5 \times 10^6}{2\pi \times 1440}$

$\boxed{P = \frac{2\pi N T}{60}}$

$(M_t)_A = 33157.27\ N\text{-}mm$

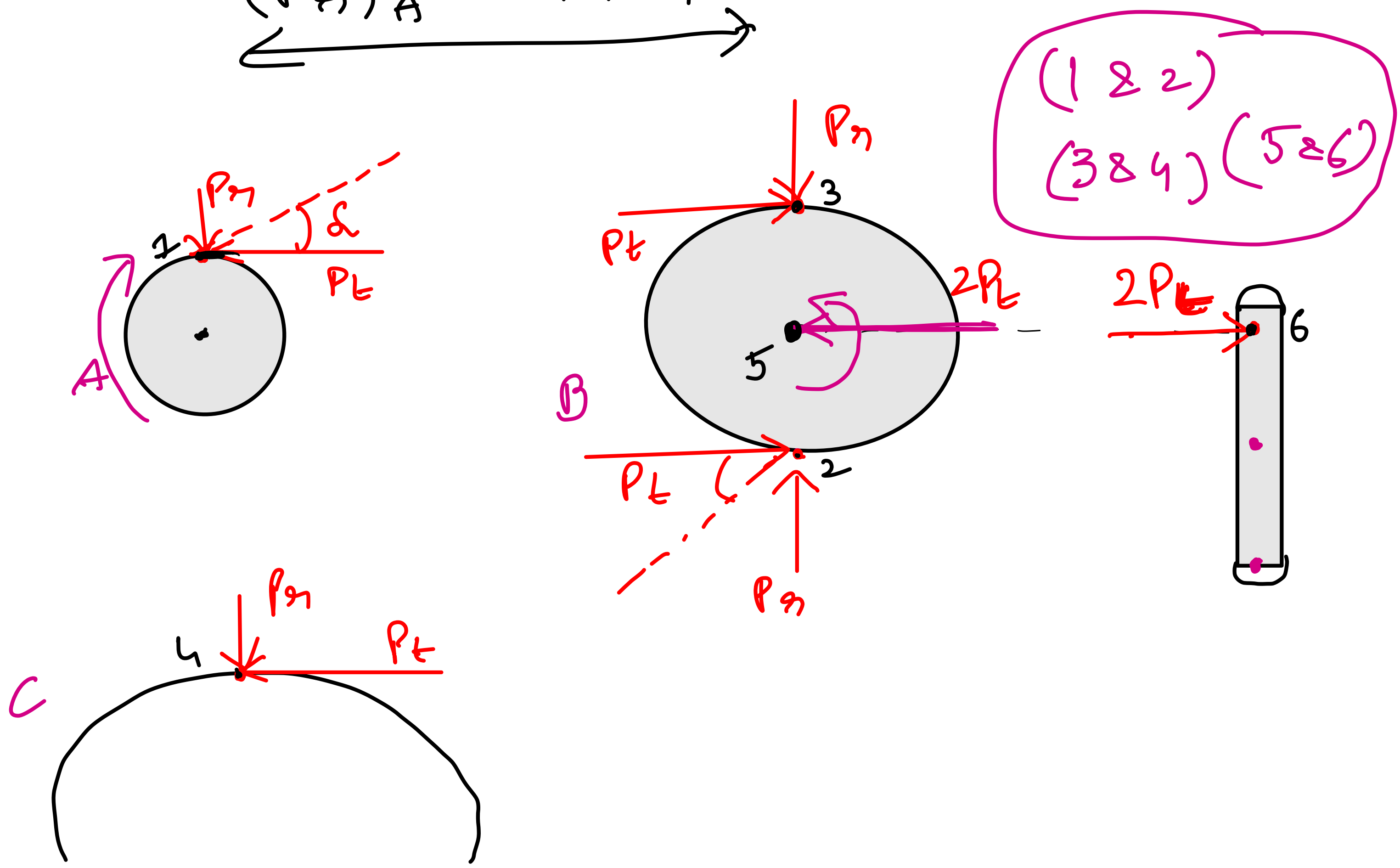
$M_t = P_t \times r_A$

$$(P_t)_A = \frac{2\times(M_t)_A}{D_A} = \frac{2\times 33157.27}{120}$$

$$(P_t)_A = 552.62 \text{ N.}$$

$$(P_n)_A = P_t \tan\alpha = 552.62\times\tan 20^\circ$$

$$(P_n)_A = 201.14 \text{ N}$$



Torque acting (or) torque transferred by arm

$$(M_t)_D = (2P_t)(L_D)$$

$$= 2\times(552.62)\times 180$$

$$(M_t)_D = 198943.2 \text{ N-mm}$$

$$\boxed{(M_t)_D = 198.94 \text{ N-mm}}$$

SELECTION OF MATERIAL

The desirable properties of gear material are as follows:

1. The load carrying capacity of the gear tooth depends upon the ultimate tensile strength or the yield strength of the material. When the gear tooth is subjected to fluctuating forces, the endurance strength of the tooth is the deciding factor. The gear material should have sufficient strength to resist failure due to breakage of the tooth.
2. In many cases, it is ***'wear rating'*** rather than '*strength rating*' which decides the dimensions of the gear tooth. The resistance to wear depends upon alloying elements, grain size, percentage of carbon, and surface hardness. The gear material should have sufficient surface endurance strength to avoid failure due to destructive pitting.
3. For high-speed power transmission, the sliding velocities are very high and the material should have low coefficient of friction to avoid failure due to scoring.
4. The amount of thermal distortion or warping during the heat treatment process is a major problem in gear applications. Due to warping, the load gets concentrated at one corner of the gear tooth. **Alloy steels** are superior to plain carbon steels in this respect, due to consistent thermal distortion.

   - Gears are made of cast iron, steel, bronze and phenolic resins.

   - Large size gears are made of **grey cast iro**n of Grades FG 200, FG 260 or FG 350. They are cheap and generate less noise compared with steel gears. They have good wear resistance. Their main drawback is poor strength.

   - **Case-hardened steel** gears offer the best combination of a wear- resisting hard surface together with a ductile and shock-absorbing core.
   - The plain carbon steels used for medium duty applications are **50C8, 45C8, 50C4 and 55C8**. For heavy duty applications, alloy steels **4OCrl, 30Ni4Cr1 and 4ONi3Cr65Mo55** are used.

– For planetary gear trains, **alloy steel 35NilCr60** is recommended.

– Although steel gears are costly, they have higher load carrying capacity. Bronze is mainly used for worm wheels due to its low coefficient of friction and excellent conformability. It is also suitable where resistance to corrosion is an important consideration in applications like water pumps. Their main drawback is excessive cost.

NUMBER OF TEETH

As the number of teeth decreases, a point is reached when there is interference and the standard tooth profile requires modification.

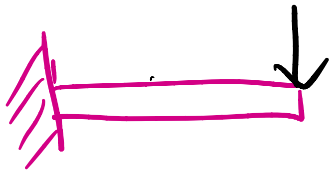
$$T_{min} = \frac{2}{\sin^2 \alpha}$$

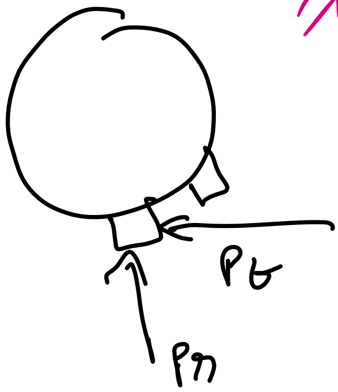
In practice, giving a slight radius to the tip of tooth can further reduce the value of z_{min}.

Pressure angle (α)	14.5°	20°	25°
$z_{min.}$ (theoretical)	32	17	11
$z_{min.}$ (practical)	27	14	9

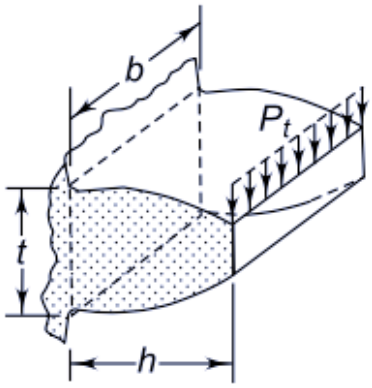
There is a concept of **'*hunting*'** tooth for uniform distribution of tooth wear. Suppose (zp = 20) and (zg = 40), then after every two revolutions of the pinion, the same pair of teeth will engage. If however, we take (zp = 20) and (zg = 41), the pinion will rotate 41 times before the same pair of teeth will engage again. This extra tooth is called the *hunting tooth*. It results in more even distribution of wear. For the provision of hunting tooth, it should be permissible to alter the velocity ratio slightly.

BEAM STRENGTH OF GEAR TOOTH





Gear tooth is treated as a cantilever beam



The tangential component (P_t) causes the bending moment about the base of the tooth.

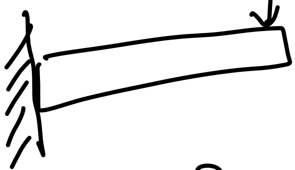
The Lewis equation is based on the following assumptions:

1 The effect of the radial component (P_r), which induces compressive stresses, is neglected.

2 It is assumed that the tangential component (P_t) is uniformly distributed over the face width of the gear. This is possible when the gears are rigid and accurately machined.

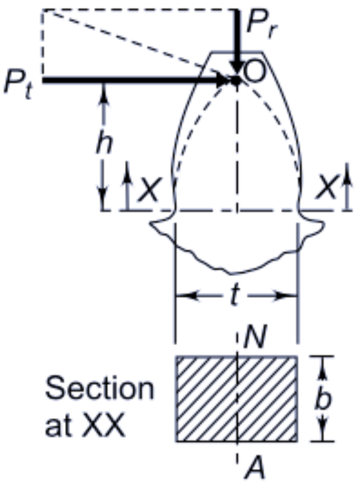
3 The effect of stress concentration is neglected.

4 It is assumed that at any time, only one pair of teeth is in contact and takes the total load.



P_t → moment at setup

σ_b → bending stress



10 *Gear Tooth as Parabolic Beam*

$$m = P_t \times h$$

$$\frac{m}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$m = \frac{\sigma_b \times I}{y}$$

$$I = \frac{bd^3}{12} = \frac{bt^3}{12}$$

$$y = t/2$$

Cross-section of the tooth varies from the free end to the fixed end. Therefore, a parabola is constructed within the tooth profile and shown by a dotted line in Fig.

The advantage of parabolic outline is that it is a beam of uniform strength.

$$\sigma_b = \frac{My}{I} = \frac{(P_t \times h) \times \cancel{t}/\cancel{2}}{\frac{b t^{\cancel{3}2}}{\cancel{12}\,6}}$$

$$P_t = \frac{b \times \sigma_b \times t^2}{6h}$$

$m \rightarrow$ module $= D/T$

multiplying & dividing by m

$$P_t = m b \sigma_b \times \left(\frac{t^2}{6hm}\right)$$

$$\frac{t^2}{6hm} = Y$$

$Y \rightarrow$ Lewis factor

$$\boxed{P_t = m b \sigma_b Y}$$

$P_t \rightarrow$ Beam strength of tooth when $\sigma_b = (\sigma_b)_{max}$

Equation (a) gives the relationship between the tangential force (P_t) and the corresponding stress σ_b. When the tangential force is increased, the stress also increases.

When the stress reaches the permissible magnitude of bending stresses, the corresponding force (P_t) is called the beam strength. ***Therefore, the beam strength (S_b) is the maximum value of the tangential force that the tooth can transmit without bending failure.***

$$\boxed{S_b = mb(\sigma_b)_{max} Y}$$

$$\boxed{S_b \geq P_{eff}}$$

$$S_b = mb \times (\sigma_b \times y)$$

$S_b \rightarrow$ Beam strength of tooth

Table 17.3 *Values of the Lewis form factor Y for 20° full-depth involute system*

z	Y	z	Y	z	Y
15	0.289	27	0.348	55	0.415
16	0.295	28	0.352	60	0.421
17	0.302	29	0.355	65	0.425
18	0.308	30	0.358	70	0.429
19	0.314	32	0.364	75	0.433
20	0.320	33	0.367	80	0.436
21	0.326	35	0.373	90	0.442
22	0.330	37	0.380	100	0.446
23	0.333	39	0.386	150	0.458
24	0.337	40	0.389	200	0.463
25	0.340	45	0.399	300	0.471
26	0.344	50	0.408	Rack	0.484

$(\sigma_b \times y)$ ↓ This decides the strength of gears which is weak.

It is observed that m and b are same for pinion as well as for gear. When different materials are used, the product (σ_b * Y) decides the weaker between pinion and gear. The Lewis form factor Y is always less for a pinion compared with gear.

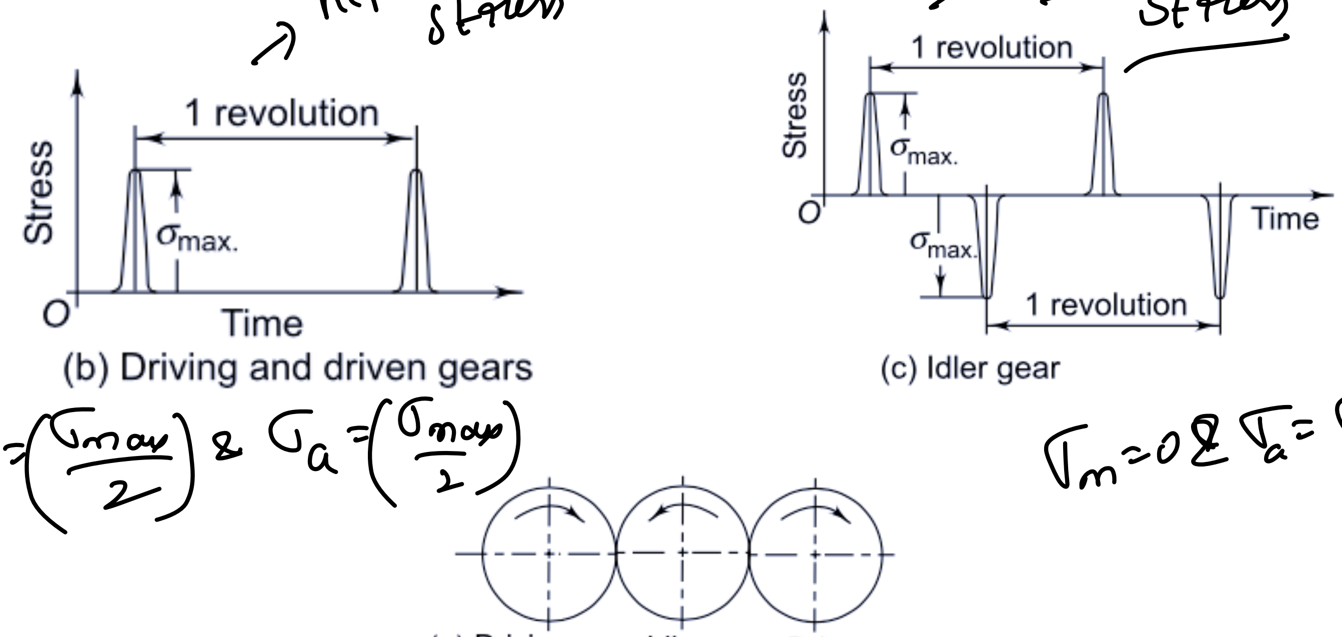
When the same material is used for the pinion and gear, the pinion is always weaker than the gear.

PERMISSIBLE BENDING STRESS

The tooth of the gear is subjected to fluctuating bending stress as it comes in contact with the meshing tooth. The stress–time diagrams for gear teeth are illustrated in Fig

Repeated stress

Reversed Stress



(b) Driving and driven gears

(c) Idler gear

$\sigma_m = \left(\frac{\sigma_{max}}{2}\right)$ & $\sigma_a = \left(\frac{\sigma_{max}}{2}\right)$

$\sigma_m = 0$ & $\sigma_a = \sigma_{max}$

(a) Driving gear Idler gear Driven gear

Since the teeth are subjected to fluctuating stresses, endurance limit stress (S_e) is the criterion of design. Therefore, the maximum bending stress is equal to the endurance limit stress of the gear tooth. The endurance limit stress of the gear tooth depends upon the following factors:

- (i) Surface finish of the gear tooth
- (ii) Size of the gear tooth
- (iii) Reliability used in design
- (iv) Stress concentration in the gear tooth
- (v) Gears rotating in one direction or both directions
- (vi) Gears tooth subjected to stress in one direction or both directions

$\sigma_b = \sigma_e = \left(\frac{1}{3}\right)\sigma_{ut}$

In practice, it is difficult to get the above- mentioned data for each and every case of gear design.

Earle Buckingham has suggested that the endurance limit stress of gear tooth is approximately one-third of the ultimate tensile strength of the material.

In case of bronze gears, the endurance limit stress is taken as 40% of the ultimate tensile strength.

Only for Bronze gears $\quad \sigma_e = 0.4 \times \sigma_{ut}$

Electric motors $\leftarrow$ $\dfrac{\text{Starting torque}}{\text{rated torque}}$

EFFECTIVE LOAD ON GEAR TOOTH

$$M_t = \frac{60 \times P \times 10^6}{2\pi n} \text{ N-mm}$$

$$P_t = \frac{2 M_t}{d}$$

Service factor $C_S = \dfrac{\text{max torque}}{\text{rated torque}}$

$$C_S = \frac{(M_t)_{max}}{M_t} = \frac{(P_t)_{max}}{P_t}$$

$$\boxed{(P_t)_{max} = C_S P_t}$$

The value of the tangential component, therefore, depends upon the rated power and rated speed. In practical applications, the torque developed by the source of power varies during the work cycle. Similarly, the torque required by the driven machine also varies. The two sides are balanced by means of a flywheel.

- In gear design, the maximum force (due to maximum torque) is the criterion.

- This is accounted by means of a service factor. The service factor C_S is defined as

Table 17.4 *Service factor for speed reduction gearboxes*

Working characteristics of Driving machine (Table 17.5)	*Working characteristics of Driven machine (Table 17.6)*		
	Uniform	*Moderate shock*	*Heavy shock*
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.5	1.75	2.25

Table 17.5 *Examples of Driving machines with different working characteristics*

Characteristic of operation	*Driving machines*
Uniform	Electric motor, steam turbine, gas turbine
Light shock	Multi-cylinder internal combustion engine
Medium shock	Single cylinder internal combustion engine

Table 17.6 *Examples of Driven machines with different working characteristics*

Characteristic of operation	*Driven machines*
Uniform	Generator, belt conveyor, platform conveyor, light elevator, electric hoist, feed gears of machine tools, ventilators, turbo-blower, mixer for constant density material
Medium shock	Main drive to machine tool, heavy elevator, turning gears of crane, mine ventilator, mixer for variable density material, multi-cylinder piston pump, feed pump
Heavy shock	Press, shear, rubber dough mill, rolling mill drive, power shovel, heavy centrifuge, heavy feed pump, rotary drilling apparatus, briquette press, pug mill

There are two methods to account for the dynamic load—approximate estimation by the *velocity factor* in the preliminary stages of gear design and precise calculation by *Buckingham's equation* in the final stages of gear design.

It is difficult to calculate the exact magnitude of dynamic load in the preliminary stages of gear design. To overcome this difficulty, a velocity factor C_v is used.

i) For ordinary and commercially cut gears made with form cutters and with $v < 10$ m/s,

$$C_v = \frac{3}{3+v}$$

ii) For accurately hobbed and generated gears with $v < 20$ m/s,

$$C_v = \frac{6}{6+v}$$

iii) For precision gears with shaving, grinding and lapping operations and with $v > 20$ m/s,

$$C_v = \frac{5.6}{5.6+\sqrt{v}}$$

$$(P_t)_{max} = C_s \times P_t$$

$v \rightarrow$ pitch line velocity

$$V = \frac{\pi d n}{60 \times 10^3}$$

$$\boxed{P_{eff} = \frac{C_s P_t}{C_v}}$$

Buckingham's method:

$$P_{eff} = C_s P_t + (P_d)$$

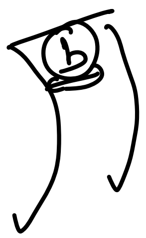
$P_d \Rightarrow$ dynamic load (or) incremented dynamic load.

2 → 10N
4 → 35N

$$P_d = \frac{21V\left(Ceb + P_t\right)}{21V + \sqrt{(Ceb + P_t)}}$$

V → Pitch line vel

C → deformation factor (N/mm^2)



e → sum of errors b/n meshing tooth

b → face width

$$C = \frac{K}{\left(\frac{1}{E_p} + \frac{1}{E_g}\right)}$$

K → constant (form of tooth)

E_p, E_g → Elastic modulus of pinion & gear

K = 0.107 → 14.5° full depth

K = 0.111 → 20° " "

K = 0.115 → 20° stub tooth

ESTIMATION OF MODULE BASED ON BEAM STRENGTH

$$m = D/T \qquad S_b \geq P_{eff} \qquad m_t = P/\omega$$

$$\boxed{S_b = (P_{eff})(fos) \quad \text{— (a)}} \qquad fos = 1.5 - 2.0$$

$$P_t = \frac{2m_t}{d} = \frac{2m_t}{m \times T} = \frac{2}{mT}\left[\frac{60 \times P \times 10^6}{2\pi n}\right]$$

$$P_{eff} = \frac{C_s}{C_v} \times P_t = \frac{C_s}{C_v}\left[\frac{\cancel{2}}{mT}\left[\frac{60 \times P \times 10^6}{\cancel{2}\pi n}\right]\right]$$

$$P_{eff} = \frac{60 \times 10^6}{\pi}\left[\frac{P \times C_s}{mTnC_v}\right] \quad \text{— (1)}$$

$$(\sigma_b)_{max} = \frac{\sigma_{ut}}{3}$$

$$S_b = mb\sigma_b Y = m^2\left(\frac{b}{m}\right)\left(\frac{\sigma_{ut}}{3}\right)Y \quad \text{— (2)}$$

$$m^2\left(\frac{b}{m}\right)\left(\frac{\sigma_{ut}}{3}\right)Y = \frac{60 \times 10^6}{\pi}\left[\frac{P \times C_s \times fos}{mTnC_v}\right]$$

$$\boxed{m = \left[\frac{60 \times 10^6}{\pi}\left\{\frac{P \times C_s \times fos}{TnC_v\left(\frac{b}{m}\right)\left(\frac{\sigma_{ut}}{3}\right)Y}\right\}\right]^{1/3}}$$

WEAR STRENGTH OF GEAR TOOTH

The failure of the gear tooth due to pitting occurs when the contact stresses between two meshing teeth exceed the surface endurance strength of the material. Pitting is a surface fatigue failure, characterized by small pits on the surface of the gear tooth. In order to avoid this type of failure, the proportions of the gear tooth and surface properties, such as surface hardness, should be selected in such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth.

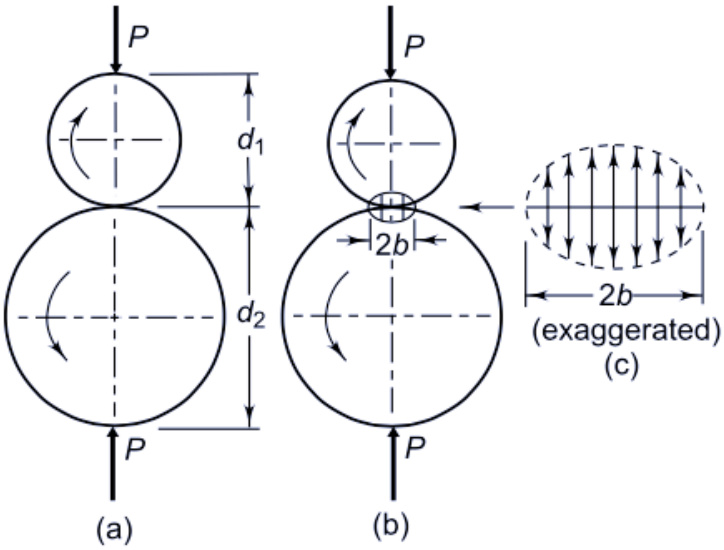


Fig. 17.42 *Contact Stresses*

$$\sigma_c = \frac{2P}{\pi b l}$$

and $$b = \left[\frac{2P(1-\mu^2)\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}{\pi l \left(\frac{1}{d_1}+\frac{1}{d_2}\right)} \right]^{1/2}$$

where,

σ_c = maximum value of the compressive stress (N/mm²)

P = force pressing the two cylinders together (N)

b = half width of deformation (mm)

l = axial length of the cylinder (mm)

d_1, d_2 = diameters of the two cylinders (mm)

E_1, E_2 = modulii of elasticity of two cylinder materials (N/mm²)

μ = Poisson's ratio

Substituting value of b in σ_c

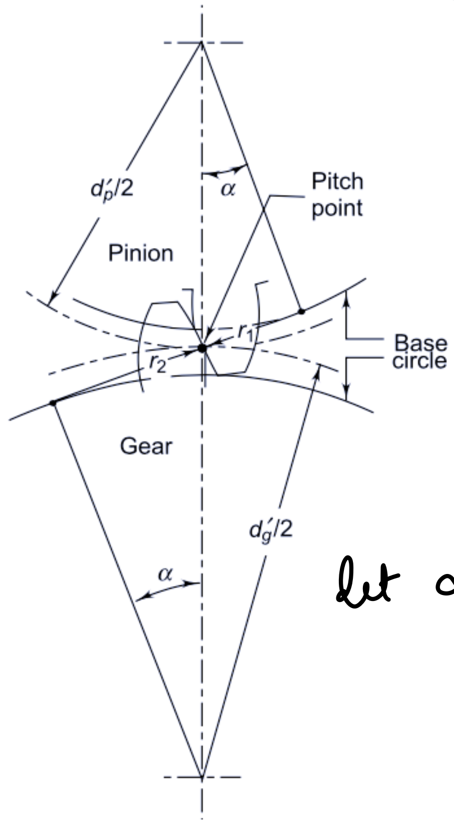
$$\sigma_c^2 = \frac{1}{\pi(1-\mu^2)}\left(\frac{P}{l}\right)\left\{\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}\right\}$$

where r_1, r_2 are the radii of two cylinders.

Substituting ($\mu = 0.3$),

$$\sigma_c^2 = 0.35\left(\frac{P}{l}\right)\left\{\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}\right\} \quad -(3)$$

Material	*Modulus of elasticity (N/mm²)*	*Poisson's ratio*
Steel	206 000	0.3
Cast steel	202 000	0.3
Spheroidal cast iron	173 000	0.3
Cast tin bronze	103 000	0.3
Tin bronze	113 000	0.3
Grey cast iron	118 000	0.3



using radius of curvature instead of radius of cylinder

$$r_1 = \frac{d_p}{2}\sin\alpha \; ; \quad r_2 = \frac{d_g}{2}\sin\alpha$$

$$\frac{1}{r_1}+\frac{1}{r_2} = \frac{2}{\sin\alpha}\left[\frac{1}{d_p}+\frac{1}{d_g}\right] \quad \text{— (a)}$$

let define a ratio factor $\left[T_g = \frac{d_g}{m}\right]$

$$Q = \frac{2T_g}{T_g+T_p} = \frac{2d_g}{d_g+d_p} \quad \text{— (b)}$$

$$\frac{1}{d_p}+\frac{1}{d_g} = \frac{d_p+d_g}{d_p d_g} = \frac{2}{Q d_p} \quad \text{— (c)}$$

using (c) in a

$$\frac{1}{r_1}+\frac{1}{r_2} = \frac{4}{Q d_p \sin\alpha}$$

The force acting along pitch line P_N

$$P_N = \frac{P_t}{\cos\alpha}$$

$l = b$ (face width)

$$\sigma_c^2 = \frac{1.4 P_t}{bQ d_p \sin\alpha\cos\alpha\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}$$

Let $$K = \frac{\sigma_c^2 \sin\alpha\cos\alpha\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}{1.4}$$

$$K = \frac{P_t}{b\,Q\,d_p} \Rightarrow \boxed{P_t = b\,Q\,d_p\,K}$$

when $\sigma_c \to$ max values, then P_t becomes the wear strength

$$\boxed{S_w = b\,Q\,d_p\,K}$$

Buckingham's eqn of wear

$S_w \to$ Wear strength of gear

$\sigma_c \to$ surface endurance strength

$$Q = \frac{2T_g}{T_g - T_p} \longrightarrow \text{For internal gear}$$

$$\boxed{S_b = m\,b\,\sigma_b\,Y}$$

lets take the case of steel

$$E_1 = E_2 = 206,000 \text{ N/mm}^2$$

$$\alpha = 20^\circ$$

$$\sigma_c = 0.27 \times 9.81 \times (BHN) \text{ N/mm}^2 \longrightarrow \text{G.Niemann eqn}$$

BHN $\to$ Brinell Hardness Numbers

$$K = \frac{\sigma_c^2 \sin\alpha \cos\alpha \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}{1.4}$$

$$\boxed{K = 0.16\left(\frac{BHN}{100}\right)^2}$$

ESTIMATION OF MODULE BASED ON WEAR STRENGTH

In order to avoid failure of gear tooth due to pitting,

$$S_w > P_{eff}$$

Introducing a factor of safety,

$$S_w = P_{eff}\,(fs)$$

(a) $$P_{eff} = \frac{60\times10^6}{\pi}\left\{\frac{(\text{kW})C_s}{mznC_v}\right\}$$

$$S_w = bQd_p'K = m\left(\frac{b}{m}\right)Q(mz_p)K \longleftrightarrow S_w = m^2\left(\frac{b}{m}\right)Qz_pK$$

$$m = \left[\frac{60\times10^6}{\pi}\left\{\frac{(\text{kW})C_s(fs)}{z_p^2 n_p C_v\left(\frac{b}{m}\right)QK}\right\}\right]^{1/3}$$

Problem: It is required to design a pair of spur gears with 20° full-depth involute teeth based on the Lewis equation. The velocity factor is to be used to account for dynamic load. The pinion shaft is connected to a 10 kW, 1440 rpm motor. The starting torque of the motor is 150% of the rated torque. The speed reduction is 4 : 1. The pinion as well as the gear is2made of plain carbon steel 40C8 (σ_{ut} = 600 N/mm^2). The factor of safety can be taken as 1.5. Design the gears, specify their dimensions and suggest suitable surface hardness for the gears.

Given data: $\delta = 20°$; $P = 10\,kW$; $n = 1440\,rpm$; $i = 4$

Starting torque = 150% of rated torque; FOS = 1.5

Both the gears are made up of same material

For 20° press angle $T_{min} = 18 = T_p$

$$\sigma_c = \sigma_b = \sigma_{ut}/3 = 600/3 = 200\ N/mm^2$$

$$i = \frac{T_g}{T_p} = 4 \Rightarrow T_g = 4 \times 18 = 72 \text{ teeth}$$

$$M_t = \frac{60 \times P \times 10^6}{2\pi n} \text{ N-mm} = \frac{60 \times 10 \times 10^6}{2\pi \times 1440} = 66314.55 \text{ N-mm}$$

$$C_s = \frac{\text{Starting torque}}{\text{rated torque}} = 1.5$$

1. Beam strength ✓

2. wear "

$$S_b = m b \sigma_b Y = P_{eff}$$

$$P_{eff} = \frac{C_s P_t}{C_v} \qquad P_t = \frac{2M_t}{d}$$

From data book $Y = 0.377$ for $T = 18$ teeth

$C_v \Rightarrow$ Assuming pitch vel $= 5$ m/s

$$C_v = \frac{3}{3+V} = \frac{3}{3+5} = \frac{3}{8}$$

lets assume the ratio $(b/m) = 10$

$$m = \left[\frac{60 \times 10^6}{\pi} \left\{\frac{P C_s (fos)}{T_p n_p C_v \left(\frac{b}{m}\right)\left(\frac{\sigma_{ut}}{3}\right) Y}\right\}\right]^{1/3}$$

$$m = \left\{\frac{60 \times 10^6}{\pi} \times \frac{10 \times 1.5 \times 1.5}{18 \times 1440 \times (3/8)(10)\left(\frac{600}{3}\right) \times 0.377}\right\}^{1/3}$$

$m = 3.88$ mm $\simeq 4$ mm

module $m = 4mm$

$$d_p = mT_p = 4\times18 = 72\,mm$$

$$d_g = mT_g = 4\times72 = 288\,mm$$

$$\left(b/m\right) = 10 \Rightarrow b = 10m = 40\,mm$$

<u>$S_b = (P_{eff})\times FOS$</u>

<u>Checking for forces acting:</u>

$$P_t = \frac{2M_t}{d_p} = \frac{2\times66,314.55}{72} = 1842.07\,N$$

S_b & FOS

$$P_{eff} = P_t\times\frac{C_s}{C_v}$$

$$S_b = m\,b\,\sigma_b\,Y$$

Pitch line Velocity $V = \dfrac{\pi d n}{60\times10^3}$

$$= \frac{\pi d_p n_p}{60\times10^3} = \frac{\pi\times72\times1440}{60\times10^3}$$

$$V = 5.42\,m/s$$

$$C_v = \frac{3}{3+V} = \frac{3}{3+5.42} = 0.3562$$

$$P_{eff} = \frac{C_s}{C_v}\times P_t = \frac{1.5}{0.3562}\times1842.07 = 7757.72\,N$$

To design

$$S_b = (P_{eff}) \times FOS$$

$$S_b = 4 \times 40 \times 200 \times 0.377$$

$$= 12064 \text{ N}$$

$$12064 = 7757.72 \times (FOS)$$

$$\boxed{FOS = 1.555} \rightarrow \text{obtained FOS.}$$

This design is safe for a module of 4 mm.

Hardness :

wear strength $S_w = b Q d_p K$

$$Q = \text{Ratio factor} = \frac{2T_g}{T_g + T_p} = \frac{2 \times 72}{72 + 18} = 1.6$$

$$K = 0.16 \left(\frac{BHN}{100}\right)^2$$

wear strength will be equal to $(P_{eff} \times FOS)$

$$S_w = (P_{eff} \times FOS)$$

$$= (7757.72 \times 1.5)$$

$$S_w = 11{,}636.58$$

$S_w = bQdpK \Rightarrow$

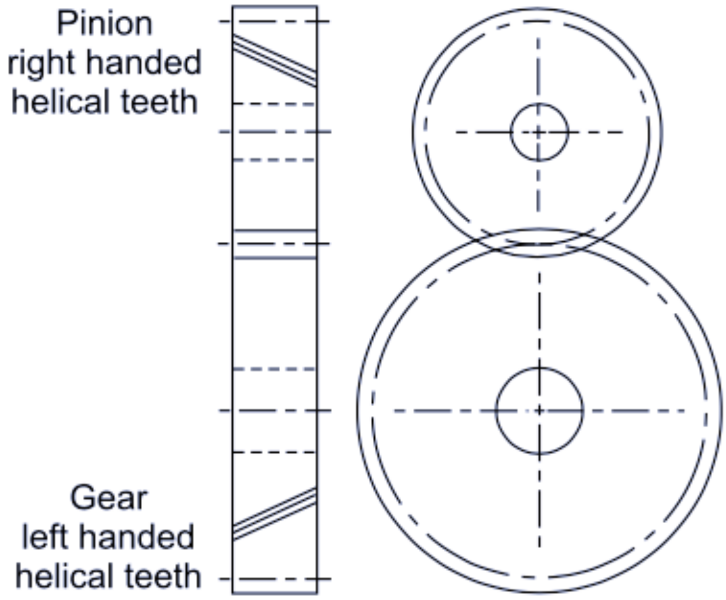
$$11{,}636.58 = (40) \times 1.6 \times 72 \times 0.16 \times \left(\frac{BHN}{100}\right)^2$$

$$\left(\frac{BHN}{100}\right)^2 = \frac{11{,}636.58}{40 \times 1.6 \times 72 \times 0.16}$$

$$BHN = 379.27 \Rightarrow \boxed{BHN = 380}$$

Assignment Problem: It is required to design a pair of spur gears with 20° full-depth involute teeth consisting of a 20-teeth pinion meshing with a 50 teeth gear. The pinion shaft is connected to a 22.5 kW, 1450 rpm electric motor. The starting torque of the motor can be taken as 150% of the rated torque. The material for the pinion is plain carbon steel Fe 410 (σ_{ut} = 410 N/mm^2), while the gear is made of grey cast iron FG 200 (σ_{ut} =200 N/mm2). The factor of safety is 1.5. Design the gears based on the Lewis equation and using velocity factor to account for the dynamic load.

HELICAL GEARS



Top view

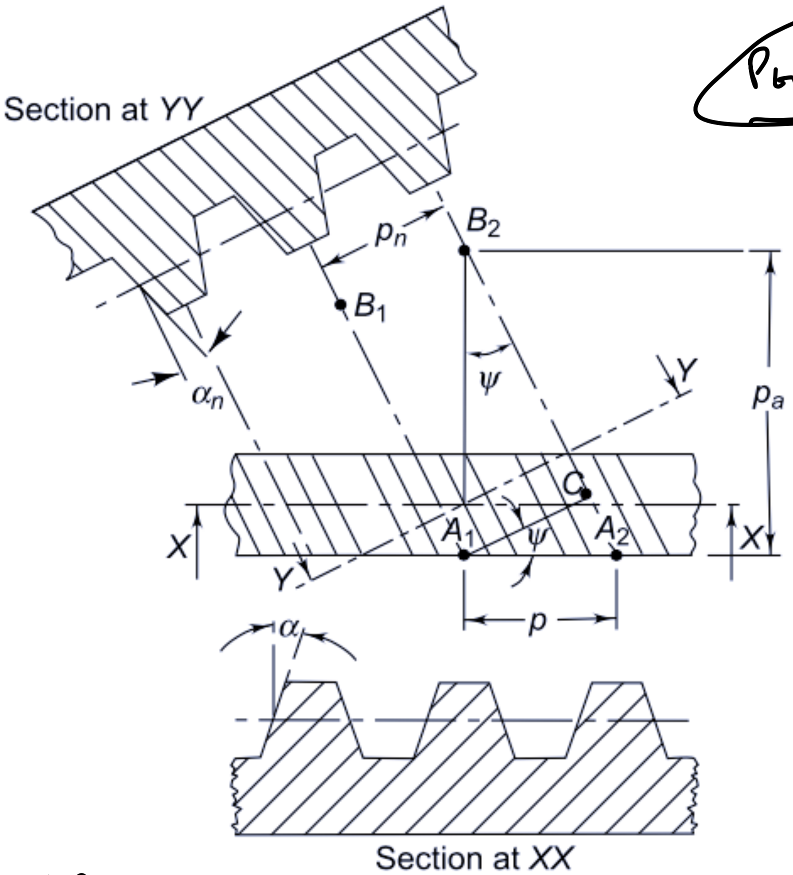
$$\boxed{P_{t_1} \times P_{t_2} = \pi}$$

$$\cos\psi = \frac{P_{t_1}}{P_{n_1}}$$

$$P_{n_1} = \frac{P_{t_1}}{\cos\psi}$$

$$P_{t_2} = \frac{\pi}{P_{t_1}}$$

$$P_{n_2} = \frac{P_{t_2}}{\cos\psi}$$

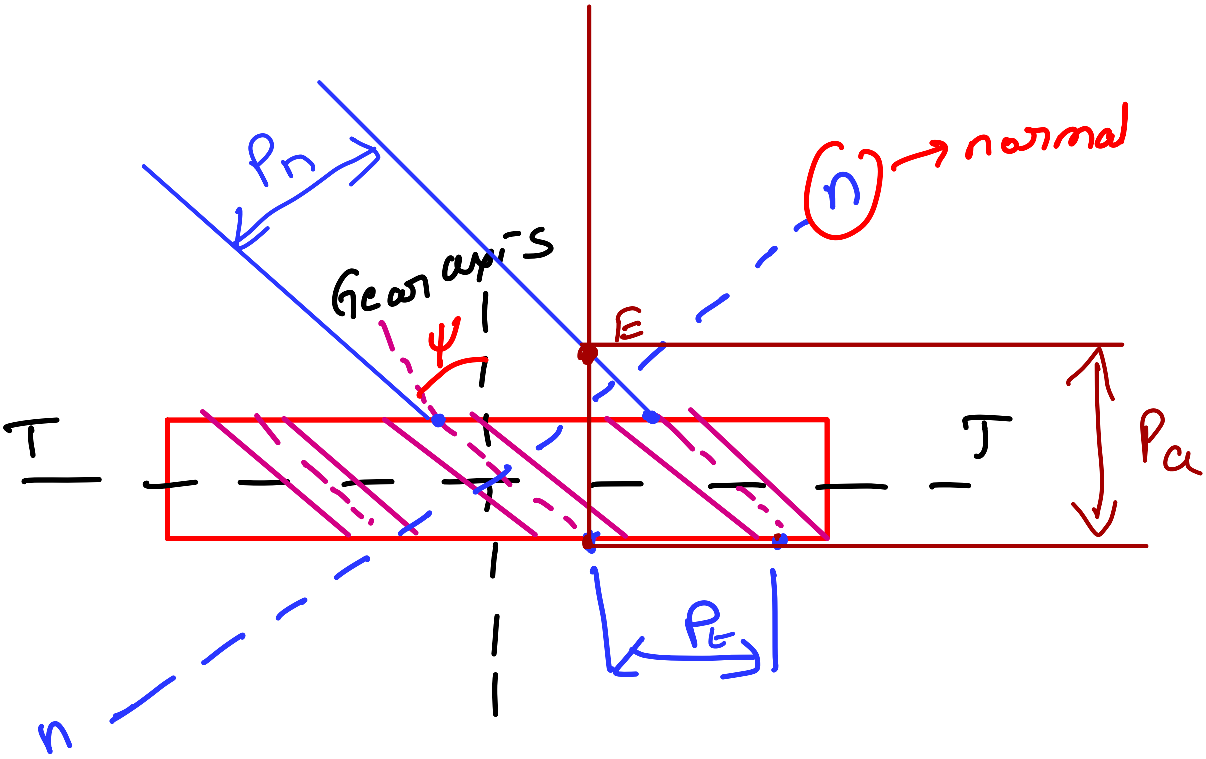


$P = \pi D/T$ $\quad$ $P_2 = T/D$

Terminologies:

1) Pitch
   - circular
     - Transverse (P_{t_1})
     - Normal (P_{n_1})
   - axial (P_a)
   - diametral
     - Transverse (P_{t_2})
     - Normal (P_{n_2})

2) module
   - Normal (m_n)
   - transverse (m)



3) Pressure Angle
   - Normal (α_n)
   - Transverse (α)

module : $P_{t2} = T/D$ $m = D/T$

$$P_{t_2} = \frac{1}{m}$$

$$m = \frac{1}{P_{t_2}} \quad \& \quad m_n = \frac{1}{P_{n_2}} = \frac{1}{P_{t_2}/\cos\psi}$$

$$m_n = \frac{\cos\psi}{P_{t_2}} \Rightarrow \boxed{m_n = m\cos\psi}$$

Axial pitch :

$$\tan\psi = \frac{P_{t_1}}{P_a} \Rightarrow \boxed{P_a = \frac{P_{t_1}}{\tan\psi}}$$

$\tan\alpha_n = \cos\psi \tan\alpha$ → Normal press angle

$P_{t_1} = \pi d / T$

Normal press angle $\simeq 20^\circ$

$$d = \frac{T P_{t_1}}{\pi} = \frac{T}{P_{t_2}} = T \times m = \frac{T \times m_n}{\cos\psi}$$

$$\boxed{d = \frac{T m_n}{\cos\psi}}$$ → pitch circle dia

Centre distance b/n pinion & gear:

$$a = \frac{d_1}{2} + \frac{d_2}{2} = \frac{T_1 m_n}{2\cos\psi} + \frac{T_2 m_n}{2\cos\psi}$$

$$\boxed{a = \frac{m_n (T_p + T_g)}{2\cos\psi}}$$

Speed ratio $\quad i = \frac{\omega_p}{\omega_g} = \frac{T_g}{T_p} = \frac{d_g}{d_p} = \frac{N_p}{N_g}$

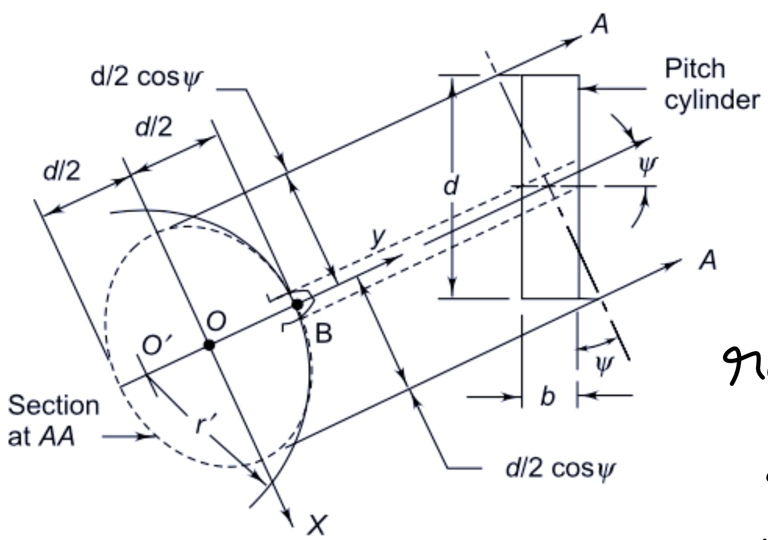
$i > 1$

VIRTUAL NUMBER OF TEETH

Major & minor axis

$$\text{Semi-major axis} = \frac{d}{2\cos\psi}$$

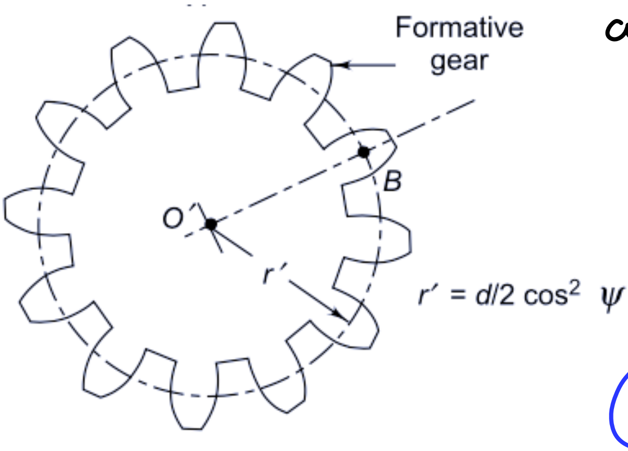
$$\text{Semi-minor axis} = \frac{d}{2}$$



radius of curvature

$$r' = a^2/b$$

$$a = \frac{d}{2\cos\psi} \quad \& \quad b = \frac{d}{2}$$



$$r' = \frac{d}{2\cos^2\psi}$$

$$d' = 2 \times r' = \frac{d}{\cos^2\psi}$$

Fig. 18.3 *Formative Gear*

d': pitch dia of imaginative spur gear

d: pitch circle of helical gear

$$P_t = \frac{\pi D}{T} \Rightarrow T = \frac{\pi D}{P_n}$$

$$T' = \frac{\pi \times d/\cos^2\psi}{P_n} = \frac{\dfrac{\pi \times d}{\cos^2\psi}}{\pi m_n} = \frac{d}{m_n \cos^2\psi}$$

$$\boxed{T' = \frac{T}{\cos^3\psi}}$$

T': virtual no. of teeth

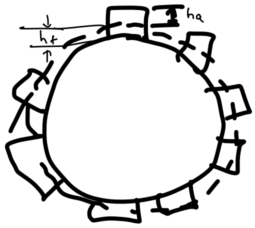
T: Actual no. of teeth

TOOTH PROPORTIONS

In helical gears, the normal module mn should be selected from standard values. The first preference values of the normal module are

m_n (in mm) = 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8 and 10

The standard proportions of the addendum and the dedendum are,



addendum (h_a) = mn $\quad h_a = m_n$

dedendum (h_f) = 1.25 mn $\quad h_f = 1.25 m_n$

clearance (c) = 0.25 mn $\quad c = 0.25 m_n$

Addendum circle dia (d_a) = $d + 2h_a$

$$d_a = \frac{T m_n}{\cos\psi} + 2m_n$$

$$d_a = m_n\left[\frac{I}{\cos\psi} + 2\right]$$

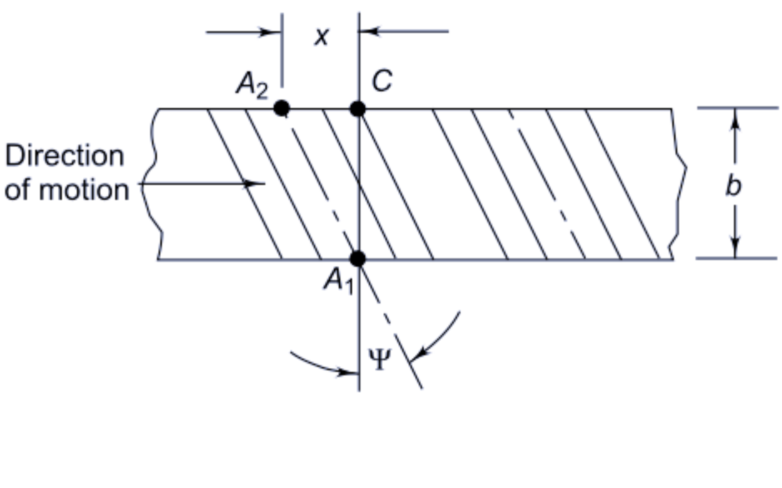
Similarly, dedendum circle dia (d_f)

$$= d - 2h_f$$

$$d_f = \frac{T m_n}{\cos\psi} - 2 \times 1.25 m_n$$

$$d_f = m_n\left[\frac{I}{\cos\psi} - 2.5\right]$$

Normal press angle = 20° & ψ = 15°–25°



The gear rotates from left to right as indicated by the arrow. For this rotation, the point $A1$ will be the first point to come in contact with its meshing tooth on the other gear. It is called the 'leading' edge of the tooth. Also, the point $A2$ will be the last point to come in contact with its meshing tooth on the other gear. It is called the 'trailing' edge of the tooth. In order that the contact on the face of the tooth shall always contain at least one point, the leading edge of the tooth should be advanced ahead of the trailing end by a distance greater than the circular pitch.

$$x \geq P$$

From Δ^{le} $A_1 A_2 C$

$$\tan\psi = \frac{A_2C}{A_1C} = \frac{x}{b}$$

$\cos\theta \times \tan\theta =$

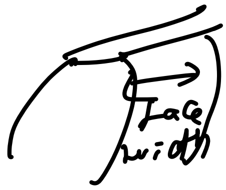
$$x = b\tan\psi$$

$$b\tan\psi \geq P$$

$$b \geq \frac{P}{\tan\psi}$$

$$\frac{P}{\tan\psi} = \frac{\pi m}{\tan\psi} = \frac{\pi m_n}{\cos\psi\tan\psi} = \frac{\pi m_n}{\sin\psi}$$

$$\boxed{b \geq \frac{\pi m_n}{\sin\psi}} \rightarrow \text{minimum face width}$$



FORCE ANALYSIS

The resultant force P acting on the tooth of a helical gear is resolved into three components, P_t, P_r and P_a

P_t = tangential component (N)

P_r = radial component (N)

P_a = axial or thrust component (N)

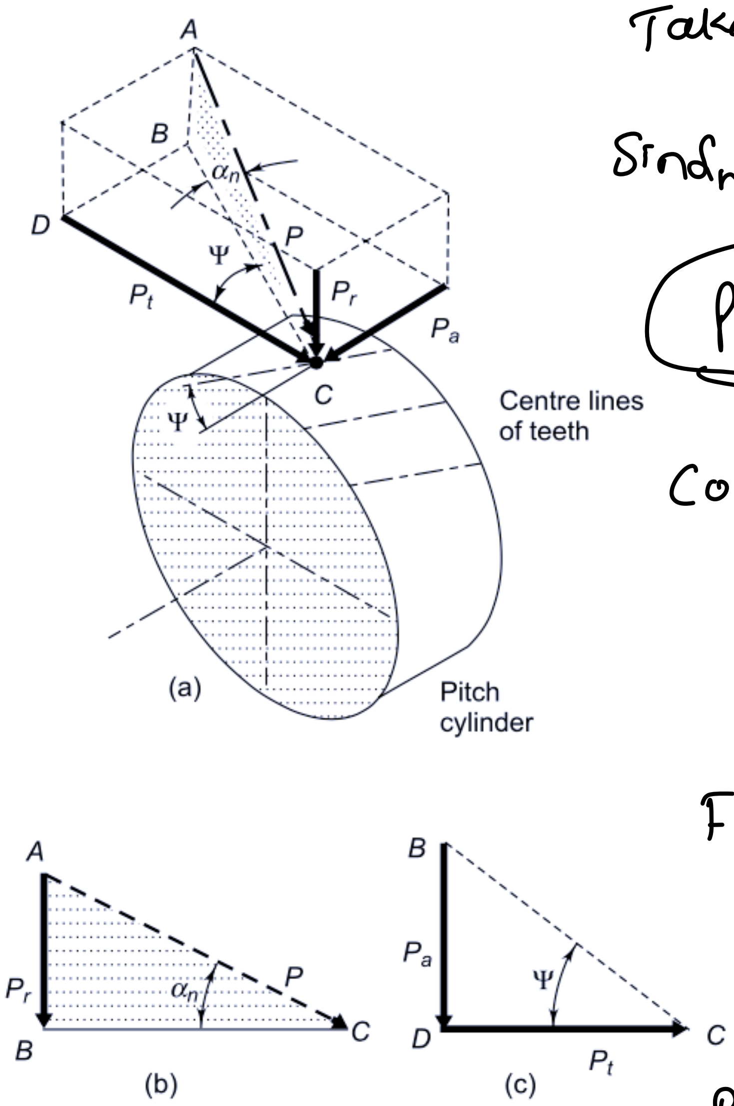


Fig. 18.5 *Components of Tooth Force*

Take Δ^{le} ABC

$$\sin\alpha_n = \frac{AB}{AC} = \frac{P_r}{P}$$

$$P_r = P\sin\alpha_n \quad \text{— (1)}$$

$$\cos\alpha_n = \frac{BC}{AC} = \frac{BC}{P}$$

$$BC = P\cos\alpha_n \quad \text{— (2)}$$

From Δ^{le} BDC

$$\sin\psi = \frac{BD}{BC} = \frac{P_a}{BC}$$

$$P_a = BC \times \sin\psi$$

$$P_a = P\cos\alpha_n\sin\psi \quad \text{— (3)}$$

$$\cos\psi = \frac{CD}{BC} = \frac{P_t}{P\cos\alpha_n} \Rightarrow P_t = P\cos\alpha_n\cos\psi \quad \text{— (4)}$$

(3)/(4) $\Rightarrow$ $$\frac{P_a}{P_t} = \frac{P\cos\alpha_n \sin\psi}{P\cos\alpha_n \cos\psi}$$

$$\boxed{P_a = P_t \tan\psi} \quad \text{—— (a)}$$

$\alpha_n = 20^\circ$

(1)/(4) $\Rightarrow$ $$\frac{P_r}{P_t} = \frac{\cancel{P}\sin\alpha_n}{\cancel{P}\cos\alpha_n \cos\psi}$$

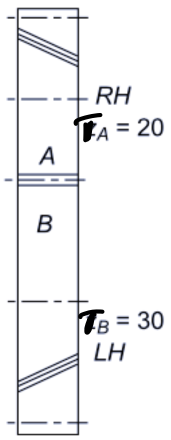
$$\boxed{P_r = P_t\left[\frac{\tan\alpha_n}{\cos\psi}\right]} \quad \text{—— (b)}$$

$$M_t = P_t \times \frac{d}{2}$$

$$\boxed{P_t = \frac{2M_t}{d}}$$

$$\boxed{M_t = \frac{60 \times (\text{Power}) \times 10^6}{2\pi n_p}}$$

Problem: A pair of parallel helical gears is shown in Fig.. A 5 kW power at 720 rpm is supplied to the pinion A through its shaft. The normal module is 5 mm and the normal pressure angle is 20°. The pinion has right-hand teeth, while the gear has left-hand teeth. The helix angle is 30°. The pinion rotates in the clockwise direction when seen from the left side of the page. Determine the components of the tooth force and draw a free-body diagram showing the forces acting on the pinion and the gear.



Given data: $P_0 = 5\,kW$ $\quad N = 720\,RPM$

$$m_n = 5\,mm \;;\; \alpha_n = 20^\circ \;;\; \psi = 30^\circ$$

$$P_t = \frac{2 M_t}{d}$$

$$M_t = \frac{60 \times P \times 10^6}{2 \pi N} \; N\text{-}mm$$

$$= \frac{60 \times 5 \times 10^6}{2 \pi \times 720} = 66{,}314.55 \; N\text{-}mm$$

$d = m \times T$

$= \frac{m_n}{\cos\psi} \times T$

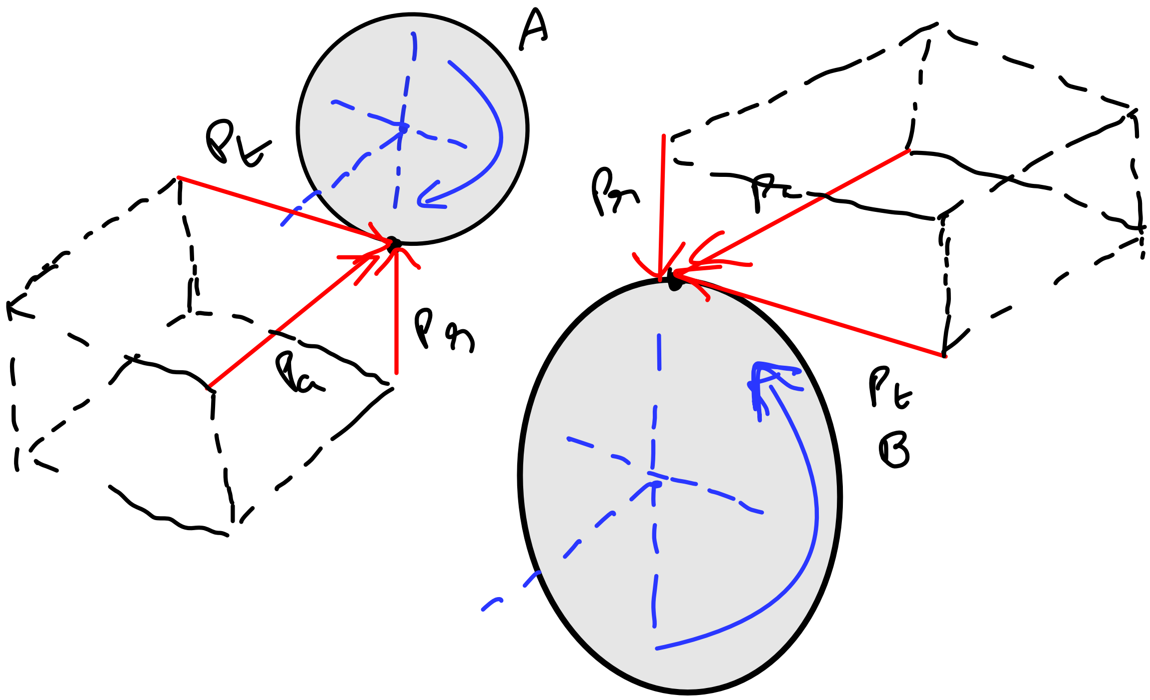
$$d_A = \frac{T_A m_n}{\cos \psi} = \frac{20 \times 5}{\cos 30}$$

$$d_A = 115.47\,mm$$

$$P_t = \frac{2 \times M_t}{d_A} = 1148.6\,N$$

$$P_a = P_t \tan\psi = P_t \times \tan 30^\circ = 663.145\,N$$

$$P_r = P_t \left(\frac{\tan \alpha_n}{\cos \psi}\right) = P_t \times \frac{\tan 20}{\cos \psi} = 482.72\,N$$

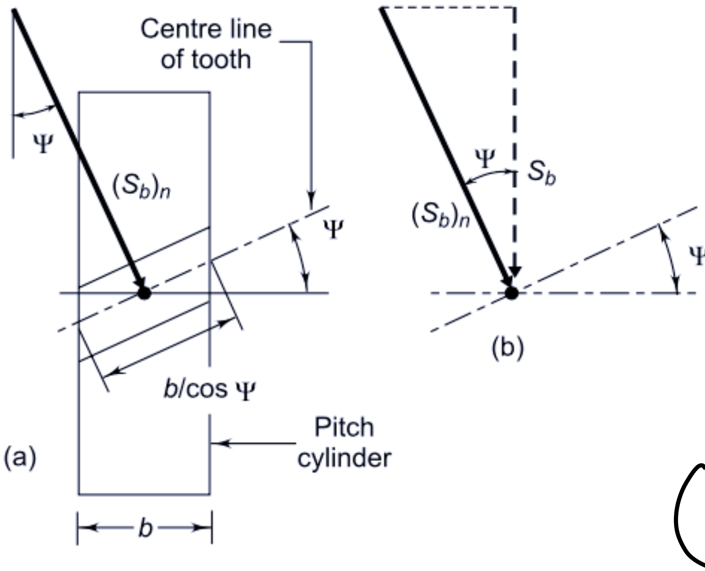

A
Pt
Pn
Pa
Pn
Pt
Pt
B

BEAM STRENGTH OF HELICAL GEARS

In order to determine beam strength, the helical gear is considered to be equivalent to a formative spur gear,

The pitch circle diameter of this gear is d', the number of teeth is z' and the module m_n.

$\cancel{S_b = mb\sigma_b Y}$ $\quad S_b = m b \sigma_b Y$



$$b = \frac{b}{\cos\psi}$$

$Y \rightarrow$ Lewis factor for z' no. of teeth

$$(S_b)_n = m_n \frac{b}{\cos\psi} \sigma_b \times Y$$

$S_b = (S_b)_n$ = beam strength perpendicular to the tooth element

$m = m_n$ = normal module

$$(S_b)_n = \frac{m_n b \sigma_b Y}{\cos\psi}$$

$S_b \rightarrow$ component of $(S_b)_n$ in plane of rotation

$$\boxed{S_b = (S_b)_n \cos\psi}$$

$$\boxed{S_b = m_n b \sigma_b Y}$$

z'

Therefore, beam strength (Sb) indicates the maximum value of tangential force that the tooth can transmit without bending failure. It should be always more than the effective force between the meshing teeth.

EFFECTIVE LOAD ON GEAR TOOTH

$S_b = (P_{eff})\ FOS$ $\quad M_t = \frac{60 \times P \times 10^6}{2 \pi N}$ $\quad P_t = \frac{2 M_t}{d}$

According to vel factor method

$P_{eff} = \frac{C_S P_t}{C_V}$ $\quad C_V = \text{vel factor} = \frac{5.6}{5.6 + \sqrt{V}}$

According to Buckingham's method!

$$P_d = \frac{21v(Ceb\cos^2\psi + P_t)\cos\psi}{21v + \sqrt{(Ceb\cos^2\psi + P_t)}} \qquad (18$$

where,

P_d = dynamic load or incremental dynamic load (N)

v = pitch line velocity (m/s)

C = deformation factor (N/mm²)

e = sum of errors between two meshing teeth (mm)

b = face width of tooth (mm)

P_t = tangential force due to rated torque (N)

ψ = helix angle (degrees)

$(P_{eff} = C_S P_t + P_d)$

$$\boxed{S_b = (P_{eff})\ FOS}$$

WEAR STRENGTH OF HELICAL GEARS

$$S_w = bQd'_pK \qquad \text{(a)}$$

$S_w = (S_w)_n$ = wear strength perpendicular to the tooth element

$b = \dfrac{b}{\cos\psi}$ = face width along the tooth element

$d'_p = \dfrac{d_p}{\cos^2\psi}$ = pitch circle diameter of the formative pinion.

Substituting these values in Eq. (a),

$$(S_w)_n = \frac{bQd_pK}{\cos^3\psi} \qquad \text{(b)}$$

The component of $(S_w)_n$ in the plane of rotation is denoted by S_w. Therefore,

$$S_w = (S_w)_n \cos\psi \qquad \text{(c)}$$

From (b) and (c),

$$S_w = \frac{bQd_pK}{\cos^2\psi} \qquad (18.24)$$

Therefore, wear strength (S_w) indicates the maximum tangential force that the tooth can transmit without pitting failure. It should be always more than the effective force between the meshing teeth.

$$Q = \frac{2z'_g}{z'_g + z'_p} \qquad \text{(d)}$$

Substituting

$$z'_g = \frac{z_g}{\cos^3\psi} \quad \text{and} \quad z'_p = \frac{z_p}{\cos^3\psi}$$

$$K = \frac{\sigma_c^2 \sin\alpha_n \cos\alpha_n \left[\frac{1}{E_1} + \frac{1}{E_2}\right]}{1.4}$$

in Eq. (d), we have

$$Q = \frac{2z_g}{z_g + z_p} \qquad (18.25)$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2$$

Similarly, for a pair of internal helical gears, it can be proved that

$$Q = \frac{2z_g}{z_g - z_p} \qquad (18.26)$$

$$\boxed{S_w = (P_{eff})\, FOS}$$

Problem: A pair of parallel helical gears consists of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 rpm. The normal pressure angle is 20°, while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as the gear is made of steel 4OC8 (S_{ut} = 600 N/mm^2) and heat treated to a surface hardness of 300 BHN. The service factor and the factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of gears.

Given data: $T_p = 20$; $T_g = 100$; $N = 720$ RPM; $\alpha_n = 20^\circ$

$\psi = 25^\circ$; $b = 40$ mm ; $m_n = 4$ mm; BHN = 300

$\sigma_{ut} = 600$ N/mm^2 ; $C_s = 1.5$; FOS = 2

Pow $\leftarrow$ m_t $\leftarrow$ P_t $\leftarrow$ P_{eff} $\leftarrow$ S_b (or) S_w

$$S_b = (P_{eff}) \times FOS \qquad P_{eff} = \frac{C_s P_t}{C_V}$$

$$S_b = m_n b \sigma_b Y \qquad Y \rightarrow \text{For } T' \text{ no. of teeth}$$

$$T' = \frac{T_p}{\cos^3 \psi} = \frac{20}{\cos^3 (25)} = 26.86$$

$$\text{For } T' = 26.86, \quad Y = 0.344 + \frac{(0.348 - 0.344)(26.87 - 26)}{(27 - 26)}$$

For 26 → 0.344
27 → 0.348

$$Y = 0.3474$$

$$\sigma_b = \frac{\sigma_{ut}}{3} = \frac{600}{3} = 200 \text{ N/mm}^2$$

$$S_b = m_n\, b\, \sigma_b\, y$$
$$= 4(40)(200)(0.3474)$$
$$S_b = 11{,}116.8 \text{ N}$$

Strength, Force, Load, weight → same units

Wear strength:

$$S_w = \frac{b\, Q\, d_p\, K}{\cos^2\psi}$$

$$Q = \frac{2T_g}{T_g + T_p} = \frac{2 \times 100}{100+20} = 1.667$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2$$
$$= 0.16\left(\frac{300}{100}\right)^2$$
$$K = 1.44 \text{ N/mm}^2$$

$$d_p = \frac{T_p\, m_n}{\cos\psi} = \frac{20 \times 4}{\cos 25} = 88.27 \text{ mm}$$

$$S_w = \frac{(40)(1.667)(88.27) \times 1.44}{\cos^2(25)}$$

$$S_w = 10{,}318 \text{ N}$$

Since $S_w < S_b$ → wear strength is the criterion for design.

$$S_w = (P_{eff})\, FoS$$

$$P_{eff} = \frac{S_w}{FoS} = \frac{10{,}318}{2} = 5159 \text{ N}$$

$$P_{eff} = \frac{C_s\, P_t}{C_v}$$

$$C_v = \frac{5.6}{5.6 + \sqrt{V}} = 0.754$$

$$P_t = \frac{C_v \times P_{eff}}{C_s}$$
$$= \frac{0.754 \times 5159}{1.5}$$
$$P_t = 2593.25 \text{ N}$$

$$V = \frac{\pi\, d_p\, N_p}{60 \times 10^3} = \frac{\pi \times 88.27 \times 720}{60 \times 10^3}$$
$$V = 3.327 \text{ m/s}$$

$$P_t = \frac{2M_t}{d_p} \Rightarrow M_t = \frac{P_t \times d_p}{2} = \frac{2593.25 \times 88.27}{2}$$

$$M_t = 114.45 \times 10^3 \text{ N-mm}$$

$$\text{Power} = (M_t) \times \omega$$

$$= \frac{M_t \times 2\pi N}{60 \times 10^3 \times 10^3}$$

$$\text{Power} = \frac{2\pi \times 720 \times 114.45 \times \cancel{10^3}}{60 \times \cancel{10^3} \times 10^3} \text{ KW}$$

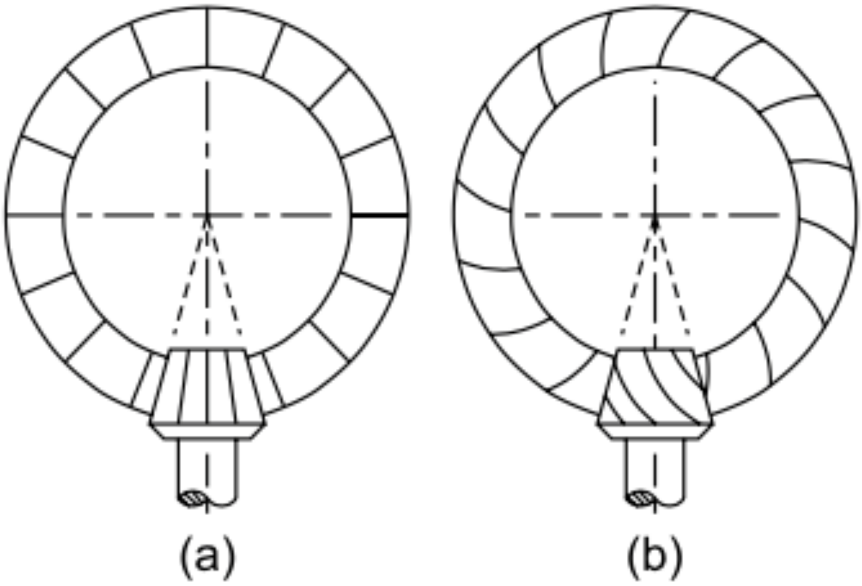
$$\boxed{\text{Power} = 8.63 \text{ KW}}$$

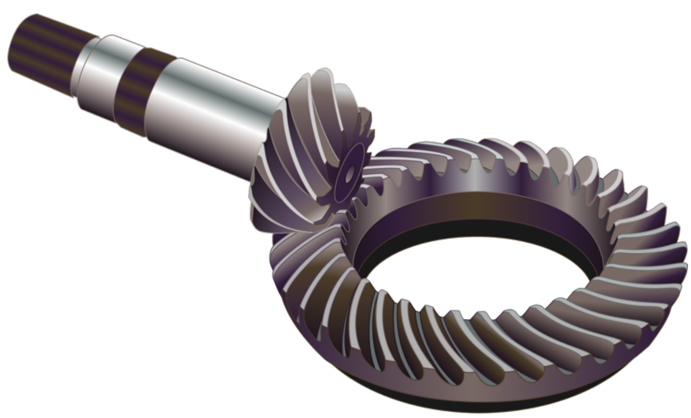
Assignment Problem: A pair of parallel helical gears consists of 24 teeth pinion rotating at 5000 rpm and supplying 2.5 kW power to a gear. The speed reduction is 4 : 1. The normal pressure angle and helix angle are 20° and 23° respectively. Both gears are made of hardened steel (S_{ut} = 750 N/mm^2). The service factor and the factor of safety are 1.5 and 2 respectively. The gears are finished to meet the accuracy of Grade 4.

1. In the initial stages of gear design, assume that the velocity factor accounts for the dynamic load and that the face width is ten times the normal module. Assuming the pitch line velocity to be 10 m/s, estimate the normal module.
2. Select the first preference value of the normal module and calculate the main dimensions of the gears.
3. Determine the dynamic load using Buckingham's equation and find out the effective load for the above dimensions. What is the correct factor of safety for bending?
4. Specify surface hardness for the gears, assuming a factor of safety of 2 for wear consideration.

BEVEL GEARS

Bevel gears are used to transmit power between two intersecting shafts. There are two common types of bevel gears—*straight* and *spiral*, as shown in Fig. 19.1. The elements of the teeth of the straight bevel gears are straight lines, which converge into a common apex point. The elements of the teeth of the spiral bevel gears are spiral curves, which also converge into a common apex point. Involute profile is used for the form of the teeth for both types of gears.





Straight bevel gears are easy to design and manufacture and give reasonably good service when properly mounted on shafts. However, they create noise at high-speed conditions. Spiral bevel gears, on the other hand, are difficult to design and costly to manufacture, for they require specialized and sophisticated machinery for their manufacture. Spiral bevel gears have smooth teeth engagement, which results in quiet operation, even at high speeds. They have better strength and are thus used for high speed–high power transmission.

In some cases, bevel gears are classified on the basis of pitch angle. Three types of bevel gears that are based on pitch angle are as follows:

1. When the pitch angle is less than 90°, it is called external bevel gear.
2. When the pitch angle is equal to 90°, it is called crown bevel gear.
3. When the pitch angle is more than 90°, it is called internal bevel gear.

Miter Gears : When two identical bevel gears are mounted on shafts, which are intersecting at right angles, they are called 'miter' gears.

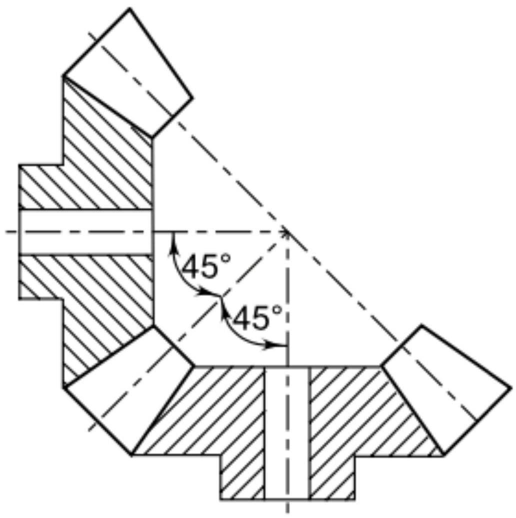
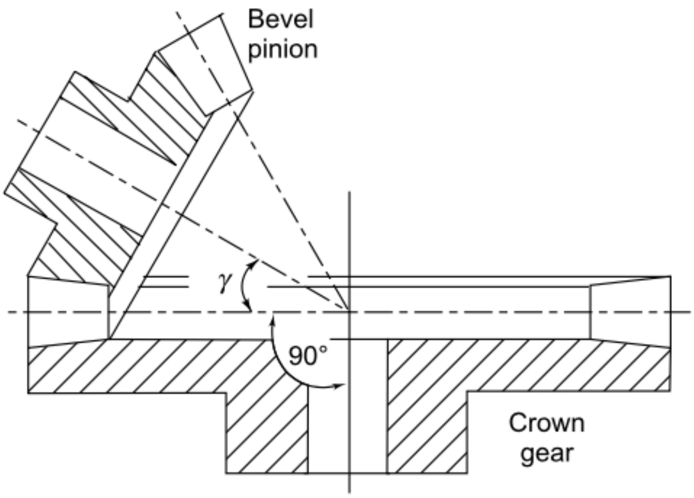


Fig. 19.2 *Miter Gears*

1. The pitch angles of pinion and gear of miter gears are same and each is equal to 45°.
2. The pinion and gear of miter gears rotate at the same speed.
3. The pinion and gear have same dimensions, namely, addendum, dedendum, pitch circle diameter, number of teeth and module.
4. The pinion and gear of miter gears are always mounted on shafts, which are perpendicular to each other.

Crown Gear In a pair of bevel gears, when one of the gears has a pitch angle of 90° then that gear is called 'crown' gear. Such bevel gears are mounted on shafts, which are intersecting at an angle that is more than 90°. The crown gear is equivalent to the rack in spur gearing. The pitch cone of the crown gear becomes plane

a. The pitch angle of crown gear is 90°.
b. The bevel pinion and crown gear are always mounted on shafts, which are intersecting at angle more than 90°.



Internal Bevel Gears When the teeth of bevel gear are cut on the inside of the pitch cone, it is called internal bevel gear. In this case, the pitch angle of internal gear is more than 90° and the apex point is on the backside of the teeth on that gear. Internal bevel gears are used in **planetary gear trains.**

Skew Bevel Gears When two straight bevel gears are mounted on shafts, which are non-parallel and non-intersecting, they are called ‘skew’ bevel gears. The apex point of pinion is offset with respect to that of gear.

a. Skew bevel gears have straight teeth.
b. Skew bevel gears are mounted on non- parallel and non-intersecting shafts.

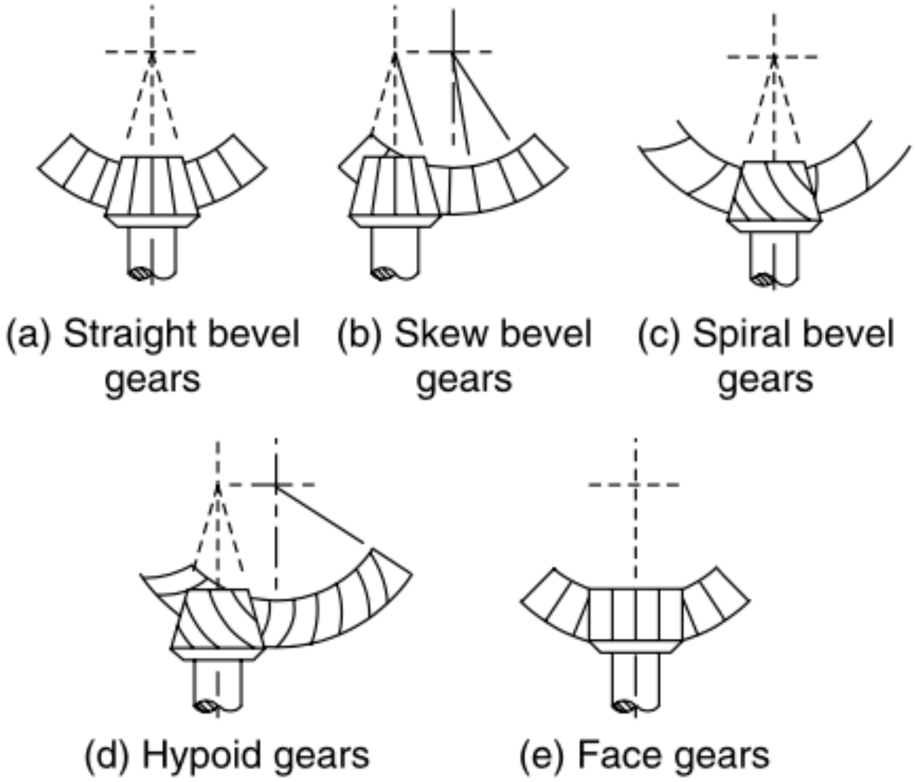
Hypoid Gears Hypoid gears are similar to spiral bevel gears that are mounted on shafts, which are non-parallel and non-intersecting. Hypoid gears are based upon pitch surfaces, which are hyperboloids of revolution.
When two hyperboloids are rotated together, the resultant motion is a combination of rolling and sliding. The sliding is along the length of teeth and results in increased friction. On this account, Extreme Pressure (EP) lubricants are used for these gears. Sliding friction reduces the efficiency of hypoid gears. The efficiency of bevel gear drive is 98 to 99 per cent, whereas that of hypoid gears is 96 to 98 per cent.

Hypoid gears have the following characteristics:

1 Hypoid gears have curved teeth.
2 Hypoid gears are mounted on non-parallel and non-intersecting shafts.

Hypoid bevel gears are mainly used in **automobile differentials**. These gears allow the drive shaft to be placed well below the centreline of the rear axle and thereby lower the centre of gravity of the vehicle. Another advantage of hypoid gears is that the offset of the shaft is so great that the shafts may continue past each other. Therefore, multiple power take-offs from a single shaft with several pinions is possible. Hypoid gears give noiseless operation even at high speeds.

Zerol Gears Zerol gears are spiral bevel gears with zero spiral angle. These gears theoretically give more gradual contact and a slightly larger contact ratio.



Face Gears Face gears consist of a spur or helical pinion mating with a conjugate gear of disk form. Face gears have the following characteristics:

1. The pinion of face gears is either a spur gear or a helical gear.
2. Face gears are mounted on intersecting shafts that are at right angles to each other.
3. The teeth of face gears can be straight or curved.

Face gears have the following advantages:

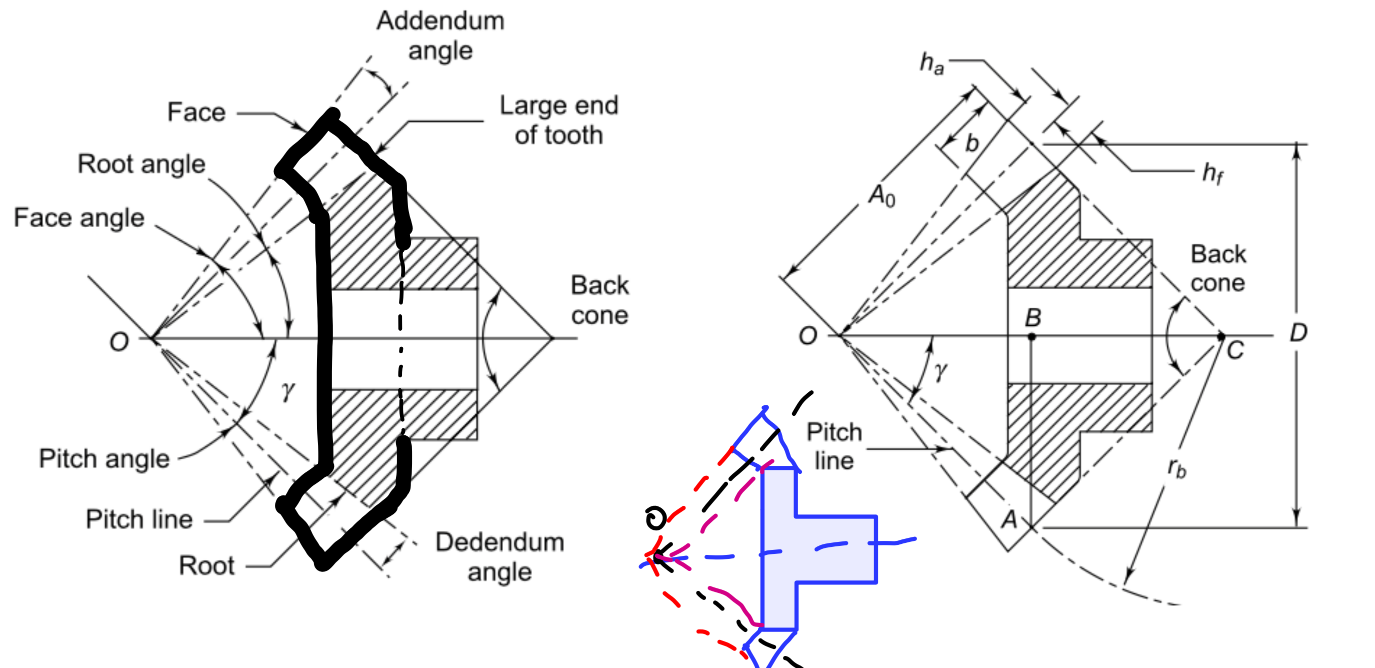
a. They can be cut with spur gear cutters and gear shapers.
b. The axial position of the pinion is not critical as in case of bevel pinion.

The disadvantage is that the width of the tooth face is small.

TERMINOLOGY OF BEVEL GEARS

A bevel gear is in the form of the frustum of a cone.

(i) Pitch Cone Pitch cone is an imaginary cone, the surface of which contains the pitch lines of all teeth in the bevel gear.



(ii) Cone centre The apex of the pitch cone is called the cone centre. It is denoted by *O*.

(iii) Cone Distance Cone distance is the length of the pitch-cone element. It is also called pitch-cone radius. It is denoted by A_0.

(iv) Pitch Angle The angle that the pitch line makes with the axis of the gear, is called the pitch angle. It is denoted by γ. The pitch angle is also called centre angle.

(v) Addendum Angle It is the angle subtended by the addendum at the cone centre. It is denoted by α.

(vi) Dedendum Angle It is the angle subtended by the dedendum at the cone centre. It is denoted by δ.

(vii) Face Angle It is the angle subtended by the face of the tooth at the cone centre.

Face angle = pitch angle + addendum angle = $\gamma + \alpha$.

(viii) Root Angle It is the angle subtended by the root of the tooth at the cone centre.

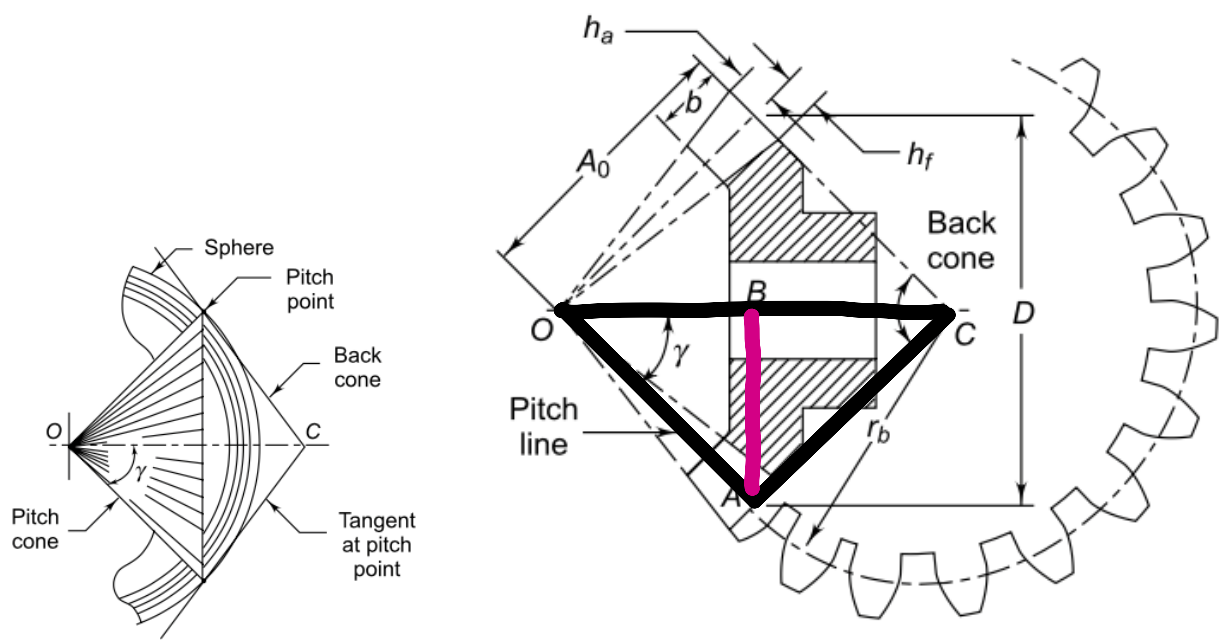
Root angle = pitch angle – dedendum angle = $\gamma - \delta$.

(ix) Back Cone The back cone is an imaginary cone and its elements are perpendicular to the elements of the pitch cone.

(x) Back Cone Distance It is the length of the back cone element. It is also called back cone radius. It is denoted by r_b.

It is observed from the figure that the cross- section of the tooth decreases in size as it approaches towards the apex point O. Therefore, the pitch circle diameter, module, addendum, and dedendum decreases and there is no single value for these parameters. In practice, these dimensions are measured at the largest tooth section called the large end of the tooth. The dimensions of the bevel gear are always specified and measured at the large end of the tooth. The addendum h_a, the dedendum h_f and the pitch circle diameter D are specified at the large end of the tooth

The analysis for a bevel gear will show that a true section of involutes profile of tooth lies on the surface of a sphere. It is not possible to represent on a plane surface, the exact profile of a bevel gear tooth that actually lies on the surface of a sphere. Since the beam strength and wear strength equations are based on upon tooth profile, it is necessary to approximate the tooth profile as accurately as possible. The approximation is called '***<u>Tredgold's approximation</u>***'.



(a) Back cone

(b) Equivalent spur gear

An imaginary spur gear is considered in a plane perpendicular to the tooth at the large end. *rb* is the pitch circle radius of this imaginary spur gear and *z¢* is the number of teeth on this gear. The gear is called the formative gear and the number of teeth *z¢* on this gear is called the *virtual* or the ***formative number of teeth.***

$$r' = r_b$$

$$T = \frac{D}{m}$$

$$T' = \frac{D'}{m} = \frac{2r'}{m} = \frac{2r_b}{m}$$

m → module @ large end of tooth

$$T = \frac{D}{m}$$

$$\frac{T'}{T} = \frac{D'/m}{D/m} = \frac{2r_b}{D}$$

consider $\triangle^{le}$ BCA

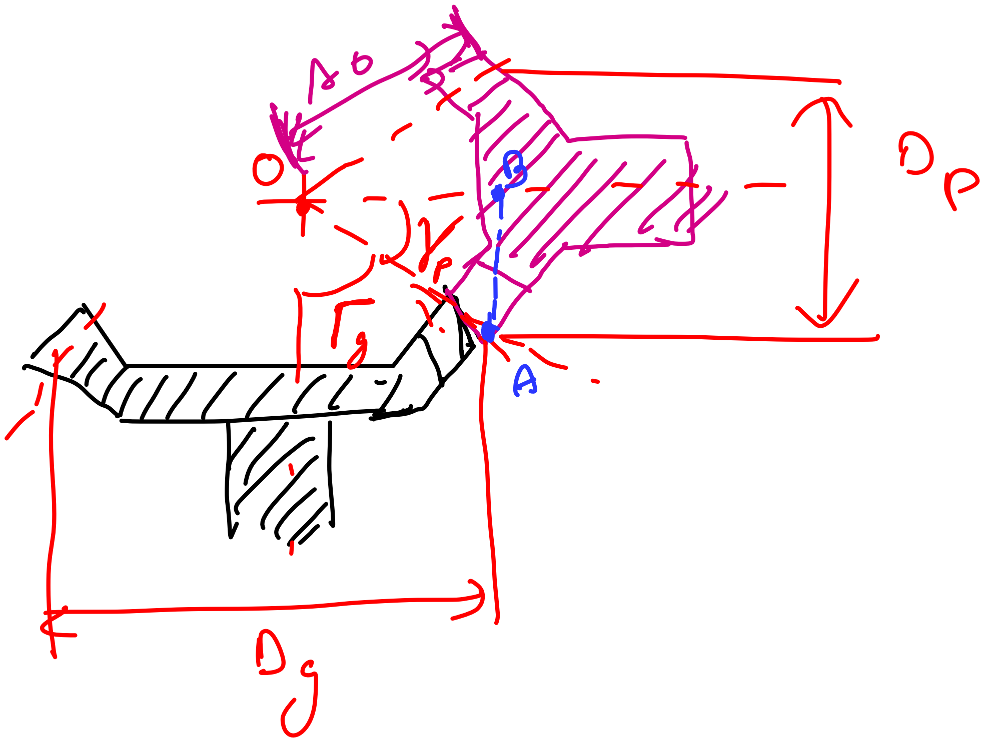
$$\sin\angle BCA = \frac{AB}{AC} = \frac{D/2}{r_b}$$

$$\sin(90-\gamma) = \frac{D/2}{r_b} \Rightarrow r_b = \frac{D}{2\cos\gamma}$$

using this in "T'" value

$$\frac{T'}{T} = \frac{2r_b}{D} = \frac{\cancel{2}\times \cancel{D}}{\cancel{2}\cos\gamma} \times \frac{1}{\cancel{D}} = \frac{1}{\cos\gamma}$$

$$\boxed{T' = \frac{T}{\cos\gamma}}$$



$$\tan\gamma = \frac{AB}{OB} = \frac{D_p/2}{D_g/2} = D_p/D_g = \frac{mT_p}{mT_g}$$

$$\boxed{\tan\gamma = \frac{T_p}{T_g}}$$

And $\gamma + \Gamma = \pi/2$ (or) 90°

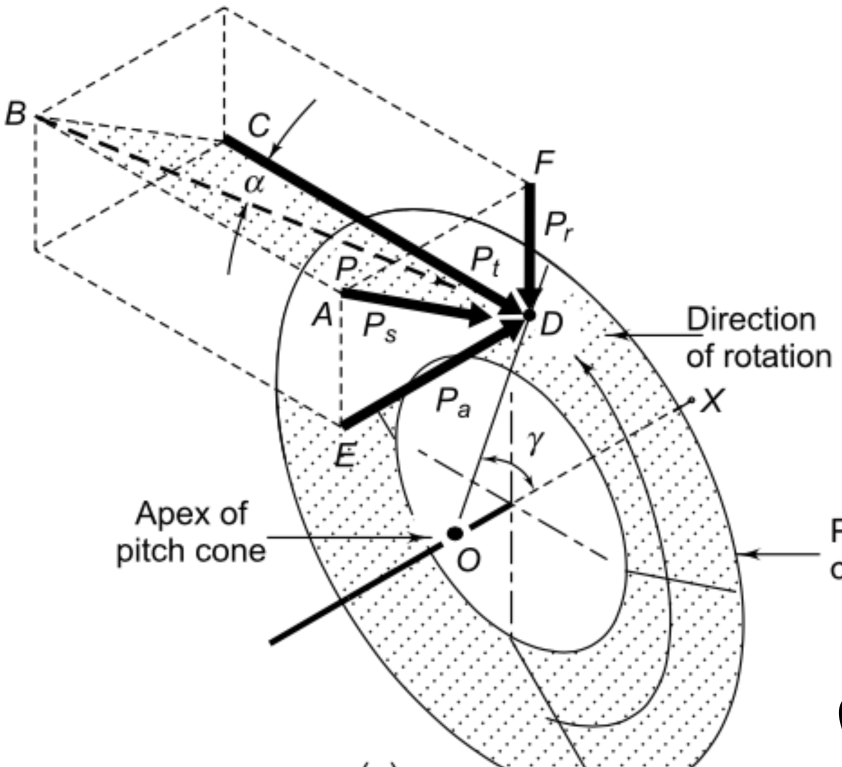
$$\boxed{\tan\Gamma = \frac{T_g}{T_p}}$$ (or) $\tan\gamma_g$

$$A_0 = OA = \sqrt{(AB)^2 + (OB)^2}$$

$$A_0 = \sqrt{\frac{D_p^2 + D_g^2}{4}} = \text{cone distance}$$

FORCE ANALYSIS

In force analysis, it is assumed that the resultant tooth force between two meshing teeth of a pair of bevel gears is concentrated at the midpoint along the face width of the tooth.



(a)

From $\triangle^{le}$ BCD

$$\tan\delta = \frac{BC}{CD}$$

$$\tan\delta = \frac{P_s}{P_t}$$

$$\boxed{P_s = P_t \tan\delta}$$

$P_s \rightarrow$ seperating force.

$\delta \rightarrow$ Press angle

The resultant force has following three components:

P_t = tangential or useful component (N)

P_r = radial component (N)

P_a = axial or thrust component (N)

From diagram

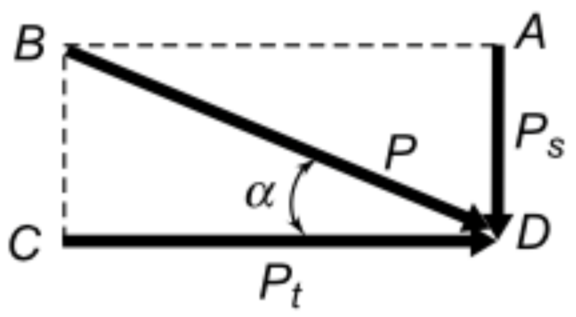
$AD \perp OD$

&

$FD \perp OX$

From this the angle b/n AD & FD

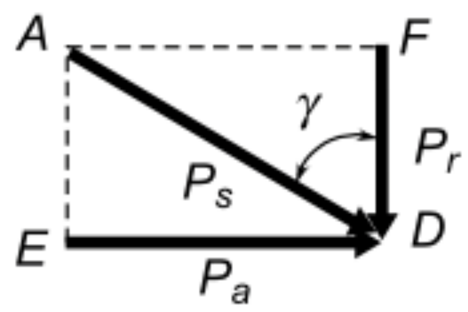
$= \gamma$



where,

Ps = separating component (N)

α = pressure angle (degrees)



From Δ^{le} ADF in plane DEAF

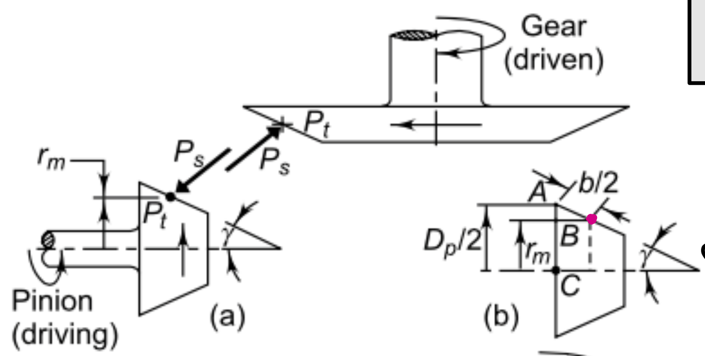
$$\cos\gamma = \frac{P_r}{P_s} \Rightarrow P_r = P_s \cos\gamma$$

Δ^{le} ADE

$$\cos(90-\gamma) = \frac{P_a}{P_s} \Rightarrow P_a = P_s \sin\gamma$$

$$\boxed{\begin{aligned} P_r &= P_t \tan\alpha \cos\gamma \\ P_a &= P_t \tan\alpha \sin\gamma \end{aligned}}$$

$$P_t = \frac{2M_t}{d} = \frac{M_t}{r}$$



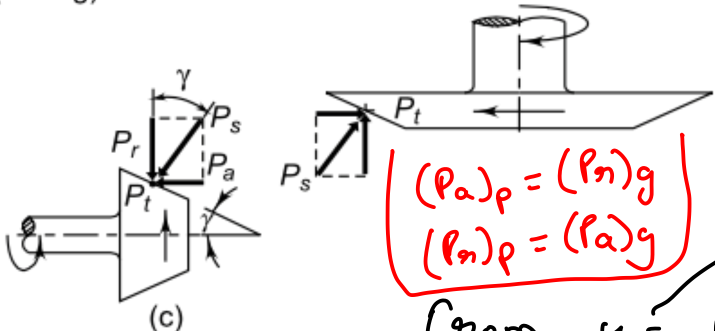
$$P_t = \frac{M_t}{r_m}$$

(x)

(·)

$M_t \rightarrow$ torque transmitted

$r_m \rightarrow$ radius of pinion @ m.p of face width



$$(P_a)_p = (P_r)_g$$
$$(P_r)_p = (P_a)_g$$

From the figure

$$r_m = BC = AC - AB$$

$$r_m = \left[\frac{D_p}{2} - \frac{b}{2}\sin\gamma\right]$$

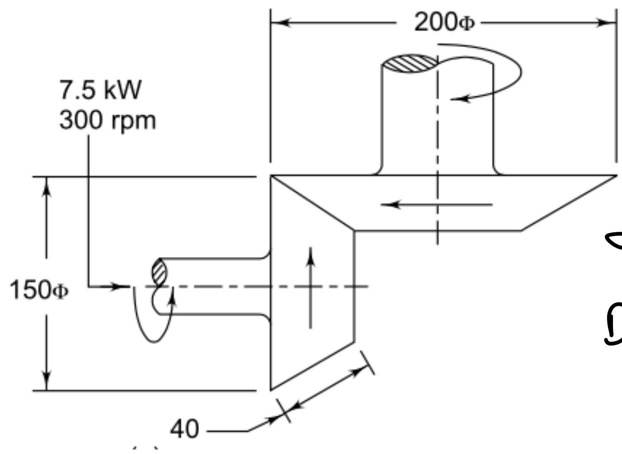
$$AC = D_p/2$$

$$\sin\gamma = \frac{AB}{b/2}$$

$$AB = \frac{b}{2}\sin\gamma$$

$b \rightarrow$ face width

Problem: A pair of bevel gears transmitting 7.5 kW at 300 rpm is shown in Fig.. The pressure angle is 20°. Determine the components of the resultant gear tooth force and draw a free-body diagram of forces acting on the pinion and the gear.



Given data:

$P = 7.5\,\text{kW}$; $N = 300\,\text{RPM}$

$\alpha = 20^\circ$; $b = 40\,\text{mm}$; $D_g = 200\,\text{mm}$

$D_p = 150\,\text{mm}$

1) Torque transmitted

$$M_t = \frac{60 \times P \times 10^6}{2\pi N_p} = \frac{60 \times 7.5 \times 10^6}{2\pi \times 300} = 238.73 \times 10^3\ \text{N-mm}$$

$$P_t = \frac{M_t}{r_m}$$

$$P_t = \frac{238.73 \times 10^3}{63}\ \frac{\cancel{\text{N-mm}}}{\cancel{\text{mm}}}$$

$$P_t = 3789.36\ \text{N}$$

$$r_m = \frac{D_p}{2} - \frac{b}{2}\sin\gamma$$

$$\tan\gamma = \frac{T_p}{T_G} = \frac{D_p}{D_g} = \frac{150}{200} = 0.75$$

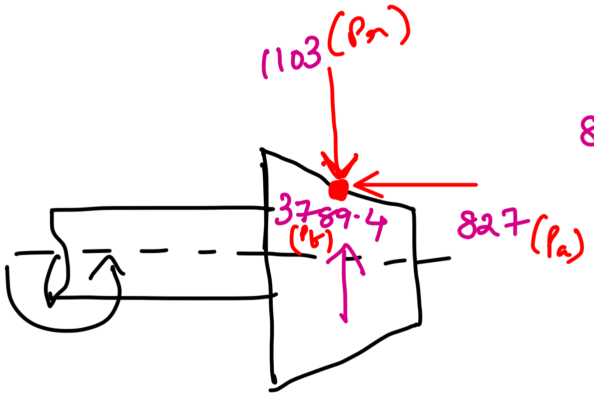
$$\gamma = 36.86^\circ$$

$$r_m = \frac{150}{2} - \frac{40}{2}\sin(36.86^\circ)$$

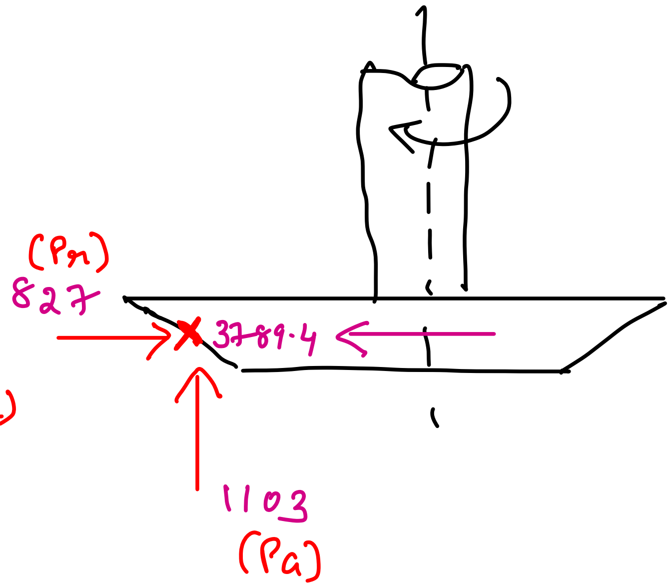
$$r_m = 63\ \text{mm}$$

$$P_r = P_t \tan\alpha \cos\gamma = (3789.63)\tan(20^\circ)\cos(36.87) = 1103\ \text{N}$$

$$P_a = P_t \tan\alpha \sin\gamma = (3789.63)\tan(20)\sin(36.87) = 827\ \text{N}$$



1103 (Pr)
3789.4
(Pt)
827 (Pa)



(Pr)
827
3789.4
1103
(Pa)

BEAM STRENGTH OF BEVEL GEARS

In order to determine the beam strength of the tooth of a bevel gear, it is considered to be equivalent to a formative spur gear in a plane perpendicular to the tooth element.

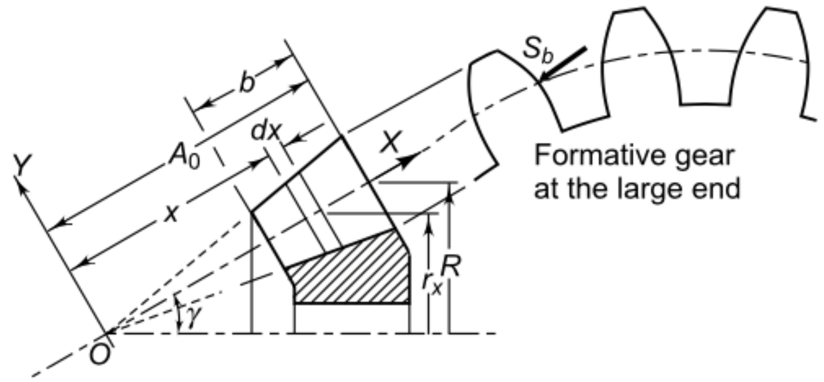


Fig. 19.17 *Beam Strength of Bevel Gear Tooth*

Consider an elemental section of the tooth at a distance x from the apex O and having a width dx.

applying Lewis eqn:

$$\delta(S_b) = m_x b_x \sigma_b Y$$

From the figure

$\delta(S_b)$ = beam strength of the elemental section (N)
m_x = module of the section (mm)
b_x = face width of elemental section (mm)
Y = Lewis form factor based on virtual number of teeth

$$\left(\frac{r_x}{x}\right) = \left(\frac{R}{A_0}\right)$$

$$r_x = \frac{xR}{A_0}$$

module @ the elemental section

$$m = \frac{D}{T} = \frac{2r}{T}$$

$$m_x = \frac{2r_x}{T} = \frac{2xR}{A_0 T} \quad \text{— (1)}$$

module @ the large end of the tooth:

$$m = \frac{2R}{T}$$

$$m_x = \frac{mx}{A_0}$$

$$b_x = dx$$

$$\delta(S_b) = \frac{mx}{A_0} dx \times \sigma_b \times Y$$

Integrating this eqn to get "S_b" along the whole tooth:

$$\int \delta(S_b) = S_b = \int \frac{m \sigma_b Y x dx}{A_0}$$

multiplying r_x to both ends:

$$\int r_x \, \delta(S_b) = \frac{m \sigma_b Y}{A_0} \int \frac{x R \times x dx}{A_0}$$

$$\int (M_t)_x = \frac{m \sigma_b Y R}{A_0^2} \int_{(A_0-b)}^{A_0} x^2 dx \qquad A_0^3 - (A_0-b)^3$$

$$M_t = \frac{m \sigma_b Y R}{A_0^2} \left[\frac{x^3}{3} \right]_{(A_0-b)}^{A_0}$$

$$(M_t) = m b \sigma_b Y R \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2} \right] \quad - (a)$$

Lets assume "S_b" is the tangential force exerted @ the large end of the tooth.

$$M_t = S_b \times R \quad — (b)$$

Equating (a) & (b)

$$S_b \times \cancel{R} = m b \sigma_b y \cancel{R} \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right]$$

$$S_b = m b \sigma_b y \left[1 - \frac{b}{A_0} + \frac{b^2}{3A_0^2}\right]$$

$\boxed{b \leq \frac{1}{3}(A_0)}$ => which $\frac{b^2}{3A_0^2} \leq \frac{1}{27}$ (which is v. small)

we will ignore that term

$$\boxed{S_b = m b \sigma_b Y \left[1 - \frac{b}{A_0}\right]} \quad — (c)$$

$\left[1 - \frac{b}{A_0}\right] \rightarrow$ bevel factor

$S_b = P_{eff} \times (FOS)$

$$P_t = \frac{2M_t}{D}$$

$b = 10m$ (or) $\frac{A_0}{3}$ (whichever is smaller)

WEAR STRENGTH OF BEVEL GEARS

In order to determine the wear strength, the bevel gear is considered to be equivalent to a formative spur gear in a plane which is perpendicular to the tooth at the large end. Applying Buckingham's equation to these formative gears,

$$S_w = b\,Q\,d_p'\,K$$

where,

b = face width of gears (mm)

Q = ratio factor

d_p' = pitch circle diameter of formative pinion (mm)

K = material constant (N/mm^2)

$$d_p' = 2r_b$$

$$d_p' = \frac{D_p}{\cos\gamma}$$

$$\boxed{S_w = \frac{b\,Q\,D_p\,K}{\cos\gamma}}$$

75%

$$S_w = \frac{0.75\,b\,Q\,D_p\,K}{\cos\gamma}$$

$$Q = \frac{2T_g'}{T_g' + T_p'}$$

$$\gamma + \Gamma = 90^\circ$$

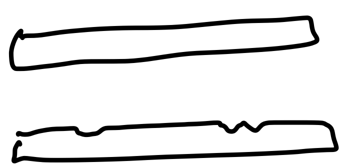
$$T_p' = \frac{T_p}{\cos\gamma}$$

$$T_g' = \frac{T_g}{\cos\Gamma} = \frac{T_g}{\cos(90-\gamma)}$$

$$T_g' = \frac{T_g}{\sin\gamma}$$

$$Q = \frac{\dfrac{2T_g}{\sin\gamma}}{\dfrac{T_g}{\sin\gamma} + \dfrac{T_p}{\cos\gamma}} = \frac{2T_g}{T_g + T_p\tan\gamma}$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2$$



Effective load on Tooth:

$$m_t = \frac{60 \times P \times 10^6}{2\pi N}$$

$$P_t = \frac{2 m_t}{D}$$

Vel factor:

$$P_{eff} = \frac{C_s P_t}{C_v}$$

$$C_v = \begin{cases} \dfrac{6}{6+v} & \rightarrow \text{For cut teeth} \\[2ex] \dfrac{5.6}{5.6+\sqrt{v}} & \rightarrow \text{Generated teeth} \end{cases}$$

$$v = \frac{\pi d N}{60 \times 10^3}$$

Buckingham method:

$$P_d = \frac{21v\,(C e b + P_t)}{21v + \sqrt{(C e b + P_t)}} \rightarrow \text{Buckingham dynamic Load}$$

$$P_{eff} = C_s P_t + P_d$$

$$S_b \text{ (or) } S_w = P_{eff} \times (FOS)$$

Module (m) (mm)	*Class - 1*	*Class - 2*	*Class - 3*
Up to 4	0.050	0.025	0.0125
5	0.056	0.025	0.0125
6	0.064	0.030	0.0150
7	0.072	0.035	0.0170
8	0.080	0.038	0.0190
9	0.085	0.041	0.0205
10	0.090	0.044	0.0220

Problem: A pair of bevel gears, with 20° pressure angle, consists of a 20 teeth pinion meshing with a 30 teeth gear. The module is 4 mm, while the face width is 20 mm. The material for the pinion and gear is steel 50C4 (S_{ut} = 750 N/mm^2). The gear teeth are lapped and ground (Class-3) and the surface hardness is 400 BHN. The pinion rotates at 500 rpm and receives 2.5 kW power from the electric motor. The starting torque of the motor is 150% of the rated torque. Determine the factor of safety against bending failure and against pitting failure.

Given data: $\phi = 20^\circ$ $T_p = 20$; $T_g = 30$; $m = 4$ mm

$b = 20$ mm ; $\sigma_{ut} = 750$ N/mm^2.

BHN = 400 ; $N_p = 500$ RPM ; $P = 2.5$ kW.

Starting torque = 150% Rated torque $\Rightarrow C_s = 1.5$

Since both pinion & gear are of same material. Pinion is the weakest.

$\sigma_b \times Y$

$$\tan\gamma = \frac{T_p}{T_G} = \frac{20}{30} \Rightarrow \gamma = 33.69^\circ$$

Formative gear teeth:

$$T_p' = \frac{T_p}{\cos\gamma} = \frac{20}{\cos(33.69)} = 24.036$$

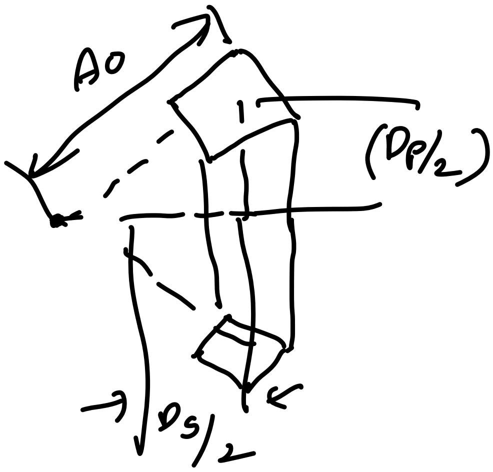
24 — 0.337

25 — 0.340

$$Y = 0.337 + \frac{(0.340 - 0.337)(24.036 - 24)}{(25 - 24)}$$

$$Y = 0.3371$$

$$S_b = m b \sigma_b y \left[1 - \frac{b}{A_o}\right]$$



$$\sigma_b = \frac{S_{ut}}{3} = \frac{750}{3} = 250 \ N/mm^2$$

$$A_o = \sqrt{\left(\frac{D_p}{2}\right)^2 + \left(\frac{D_g}{2}\right)^2}$$

$m = 4$
$T_p = 20$
$T_g = 30$

$$\left.\begin{aligned} D_p &= mT_p = 80mm \\ D_g &= mT_g = 120mm \end{aligned}\right\}$$

$$A_o = 72.11 \ mm$$

$$S_b = (4)(20)(250)(0.3371)\left[1 - \frac{20}{72.11}\right]$$

$$\underline{S_b = 4872.07} \ N$$

Wear strength:

$$S_w = \frac{0.75 \, b \, Q \, D_p K}{\cos \gamma}$$

$$K = 0.16\left(\frac{BHN}{100}\right)^2$$

$$= 0.16\left(\frac{400}{100}\right)^2$$

$$K = 2.56 \ N/mm^2$$

$$Q = \frac{2T_g}{T_g + T_p \tan \gamma} = \frac{2 \times 30}{30 + 20 \tan(33.69)}$$

$$Q = 1.384$$

$$S_w = \frac{0.75\,(20)\,(1.384)\,(80)\,(2.56)}{\cos(33.69)}$$

$$\underline{S_w = 5109.84\ N}$$

$$\boxed{S_b(\text{or})\,S_w} = \underline{P_{eff}} \times (F.O.S)$$

$$P_{eff} = P_t C_s + P_d$$

$$m_t = \frac{60 \times P \times 10^6}{2\pi N}$$

$$P = 2.5$$

$$N = 500\ \text{RPM}$$

$$P_t = \frac{2 m_t}{d_p}$$

$$m_t = 47.74 \times 10^3\ \text{N-mm}$$

$$= \frac{2 \times (47.74 \times 10^3)}{80}$$

$$\underline{P_t = 1193.5\ N}$$

$$P_d = \frac{21v\,(Ceb + P_t)}{21v + \sqrt{(Ceb + P_t)}}$$

$$C = 11{,}400\ N/mm^2$$

$$e = 0.0125$$

(from table)

$$V = \frac{\pi d_p n_p}{60 \times 10^3} = \frac{\pi \times 80 \times 500}{60 \times 10^3} = 2.09\ m/s$$

$$P_d = \frac{21(2.09)\left[(11,400\times 0.0125\times 20)+1193.5\right] = x}{21(2.09)+\sqrt{x}}$$

$$P_d = \frac{21(2.09)\times 4043.5}{\left[21(2.09)+63.58\right]}$$

$\sqrt{x} = 63.58$

$x = 4043.5$

$$P_d = 1651.33 \text{ N}$$

$$P_{eff} = P_t \times C_s + P_d$$

$$= (1193.5)\times 1.5 + \underline{1651.53}$$

$$= 3441.78 \text{ N}$$

$$(FOS)_{bending} = \frac{S_b}{P_{eff}} = \frac{4872}{3441.78} = \underline{1.41}$$

$$(FOS)_{pitting} = \frac{S_w}{P_{eff}} = \frac{5109}{3441.78} = \underline{1.48}$$

1.5

WORM GEARS

Worm gear drives are used to transmit power between two non-intersecting shafts, which are, in general, at right angles to each other.

The worm is a threaded screw, while the worm wheel is a toothed gear. The teeth on the worm wheel envelope the threads on the worm and give line contact between mating parts. The advantages of worm gear drives are as follows:

1. The most important characteristic of worm gear drives is their high speed reduction. A speed reduction as high as 100 : 1 can be obtained with a single pair of worm gears.
2. The worm gear drives are compact with small overall dimensions, compared with equivalent spur or helical gear drives having same speed reduction.
3. The operation is smooth and silent.
4. Provision can be made for self locking operation, where the motion is transmitted only from the worm to the worm wheel. This is advantageous in applications like cranes and lifting devices.

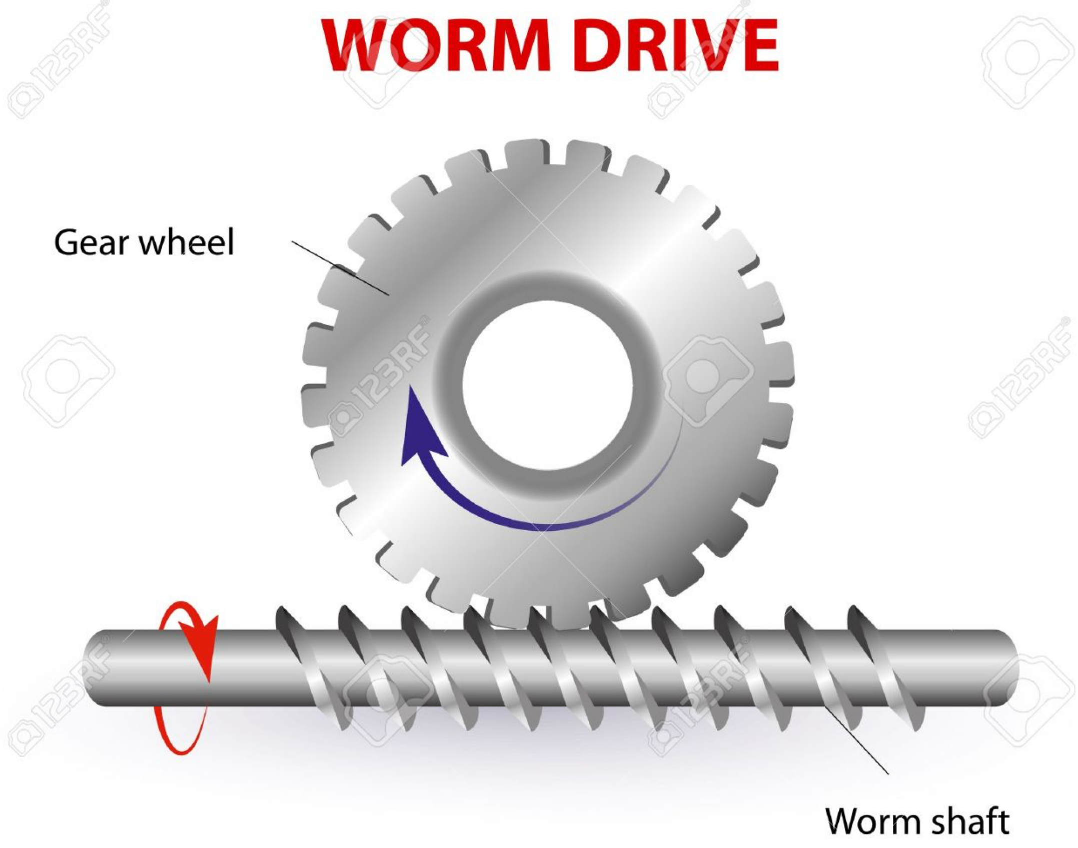
The drawbacks of the worm gear drives are as follows:

(i) The efficiency is low compared with other types of gear drives.

(ii) The worm wheel, in general, is made of phosphor bronze, which increases the cost.

(iii) Considerable amount of heat is generated in worm gear drives, which is required to be dissipated by a lubricating oil to the housing walls and finally to the surroundings.

(iv) The power transmitting capacity is low. Worm gear drives are used for up to 100 kW of power transmission.



The relation between no.of starts and the efficiency of worm gear is:

(i) Single-threaded worm gives large speed reduction, however, the efficiency is low. The large velocity ratio is obtained at the cost of efficiency.

(ii) Multi-threaded worm gives high efficiency, however, the speed reduction is low. The high efficiency is obtained at the cost of speed reduction.

- ***Manually Operated Intermittent Mechanisms*** In these applications, large mechanical advantage is required and efficiency is of minor importance. The examples of these mechanisms include steering mechanism and opening and closing of gate valves by means of hand wheels.
- ***Motorized Operated Intermittent Mechanisms*** In these applications, a small capacity low-cost motor drives the mechanism and the efficiency is of minor importance. The examples of these mechanisms include drive for small hoists and opening and closing of large gate valves by means of electric motor.
- ***Motorized Continuous Operations*** In these applications, worm gear drives are used in place of other gear drives due to space limitations and silent operation. The efficiency

is more important in these applications. Multi-threaded worms are used in these applications to obtain higher efficiency. The examples of this type include drives for machine tools and elevators.

- ***Motorized Speed Increasing Applications*** In these applications, worm gear drives are preferred due to high velocity ratio and silent operation. The efficiency is more important in these applications. Speed increasing applications include drives for automotive supercharger and centrifugal cream charger. In order to increase efficiency, the automotive supercharger is provided with six- threaded worm having lead angle of about 45°.

TERMINOLOGY OF WORM GEARS

A pair of worm gears is specified and designated by four quantities in the following manner:

$$z_1/z_2/q/m$$

where,

z_1 = number of starts on the worm

z_2 = number of teeth on the worm wheel

q = diametral quotient

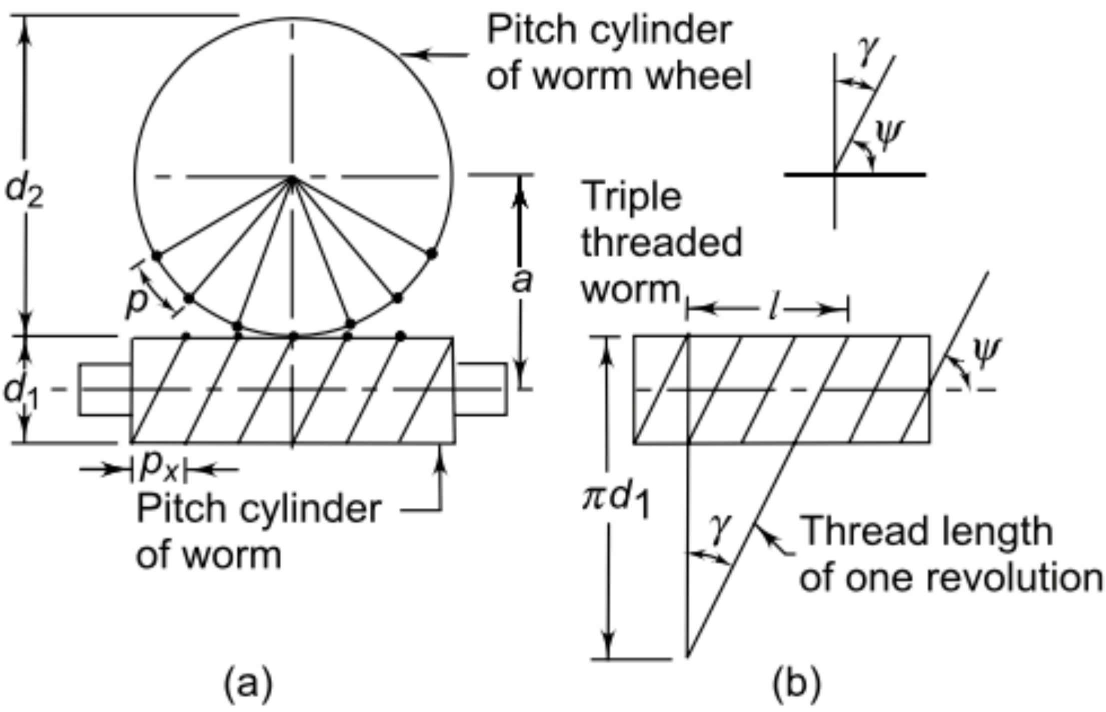
m = module (mm)

The diametral quotient is given by,

$$q = \left(\frac{d_1}{m}\right)$$

$q = d_1/m$

where d1 is the pitch circle diameter of the worm.



(i) Axial Pitch The axial pitch (p_x) of the worm is defined as the distance measured from a point on one thread to the corresponding point on the adjacent thread, measured along the axis of the worm.

(ii) Lead The lead (l) of the worm is defined as the distance that a point on the helical profile will move when the worm is rotated through one revolution. It is the thread advance in one turn. For single-start threads, the lead is equal to the axial pitch. For double-start threads, the lead is twice the axial pitch, and so on. Therefore,

$$l = p_x z_1$$

The recommended number of starts on the worm is as follows:

Velocity ratio	*Number of starts*
20 and above	Single-start
12–36	Double-start
8–12	Triple-start
6–12	Quadruple-start
4–10	Sextuple-start

The pitch circle diameter of the worm wheel is given by

$d_2 = mz_2$

As seen in the figure, the axial pitch of the worm should be equal to the circular pitch of the worm wheel.

When one thread of the worm is developed, it becomes the hypotenuse of a triangle as shown in Fig. The base of this triangle is equal to the lead of the worm, while the altitude is equal to the circumference of the worm. There are two angles related to this triangle, namely, lead angle and helix angle.

$$P_x = P_c \Rightarrow P_x = \frac{\pi d_2}{z_2} = \frac{\pi (m z_2)}{z_2}$$

Axial pitch of worm $\boxed{P_x = \pi m}$

Lead $\boxed{l = \pi m z_1}$

(iii) Lead Angle The lead angle (γ) is defined as the angle between a tangent to the thread at the pitch diameter and a plane normal to the worm axis.

$$\tan \gamma = \frac{l}{\pi d_1} \Rightarrow \tan \gamma = \frac{\cancel{\pi} m z_1}{\cancel{\pi} d_1} = \frac{\cancel{m} z_1}{\cancel{m} q}$$

$$\boxed{\tan \gamma = \frac{z_1}{q}}$$

(iv) Helix Angle The helix angle (y) is defined as the angle between a tangent to the thread at the pitch diameter and the axis of the worm. The worm helix angle is the complement of the worm lead angle.

$\gamma + \psi = \pi/2$

$\psi = 24^\circ \rightarrow 4$

$\gamma = 66^\circ$

The helix angle should be limited to 6° per thread. For example, if y = 30° then the worm should have at least five threads.

(v) Pressure Angle The tooth pressure angle (a) is measured in a plane containing the axis of the worm and it is equal to one-half of the thread angle. The pressure angle should not be less than 20° for single and double start worms and 25° for triple and multi-start worms.

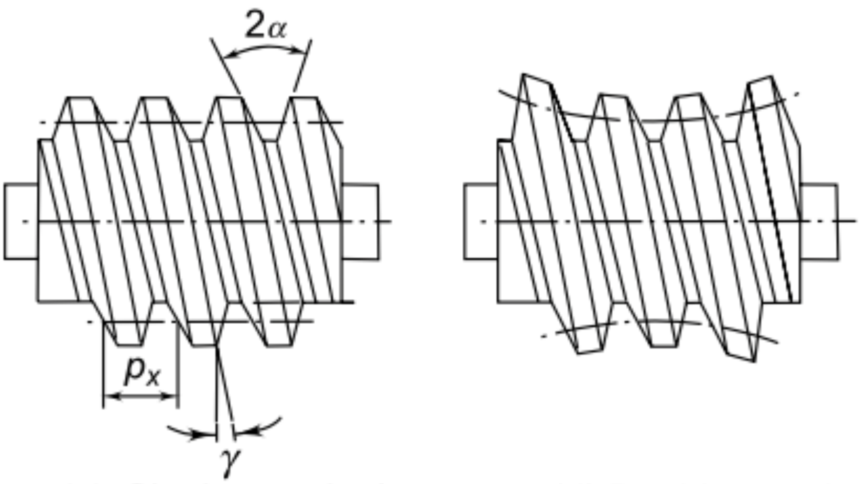
$a = \frac{1}{2}(d_1 + d_2) \Rightarrow a = \frac{1}{2}m(q + z_2)$

$d_1 = mq$ & $d_2 = mz_2$

When the worm wheel is rotated through one revolution, the worm will complete z_2 revolutions for single-start threads. For double-start threads, the number of revolutions of the worm will be ($z_2/2$). The speed ratio (i) is, therefore, given by,

$$i = \left(\frac{z_2}{z_1}\right)$$

There are two classes of worm gear drives in common use, namely, single enveloping and double enveloping



(c) Single enveloping (d) Double enveloping

(i) Single-enveloping Worm Gear Drive A single- enveloping worm gear set is one in which the gear wraps around or partially encloses the worm. This results in line contact between the threads of the worm and the teeth of the worm wheel. In this case, the worm is also called '*cylindrical*' or '*straight*' *cylindrical* worm. The single-enveloping worm gear drive is more widely used.

(i) Double-enveloping Worm Gear Drive A double- enveloping gear set is one in which the gear wraps around the worm and the worm also wraps around the gear. This results in area contact between the threads of the worm and the teeth of the worm wheel. In this case, the worm is also called '*hourglass*' worm. This drive is also called '*cone*' gearing.
Double-enveloping worm gear drive has the following advantages:

a. The contact pressure between the threads of the worm and the teeth of the worm wheel is low. This reduces wear.
b. The drive occupies less space for a given capacity. Double-enveloping worm gear drive needs only about two-thirds of the space and has about one-third of the weight compared with a single-enveloping worm gear drive.

The main drawback of double-enveloping worm gear drive is the requirement of precise alignment. It is much more critical than in case of single- enveloping worm gear drive.

PROPORTIONS OF WORM GEARS

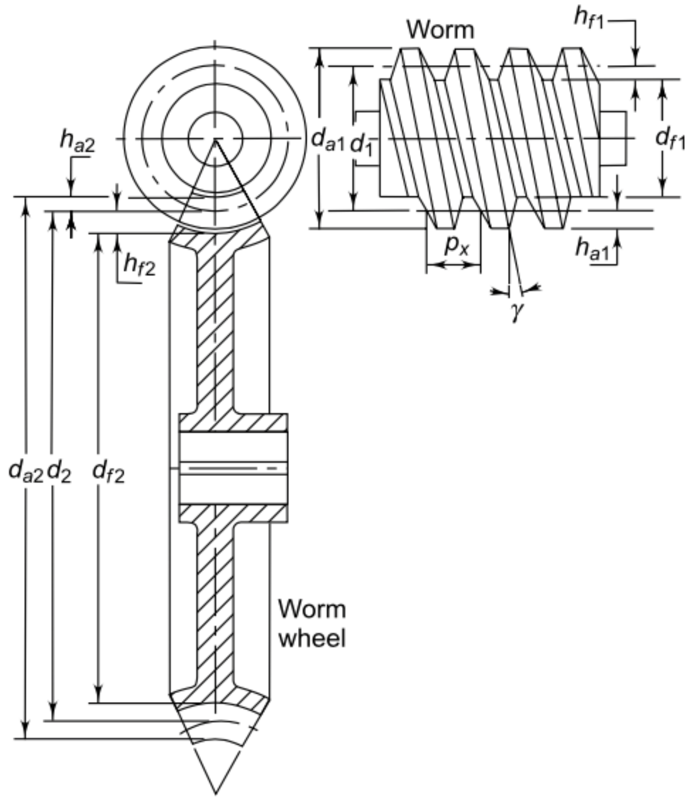


Fig. 20.2 *Dimension of Worm Gears*

$$h_{a1} = m$$
$$h_{f1} = (2.2 \cos \gamma - 1)\text{m}$$
$$c = 0.2\, m \cos \gamma$$

where,

$$h_{a1} = \text{addendum (mm)}$$
$$h_{f1} = \text{dedendum (mm)}$$
$$c = \text{clearance (mm)}$$

$$d_{a1} = d_1 + 2h_{a1} = qm + 2m$$
$$\therefore \quad d_{a1} = m(q + 2) \tag{20.14}$$
$$d_{f1} = d_1 - 2h_{f1} = qm - 2m\,(2.2 \cos \gamma - 1)$$
$$\therefore \quad d_{f1} = m(q + 2 - 4.4 \cos \gamma) \tag{20.15}$$

where

$$d_{a1} = \text{outside diameter of the worm (mm)}$$
$$d_{f1} = \text{root diameter of the worm (mm)}$$

For Worm wheel:

$$h_{a2} = m\,(2\cos\gamma - 1) \qquad (20.16)$$
$$h_{f2} = m\,(1 + 0.2\cos\gamma) \qquad (20.17)$$

where,

h_{a2} = addendum at the throat (mm)
h_{f2} = dedendum in the median plane (mm)

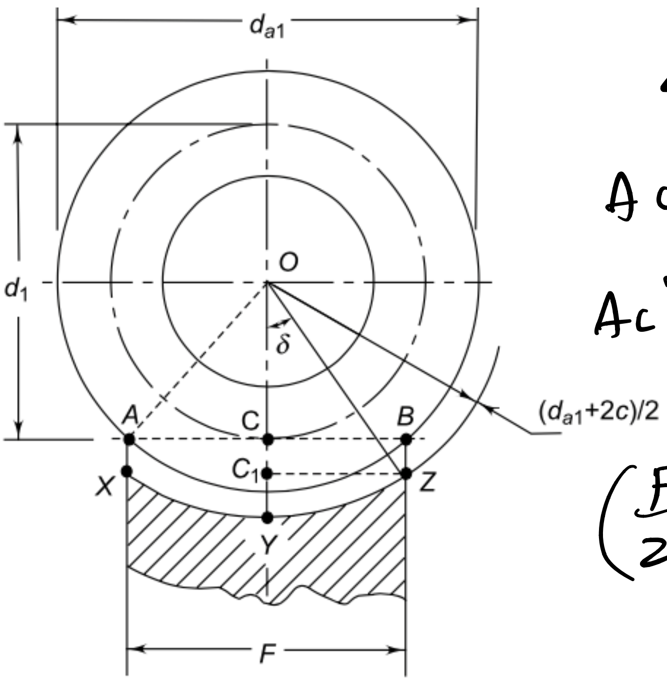
The dimensions of the worm wheel are as follows:

$$d_{a2} = d_2 + 2h_{a2} = mz_2 + 2m(2\cos\gamma - 1)$$
$$\therefore \quad d_{a2} = m\,(z_2 + 4\cos\gamma - 2) \qquad (20.18)$$
$$d_{f2} = d_2 + 2h_{f2} = mz_2 - 2m(1 + 0.2\cos\gamma)$$
$$\therefore \quad d_{f2} = m(z_2 - 2 - 0.4\cos\gamma) \qquad (20.19)$$

where,

d_{a2} = throat diameter of the worm wheel (mm)
d_{f2} = root diameter of the worm wheel (mm)

The effective face width *F* of the worm wheel is shown in Fig. It is obtained by drawing a tangent *AB* to the pitch circle diameter of the worm. *A* and *B* are the points of intersection of this tangent and the outside diameter of the worm.



In Δ^{le} AOC

$$AC^2 = AO^2 - OC^2$$

$$AC^2 = \left(\frac{d_{a1}}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2$$

$$\left(\frac{F}{2}\right)^2 = \left(\frac{d_{a1}}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2$$

where F → Face width
d_{a1} → addendum circle dia
d_1 - pitch dia

$$\left(\frac{F}{2}\right)^2 = \left(\frac{m(q+2)}{2}\right)^2 - \left(\frac{qm}{2}\right)^2$$

$$F^2 = m^2(q+2)^2 - (qm)^2$$

$$F^2 = m^2\left[(q+2)^2 - q^2\right]$$

$$= m^2\left[\cancel{q^2} + 4 + 4q - \cancel{q^2}\right]$$

$$F^2 = 4m^2(q+1)$$

$$\boxed{F = 2m\sqrt{(q+1)}}$$

Δ^{le} OZC_1

$$\sin\delta = \frac{C_1Z}{OZ} = \frac{F/2}{\frac{(d_{a_1}+2c)}{2}}$$

$x = 90$

$$\delta = \sin^{-1}\left[\frac{F}{d_{a_1}+2c}\right]$$

$$l_r = (d_{a_1}+2c)\sin^{-1}\left[\frac{F}{d_{a_1}+2c}\right]$$

$l_r \to$ length of root of worm wheel.

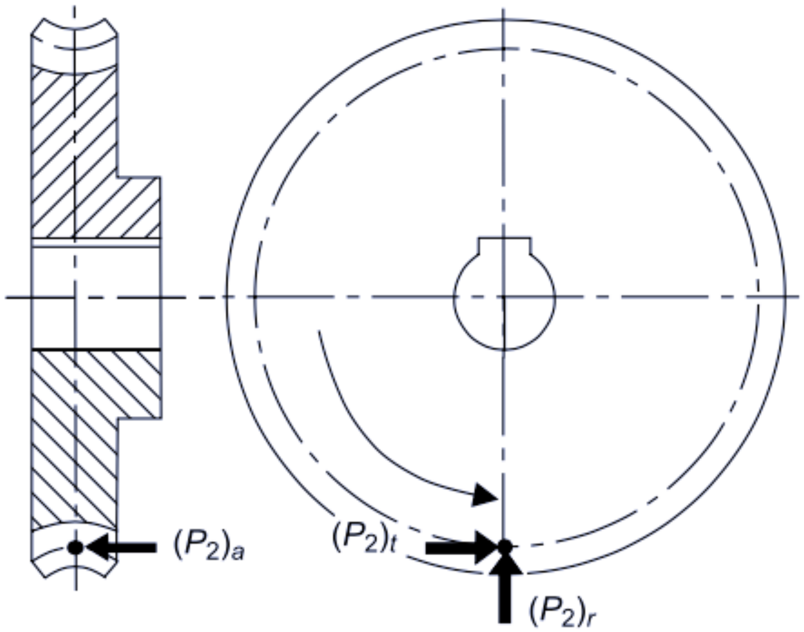
FORCE ANALYSIS

Assumptions in this analysis are:

I. The worm is the driving element, while the worm wheel is the driven element.

II. The worm has right-handed threads.

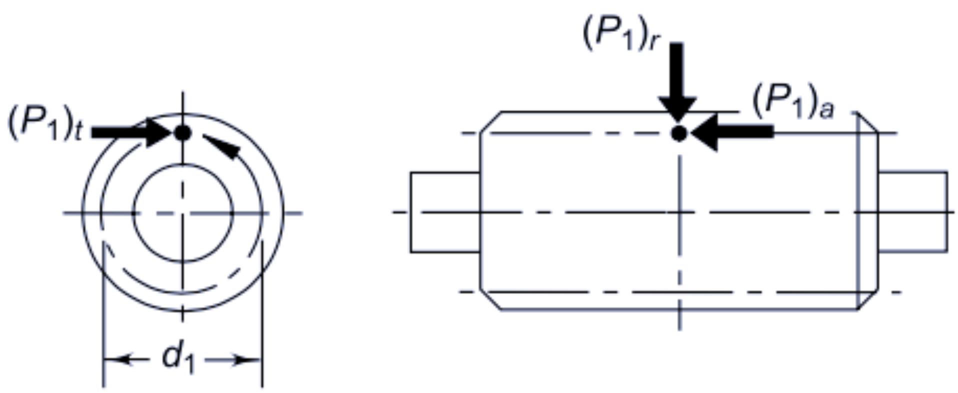
III. The worm rotates in anti-clock wise directions as shown

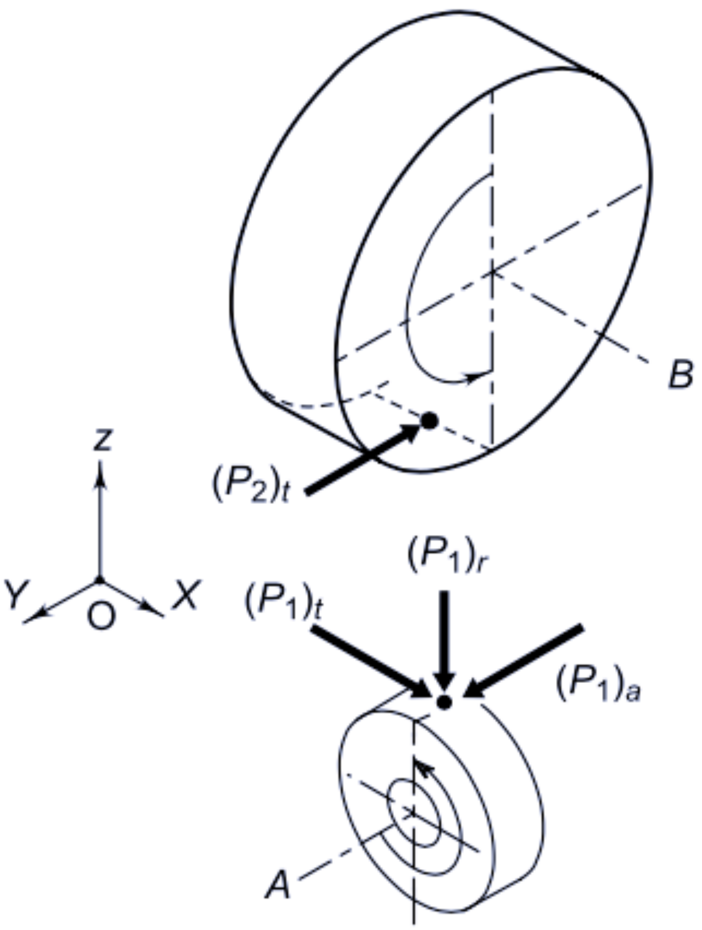


$$(P_2)_t = (P_1)_a$$

$$(P_2)_a = (P_1)_t$$

$$(P_2)_r = (P_1)_r$$





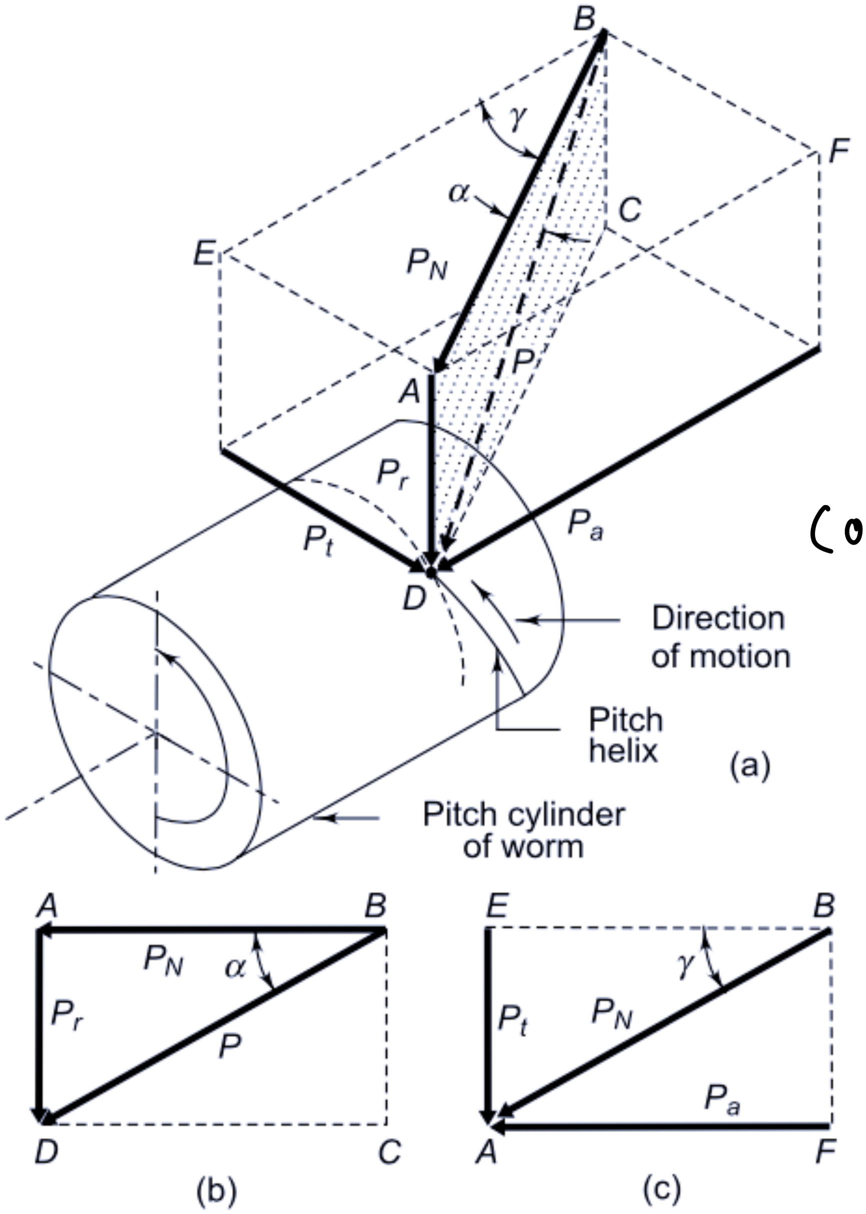
(i) Tangential Component $(P_1)_t$ The worm is the driving element. It is rotating in an anti-clockwise direction, when viewed from *A*. For the driving element, the direction of tangential component is opposite to the direction of rotation. Therefore, $(P_1)_t$ will act in the positive *X* direction at the point of contact.

(ii) Axial Component $(P_1)_a$ The worm has right- hand threads and when the right-hand thumb rule is applied, by keeping the fingers in the direction of rotation, the thumb will be projecting along the positive *Y*-axis. Therefore, if we treat the worm as 'screw' and the worm wheel as 'nut', the screw will have a tendency to move in the direction of the thumb or along the positive *Y*-axis. The nut or the worm wheel will have a tendency to move in the opposite direction, i.e., along the negative *Y*-axis. Therefore, the worm wheel will rotate in the anti-clockwise direction when observed from the bearing B. The worm wheel is the driven member and the direction of $(P_2)_t$ will be the same as the direction of rotation or along the negative *Y*-axis. Since, $(P_1)_a$ and $(P_2)_t$ are equal and opposite, the axial component $(P_1)_a$ will act in the positive *Y* direction at the point of contact.

(iii) Radial Component $(P_1)_r$ The radial component always acts towards the centre of gear. Therefore, $(P_1)_r$ will act towards the centre of the worm or along the negative Z-direction at the point of contact.

The resultant force acting on the worm consists of two components—components of normal reaction between the meshing teeth and components of frictional force. The two components are superimposed to get the resultant components.

The components of the normal reaction P acting on the worm are shown in Fig. Here, α is the normal pressure angle, while γ is the lead angle.



Considering plane ABCD

$P_N = P\cos\alpha$ — (1)

$P_r = P\sin\alpha$ — (2)

Considering plane AEBF

$P_a = P_N\cos\gamma$ — (3)

$P_t = P_N\sin\gamma$ — (4)

From eqn (1), (2), (3) & 4

$P_a = P\cos\alpha\cos\gamma$ — (5)

$P_t = P\cos\alpha\sin\gamma$ — (6)

$$\boxed{P = \frac{P_t}{\cos\alpha\sin\gamma}}$$ (a)

The frictional force is significant in worm gear drives, because there is sliding motion between the threads of the worm and the teeth of the worm wheel, as compared with the rolling motion between the teeth of the pinion and gear in other types of gears.

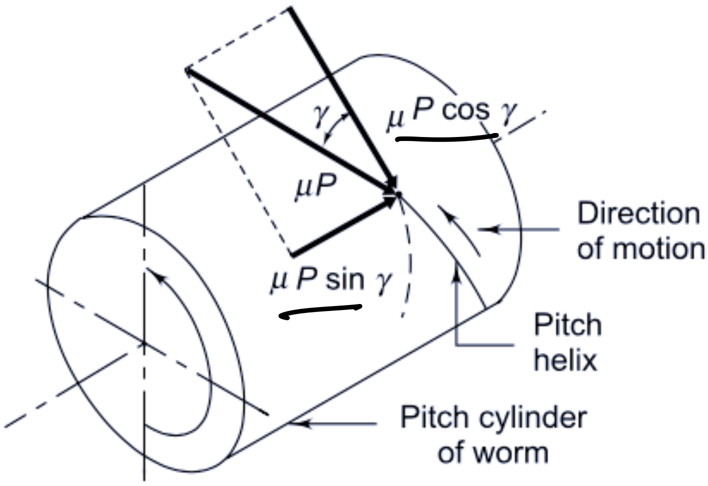


Fig. 20.7 *Components of Frictional Force*

1. Component ($\mu P \cos \gamma$) in the tangential direction. The direction of this component is same as that of Pt.
2. Component ($\mu P \sin \gamma$) in the axial direction. The direction of this component is opposite to that of Pa.

Final forces

$(P_1)_t = (P_t)$ + Frictional force in tangential dirn

$$P = \frac{(P_1)_t}{\cos\alpha \sin\gamma + \mu \cos\gamma} \quad (b)$$

$= P\cos\alpha \sin\gamma + \mu P \cos\gamma$

$$(P_1)_t = P\left[\cos\alpha \sin\gamma + \mu \cos\gamma\right] \quad —(7)$$

$(P_1)_a = (P_a)$ − Frictional force in axial dirn

$= P\cos\alpha \cos\gamma - \mu P \sin\gamma$

$$= P\left[\cos\alpha \cos\gamma - \mu \sin\gamma\right] \quad —(8)$$

$P_t \rightarrow P_a, P_r$

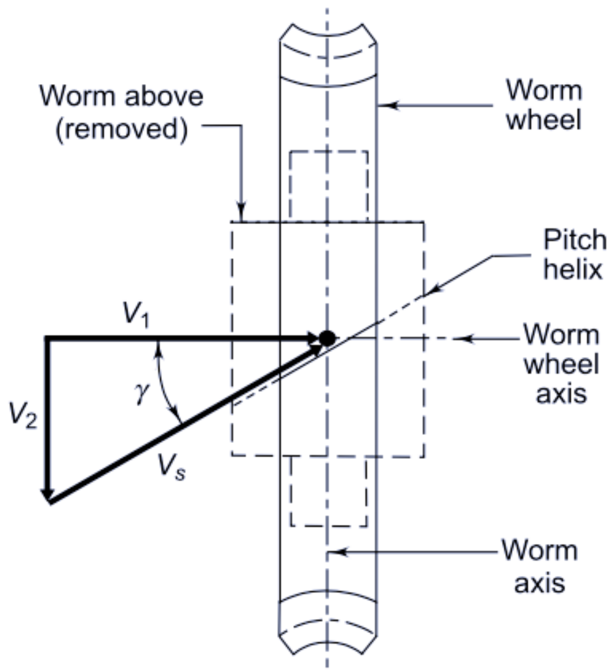
$$\boxed{(P_1)_t = \frac{2M_t}{d_1} \qquad \text{From } (\beta)}$$

$$(P_1)_a = (P_1)_t \quad \frac{\cos\delta\cos\gamma - \mu\sin\gamma}{\cos\delta\sin\gamma + \mu\cos\gamma}$$

$$(P_1)_r = (P_1)_t \times \left(\frac{\sin\delta}{\cos\delta\sin\gamma + \mu\cos\gamma}\right)$$

FRICTION IN WORM GEARS

The coefficient of friction in worm gear drives depends upon the rubbing speed. The rubbing speed is the relative velocity between the worm and the wheel.



$$V_1 = \frac{\pi d_1 n_1}{60 \times 10^3} \text{ m/s}$$

$$V_s = \frac{V_1}{\cos\gamma}$$

$$V_s = \frac{\pi d_1 n_1}{60{,}000 \cos\gamma} \text{ m/s}$$

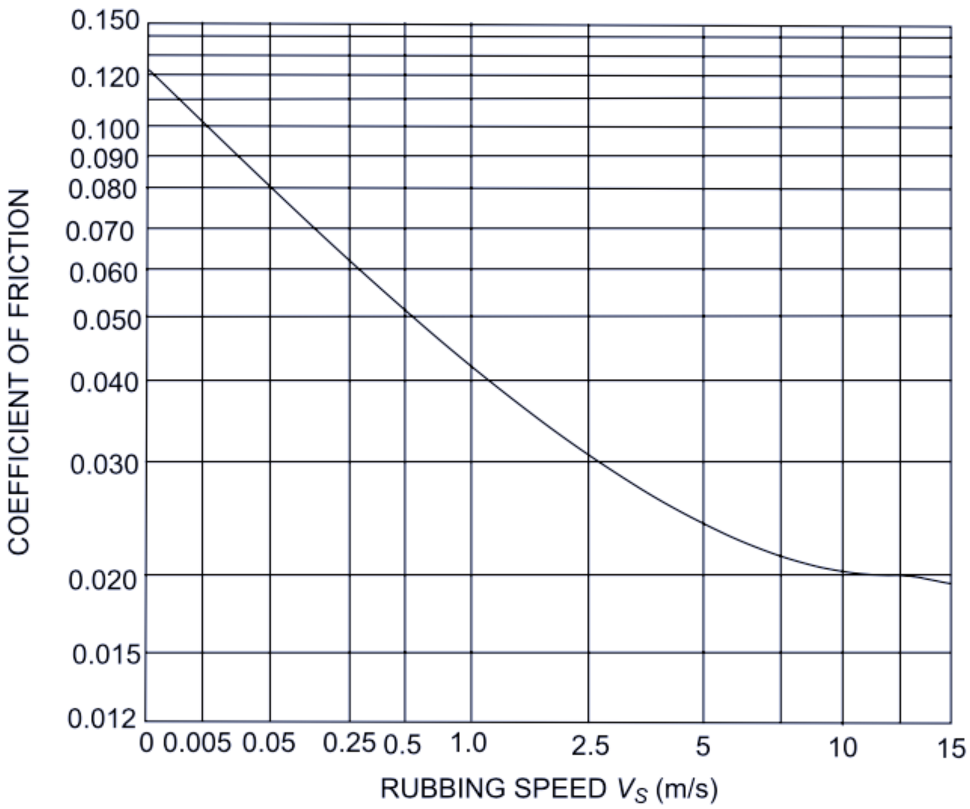


Fig. 20.9 *Coefficient of Friction of Worm Gears*

The values of the coefficient of friction in this figure are based on the following two assumptions:

(i) The worm wheel is made of phosphor- bronze, while the worm is made of case-hardened steel.

(ii) The gears are lubricated with a mineral oil having a viscosity of 16 to 130 centi-Stokes at 60°C.

Efficiency of worm drive:

$$\eta = \frac{\text{power o/p}}{\text{power i/p}} = \frac{(\text{Force} \times \text{dist} \times \text{speed})_{o/p}}{(\text{Force} \times \text{dist} \times \text{speed})_{i/p}}$$

$$\frac{(\text{speed})_{o/p}}{(\text{speed})_{i/p}} = \frac{\frac{\cancel{2\pi} N_2}{\cancel{60}}}{\frac{\cancel{2\pi} N_1}{\cancel{60}}} \Rightarrow N_2/N_1 = \frac{1}{i}$$

$i \rightarrow$ speed ratio

$\frac{d_2}{2} \rightarrow$ o/p dist

$d_1/2 \rightarrow$ i/p dist

$$\frac{d_2}{d_1} = \frac{m z_2}{m q} = \frac{z_2}{q}$$

$$\frac{d_2}{d_1} = \frac{z_2/z_1}{q/z_1} = i \tan\gamma$$

$$\eta = \frac{(P_2)_t}{(P_1)_t} \times \cancel{i} \tan\gamma \times \frac{1}{\cancel{i}}$$

$$\eta = \frac{(P_2)_t}{(P_1)_t} \times \tan\gamma = \frac{(P_1)_a}{(P_1)_t} \times \tan\gamma$$

$$\eta = \tan\gamma \times \frac{(\cos\alpha \cos\gamma - \mu \sin\gamma)}{(\cos\alpha \sin\gamma + \mu \cos\gamma)}$$

$$\boxed{\eta = \frac{\cos\alpha - \mu \tan\gamma}{\cos\alpha + \mu \cot\gamma}}$$

The efficiency of spur or helical gears is very high and virtually constant in the range of 98% to 99%. On the other hand, the efficiency of worm gears is low and varies considerably in the range of 50% to 98%. In general, the efficiency is inversely proportional to speed ratio, provided the coefficient of friction is constant.

In general, the worm is the driver and the worm wheel is the driven member and the reverse motion is not possible. This is called '*self-locking*' drive, because the worm wheel cannot drive the worm. As for screw threads, the criterion for self-locking is the relationship between the coefficient of friction and lead angle. A worm gear drive is said to be self-locking if the coefficient of friction is greater than tangent of lead angle, i.e., the friction angle is more than the lead angle.

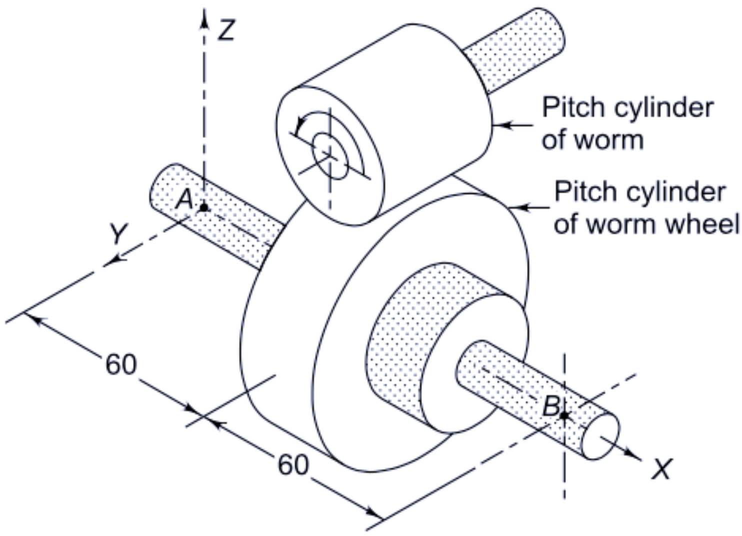
$\mu \geq \tan\gamma$ $\quad$ $\underline{\tan\phi \geq \tan\gamma}$ $\quad$ $\phi \geq$ friction angle

There is another term, '*reversible*' or '*overrunning*' or '*back-driving*' worm gear drive. In this type of drive, the worm and worm wheel can drive each other. In general, the worm is the driver and the worm wheel is the driven member. If the driven machinery has large inertia and if the driving power supply is cut off suddenly, the worm is freely driven by the worm wheel. This prevents the damage to the drive and source of power. A worm-gear drive is said to be reversible if the coefficient of friction is less than tangent of lead angle, i.e., the friction angle is less than the lead angle

Problem: 5 kW of power at 720 rpm is supplied to the worm shaft, as shown in Fig.The worm gear drive is designated as,

2/40/10/5

The worm has right-hand threads and the pressure angle is 20°. The worm wheel is mounted between two bearings A and B. It can be assumed that the bearing A is located at the origin of the co-ordinate system and the bearing B takes complete thrust load. Determine the reactions at the two bearings.



Given data:

$P = 5\ kW$; $N = 720\ RPM$

$\alpha = 20°$;

$z_1/z_2/q/m = 2/40/10/5$

$z_1 = 2$; $z_2 = 40$ teeth; $q = 10$

$m = 5\ mm$

Reaction @ bearing $\Leftarrow$ worm wheel forces $\Leftarrow$ worm forces

P_t, P_n, P_a → forces acting

$$M_t = \frac{60 \times P \times 10^6}{2\pi N} = \frac{60 \times 5 \times 10^6}{2\pi \times 720} = 66.314 \times 10^3\ N\text{-}mm$$

$$P_t = \frac{2M_t}{d_1}$$

$d_1 = mq = 5 \times 10 = 50\ mm$

$d_2 = z_2 m = 40 \times 5 = 200\ mm$

$$\tan\gamma = \frac{z_1}{q} \Rightarrow \gamma = \tan^{-1}\left(\frac{2}{10}\right) \Rightarrow \gamma = 11.31°$$

$$(P_1)_t = \frac{2\times 66.314\times 10^3}{50} \Rightarrow \boxed{(P_1)_t = 2652.6\ N}$$

$$(P_1)_a = (P_1)_t\left[\frac{\cos\alpha\cos\gamma - \mu\sin\gamma}{\cos\alpha\sin\gamma + \mu\cos\gamma}\right]$$

$$(P_1)_r = (P_1)_t \times \left(\frac{\sin\alpha}{\cos\alpha\sin\gamma + \mu\cos\gamma}\right)$$

$$\alpha = 20^\circ;\quad \gamma = 11.31^\circ;\quad \mu = 0.035$$

$$V_s = \frac{V_1}{\cos\gamma}$$

720

$$V_1 = \frac{\pi d_1 N_1}{60\times 10^3}$$

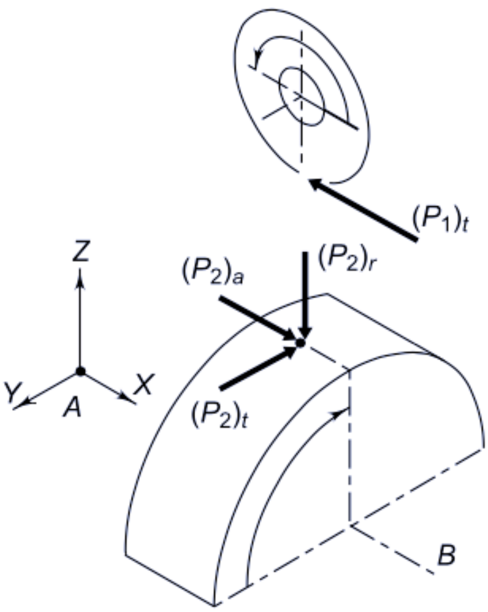
$$V_1 = 1.88\ ms^{-1}$$

$$V_s = 1.92\ ms^{-1}$$

From graph for $V_s = 1.92$; $\underline{\mu = 0.035}$

$$\underline{(P_1)_a = 11{,}061\ N}$$

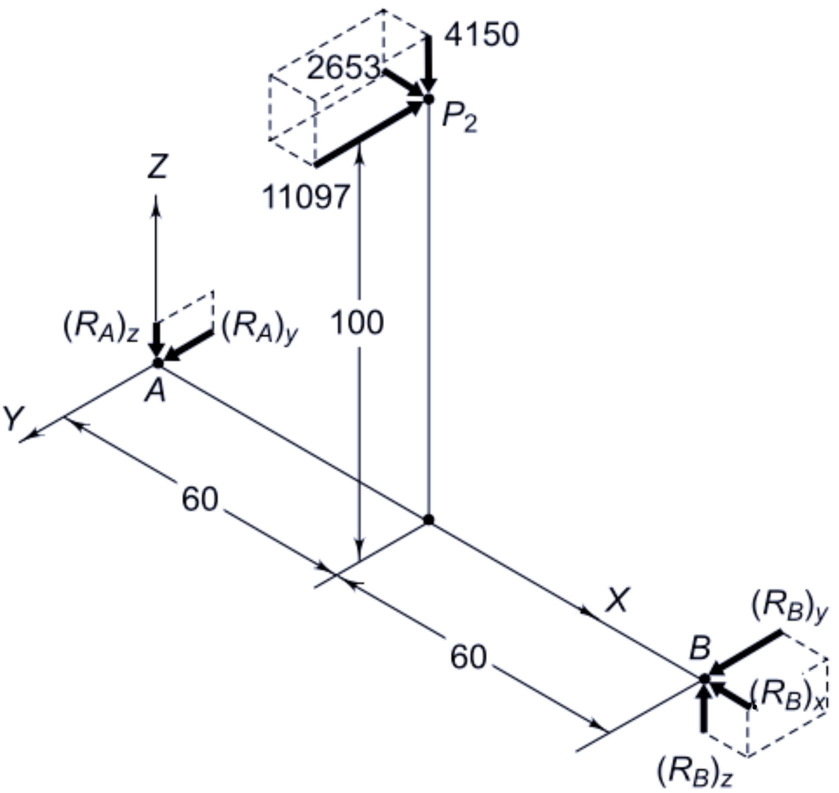
$$\underline{(P_1)_r = 4153\ N}$$

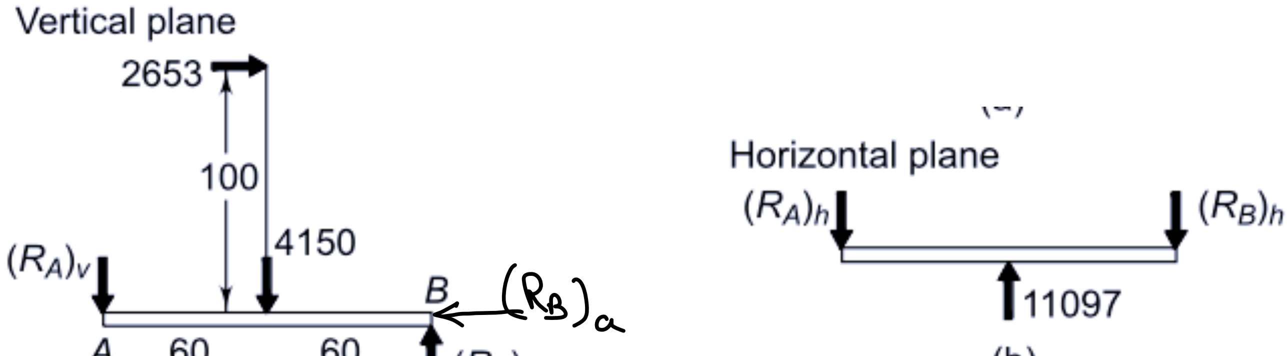


$$(P_1)_t = (P_2)_a = 2652.6$$

$$(P_1)_a = (P_2)_t = 11{,}061$$

$$(P_1)_r = (P_2)_r = 4153$$





(b)

Taking moment about "A"

$$R_B \times 120 - 4150 \times 60 - 2653 \times 100 = 0$$

$$(R_B)_v = \frac{(4150 \times 60) + (2653 \times 100)}{120}$$

$$(R_B)_v = 4285.83 \text{ N} = (R_B)_z$$

Equilibrium of forces

$$(R_A)_v + 4150 = (R_B)_v$$

$(R_A)_v = 135.83 \text{ N} = (R_A)_z$

Equilibrium of forces

$(R_B)_a = (R_B)_x = 2053 \text{ N}$

From figure

$$\underbrace{(R_A)_h = (R_B)_h = \frac{11{,}061}{2} = 5530.5 \text{ N}}$$

$$(R_A)_y = (R_B)_y =$$

STRENGTH RATING OF WORM GEARS

Since the teeth of worm wheel are weaker than the threads of worm, the design for strength can be based on Lewis' equation as applied to worm wheel teeth. In this case, it is not necessary to design the worm on the basis of strength.

$$(M_t)_1 = 17.65\ X_{b1} S_{b1} m l_r\, d_2 \cos\gamma$$

$$(M_t)_2 = 17.65\ X_{b2} S_{b2} m l_r\, d_2 \cos\gamma$$

where,

$(M_t)_1, (M_t)_2$ = permissible torque on the worm wheel (N-mm)

X_{b1}, X_{b2} = speed factors for strength of worm and worm wheel

S_{b1}, S_{b2} = bending stress factors of worm and worm wheel

m = module (mm)

l_r = length of the root of worm wheel teeth (mm) [Eq. (20.21)]

d_2 = pitch circle diameter of worm wheel (mm) g = lead angle of the worm

Table 20.2 *Values of bending stress factor S_b*

Material	S_b
Phosphor-bronze (centrifugally cast)	7.00
Phosphor-bronze (sand-cast and chilled)	6.40
Phosphor-bronze (sand-cast)	5.00
0.4% Carbon steel-normalized (40C8)	14.10
0.55% Carbon steel-normalized (55C8)	17.60
Case-hardened carbon steels (10C4, 14C6)	28.20
Case-hardened alloy steels (16Ni80Cr60 and 20Ni2Mo25)	33.11
Nickel-chromium steels (13Ni3Cr80 and 15Ni4Crl)	35.22

WEAR RATING OF WORM GEARS

The maximum permissible torque that the worm wheel can withstand without pitting failure, is given by the lower of the following two values:

$$(M_t)_3 = 18.64\, X_{c1}\, S_{c1}\, Y_Z\, (d_2)^{1.8}\ \text{m}$$

$$(M_t)_4 = 18.64\, X_{c2}\, S_{c2}\, Y_Z\, (d_2)^{1.8}\ \text{m}$$

where,

$(M_t)_3$, $(M_t)_4$ = permissible torque on the worm wheel (N-mm)

X_{c1}, X_{c2} = speed factors for the wear of worm and worm wheel

S_{c1}, S_{c2} = surface stress factors of the worm and worm wheel

Y_z = zone factor

Table 20.3 *Values of the Surface Stress Factor S_c*

	Materials	*Values of S_c when running with*			
		A	*B*	*C*	*D*
A	Phosphor-bronze (centrifugally cast)	–	0.85	0.92	1.55
	Phosphor-bronze (sand cast and chilled)	–	0.63	0.70	1.27
	Phosphor-bronze (sand-cast)	–	0.47	0.54	1.06
B	0.4% carbon steel-normalized (40C8)	1.1	–	–	–
C	0.55% carbon steel-normalized (55C8)	1.55	–	–	–
D	Case-hardened carbon steel (10C4 , 14C6)	4.93	–	–	–
	Case-hardened alloy steel (16Ni80Cr60 , 20Ni2Mo25)	5.41	–	–	–
	Nickel–chromium steel (13Ni3Cr80 , 15Ni4Cr1)	6.19	–	–	–

Table 20.4 *Values of the zone factor Y_z*

z_1	$q = 8$	$q = 9$	$q = 10$	$q = 12$	$q = 16$	$q = 20$
1	1.084	1.128	1.143	1.202	1.374	1.508
2	1.114	1.214	1.231	1.280	1.418	1.575
4	1.204	1.380	1.460	1.515	1.634	1.798

THERMAL CONSIDERATIONS

The efficiency of a worm gear drive is low and the work done by friction is converted into heat. When the worm gears operate continuously, considerable amount of heat is generated. The rate of heat generated (H_g) is given by,

$$\mathbf{H_g = 1000\ (1 - \eta)\ (Power)}$$

where

H_g = rate of heat generation (W)

η = efficiency of worm gears (fraction)

Power in KW

The heat is dissipated through the lubricating oil to the housing wall and finally to the surrounding air. The rate of heat dissipated (H_d) by the housing walls to the surrounding air is given by,

$$\mathbf{H_d = k\ (t - t_o)A}$$

where

H = the rate of heat dissipation (W)

k = overall heat transfer coefficient of housing walls (W/m^2 °C)

t = temperature of the lubricating oil (°C)

t_o = temperature of the surrounding air (°C)

A = effective surface area of housing (m^2)

$$\mathrm{kW} = \frac{k(t - t_o)A}{1000(1-\eta)}$$

$$t = t_o + \frac{1000(1-\eta)\,\mathrm{kW}}{kA}$$

Unit 1

Design of IC Engine Components

(i) Cylinder and cylinder liner
(ii) Piston, piston rings and gudgeon pin
(iii) Connecting rod with big and small ends
(iv) Crankshaft, crank and crank pin
(v) Valve gear mechanism

Engine design is a specialized subject and it differs from machine design.

CYLINDER AND CYLINDER LINER

There are two types of cooling systems—air-cooling and water-cooling. Small, single-cylinder engines are usually air-cooled. Such cylinders are provided with fins over the outer surface of the cylinder. Excess heat of combustion is transmitted by the cylinder wall to the surroundings through the fins. The fins increase the surface area of the cylinder wall and improve the overall heat transfer coefficient. Air- cooled engines are mainly used on scooters and motorcycles.

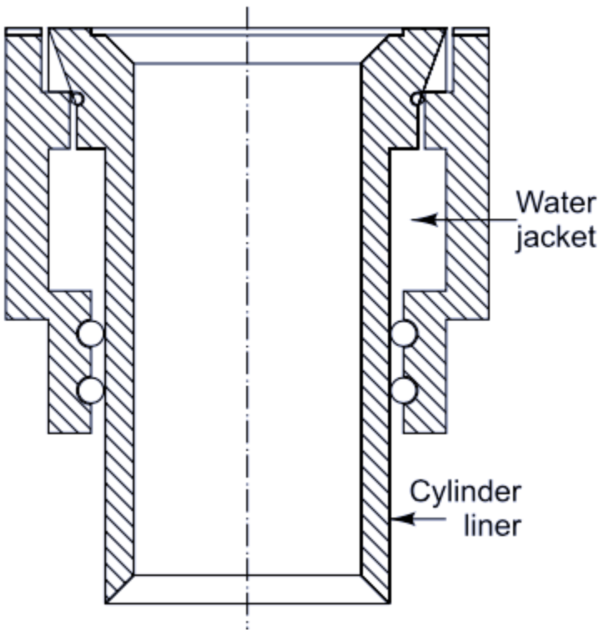
In small engines, the cylinder and frame is made of one-piece casting. In large engines, a separate cylinder liner is used. The cylinder liner, water jacket and frame are manufactured separately and then assembled.

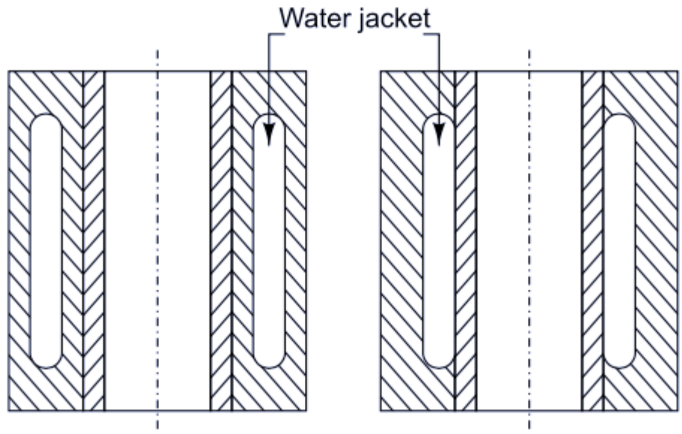
The use of separate cylinder liner has the following advantages:

(i) Cylinder liners are more economical because they can be easily replaced after being worn out. It is not necessary to replace the complete assembly of cylinder, jacket and frame.

(ii) Instead of using better-grade material for all parts of the cylinder assembly, only the cylinder liner is made of better-grade wear resistant cast iron. The frame and jacket can be made of ordinary cast iron.

(iii) Use of cylinder liner allows for longitudinal expansion.





(a) Dry liner (b) Wet liner

The desirable properties of materials for cylinders and cylinder liners are as follows:

1. It should be strong enough to with stand high gas pressure during the combustion of fuel.
2. It should be strong enough to withstand thermal stresses due to heat transfer through the cylinder wall.
3. It should be hard enough to resist wear due to piston movement. It should have good surface finish to reduce friction and wear during the piston movement.
4. It should be corrosion resistant.

Cylinders and cylinder liners are usually made of **grey cast iron** with homogeneous and close grained structure. They are **centrifugally cast**. For heavy- duty cylinders, **nickel cast iron and nickel chromium cast iron** are used. In some cases, cast steel and aluminium alloys are used for cylinders.

BORE AND LENGTH OF CYLINDER

In engine terminology, 'bore' means the inner diameter of the cylinder.

$$\text{IP} = \frac{\text{BP}}{\eta} \qquad (25.1)$$

where,

IP = indicated power or power produced inside the cylinder (W)

BP = brake power or power developed at the crankshaft (W)

η = mechanical efficiency (in fraction)

$$IP = \frac{P_m L A n}{60}$$

$P_m \rightarrow$ Mean effective pressure

$N \rightarrow$ Engine speed

$n \rightarrow$ No. of working strokes

$l \rightarrow$ length of stroke

$A \rightarrow$ Area of cross section of cylinder

$n = N \rightarrow$ 2-stroke

$n = N/2 \rightarrow$ 4-stroke

$$A = \frac{\pi D^2}{4}$$

$D \rightarrow$ Dia of cylinder (or) bore

$$l = 1.5D \text{ (mm)} = \frac{1.5D}{1000} \text{ m}$$

The (l/D) ratio for the cylinder is usually assumed

from 1.25 to 2. In examples where (l/D) ratio is not specified, it is assumed as 1.5.

The length of the cylinder is more than the length of the stroke. There is clearance on both sides of the stroke. The total clearance on two sides is taken as 15% of the stroke length.

$l \rightarrow$ length of stroke

Clearance = $0.15\,l$

Length of cylinder $L = l + 0.15\,l = \underline{1.15\,l}$

THICKNESS OF CYLINDER WALL

$$t = \frac{P_{max}\,D}{2\,\sigma_c} + C$$

$C \rightarrow$ Reboring allowance

$t \rightarrow$ thickness of cylinder wall

$P_{max} = 10\,P_m$

$D \rightarrow$ bore (mm)

$\sigma_c \rightarrow$ permissible circumferential stress (hoop stress)

A. In examples where maximum gas pressure inside the cylinder is not specified, it is assumed as 10 times of the indicated mean effective pressure,

$$p_{max.} = 10\,(p_m)$$

B. The circumferential hoop stress (σ_c)is the allowable tensile stress (σ_t). Since the cylinder material is brittle,

$$\sigma_c = \sigma_t = \frac{(\sigma_{ut})}{(fs)}$$

C. In examples where ultimate tensile strength of cylinder material and factor of safety are not specified, the allowable circumferential stress (σ_c) is taken as 35 to 100 N/mm^2.

D. The reboring allowance is taken from Table below. Reboring is required to compensate uneven wear on the inner wall of the cylinder. *Reboring allowance is additional metal thickness over and above that required to withstand maximum gas pressure inside the cylinder*. It is provided to compensate for reboring at intervals during the lifetime of the cylinder.

Table 25.1 *Reboring allowance for IC engine cylinders*

D	75	100	150	200	250	300	350	400	450	500
C	1.5	2.4	4.0	6.3	8.0	9.5	11.0	12.5	12.5	12.5

(*Note:* *D* and *C* are in mm)

Empirical Relationships There are some empirical relationships used in cylinder design. They are as follows:

1. The thickness of cylinder wall varies from 5 to 25 mm depending upon the cylinder bore. It can be calculated by using the following empirical equation:

$$t = 0.045\, D + 1.6 \text{ (mm)}$$

2. Thickness of dry liner = **0.03*D* to 0.035*D* (mm)**
3. Thickness of water jacket wall = **(1/3) *t* to (3/4) *t*,**
4. Thickness of water jacket wall = **0.032*D* + 1.6 (mm)**
5. Water space between outer cylinder wall and inner jacket wall = **9 mm for 75 mm** cylinder bore **to 75 mm for 750 mm** cylinder bore,
6. Water space between outer cylinder wall and inner jacket wall = **0.08*D* + 6.5 mm**
7. Thickness of cylinder flange = 1.2 t to 1.4 t Also,
8. Thickness of cylinder flange = 1.25 d to 1.5 d (d = nominal diameter of bolt or stud)
9. Radial distance between outer diameter of flange and pitch circle diameter of studs = (d + 6) to (1.5d) mm

STRESSES IN CYLINDER WALL

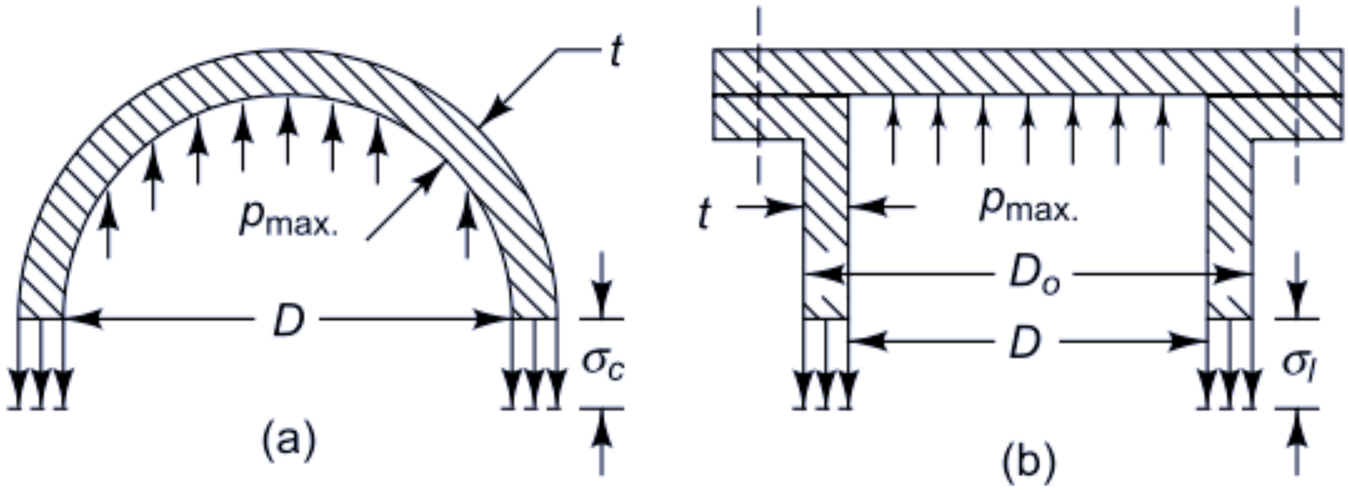


Fig. 25.3 *Stresses in thin cylinder*

$$D \times l \times P_{max} = 2 \sigma_c t \times l$$

$$\boxed{\sigma_c = \frac{P_{max} D}{2t}}$$

$$P_{max}\left(\frac{\pi}{4} D^2\right) = \sigma_l \left[\frac{\pi}{4}\left(D_o^2 - D^2\right)\right]$$

$$\sigma_l = \frac{P_{max} D^2}{\left(D_o^2 - D^2\right)}$$

Apparent stress

$\mu \rightarrow$ poisson's ratio.

Net stress:

$$(\sigma_c)_{net} = \sigma_c - \mu \sigma_l$$

$$(\sigma_l)_{net} = \sigma_l - \mu \sigma_c$$

CYLINDER HEAD

In most of the IC engines, a separate cylinder head or cylinder cover is provided. The cylinder cover accommodates the following parts:

(i) Inlet and exhaust valves

(ii) Air and gas ports

(iii) Spark plug in case of petrol engine and atomizer in case of diesel engine

The shape of the cylinder head becomes complicated due to accommodation of the above units. In general, a box type section with considerable thickness is used for the cylinder head. Calculating the various dimensions of the actual cylinder head is a difficult exercise. However, in the preliminary stages of design, the cylinder head is assumed as a flat circular plate and its thickness is calculated by the following equation:

$$t_h = D\sqrt{\frac{Kp_{\text{max.}}}{\sigma_c}} = \sqrt{\frac{K P_{max} \times D^2}{\sigma_c}} \qquad (25.9)$$

where,

t_h = thickness of cylinder head (mm)

K = constant (K = 0.162)

σ_c = allowable circumferential stress (N/mm^2)

Note

1. The circumferential hoop stress (σ_c) is the allowable tensile stress (σ_t). Since the material of the cylinder head is brittle,

$$\sigma_c = \sigma_t = \frac{(\sigma_{ut})}{(fs)}$$

2. In examples where ultimate tensile strength of cylinder head material and factor of safety are not specified, the allowable circumferential stress (σ_c) is taken as 30 to 50 N/mm^2.

DESIGN OF STUDS FOR CYLINDER HEAD

1. **The number of studs** The number of studs (z) should be between the following limits,

$$\text{Minimum number of studs} = 0.01D + 4$$

$$\text{Maximum number of studs} = 0.02D + 4$$

2. ***Diameter of Studs*** The core diameter of studs is obtained by equating the maximum gas force acting on the cylinder cover to the resisting force offered by all studs.

$$\text{Gas force acting on cylinder cover} = \left(\frac{\pi D^2}{4}\right) p_{\text{max.}} \quad \text{(a)}$$

$$\text{Resisting force offered by all studs} = z\left(\frac{\pi d_c^2}{4}\right)\sigma_t$$

Equating (a) and (b),

$$\left(\frac{\pi D^2}{4}\right) p_{\text{max.}} = z\left(\frac{\pi d_c^2}{4}\right)\sigma_t$$

where,

d_c = core or minor diameter of studs (mm)

z = number of studs

s_t = allowable tensile stress for stud material (N/mm^2)

Note

(i) The studs are made of steel and since the material is ductile,

$$\sigma_t = \frac{S_{yt}}{(fs)}$$

(ii) In examples where the yield strength of stud material and factor of safety are not specified, the allowable tensile stress (σ_t) is taken as 35 to 70 N/mm^2.

The nominal diameter of studs is obtained by the following relationship:

$$d = \frac{d_c}{0.8} \qquad (25.12)$$

3. ***Pitch of Studs*** The pitch circle diameter of the studs is obtained by the following empirical relationship:

$$D_p = D + 3d$$

$$\text{Pitch of studs} = \frac{\pi D_p}{z}$$

In order to obtain a leakproof joint, the pitch of studs should be between the following two limits:

$$\text{Minimum pitch} = 19 * \sqrt{d}$$

$$\text{Maximum pitch} = 28.5 * \sqrt{d}$$

Problem: The cylinder of a four-stroke diesel engine has the following specifications:

Brake power = 3.75 kW

Speed = 1000 rpm

Indicated mean effective pressure = 0.35 MPa Mechanical efficiency = 80%

Determine the bore and length of the cylinder liner.

Given data: $BP = 3.75\,kW = 3750\,W$; $N = 1000$ RPM ;

$P_m = 0.35\,MPa$; $\eta = 80\%$ $\quad P_m = 0.35\,N/mm^2$

Assuming an (l/D) ratio of 1.5.

Area of cylinder $A = \left(\frac{\pi D^2}{4}\right)$ (mm^2)

$l = 1.5 \times D = \left(\frac{1.5\,D}{1000}\right)$ (m) $\quad I.P = \frac{P_m L A n}{60 \times 10^3}$

For 4-stroke engine

$$n = N/2 = \frac{1000}{2} = 500 \text{ strokes/min}$$

1000 rpm

$$I.P = \frac{B.P}{\eta} = \frac{3750}{0.8} = 4687.5\,W$$

$$I.P = \frac{P_m L A n}{60} = \frac{0.35 \times \frac{1.5\,D}{1000} \times \frac{\pi D^2}{4} \times 500}{60}$$

$$4687.5 = \frac{0.35 \times 1.5 \times \pi \times 500}{1000 \times 60 \times 4} \times D^3$$

$\boxed{D = 111\,mm}$; $\boxed{l = 1.5D = 166\,mm}$

Assuming a clearance of 15% $L = l + 0.15\,l = 1.15\,l$

Length of cylinder $\boxed{L = 190\,mm}$

Problem: The cylinder of a four-stroke diesel engine has the following specifications:

Brake power = 7.5 kW

Speed = 1400 rpm

Indicated mean effective pressure = 0.35 MPa Mechanical efficiency = 80%

Maximum gas pressure = 3.5 MPa

The cylinder liner and head are made of grey cast iron FG 260 (S_{ut} = 260 N/mm^2 and m = 0.25). The studs are made of plain-carbon steel 40C8 (S_{yt} = 380 N/mm^2). The factor of safety for all parts is 6.

Calculate:

(i) bore and length of the cylinder liner

(ii) thicknessofthecylinderliner (iii) thicknessofthecylinderhead

(iv) size, number and pitch of studs

Given data: $B\cdot P = 7500 W$; $P_m = 0.35 N/mm^2$; $\eta_m = 0.8$

$N = 1400$ RPM ; $P_{max} = 3.5 N/mm^2$; FOS = 6

$\sigma_{yt} = 380 N/mm^2 \rightarrow$ Studs

$\sigma_{ut} = 260 N/mm^2 \rightarrow$ cylinder liner & head

$\mu = 0.25 \rightarrow$ poisson ratio

Assume $l/D = 1.5$

For 4-stroke engine

$$l = \frac{1.5 D}{1000} \ (m) \quad ; \quad n = N/2 = \frac{1400}{2} = 700 \, s/min$$

$$I\cdot P = \frac{B\cdot P}{\eta} = \frac{7500}{0.8} = 9375 \ W$$

$$I.P = \frac{P_m L A n}{60} \Rightarrow 9375 = \left(\frac{0.35}{60}\right)\left(\frac{1.5D}{1000}\right)\left(\frac{\pi D^2}{4}\right) \times 700$$

$$D^3 = \frac{9375 \times 60 \times 1000 \times 4}{0.35 \times 1.5 \times \pi \times 700}$$

$$\underline{D = 125\,mm}$$

$$l = 1.5D = 187.5\,mm$$

$$L = 1.15 l = 216\,mm$$

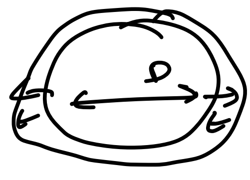
Thickness of cylinder liner:

From table for $D = 125\,mm$, $C = 3.2$

$$t = \frac{P_{max} D}{2\sigma_c} + C$$

$$\sigma_c = \sigma_t = \frac{\sigma_{ut}}{FOS} = \frac{260}{6} = 43.33\,N/mm^2$$

$$t = \left(\frac{3.5 \times 125}{2 \times 43.33}\right) + 3.2$$



$$t = 8.27\,mm \simeq \underline{10}\,mm$$

Apparent stress:

$$\sigma_c = \frac{P_{max} D}{2t} = \underline{21.88}\,N/mm^2 < 43.33$$

$$\sigma_l = \frac{P_{max} D^2}{(D_o^2 - D^2)} \qquad D_o = D + 2t = 145\,mm$$

$$\sigma_l = \frac{3.6 \times (125)^2}{145^2 - 125^2} = \underline{10.15\,N/mm^2}$$

Net stress:

$$(\sigma_c)_{net} = \sigma_c - \mu\sigma_d = 19.35\ N/mm^2$$

$$(\sigma_d)_{net} = \sigma_d - \mu\sigma_c = 4.66\ N/mm^2$$

Thickness of cylinder head:

$$\sigma_c = \sigma_t = \frac{\sigma_{ut}}{F_{OS}} = \frac{260}{6}$$

$$t_h = D\sqrt{\frac{KP_{max}}{\sigma_c}}$$

$$= (125)\sqrt{\frac{0.162 \times 3.5}{43.33}} = 14.4mm \simeq 15mm$$

No. of studs:

$$\text{min num} = 0.01D + 4$$
$$= 5.25$$

$$\text{max num} = 0.02D + 4$$
$$= 6.5$$

No. of studs $Z = 6$

Nominal dia of stud:

$$\sigma_t = \frac{\sigma_{yt}}{F_{OS}} = \frac{380}{6} = 63.33\ N/mm^2$$

$$\left(\frac{\pi}{4}D^2\right)P_{max} = Z\left(\frac{\pi}{4}d_c^2\right) \times \sigma_t$$

$$\left(\frac{\pi}{4} \times (125)^2\right)(3.5) = 6\left(\frac{\pi}{4} \times d_c^2\right) \times 63.33$$

$$d_c = 12mm$$

$$d = \frac{d_c}{0.8} = \frac{12}{0.8} = 15\ mm$$

pitch of studs

$$D_p = D + 3d$$

$$= 125 + 3(15) = 170\,mm$$

$$p = \frac{\pi D_p}{z}$$

$$= \frac{\pi \times 120}{6} = 89\,mm$$

min pitch $= 19 \times \sqrt{d} = 19\sqrt{15} = 73.6\,mm$

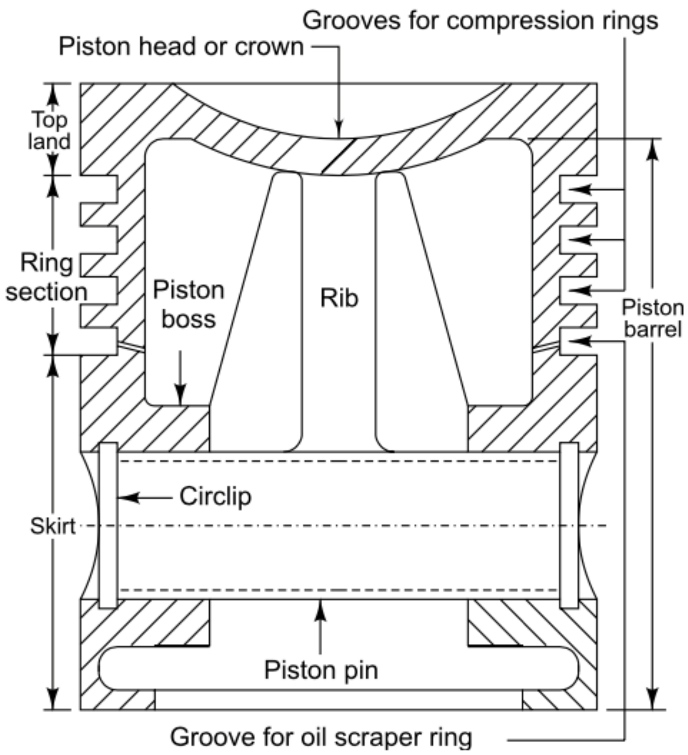
max pitch $= 28.5\sqrt{d} = 28.5\sqrt{15} = 110\,mm$

Hence the pitch chosen is satisfactory.

PISTON

The piston is a reciprocating part of IC engine that performs a number of functions. The main functions of the piston are as follows:

1. It transmits the force due to gas pressure inside the cylinder to the crankshaft through the connecting rod.
2. It compresses the gas during the compression stroke.
3. It seals the inside portion of the cylinder from the crankcase by means of piston rings.
4. It takes the side thrust resulting from obliquity of the connecting rod.



(i) Piston Head or Crown It is the top portion of the piston which withstands the gas pressure inside the cylinder. It has flat, concave or convex shape depending upon the construction of combustion chamber.

(ii) Piston rings They act as seal and prevent the leakage of gas past the piston. Piston rings are also called '*compression*' rings.

(iii) Oil Scraper Ring It prevents the leakage of lubricating oil past the piston into the combustion chamber.

(iv) Piston rings It is the lower part of the piston below the piston rings which acts as bearing surface for the side thrust exerted by the connecting rod.

(v) Piston Pin It connects the piston to the connecting rod. It is also called '*gudgeon*' pin or '*wrist*' pin.

PISTON MATERIALS

Commonly used materials for IC engine pistons are cast iron, cast steel, forged steel, cast aluminium alloys and forged aluminium alloy.

There are two criteria for calculating the thickness of piston head—**strength and heat dissipation**. On the basis of strength criterion, the piston head is treated as a flat circular plate of uniform thickness fixed at the outer edge and subjected to uniformly distributed gas pressure ($p_{max.}$) over the entire surface area. According to Grashoff's formula, the thickness of the piston head is given by,

$$t_h = D\sqrt{\frac{3}{16}\frac{p_{max.}}{\sigma_b}}$$

where,

t_h = thickness of piston head (mm)
D = cylinder bore (mm)
$p_{max.}$ = maximum gas pressure or explosion pressure (MPa or N/mm^2)
σ_b = permissible bending stress (N/mm^2)

The piston head absorbs the heat during combustion of fuel and transmits it to the cylinder wall. It should have sufficient thickness to quickly transfer the heat to the cylinder wall. On the basis of heat dissipation, the thickness of the piston head is given by,

$$t_h = \left[\frac{H}{12.56k(T_c - T_e)}\right] \times 10^3$$

where,

t_h = thickness of piston head (mm)
H = amount of heat conducted through piston head (W)
k = thermal conductivity factor (W/m/°C)
T_c = temperature at the center of piston head (°C)
T_e = temperature at the edge of piston head (°C)

Heat conducted through piston

$H = [C \times HCV \times m \times BP] \times 10^3$

Generally $C = 0.05$

PISTON RIBS AND CUP

The piston head is provided with a number of ribs for the following reasons:

1) Ribs strengthen the piston head against the gas pressure. They increase the rigidity and prevent distortion of piston head
2) Ribs transmit a large portion of combustion heat from the piston head to the piston rings. This reduces the temperature difference between the centre and edge of piston head.
3) The side thrust created by obliquity of connecting rod is transmitted to the piston at the piston pin. It is then transmitted to the cylinder wall through the skirt. The stiffening rib provided at the centre of boss and extending around the skirt, distributes the side thrust more uniformly and prevents distortion of the skirt.

Guidelines for ribs

(i) When the thickness of the piston head is 6 mm or less, no ribs are required. When the thickness of the piston head is more than 6 mm, a suitable number of ribs are required.

$t_h \leq 6$ mm (no ribs)

$t_h > 6$ mm (provide ribs) (25.20)

(ii) The number of ribs is given by,

Number of ribs = 4 to 6 (25.21)

(iii) The thickness of ribs is given by,

$$t_R = \left(\frac{t_h}{3}\right) \text{to} \left(\frac{t_h}{2}\right) \qquad (25.22)$$

where,

t_R = thickness of ribs (mm)

t_h = thickness of piston head (mm)

A cup provides additional space for combustion of fuel. Provision of cup at the top of the piston head depends upon the volume of combustion chamber. It also depends upon the arrangement of valves. If inlet and exhaust valves open and close at angles near the top dead centre, then there is possibility that either inlet or exhaust valve may strike the piston top due to overtaking. A spherical cavity in the form of cup is provided for this purpose.

Guidelines for piston cup

(i) When the ratio of stroke length to bore (l/D) is up to 1.5, a cup is required on the top of the piston.

$(l/D) \leq 1.5$ (cup required)

$(l/D) > 1.5$ (no cup required) (25.23)

(ii) The radius of cup is given by,

radius of cup = $0.7D$ (25.24)

PISTON RINGS

In IC engines, two types of piston rings are used, viz., *compression rings* and *oil scraper rings*. The main function of compression rings is to maintain a seal between the cylinder wall and piston and prevent leakage of gas past the piston. They also transfer heat from the piston head to the cylinder wall. Piston rings also absorb fluctuations in side thrust. Oil scraper rings or oil control rings are provided below the compression rings. They provide proper lubrication of the cylinder liner and reduce frictional losses.

(iii) Dimensions of Cross-section The compression rings have rectangular cross-section as shown in Fig. 25.6(a). The radial width of the ring is given by,

$$b = D\sqrt{\frac{3p_w}{\sigma_t}} \qquad (25.25)$$

where,

b = radial width of ring (mm)

p_w = allowable radial pressure on cylinder wall (N/mm^2)

σ_t = permissible tensile stress for ring material (N/mm^2)

Note

(i) The radial wall pressure is usually taken from 0.025 to 0.042 MPa.

(ii) The permissible tensile stress for cast iron rings is taken from 85 to 110 N/mm^2.

The axial thickness of piston ring is given by,

$$h = (0.7b) \text{ to } b \qquad (25.26)$$

where h is the axial thickness of the piston ring in mm.

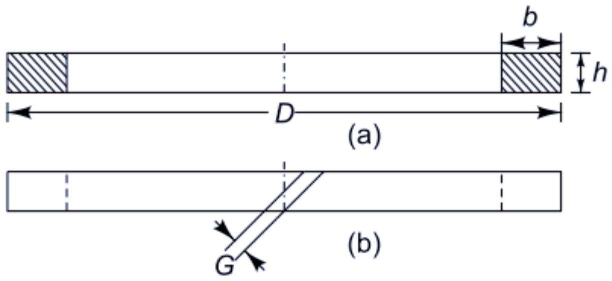
There is a limit on the minimum axial thickness. It is given by,

$$h_{\text{min.}} = \left(\frac{D}{10z}\right)$$

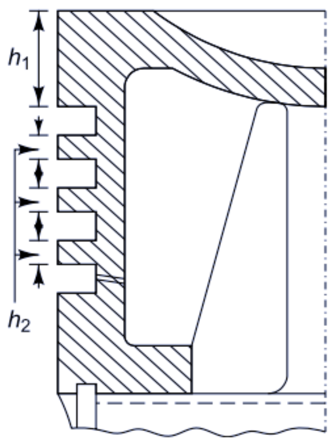
z = number of rings

(iv) Gap between Free Ends The diameter of a piston ring is slightly more than the cylinder bore (D). During the assembly, the ring is compressed diagonally and passed into the liner.

$$G = 3.5\,b \text{ to } 4\,b \text{ (before assembly)}$$
$$G = 0.002\,D \text{ to } 0.004\,D$$
$$\text{(after assembly in cylinder)} \qquad (25.27)$$



(v) Width of Top Land and Ring Lands



$$h_1 = (t_h) \rightarrow (1.2\,t_h)$$

$$h_2 = 0.75\,h \rightarrow h$$

The distance from the top of the piston to the first ring groove (h_1) is called top land.

The distance between two consecutive ring grooves (h_2) is called the width of the ring groove

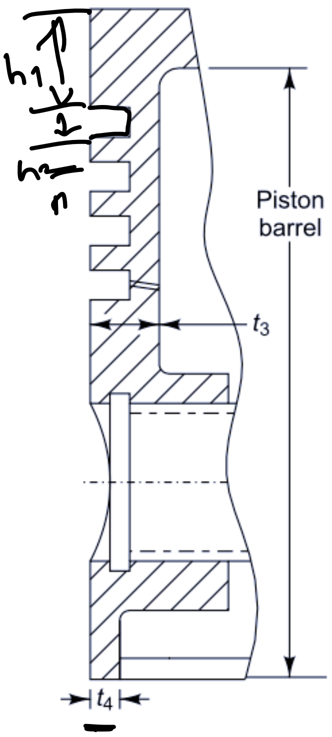
PISTON BARREL

It is the cylindrical portion of the piston below the piston head.

The thickness of the piston barrel at the top end

$$t_3 = (0.03D + b + 4.9) \text{ mm}$$

b → radial width of ring



$t_4 = (0.25 t_3)$ to $(0.35) t_3$

@ open end

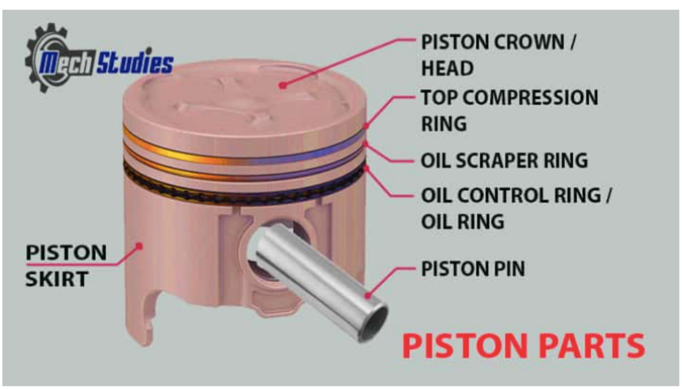
The thickness of piston barrel at the lower or open end is given by,

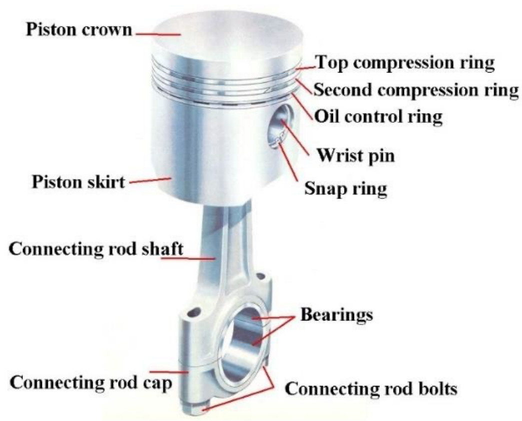
$t_4 = (0.25\ t_3)$ to $(0.35\ t_3)$

t_4 = thickness piston barrel at open end (mm)

PISTON SKIRT

The cylindrical portion of the piston between the last scrapper ring and the open end is called the piston skirt. The piston skirt acts as a bearing surface for the side thrust. The length of the skirt should be such that the bearing pressure due to side thrust is restricted to 0.25 MPa on the projected area. In high speed engines, the bearing pressure up to 0.5 MPa is allowed to reduce the weight of the reciprocating piston. The maximum side thrust will occur during expansion stroke.





Max force on piston head $= \left(\frac{\pi D^2}{4}\right) P_{max}$

Side thrust $= \mu \times F$

(Normal force) $= \mu \times \left(\frac{\pi D^2}{4}\right) \times P_{max}$

$(\mu = 0.1)$

Side thrust taken by the skirt:

$$(ST)_s = P_b D l_s$$

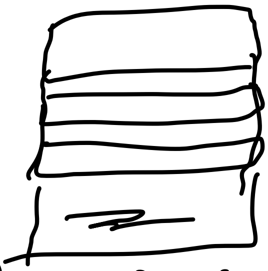
$l_s \rightarrow$ length of skirt (mm)

Equating the above 2 equations: $P_b \rightarrow$ allowable bearing press.

$$\mu\left(\frac{\pi D^2}{4}\right) P_{max} = P_b D l_s$$

Empirical relationship is

$$l_s = (0.65D) \text{ to } (0.8D)$$



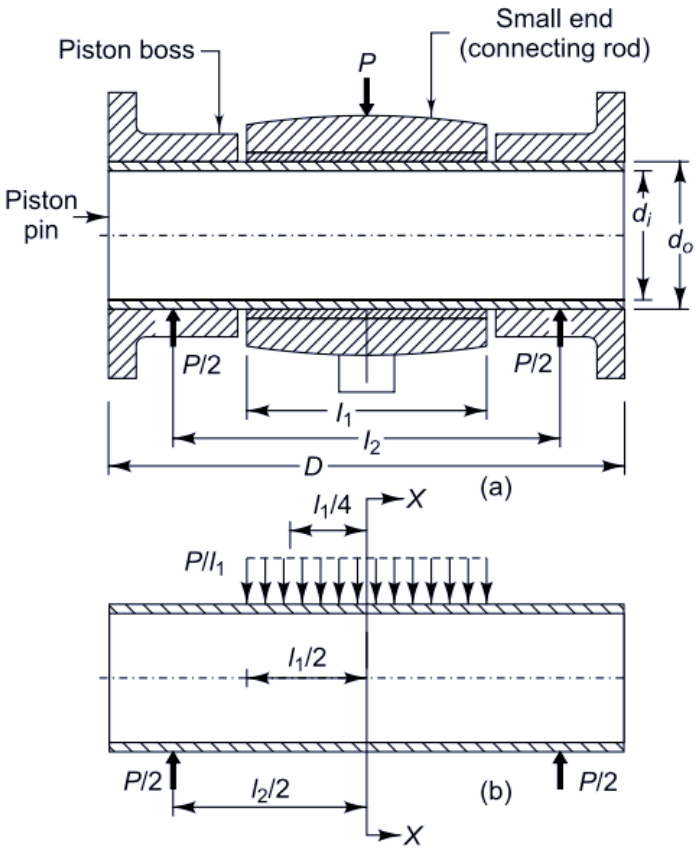
Total length of piston:

$$L = (\text{Top land}) + \left(\text{length of ring section}\right) + (\text{length of skirt})$$

$L = D$ to $1.5D$ (assumption)

PISTON PIN

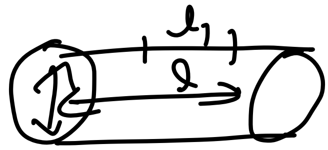
The function of the piston pin is to connect the piston to the connecting rod. It is also called '*gudgeon*' pin or '*wrist*' *pin*. It is made of hollow circular cross- section to reduce its weight. It is often tapered on the inside and the smallest diameter is at the centre of the pin. The piston pin passes through the bosses provided on the inner side of the piston skirt and a bearing bush inside the small end of the connecting rod .The end movement of the piston pin is restricted by means of circlips.



There are two types of connections between the piston pin and the small end of the connecting rod, viz., '*full-floating*' type and '*semi-floating*' type. A full-floating piston pin is free to turn both in the piston bosses as well as the bush in the small end of the connecting rod. The end movement of the pin is restricted by circlips. The semi-floating piston pin is either free to turn in the piston bosses and rigidly fixed to the small end of the connecting rod, or free to turn in the bush of the small end and rigidly fixed to the piston bosses by means of screws.

The piston pin is made of carbon steel or alloy steel. It is hardened and ground to reduce wear while turning inside the phosphor bronze bush. There are two criteria for design of the piston pin—bearing consideration and bending failure.

(i) Bearing Consideration The piston pin is partly in contact with piston bosses and partly with the bush of the connecting rod as shown in Fig. 25.9(a). The bearing area of the piston pin is approximately divided between the piston bosses and the connecting rod bush. It is assumed that the length of the pin in the connecting rod bush is 45% of the piston diameter (D) or cylinder bore. Therefore



$$l_1 = 0.45\,D$$

The outer diameter of the piston pin (d_o) is determined by equating the force acting on the piston and the resisting bearing force offered by the piston pin:

Force on piston $= \left(\frac{\pi}{4} D^2\right) P_{max}$

Resisting force by pin $= (P_b)_1 \times d_o \times l_1$

Equating these 2 equation, we can get "d_o"

Note

1. The bearing pressure at the bushing of the small end of the connecting rod $(p_b)_1$ is taken as 25 MPa.
2. The inner diameter of the piston pin is taken as 0.6 times of the outer diameter.

$$d_i = 0.6\, d_o$$

The mean diameter of the piston bosses is given by,

Mean diameter of piston bosses = 1.4 d_o (for grey cast iron piston)

Mean diameter of piston bosses = 1.5 d_o (for aluminium alloy piston)

$$M_b = \left(\frac{P}{2}\right)\times\left(\frac{l_2}{2}\right)-\left(\frac{P}{l_1}\times\frac{l_1}{2}\right)\times\left(\frac{l_1}{4}\right) \longrightarrow \text{Bending consideration}$$

$$= \left(\frac{P}{2}\right)\times\left(\frac{l_2}{2}\right)-\left(\frac{P}{2}\right)\times\left(\frac{l_1}{4}\right) \qquad \text{(a)}$$

Also,

$$l_2 = \left(\frac{D+l_1}{2}\right) \qquad \text{(b)}$$

Substituting (b) in (a),

$$M_b = \left(\frac{P}{2}\right)\times\left(\frac{1}{2}\right)\left(\frac{D+l_1}{2}\right)-\left(\frac{P}{2}\right)\times\left(\frac{l_1}{4}\right)$$

$$= \left(\frac{PD}{8}\right)+\cancel{\left(\frac{Pl_1}{8}\right)}-\cancel{\left(\frac{Pl_1}{8}\right)}$$

$$M_b = \frac{PD}{8} \qquad \sigma_b = \frac{M_b y}{I}$$

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) \; ; \; y = d_o/2$$

***Problem*:** The following data is given for a four-stroke diesel engine:

Cylinder bore = 250 mm

Length of stroke = 300 mm

Speed = 600 rpm

Indicated mean effective pressure = 0.6 MPa

Mechanical efficiency = 80%

Maximum gas pressure = 4 MPa

Fuel consumption = 0.25 kg per BP per h

Higher calorific value of fuel = 44 000 kJ/kg

Assume that 5% of the total heat developed in the cylinder is transmitted by the piston. The piston is made of grey cast iron FG 200 (S_{ut} = 200 N/mm^2 and k = 46.6 W/m/°C) and the factor of safety is 5. The temperature difference between the centre and the edge of the piston head is 220°C.

1. Calculate the thickness of piston head by strength consideration.
2. Calculate the thickness of piston head by thermal consideration.
3. Which criterion decides the thickness of piston head?
4. State whether the ribs are required.
5. If so, calculate the number and thickness of piston ribs.
6. State whether a cup is required in the top of the piston head.
7. If so, calculate the radius of the cup.

Given data:

$D = 250mm$; $l = 300mm$; $N = 600$ RPM; $P_m = 0.6$ mPa

$P_{max} = 4$ mpa; $\eta = 0.8$; $\sigma_{ut} = 200$ N/mm^2; $FoS = 5$

$C = 0.05$; $HCV = 44{,}000$ (KJ/Kg); $m = 0.25$ Kg/BP/h

$K = 46.6$ W/m°C & $\Delta T = T_C - T_E = 220°C$

Strength consideration:

$$t_h = \sqrt{\frac{3P_{max} \times D^2}{16\,\sigma_b}}$$

$$\sigma_b = \sigma_t = \frac{\sigma_{ut}}{FoS} = \frac{200}{5} = 40\ N/mm^2$$

$$t_h = \sqrt{\frac{3 \times 4 \times (250)^2}{16 \times 40}} = 35\ mm$$

Thermal consideration:

$$t_h = \left[\frac{H}{12.56 \times K \times \Delta T}\right] \times 10^3$$

$$H = [C \times HCV \times m \times BP] \times 10^3$$

$m = 0.25$ kg per BP per h

$$= \frac{0.25}{60 \times 60} = 69.44 \times 10^{-6} \text{ kg per BP per second}$$

$$B.P = \eta(IP) = 0.8\left(\frac{P_m L A n}{60}\right)$$

$$= \frac{0.8}{60}\left(0.6 \times 0.3 \times \frac{\pi}{4}(250)^2 \times 300\right)$$

$$= 35.34\ kW$$

$$H = \left[0.05 \times 44{,}000 \times 69.44 \times 10^{-6} \times 35.34\right] \times 10^{3}$$

$$H = 5398.8 \ \omega$$

$$t_h = \left(\frac{5398.8}{12.56 \times 46.6 \times 220}\right) \times 10^{3}$$

$$\boxed{t_h = 42 \text{mm}}$$

Hence thermal consideration becomes the criterion.

So, $\boxed{t_h = 42 \text{mm}}$

Since $t_h = 42 \text{ mm} > 6\text{mm} \rightarrow$ Ribs are req.

No. of ribs $= 4$. $[4 \rightarrow 6$ is allowable$]$

$$t_R = \left(\frac{t_h}{3}\right) \text{ to } \left(\frac{t_h}{2}\right) = 14 \text{ to } 21$$

$\boxed{t_R = 18 \text{ mm}}$ $\rightarrow$ thickness of ribs

Cup:

$$l/D = \frac{300}{250} = 1.2 \rightarrow l/D \leq 1.5 \text{. Hence a cup is required}$$

Radius of cup $= 0.7D = 0.7(250) = 175 \text{mm}$.

Problem: The following data is given for the piston of a four-stroke diesel engine:

Cylinder bore = 250 mm

Maximum gas pressure = 4 MPa

Allowable bearing pressure for skirt = 0.4 MPa

Ratio of side thrust on liner to maximum gas load on piston = 0.1

Width of top land = 45 mm (h_1)

Width of ring grooves = 6 mm

Total number of piston rings = 4

Axial thickness of piston rings = 7 mm

Calculate:

(i) length of the skirt; and

(ii) length of the piston.

Given data:

$D = 250mm$; $P_{max} = 4\ N/mm^2$; $\mu = 0.1$

Side thrust = $\mu \times$ Force on piston

$$\mu = \frac{\text{Side thrust}}{\text{force on piston}} = 0.1 \ ; \ P_b = 0.4\ N/mm^2$$

$h_1 = 45mm$; $h_2 = 6$ mm ; $z = 4$; $h = 7mm$

Length of Skirt

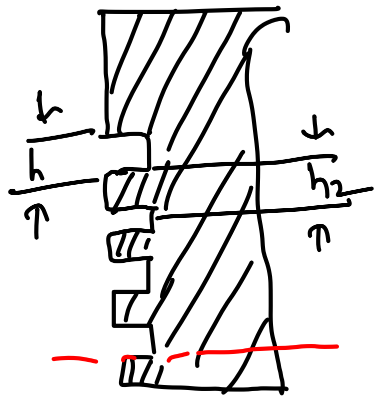
$$\mu\left(\frac{\pi}{4} D^2\right) P_{max} = (P_b)\, \cancel{D}\, l_s$$

$$l_s = \frac{\mu\left(\frac{\pi}{4} D\right) P_{max}}{P_b}$$

$$l_s = \frac{0.1 \times \frac{\pi}{\cancel{4}} \times 250 \times \cancel{4}}{0.4}$$

$$l_s = 196.34 \simeq 197 \text{ mm}$$

$h \rightarrow$ axial thickness of ring



Length of ring section

$= 4h + 3h_2$

$= (4 \times 7) + 3(6)$

$l_r = 28 + 18 = 46 \text{mm}$

Total length of the piston:

$L =$ topland + length of ring section + length of piston skirt

$$L = h_1 + l_r + l_s$$

$$L = 288 \text{mm}.$$

$L \rightarrow D$ to $1.5D$

250 mm to 375 mm

So our design(on) values are correct

Problem: The following data is given for the piston of a four-stroke diesel engine:

Cylinder bore = 250 mm

Maximum gas pressure = 4 MPa

Bearing pressure at small end of connecting rod = 15 MPa

Length of piston pin in bush of small end = 0.45D

Ratio of inner to outer diameter of piston pin = 0.6

Mean diameter of piston boss = 1.4 * outer diameter of piston pin

Allowable bending stress for piston pin = 84 N/mm^2

Calculate:

(i) outer diameter of the piston pin;

(ii) inner diameter of the piston pin;

(iii) mean diameter of the piston boss; and

(iv) Check the design for bending stresses.

Given data:

$D = 250$ mm; $P_{max} = 4$ N/mm^2; $(P_b)_1 = 15$ mPa

$l_1 = 0.45D$; $d_i = 0.6 d_o$; $\sigma_b = 84$ N/mm^2

i) Max force acting on piston:

$$F_{max} = P_{max}\left(\frac{\pi D^2}{4}\right) = \cancel{4} \times \left(\frac{\pi \times (250)^2}{\cancel{4}}\right)$$

$$F_{max} = 196349.5 \text{ N}$$

Equating this value to force transmitted by pin to conn. rod

$$= (P_b)_1 \times d_o \times l_1$$

$(P_b)_1 \times d_o \times l_1 = 196349.5$ $\quad [l_1 = 0.45D]$

$15 \times d_o \times 0.45 \times 250 = 196349.5$

$d_o = 116.4\,mm$ (or) $\underline{118\,mm}$

(ii) <u>Inner dia of piston pin:</u>

$\underline{d_i = 0.6 \times 118 \simeq 70\,mm}$

(iii) <u>Mean dia of piston boss:</u>

$= 1.4\,d_o = 1.4 \times 116.4 \simeq \underline{163\,mm}$

(iv) <u>checking for bending:</u>

$$M_b = \frac{PD}{8} = \frac{(196349.5)(250)}{8}$$

$y = \frac{d_o}{2}$

$M_b = 6135921.8\ N\text{-}mm$

$$\sigma_b = \frac{M_b\,y}{I} = \frac{61359 \times \cancel{10^3} \times \frac{118}{2}}{8338 \times \cancel{10^3}}$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}\left((118)^4 - (70)^4\right) = 8338 \times 10^3\,mm^4$$

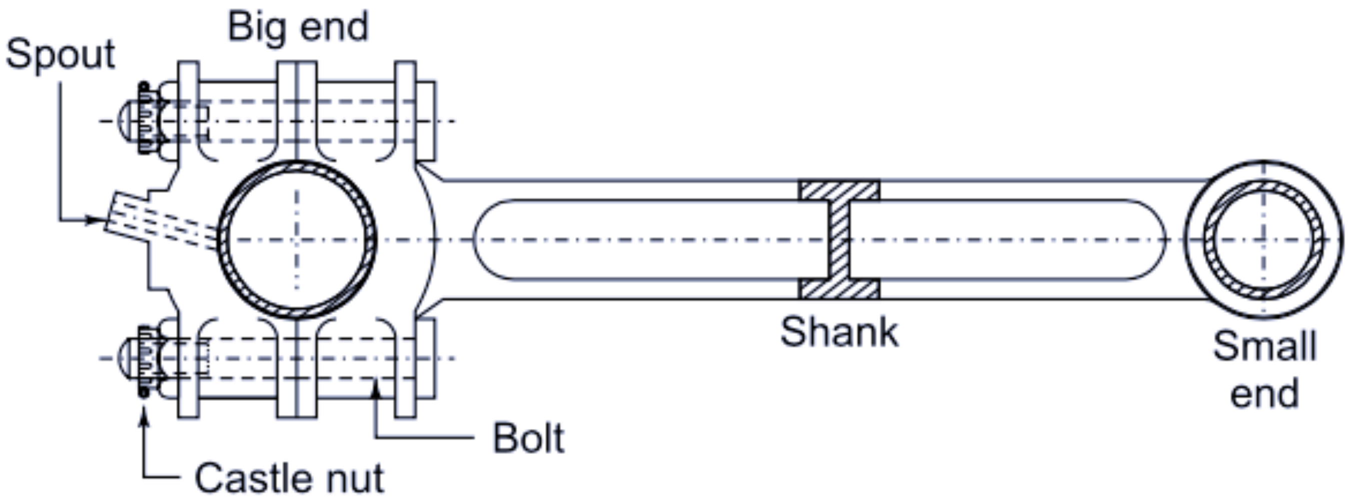
$\sigma_b = 43.41\ N/mm^2$

→ stress induced

$(\sigma_b) < (\sigma_b)_{max}$ → Hence the design & values are safe

CONNECTING ROD

The connecting rod consists of an eye at the small end to accommodate the piston pin, a long shank and a big end opening split into two parts to accommodate the crank pin. The construction of connecting rod is illustrated in Fig. **The basic function of the connecting rod is to transmit the push and pull forces from the piston pin to the crank pin**. The connecting rod transmits the reciprocating motion of the piston to the rotary motion of the crankshaft. **It also transfers lubricating oil from the crank pin to the piston pin and provides a splash or jet of oil to the piston assembly**.



The connecting rod of an IC engine is made by the drop forging process and the outer surfaces are left unfinished. Most internal combustion engines have a conventional two-piece connecting rod. The whole rod is forged in one piece; the bearing cap is cut off, faced and bolted in place for final machining of the big end. The small end of the rod is generally made as a solid eye and then machined.

The connecting rod is subjected to the force of gas pressure and the inertia force of the reciprocating part. It is one of the most heavily stressed parts of the IC engine. The materials used for the connecting rod are either medium carbon steels or alloy steels. The medium carbon steels contain 0.35 to 0.45 per cent carbon. The alloy steels include nickel chromium or chromium molybdenum steels. Medium carbon steels are used for the connecting rods of industrial engines. Alloy steels are used for connecting rods of automobile and aero engines.

There are two methods of lubrication of bearings at the two ends—splash lubrication and pressure feed lubrication.

In *splash lubrication*, a spout is attached to the big end of the connecting rod and set at an angle to the axis of the rod. The spout dips into the sump of lubricating oil during the downward motion of the connecting rod and splashes the oil as the connecting rod moves up. The splashed up oil finds its way into the small end bearing.

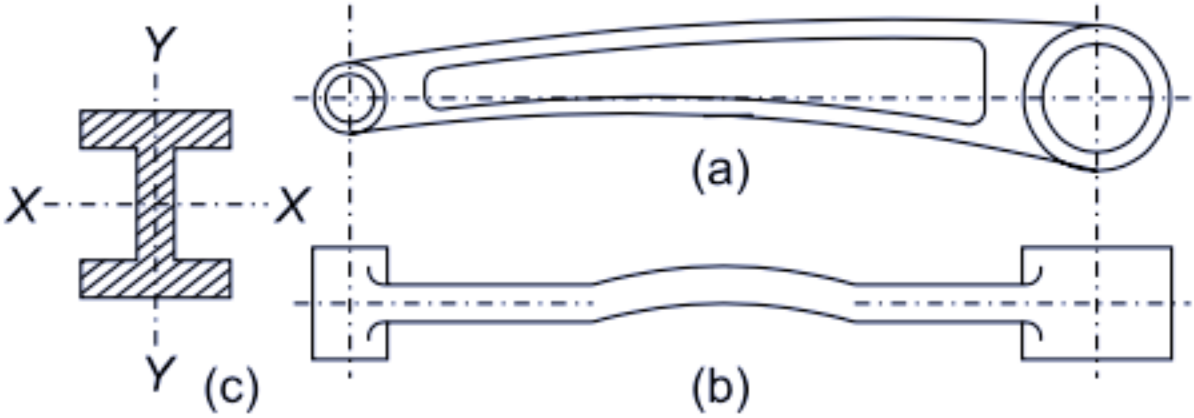
In the *pressure feed system*, oil is fed under pressure to the crank pin bearing through the holes drilled in the crankshaft. From the crank pin bearing, the oil is fed to the small end bearing through the hole drilled in the shank of the connecting rod.

The length of the connecting rod is an important consideration. When the connecting rod is short as compared to the crank radius, it has greater angular swing, resulting in greater side thrust on the piston. In high-speed engines, the ratio of the length of the connecting rod to the crank radius (L/r) is generally 4 or less. In low-speed engines, the (L/r) ratio varies from 4 to 5.

Most of the connecting rods in high-speed engines have an I-section. It reduces the weight and inertia forces. It is also easy for forging. In low-speed engines, circular cross-section is used.

BUCKLING OF CONNECTING ROD

The connecting rod is a slender engine component that has considerable length in proportion to its width and breadth. It is subjected to axial compressive force equal to maximum gas load on the piston. The compressive stress is of significant magnitude. Therefore, the connecting rod is designed as a column or a strut. The buckling of the connecting rod in two different planes—plane of motion and a plane perpendicular to the plane of motion



1. The buckling of the connecting rod in the plane of motion is shown in Fig. (a). In this plane, the ends of connecting rod are hinged in the crank pin and piston pin. Therefore, for buckling about the *XX*-axis, the end fixity coefficient (*n*) is one.
2. The buckling of the connecting rod in a plane perpendicular to the plane of motion is shown in Fig. (b). In this plane, the ends of the connecting rod are fixed due to constraining effect of bearings at the crank pin and piston pin. Therefore, for buckling about the *YY*-axis, the end fixity coefficient (*n*) is four.
3. (iii) Therefore, the connecting rod is four times stronger for buckling about the *YY*-axis as compared to buckling about the *XX*-axis.
4. (iv) If a connecting rod is designed in such a way that it is equally resistant to buckling in either plane then

$$I = mk^2$$

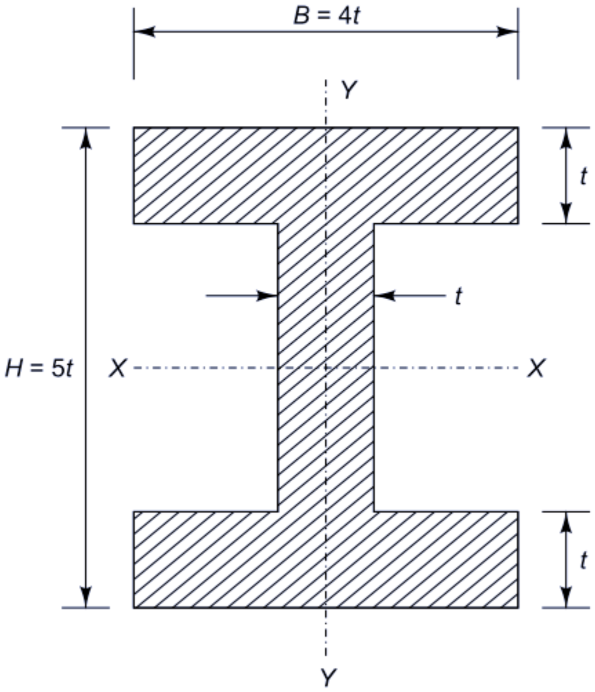
$$4\, I_{YY} = I_{XX}$$

$$4\,(\cancel{m}\, k_{YY}^2) = \cancel{m}\, k_{XY}^2$$

$$4\, K_{YY}^2 = K_{XY}^2 \Rightarrow k_{YY}^2 = \frac{1}{4} K_{XY}^2$$

I-Section

5. (v) The above relationship proves that I-section is ideally suitable for the connecting rod. On the other hand, a circular cross-section is unnecessarily strong for buckling about the *YY*-axis.



Area of cross-section

$$= 2(4t \times t) + (5t - 2t) \times t$$

$$= 11t^2$$

$$I_{xx} = \frac{1}{12}(4t)(5t)^3$$

$$- 2 \times I_{openspace}$$

$$I_{openspace} = \frac{1}{12}\left(\frac{4t - t}{2}\right)(5t - 2t)^3$$

$$= \frac{1}{12}\left(\frac{3t}{2}\right)(27t^4)$$

$$I_{xx} = \frac{1}{12}(500t^4) - \frac{1}{12}\left(\frac{2 \times 3t}{2}\right)(27t^4)$$

$$= \frac{1}{12}\left[500t^4 - 81t^4\right]$$

$$\boxed{I_{xx} = \left(\frac{419}{12}\right)t^4}$$

$$I = mk^2$$

$$I = Ak^2$$

$$K_{xx}^2 = \frac{I_{xx}}{A} = \frac{419}{12}t^4 \times \frac{1}{11t^2} = 3.17t^2$$

$$K_{xx} = 1.78t$$

$$I_{yy} = 2\left[\frac{1}{12}(t)(4t)^3\right] + \frac{1}{12}(5t-2t)(t)^3$$

$$\boxed{I_{yy} = \frac{131}{12}t^4}$$

$$K_{yy}^2 = \frac{131t^4}{12} \times \frac{1}{11t^2} = 0.992t^2$$

$$\frac{I_{xx}}{I_{yy}} = \frac{\left(\frac{419}{\cancel{12}}\right)\cancel{t^4}}{\left(\frac{131}{\cancel{12}}\right)\cancel{t^4}} = \frac{419}{131} = 3.2$$

$$\boxed{I_{xx} = 3.2 I_{yy}} \simeq \boxed{I_{xx} = 4 I_{yy}}$$

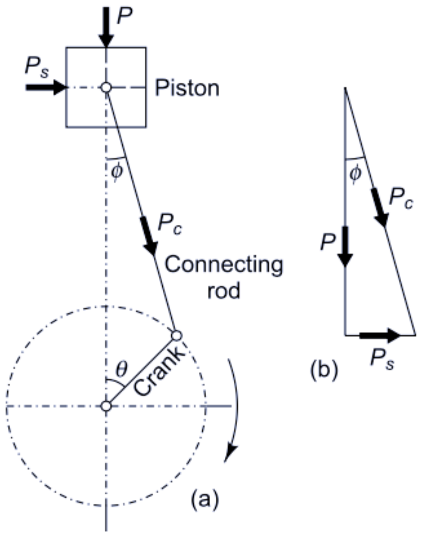
Hence I-section is most preferred for Connecting Rod.

CROSS-SECTION FOR CONNECTING ROD

P = force acting on the piston due to gas pressure (N)

P_S= side thrust on the cylinder wall (N)

P_C= force acting on the connecting rod (N)



$$P = P_c \cos\phi$$

$$P_c = \frac{P}{\cos\phi} \quad -(1)$$

Max gas load occurs shortly after TDC.

$\Rightarrow \phi = 3.3^\circ$

φ= angle of inclination of connecting rod with line of stroke

θ= angle of inclination of crank from top dead centre position

$\cos\phi = \cos 3.3 = 0.9983 \simeq 1$

$$\boxed{P_c = P}$$

$$P_c = \left(\frac{\pi D^2}{4}\right) p_{max}$$

From last section, it was fixed that I-section is gonna be used as conn. rod.

$$A = 11t^2$$

$$k_{xx} = 1.78t$$

According to Rankine's formula for buckling load

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

$P_{cr} \rightarrow$ critical buckling load

$\sigma_c \rightarrow$ comp. yield stress

$A \rightarrow$ cross-section area

$a \rightarrow$ const depending upon material & end fixity

$L \rightarrow$ Length of conn. rod

$\sigma_c \rightarrow$ carbon steel

$= 330 \ N/mm^2$

In plane of motion, both ends are hinged

$L_{eq} = L$

For steel $a = \frac{1}{7500}$

Critical buckling load

$P_{cr} = P_c(\text{Fos})$

Fos = 5 to 6

1) Calc force on conn. rod (P_c)

2) Calc critical buckling load ($P_{cr} = P_c(fs)$)

3) using Rankine formula, find thickness

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

4) Find B & H as from figure

5) B is kept constant

6) Middle Section $H = 5t$; small end (H_1) $= (0.75 - 0.9)H$

Big end (H_2) $= (1.1 - 1.25)H$

Determine the dimensions of cross-section of the connecting rod for a diesel engine with the following data:

Cylinder bore = 100 mm

Length of connecting rod = 350 mm

Maximum gas pressure = 4 MPa

Factor of safety = 6

Given data:

$D = 100\,mm$, $L = 350\,mm$; $P_{max} = 4\,N/mm^2$

$FOS = 6$

1) Force (or) Load on Conn. Rod:

$$P_c = \left(\frac{\pi}{4} D^2\right) P_{max} = \frac{\pi}{4} \times (100)^2 \times 4 = 31,415.9\,N$$

2) Critical bucking load

$$P_{cr} = P_c\,(fos) = 188495.5\,N$$

3) Calculation of t:

$$A = 11t^2;\quad K_{xx} = 1.78t;\quad a = \frac{1}{7500}$$

$$\sigma_c = 330\,N/mm^2$$

$$P_{cr} = \frac{\sigma_c (A)}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$$

$$\boxed{t = 7.63 \text{ (or) } 8\,mm}$$

4) Dimension of Cross-Section:

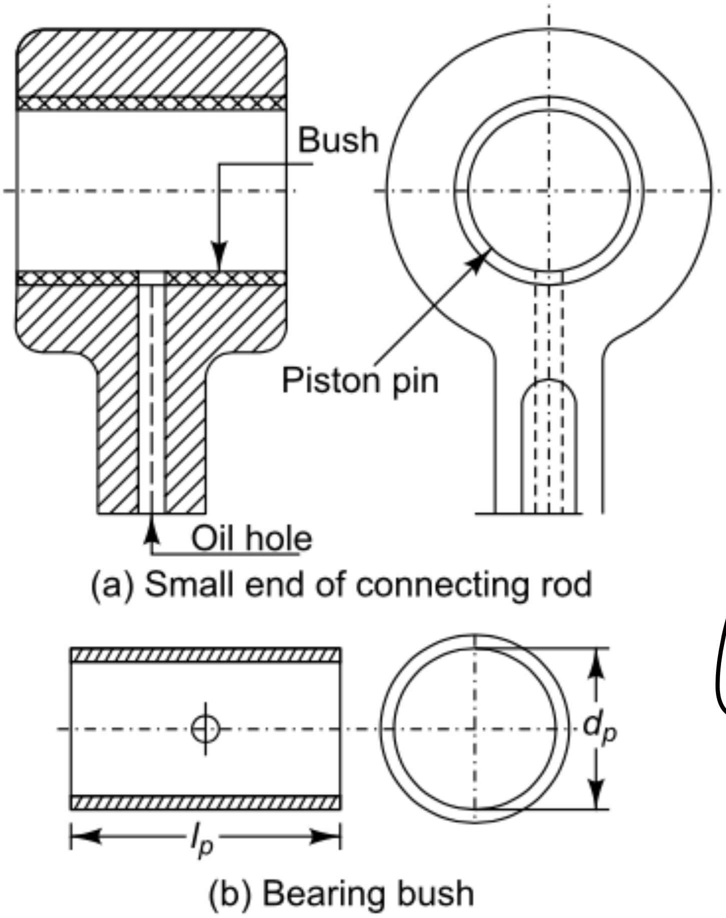
$B = 4t = 32\ mm$

$H = 5t = 40\ mm$

thickness of web $= t = 8\ mm$

thickness of flange $= t = 8\ mm$ //

BIG AND SMALL END BEARINGS



$$P = \left(\frac{\pi}{4} D^2\right)(p_{max})$$

$$P_c = d_p l_p (p_b)_p$$

$$P_c = P = \left(\frac{\pi}{4} D^2\right)(p_{max})$$

$$\left(\frac{l_p}{d_p}\right) = 1.5 \text{ to } 2$$

$$(p_b)_p = 10 \text{ to } 12.5 \text{ MPa}$$

Fig. 25.16 *Small End of Connecting Rod*

where,

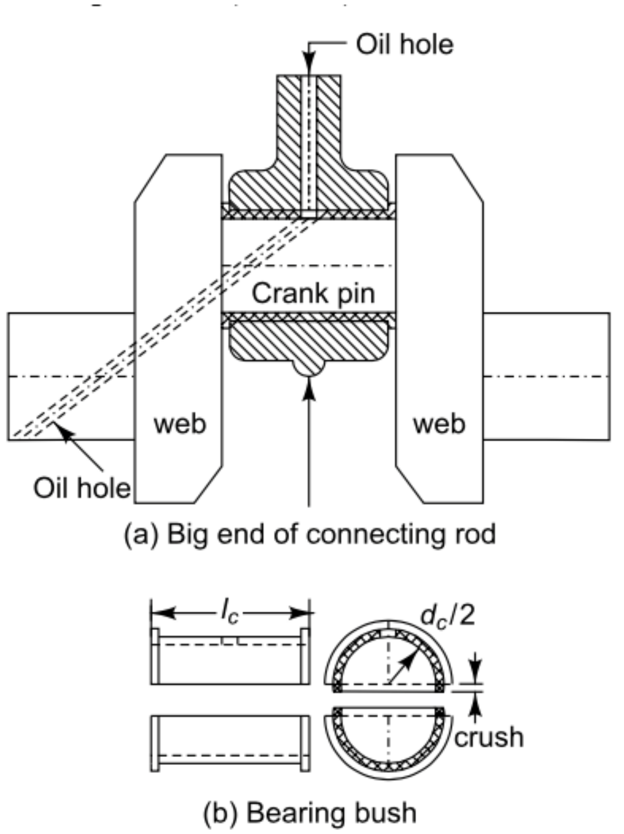
d_p = diameter of the piston pin or inner diameter of the bush on the piston pin (mm)

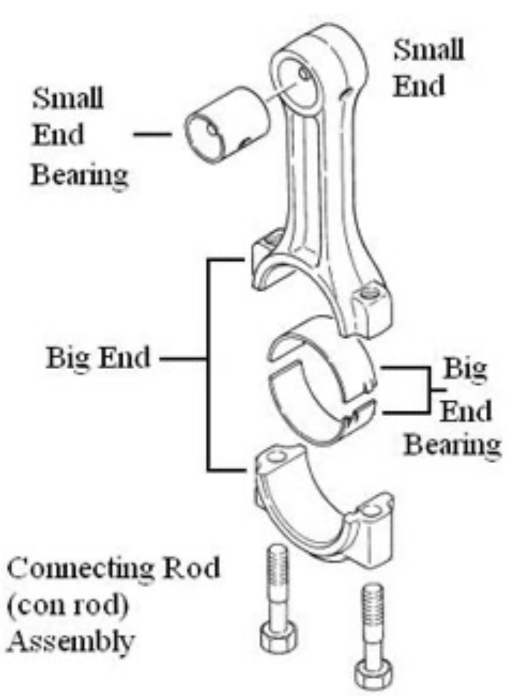
l_p = length of the piston pin or length of the bush on the piston pin (mm)

$(p_b)_p$ = allowable bearing pressure for the piston pin bush (N/mm^2)

The allowable bearing pressure for the piston pin bush is usually taken from 10 to 12.5 MPa.

The (l/d) ratio for the piston pin bush is taken from 1.5 to 2.





$$P_c = d_c \, l_c \, (P_b)_c$$

Fig. 25.17 *Big End of Connecting Rod*

The allowable bearing pressure for the crank pin bush is usually taken from 5 to 10 N/mm2. The (l/d) ratio for the crank pin bush is taken from 1.25 to 1.5.

There is a peculiar term, **'crush'**, related to big end bearings. In order to make good seating of bearing bushes in the cap and connecting rod, the sleeve height is slightly more (approximately 0.05 mm) than the half bore of the housing in which it fits. This is called bearing 'crush' .When the cap is tightened by bolts, the projecting bearing faces are squeezed in (or crushed) to form a press fit between the split bushes and cap and the big end of the connecting rod.

There is one more term called **'shim'**. The wear of the big end bearing is compensated by means of thin metallic strips between the cap and the fixed half. As wear takes place, one or more strips are removed and the cap is tightened. These strips are called shims.

$$(l_c/d_c) = 1.25 \text{ to } 1.5 .$$

Problem: Determine the dimensions of small and big end bearings of the connecting rod for a diesel engine with the following data:

Cylinder bore = 100 mm
Maximum gas pressure = 4 MPa
(l/d) ratio for piston pin bearing = 2
(l/d) ratio for crank pin bearing = 1.3
Allowable bearing pressure for piston pin bearing = 12 MPa

Allowable bearing pressure for crank pin bearing = 7.5 MPa

Given data:

$D = 100\,mm; \quad P_{max} = 4\,mpa$

$\left(l_p/d_p\right) = 2; \quad \left(l_c/d_c\right) = 1.3; \quad (P_b)_p = 12\,mpa$

$(P_b)_c = 7.5\,mpa$

1) Max bearing load:

$$P_c = \left(\frac{\pi}{4}\right) D^2 P_{max} = \frac{\pi}{\cancel{4}} \times (100)^2 \times \cancel{4} = 31.4 \times 10^3\,N$$

2) Piston pin bearing:

$$P_c = d_p l_p (P_b)_p \Rightarrow (d_p)(2d_p) \times 12 = 31.4 \times 10^3\,N$$

$$d_p = 36.3\,mm \quad (or) \quad d_p = 38\,mm$$

$$l_p = 2d_p = 76\,mm;$$

3) Crank pin bearing:

$$P_c = d_c l_c (P_b)_c \Rightarrow 31.4 \times 10^3 = d_c \times (1.3 d_c) \times 7.5$$

$$\underline{d_c = 58\,mm \;\&\; l_c = 76\,mm}$$

BIG END CAP AND BOLTS

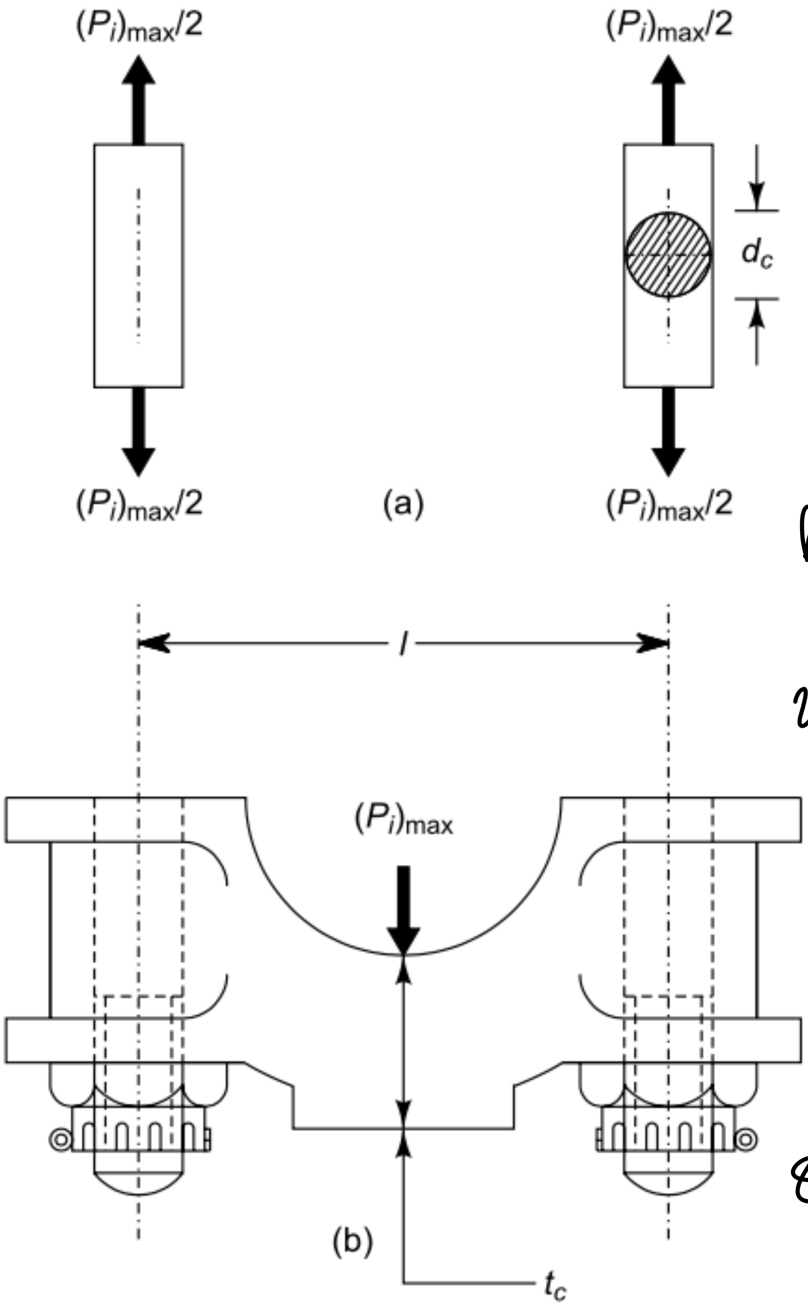


Fig. 25.18 *Forces on cap and bolts*

Max load on bolts will be inertia force when piston is @ TDC during exhaust stroke.

$$P_i = m_r \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n_1}\right]$$

$P_i \rightarrow$ inertia force on cap (or) bolts.

$\omega \rightarrow$ ang vel of engine

$m_r \rightarrow$ mass of reciprocating parts

$r \rightarrow$ radius of crank

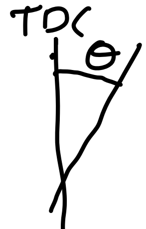
$n_1 \rightarrow \left(L/r\right) \rightarrow L \rightarrow$ length of conn. rod

$\theta \rightarrow$ angular position of crank from TDC.

$m = \left(m + \frac{m_s}{3}\right)$

$m_r = \left(\text{mass of piston assembly}\right) + \left(\frac{1}{3}\text{rd mass of conn. rod}\right)$

$r = l/2$ $l \rightarrow$ length of stroke



@ TDC, where inertia force is max

$\theta = 0 \Rightarrow \cos\theta = 1;\ \cos 2\theta = 1$

$$(P_i)_{max} = m_r \omega^2 r \left[1 + \frac{1}{n_1}\right]$$

$$(P_i)_{max} = 2\left(\frac{\pi d_c^2}{4}\right)\sigma_t$$

$d_c \rightarrow$ core dia of bolts

$\sigma_t \rightarrow$ Permissible tensile Stress for bolts

nominal dia $d = \left(\frac{d_c}{0.8}\right)$

Bending moment $= \frac{wl}{6}$

$$m_b = \frac{(P_i)_{max}\, l}{6}$$

$m_b \rightarrow$ Bending moment on cap

$l \rightarrow$ dist b/w bolt centres

$l = \left(\text{dia of crankpin}\right) + 2\left[\text{thickness of bush (3mm)}\right] + \text{nominal dia of bolt}$

$+ \text{clearance (3mm)}$

$$\sigma_b = \frac{m_b y}{I} \qquad I = \left(\frac{b_c (t_c)^3}{12}\right) \; ; \; y = \frac{t_c}{2}.$$

$b_c \rightarrow$ width of cap $= l_c$

Problem: The following data is given for the cap and bolts of the big end of connecting rod:

Engine speed = 1800 rpm

Length of connecting rod = 350 mm

Length of stroke = 175 mm

Mass of reciprocating parts = 2.5 kg

Length of crank pin = 76 mm

Diameter of crank pin = 58 mm

Thickness of bearing bush = 3 mm

Permissible tensile stress for bolts = 60 N/mm^2

Permissible bending stress for cap = 80 N/mm^2

Calculate the nominal diameter of bolts and thickness of cap for the big end.

Given data:

$N = 1800$ RPM; $L = 350$ mm; $l = 175$ mm;

$m_r = 2.5$ Kg; $l_c = 76$ mm, $d_c = 58$ mm, $\sigma_t = 60$ N/mm^2

$\sigma_b = 80$ N/mm^2

1) Inertia force:

$$(P_1)_{max} = m_r \omega^2 r \left[1 + \frac{1}{n_1}\right] \qquad r = \frac{l}{2} = \frac{175}{2}$$

$$n_1 = \left(\frac{L}{r}\right) = \frac{350}{175/2}$$

$$(P_1)_{max} = (2.5)(188.5)(r)\left[1 + \frac{1}{n_1}\right] \qquad \omega = \frac{2\pi N}{60} = 188.5 \text{ rad/s}$$

$$= 9715.8 \text{ N}$$

2) Dia of bolts:

$$(P_1)_{max} = 2 \times \left(\frac{\pi d_c^2}{4}\right) \sigma_t$$

$$d_c = 10.15 \text{ mm}$$

Nominal dia $(d) = \frac{d_c}{0.8} = 12.69$ mm either 14 mm (or) 16 mm $\rightarrow$ M16.

3) Thickness of cup:

$$\sigma_b = \frac{M_b\, y}{I} \qquad I = \frac{b_c (t_c)^3}{12}, \quad y = t_c/2$$

$$M_b = \frac{(P_1)_{max}\, l}{6} = \frac{(9715.8)(83)}{6} \qquad b_c = l_c = 76 \text{ mm}$$

$$l = (58) + 2(3) + 16 + 3$$

$l = 83$ mm $\rightarrow$ dist b/n bolt centres.

$$80 = \frac{(134{,}402.6)(t_c/2)}{(6.33\, t_c^3)}$$

$t_c = 11.5$ (or)

$$\boxed{t_c = 12 \text{ mm}}$$

WHIPPING STRESS

The small end of the connecting rod is subjected to pure translation motion while the big end is subjected to pure rotary motion. The intermediate points on the connecting rod move in elliptical orbits. The lateral oscillations of the connecting rod induce inertia forces that act all along the length of the connecting rod causing bending. This type of action is called '*whipping*'. The bending stress due to inertia force is called '*whipping stress*'.

$m_1 \rightarrow$ mass of conn-rod per unit length

$m_1 = \text{vol} \times \text{density}$

$\quad = \text{Area} \times \cancel{\text{length}} \times \rho$ $\quad 1$

$\rho \rightarrow 7800\ \text{kg/m}^3 \rightarrow$ Steel

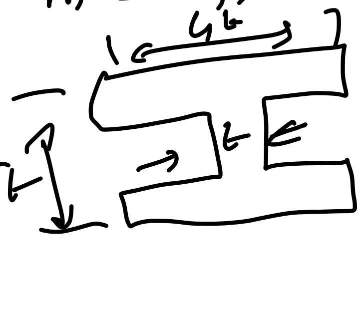
$m_1 = A\rho$

$A = 11t^2$

$m_1 = (11t^2\rho)\ \text{kg/m}$

max B.M will occur @ a dist of $l/\sqrt{3}$ from piston pin.

$$(M_b)_{max} = m\omega^2 r\left(\frac{l}{9\sqrt{3}}\right) \qquad m = m_1 \times l$$



$$(M_b)_{max} = m_1\omega^2 r\left[\frac{l^2}{9\sqrt{3}}\right]$$

$$\sigma_b = \frac{(M_b)_{max}\, y}{I} \qquad I = \left(\frac{419}{12}\right)t^4 ; \left(y = \frac{5t}{2}\right)$$

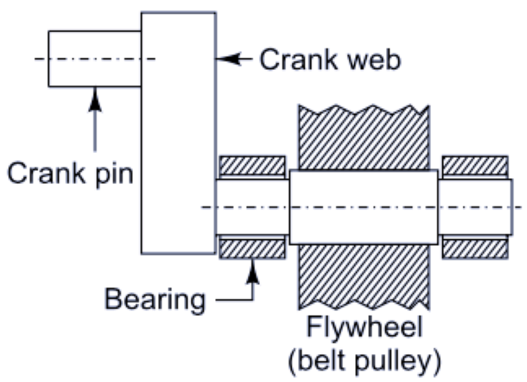
CRANKSHAFT

The crankshaft is an important part of IC engine that converts the reciprocating motion of the piston into rotary motion through the connecting rod. The crankshaft consists of three portions—crank pin, crank web and shaft. The big end of the connecting rod is attached to the *crank pin*. The *crank web* connects the crank pin to the shaft portion. The shaft portion rotates in the main bearings and transmits power to the outside source through the belt drive, gear drive or chain drive.

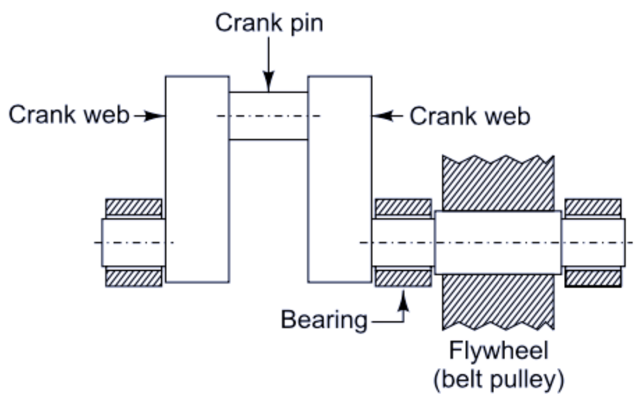
There are two types of crankshafts—*side crankshaft* and *centre crankshaft* as shown in Fig. The side crankshaft is also called the '*overhung*' crankshaft. It has only one crank web and requires only two bearings for support. It is used in medium-size engines and large-size horizontal engines.

The centre crankshaft has two webs and three bearings for support. It is used in radial aircraft engines, stationary engines and marine engines. It is more popular in automotive engines. Crankshafts are also classified as single- throw and multi-throw crankshafts depending upon the number of crank pins used in the assembly. The crankshafts illustrated in Fig have one crank pin and are called single-throw crankshafts. Crankshafts used in multi-cylinder engines have more than one crank pin. They are called multi-throw crankshafts.

A crankshaft should have sufficient strength to withstand the bending and twisting moments to which it is subjected. In addition, it should have sufficient rigidity to keep the lateral and angular deflections within permissible limits. The crankshaft is subjected to fluctuating stresses and, as such, it should have sufficient endurance limit stress. Crankshafts are made by the drop forging process.



(a) Side crankshaft



(b) Centre crankshaft

DESIGN OF CENTRE CRANKSHAFT

A crankshaft is subjected to bending and torsional moments due to the following three forces:

(i) Force exerted by the connecting rod on the crank pin

(ii) Weight of flywheel (*W*) acting downward in the vertical direction

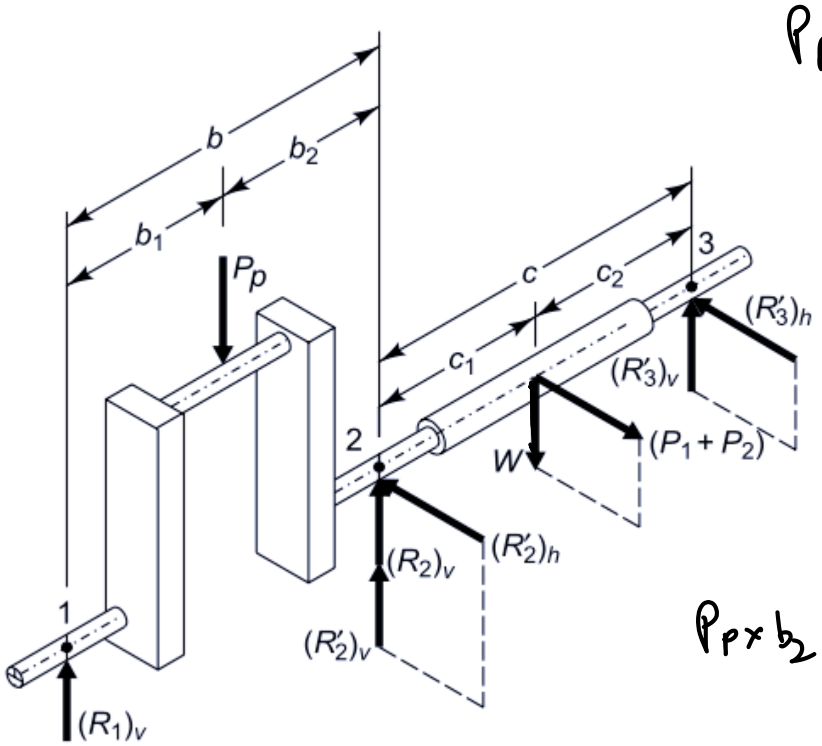
(iii) Resultant belt tensions acting in the horizontal direction ($P1 + P2$)

In the design of the centre crankshaft, two cases of crank positions are considered. They are as follows:

Case I The crank is at the top dead centre position and subjected to maximum bending moment and no torsional moment.

Case II The crank is at an angle with the line of dead centre positions and subjected to maximum torsional moment.

CENTRE CRANKSHAFT AT TOP DEAD CENTRE POSITION



$$P_p = \left(\frac{\pi D^2}{4}\right) P_{max}$$

$$P_p \times b_1 = (R_2)_v \times b$$

$$(R_2)_v = \frac{P_p \times b_1}{b} \quad -(1)$$

$$(R_1)_v + (R_2)_v = P_p$$

(or)

$$P_p \times b_2 = (R_1)_v \times b$$

$$(R_1)_v = \frac{P_p \times b_2}{b} \quad -(2)$$

Assumptions

(i) The engine is vertical and the crank is at the top dead centre position.

(ii) The belt drive is horizontal.

(iii) The crankshaft is simply supported on bearings.

$$W \times C_1 = (R_3')_v \times C \Rightarrow (R_3')_v = \frac{W \times C_1}{C} \quad -(3)$$

$$W \times C_2 = (R_2')_v \times C \Rightarrow (R_2')_v = \frac{W \times C_2}{C} \quad -(4)$$

Considering horizontal forces:

$$(P_1+P_2)C_1 = (R_3')_h \times C \Rightarrow (R_3')_h = \frac{(P_1+P_2)C_1}{C} \quad -(5)$$

$$(P_1+P_2)C_2 = (R_2')_h \times C \Rightarrow (R_2')_h = \frac{(P_1+P_2)C_2}{C} \quad -(6)$$

Resultant Reaction:

$$R_1 = (R_1)_v$$

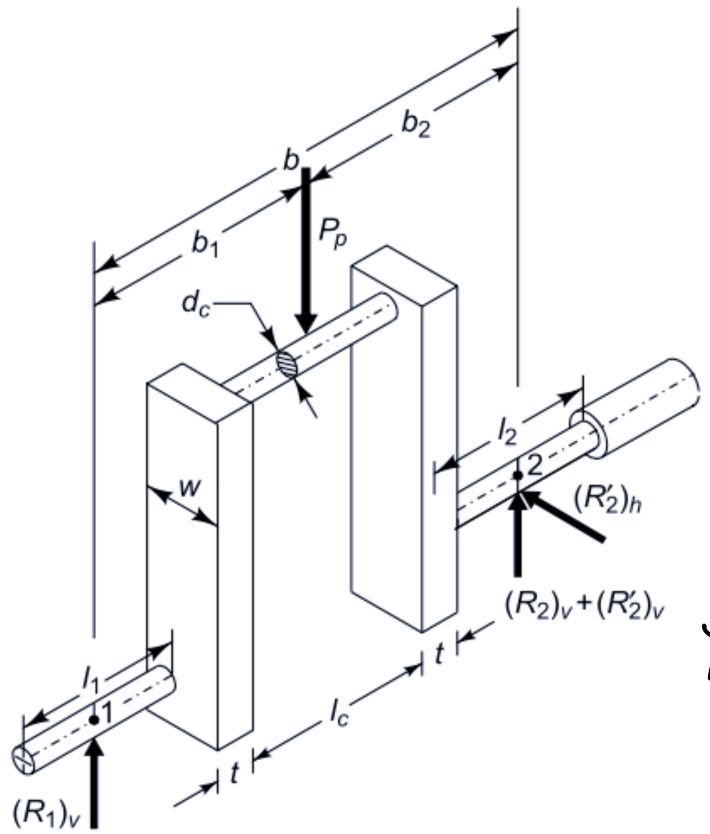
$$R_2 = \sqrt{\left[(R_2)_v + (R_2')_v\right]^2 + \left[(R_2')_h\right]^2}$$

$$R_3 = \sqrt{(R_3')_v^2 + (R_3')_h^2}$$

Empirical relationship:

$$b = 2D$$

Design of Crank Pin



$d_c \rightarrow$ dia of crankpin

$l_c \rightarrow$ length of crankpin

$\sigma_b \rightarrow$ allowable stress.

$$\boxed{\sigma_b = \frac{M_b \times y}{I}} \quad \text{— (a)}$$

$$(M_b)_c = (R_1)_v \times b_1 \quad \text{— (b)}$$

$$y = \frac{d_c}{2} \; ; \quad I = \frac{\pi}{64} d_c^4$$

from (a) $\Rightarrow$ $M_b = \frac{\sigma_b \times I}{y} = \frac{\sigma_b \times \frac{\pi}{64} d_c^4}{d_c/2}$

$$(M_b)_c = \left(\frac{\pi}{32}\right) \sigma_b d_c^3 \quad \text{— (c)}$$

Equating (b) & (c)

$$(R_1)_v \times b_1 = \frac{\pi}{32} \sigma_b d_c^3$$

$$\boxed{d_c = \left(\frac{32 (R_1)_v \times b_1}{\pi \sigma_b}\right)^{1/3}}$$

For length of crank pin :- (l_c)

$p_b \rightarrow$ allowable bearing pressure @ crank pin such

$$p_b = \frac{P_p}{d_c l_c} \Rightarrow \boxed{l_c = \frac{P_p}{p_b \times d_c}}$$

3) Design of Left hand & Right hand Crank web:

$w \rightarrow$ width of web

$t \rightarrow$ thickness of web

Empirical relationship:

$d_c \rightarrow$ dia of crank pin

$$t = 0.7 d_c$$

$$w = 1.14 d_c$$

Due to eccentric loading:

1) direct comp stress (σ_c)

2) Bending stress (σ_b)

$$\sigma_c = \frac{(R_1)_v}{wt}$$

$$m_b = (R_1)_v \times \text{distance}$$

Bending moment due to $(R_1)_v$ @ Centre plane of web)

$$m_b = (R_1)_v \times \left[b_1 - \frac{l_c}{2} - \frac{t}{2} \right]$$

$$\sigma_b = \frac{m_b \times y}{I} \qquad I = \frac{wt^3}{12} \qquad y = \frac{t}{2}$$

$$\sigma_b = \frac{6 (R_1)_v \left(b_1 - \frac{l_c}{2} - \frac{t}{2} \right)}{wt^2}$$

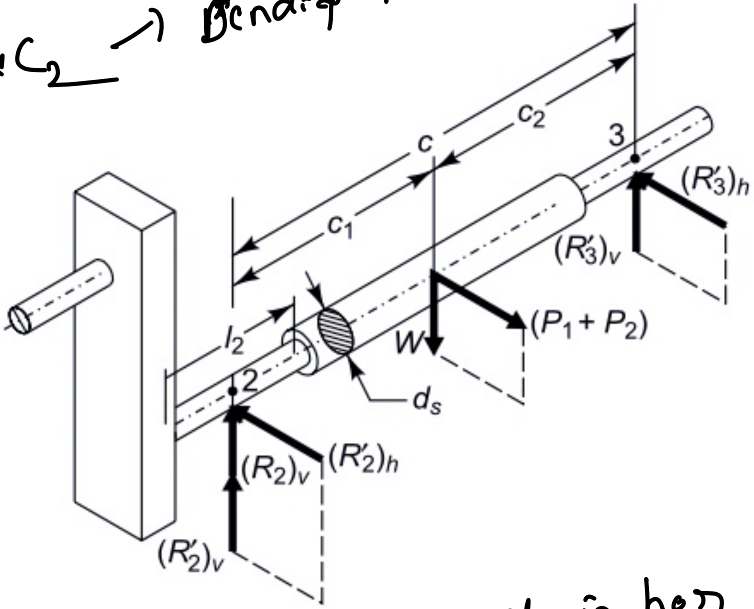
Right hand Crank web is identical to left hand Crank web.

$$\text{Total comp Stress} = (\sigma_c + \sigma_b)$$

5) Design of shaft under flywheel:

$d_s \rightarrow$ dia of shaft

$(M_b)_v = (R_3')_v \times c_2 \rightarrow$ Bending moment in vertical plane.



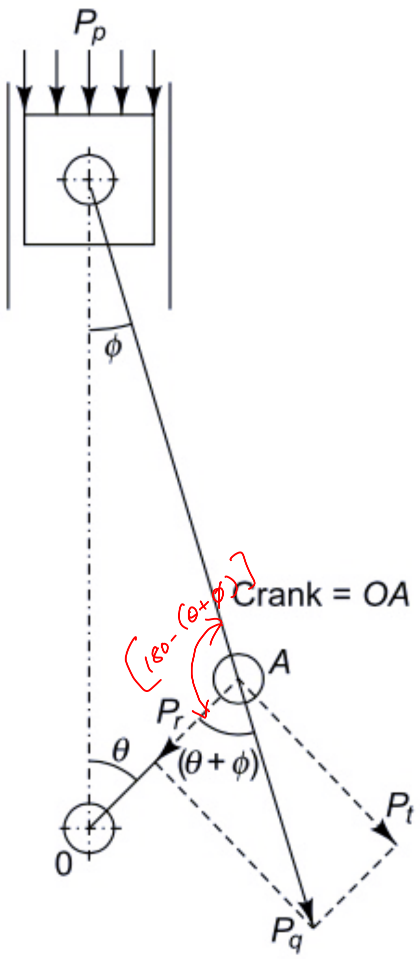
$(M_b)_h = (R_3')_h \times c_2 \rightarrow$ Bending moment in hori plane

$$M_b = \sqrt{(M_b)_h^2 + (M_b)_v^2}$$

$$= \sqrt{\left[(R_3')_h^2 + (R_3')_v^2\right] c_2^2}$$

$$\boxed{M_b = \frac{\pi}{32} \sigma_b d_s^3}$$

CENTRE CRANKSHAFT AT ANGLE OF MAXIMUM TORQUE



$\phi \rightarrow$ angle of Conn. rod with line of dead centres

$\theta \rightarrow$ angle of crank with line of dead centres

$P_q \rightarrow$ thrust on conn. rod

$P_t \rightarrow$ tangential component of force on crank pin

$P_p \rightarrow$ Gas force on piston

$$P_p = \left(\frac{\pi D^2}{4}\right) p'$$

$l/r \rightarrow$ Ratio of length of conn. rod to radius of crank.

Relation b/n θ & ϕ

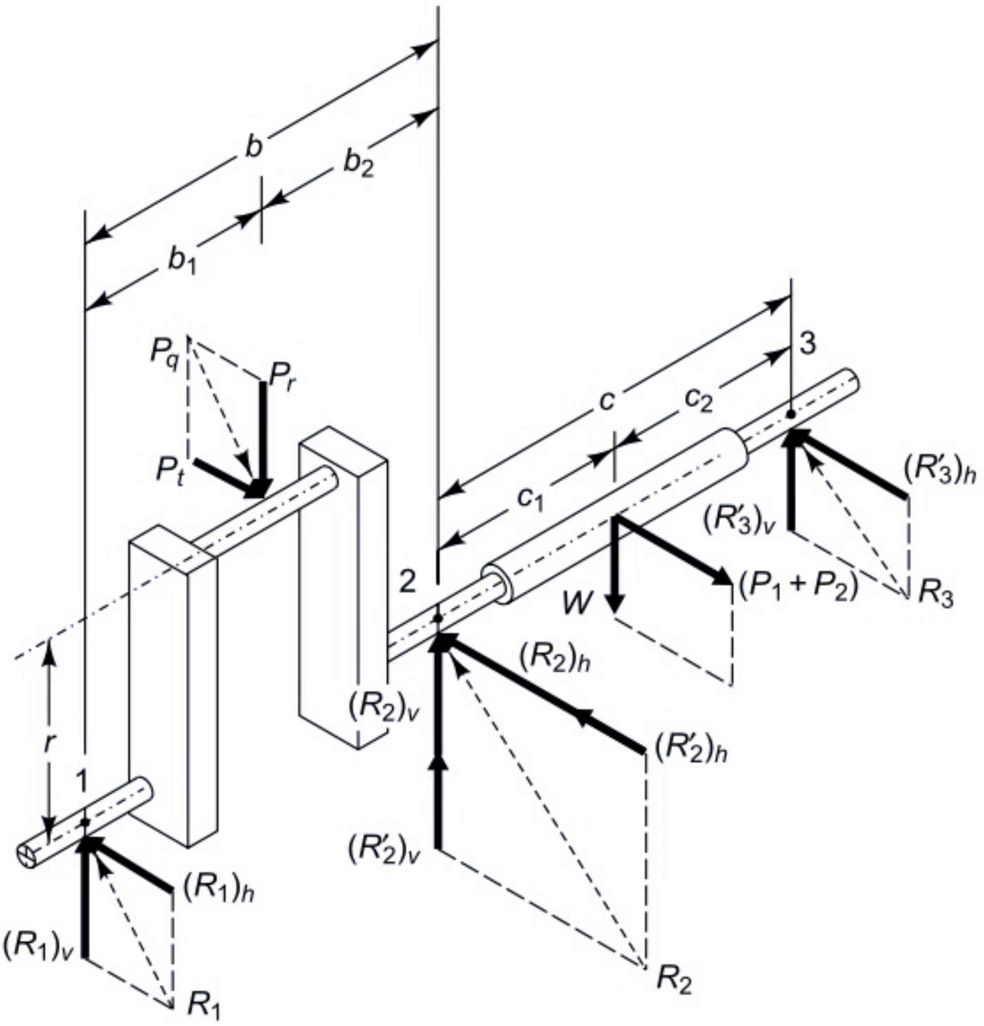
$$\sin\phi = \frac{\sin\theta}{(l/r)}$$

Thrust Component $\quad P_q = \dfrac{P_p}{\cos\phi}$

$$\sin(\theta+\phi) = \frac{P_t}{P_q}$$

$$\Rightarrow \boxed{\begin{aligned} P_t &= P_q \sin(\theta+\phi) \\ P_r &= P_q \cos(\theta+\phi) \end{aligned}}$$

Bearing Reactions:



$$P_t \times b_1 = (R_2)_h \times b \Rightarrow \boxed{(R_2)_h = \frac{P_t \times b_1}{b}}$$

$$P_t \times b_2 = (R_1)_h \times b \Rightarrow (R_1)_h = \frac{P_t \times b_2}{b}$$

$$(R_2)_v = \frac{P_r \times b_1}{b} \quad \& \quad (R_1)_v = \frac{P_r \times b_2}{b}$$

$$(R_3')_v = \frac{W \times c_1}{c} \quad \& \quad (R_2')_v = \frac{W \times c_2}{c}$$

$$(R_3')_h = \frac{(P_1+P_2)\,c_1}{c} \quad \& \quad (R_2')_h = \frac{(P_1+P_2) \times c_2}{c}$$

Resultant Reactions:

$$R_1 = \sqrt{(R_1)_v^2 + (R_1)_h^2}$$

$$R_2 = \sqrt{\left((R_2)_v + (R_2')_v\right)^2 + \left((R_2)_h + (R_2')_h\right)^2}$$

$$R_3 = \sqrt{(R_3')_v^2 + (R_3')_h^2}$$

2) Design of Crank Pin:

$$(M_b) = (R_1)_v \times b_1$$

$$(M_t) = (R_1)_h \times r$$

$$T_{eq} = \frac{\pi}{16} \tau d^3$$

$$T_{eq} = \sqrt{(M_b)^2 + (M_t)^2}$$

$$\tau = \frac{16}{\pi d^3} \sqrt{(M_b)^2 + (M_t)^2}$$

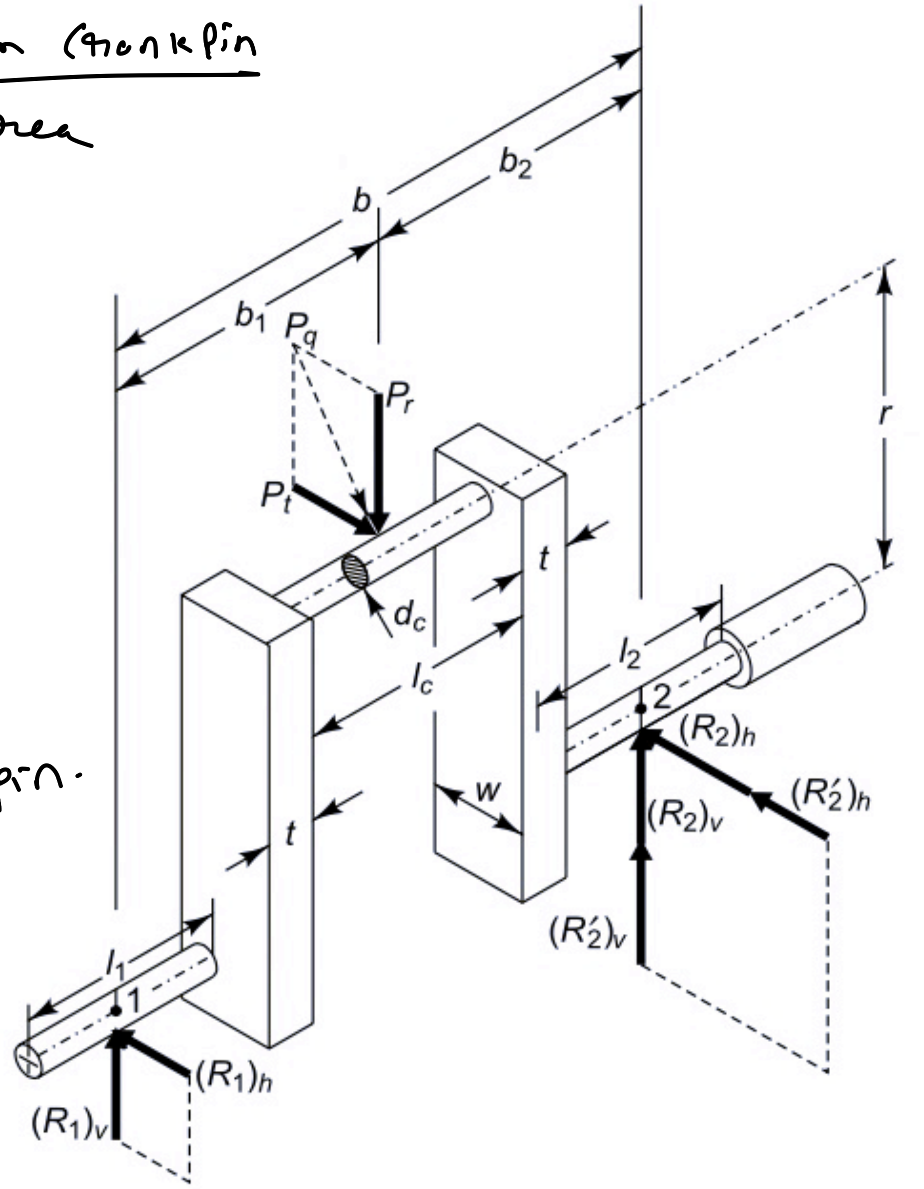
$$d_c^3 = \frac{16}{\pi \tau} \sqrt{(M_b)^2 + (M_t)^2}$$

$$P_b = \frac{\text{Force acting on crankpin}}{\text{pin area}}$$

$$P_b = \frac{P_q}{l_c \times d_c}$$

$$\boxed{l_c = \frac{P_q}{d_c \times P_b}}$$

length of crankpin.



3) Design of shaft under flywheel:

$d_s \rightarrow$ dia of shaft

$$(M_b)_s = (R_3) \times C_2$$

$$(M_t)_s = (P_t) \times r$$

$$d_s^3 = \frac{16}{\pi \tau} \sqrt{(M_b)_s^2 + (M_t)_s^2}$$

4) Shaft @ Juncture of Right-hand web:

d_{s_1} = dia of shaft @ Juncture of right hand web

$$(M_b)_v = (R_1)_v\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_r\left(\frac{l_c}{2} + \frac{t}{2}\right) \quad -(1)$$

$$(M_b)_h = (R_1)_h\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_t\left[\frac{l_c}{2} + \frac{t}{2}\right] \quad -(2)$$

$$(M_t)_{sJ} = P_t \times r$$

$$(M_b)_{sJ} = \sqrt{(M_b)_v^2 + (M_b)_h^2}$$

$$T_{eq} = \sqrt{M^2 + T^2}$$

$$d_{s_1}^3 = \frac{16}{\pi\tau}\sqrt{(M_b)_{sJ}^2 + (M_t)_{sJ}^2}$$

5) Design of Right hand web:

$$(M_b)_r = (R_2)_v\left[b_2 - \frac{l_c}{2} - \frac{t}{2}\right]$$

$$\frac{I}{y} = Z$$

$$\sigma_b = \frac{M_b \times y}{I} = \frac{M_b}{Z}$$

$$Z = \frac{\frac{wt^3}{12}}{t/2} = \frac{wt^2}{6}$$

$$(M_b)_r = (\sigma_b)_r \times Z$$

$$(M_b)_r = \frac{1}{6}(\sigma_b)_r \times w t^2$$

$$(M_b)_t = P_t\left[r - \frac{d_{s1}}{2}\right]$$

$$(M_b)_t = \frac{1}{6}(\sigma_b)_t \times t w^2$$

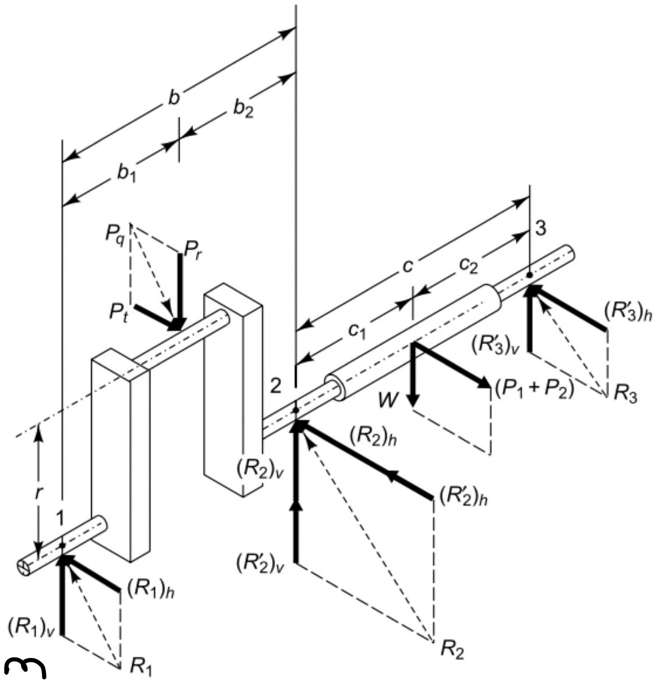
$$(\sigma_c)_d = \frac{P_r}{2wt}$$

Max comp stress

$$= (\sigma_b)_r + (\sigma_b)_t + (\sigma_c)_d$$

Design a centre crankshaft for a single-cylinder vertical engine using the following data:
Cylinder bore = 125 mm
(L/r) ratio = 4.5
Maximum gas pressure = 2.5 MPa
Length of stroke = 150 mm
Weight of flywheel cum belt pulley = 1 kN
Total belt pull = 2 kN
Width of hub for flywheel cum belt pulley= 200 mm

The torque on the crankshaft is maximum when the crank turns through 25° from the top dead centre and at this position the gas pressure inside the cylinder is 2 MPa. The belts are in the horizontal direction. Assume suitable data and state the assumptions you make.



Given data:

$D = 125\,mm$; $L/r = 4.5$;

$p_{max} = 2.5\,mpa$; $l = 150\,mm$

$W = 1000\,N$; $(P_1 + P_2) = 2000\,N$

$\theta = 25^\circ$

Crank radius $(r) = \frac{l}{2} = 75\,mm$

Case 1: max Bending moment Condition:

Thrust on conn. rod = Force acting on Piston

$$P_p = \left(\frac{\pi D^2}{4}\right) p_{max} = \left[\frac{\pi \times (125)^2}{4}\right] \times 2.5$$

$$P_p = 30.67 \times 10^3\ N$$