

OPERATIONS RESEARCH (20BSC243): 2 MARKS QUESTIONS
UNIT -1: LINEAR PROGRAMMING MODELS

1. Define Linear programming. May 2016

Linear Programming is a technique for the optimization of a linear objective function subjected to linear equality or inequality constraints.

2. Define feasible solution and optimum solution.
 - The solution which satisfies all the constraints is known as feasible solution.
 - The best feasible solution is called optimal solution.
3. **Slack variable:** it is the variable added to less than or equal to type constraint equation to convert it in to equality type equation.
4. **Surplus variable:** it is the variable used/added to greater than or equal to type constraint equation to convert it in to equality type equation.
5. **Interpretation of dual variable:** it represents the unit worth of a resource.
6. **Write the standard mathematical formulation to LPP. (May 2018)**

General form of linear programming problem:

If objective function is maximization

$$\begin{aligned} \text{Max } Z &= c_1x_1 + c_2x_2 + \dots + c_nx_n \\ \text{subjected to} \\ a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\ \dots & \\ \dots & \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m \\ x_1, x_2, \dots, x_n &\geq 0 \end{aligned}$$

If objective function is minimization

$$\begin{aligned} \text{Min } Z &= c_1x_1 + c_2x_2 + \dots + c_nx_n \\ \text{subjected to} \\ a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\geq b_2 \\ \dots & \\ \dots & \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\geq b_m \\ x_1, x_2, \dots, x_n &\geq 0 \end{aligned}$$

7. what are the characteristics or properties a liner programming problem should have? May 2017

All linear programming problems must have following five characteristics:

(a) Objective function:

There must be clearly defined objective which can be stated in quantitative way. In business problems the objective is generally profit maximization or cost minimization.

(b) Constraints:

All constraints (limitations) regarding resources should be fully spelt out in mathematical form.

(c) Non-negativity:

The value of variables must be zero or positive and not negative. For example, in the case of production, the manager can decide about any particular product number in positive or minimum zero, not the negative.

(d) Linearity:

The relationships between variables must be linear. Linear means proportional relationship between two 'or more variable, i.e., the degree of variables should be maximum one.

(e) Finiteness:

The number of inputs and outputs need to be finite. In the case of infinite factors, to compute feasible solution is not possible.

8. What are the assumptions in Linear programming problem? May 2016

Assumptions Linear Programming Problem (LPP): **A linear programming problem (LPP) or Linear Programming Model (LP model) is based the assumptions of proportionality, additivity, continuity, certainty and finite choices.**

Proportionality:

A basic assumption in LP model is that proportionality exists in the objective function and the constraint inequalities. [If number of products produced is increased then proportionally the profit (contribution increases in objective function also. Similarly, the time required to produce also increases according to increase in quantity in constraint equations].

Additivity:

Another assumption underlying in LP model is that in the objective function and the constraint inequalities both, the total of all the activities is given by the sum total of each activity conducted separately. This assumption implies that there is no interaction between the decision variables.

Continuity: The decision variables are continuous.

Certainty:

The objective function coefficients, the coefficients of the inequality/equality constraints and the constraint (resource) values are constant. i.e known with certainty.

Finite choice:

LP model assumes that a limited number of choices are available to a decision maker and the decision variables do not assume negative values.

9. List out Phases of OR. Dec 2017

Seven Steps or phases of OR .

An OR project can be split in the following seven steps:

Step 1: Formulate the problem

The OR analyst first defines the organization's problem. This includes specifying the organization's objectives and the parts of the organization (or system) that must be studied before the problem can be solved.

Step 2: Observe the system

Next, the OR analyst collects data to estimate the values of the parameters that affect the organization's problem. These estimates are used to develop (in Step 3) and to evaluate (in Step 4) a mathematical model of the organization's problem.

Step 3: Formulate a mathematical model of the problem

The OR analyst develops an idealized representation — i.e. a mathematical model — of the problem.

Step 4: Verify the model and use it for prediction

The OR analyst tries to determine if the mathematical model developed in Step 3 is an accurate representation of the reality. The verification typically includes observing the system to check if the parameters are correct. If the model does not represent the reality well enough then the OR analyst goes back either to Step 3 or Step 2.

Step 5: Select a suitable alternative

Given a model and a set of alternatives, the analyst now chooses the alternative that best meets the organization's objectives. Sometimes there are many best alternatives, in which case the OR analyst should present them all to the organization's decision-makers, or ask for

more objectives or restrictions.

Step 6: Present the results and conclusions

The OR analyst presents the model and recommendations from Step 5 to the organization's decision-makers. At this point the OR analyst may find that the decision makers do not approve of the recommendations. This may result from incorrect definition of the organization's problems or decision-makers may disagree with the parameters or the mathematical model. The OR analyst goes back to Step 1, Step 2, or Step 3, depending on where the disagreement lies.

Step 7: Implement and evaluate recommendation

Finally, when the organization has accepted the study, the OR analyst helps in implementing the recommendations. The system must be constantly monitored and updated dynamically as the environment changes. This means going back to Step 1, Step 2, or Step 3, from time to time.

Unit – 2: TRANSPORTATION PROBLEM AND ASSIGNMENT PROBLEMS

1. Define balanced and unbalanced Transportation problem? **May 2016**
If the sum of supply = sum of demand, then it is called a balanced Transportation problem. If supply is not equal to demand, then it is called unbalanced Transportation problem.
2. What is degeneracy in Transportation Problem? **Dec 2017**
If the number of occupied cells in a transportation problem is less than $m+n - 1$, then degeneracy occurs in that problem. Such a solution is called degenerate solution.
 - To overcome this, we add infinitesimally small quantity to one (or more, if the need be) the least cost independent empty cell and treat this cell as an occupied cell.
3. What do you mean by travelling salesman problem?
A traveling salesman wishes to go to a certain number of destinations in order to sell objects. He wants to travel to each destination exactly once and return home taking the shortest total route.
4. Explain North West corner rule with example?
5. Explain least cost cell method with example. **May 2017**
6. Explain Vogels Approximation method. **May 2016**
7. Write mathematical model of transportation problem?
The transportation model can also be portrayed in a tabular form by means of transportation table shown in table 3.1.

Table 3.1 Transportation Table

| Origin(i) | Destination(j) | | | | Supply(a _i) |
|-------------------------|----------------|----------|-----|----------|-------------------------|
| | 1 | 2 | ... | n | |
| 1 | x_{11} | x_{12} | | x_{1n} | a_1 |
| | c_{11} | c_{12} | | c_{1n} | |
| 2 | x_{21} | x_{22} | | x_{2n} | a_2 |
| | c_{21} | c_{22} | | c_{2n} | |
| ... | ... | ... | ... | ... | ... |
| M | x_{m1} | x_{m2} | | x_{mn} | a_m |
| | c_{m1} | c_{m2} | | c_{mn} | |
| Demand(b _j) | b_1 | b_2 | ... | b_n | $\sum a_i = \sum b_j$ |

Mathematical model:

Let a_i = quantity of product available at origin i

b_j = quantity of product required at destination j

c_{ij} = the cost of transporting one unit of product from origin i to destination j

x_{ij} = the quantity transported from origin i to destination j

Minimize (Total cost) $Z = c_{11}x_{11} + c_{12}x_{12} + \dots + c_{1n}x_{1n} +$

$c_{21}x_{21} + c_{22}x_{22} + \dots + c_{2n}x_{2n} + \dots + c_{m1}x_{m1} + c_{m2}x_{m2} + \dots + c_{mn}x_{mn}$

subjected to

$$x_{11} + x_{12} + \dots + x_{1n} = a_1$$

$$x_{21} + x_{22} + \dots + x_{2n} = a_2$$

.....

$$x_{m1} + x_{m2} + \dots + x_{mn} = a_m$$

$$x_{11} + x_{21} + \dots + x_{m1} = b_1$$

$$x_{12} + x_{22} + \dots + x_{m2} = b_2$$

.....

$$x_{1n} + x_{2n} + \dots + x_{mn} = b_n$$

$$x_1, x_2, \dots, x_n \geq 0$$

(Or)

$$\text{Minimize, Totalcost, } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i \quad \text{for } i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j \quad \text{for } j = 1, 2, \dots, n$$

$$x_{ij} \geq 0 \quad \text{for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

8. Write mathematical model of Assignment problem.

$$\text{Minimize (Total cost) } Z = c_{11}x_{11} + c_{12}x_{12} + \dots + c_{1n}x_{1n} +$$

$$c_{21}x_{21} + c_{22}x_{22} + \dots + c_{2n}x_{2n} + \dots + c_{m1}x_{m1} + c_{m2}x_{m2} + \dots + c_{mn}x_{mn}$$

subjected to

$$x_{11} + x_{12} + \dots + x_{1n} = 1$$

$$x_{21} + x_{22} + \dots + x_{2n} = 1$$

.....

.....

$$x_{m1} + x_{m2} + \dots + x_{mn} = 1$$

$$x_{11} + x_{21} + \dots + x_{m1} = 1$$

$$x_{12} + x_{22} + \dots + x_{m2} = 1$$

.....

.....

$$x_{1n} + x_{2n} + \dots + x_{mn} = 1$$

x_1, x_2, \dots, x_n are either zero or one

(Or)

$$\text{Minimize, Totalcost, } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = 1 \quad \text{for } i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = 1 \quad \text{for } j = 1, 2, \dots, n$$

$$x_{ij} = 0 \text{ or } 1; \text{ for all } i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

9. Explain Hungarian assignment method. Dec 2017, May 2017

This method is applied for balanced assignment problem and is explained below.

Step 1: locate the smallest cost element in each row of the cost table. Now subtract this smallest element from each element in that row. As a result, there shall be at least one zero in each row of this new table, called the reduced cost table.

Step 2: in the reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest element from every other entry in the column.

Step 3: draw the minimum number of horizontal and vertical lines (not the diagonal ones) that are required to cover all the zero elements. If the number of lines drawn is equal to “n” (number rows or columns) the solution is optimal, and proceed to step 6. If the number of lines drawn is smaller than “n” go to step 4.

Step 4: Select the smallest uncovered (by the lines) cost element. Subtract this element from all other elements including itself and add this element to each value located at the intersection of any two lines. The cost elements through which only one line passes remain unaltered.

Step 5: Repeat steps 3 and 4 until an optimal solution is obtained.

Step 6: given the optimal solution, make the job assignments as indicated by the zero elements. This is done as follows:

- a. Locate a row which contains only one zero element. Assign the job corresponding to this element to its corresponding person. Cross out the zeros, if any, in the column corresponding to the element.
- b. Repeat (a) for each of such rows which contain only one zero. Similarly, perform the same operation in respect of each column containing only one zero element, crossing out the zero(s), if any, in the row in which the element lies.
- c. If there is no row or column with only a single zero element left, then select a row or column arbitrarily and choose one of the jobs (or persons) and make the assignment. Now cross the remaining zeroes in the column and row in respect of which the assignment is made.
- d. Repeat steps (a) through (c) until all assignments are made
- e. Determine the total cost with reference to the original cost table.

Unit – 3: Network Models and SEQUENCING MODELS.

1. Define minimum spanning tree problem.
 - A tree is a subgraph that contains “ n” vertices and “n-1” arcs.
 - A minimum spanning tree is a tree that covers all the vertices in the graph with minimum weight.
2. What is job shop sequencing and flow shop sequencing? (May 2016)
Flow shop sequencing: in this all jobs follows same sequence of operations.
3. Distinguish CPM and PERT. (May 2016)

| Sl. No. | CPM | PERT |
|---------|--|--|
| 1 | CPM means Critical Path Method | PERT means Programme Evaluation and Review Technique |
| 2 | It is deterministic model | PERT is Probabilistic model |
| 3 | CPM is activity oriented approach | It is event oriented approach |
| 4 | It is useful for projects which are repetitive and standardized like construction activities | It is useful for projects which are new and non-repetitive like Research and Development projects. |

4. What are the three time estimates used in PERT Network? Define them. (Dec 2017, May 2017)

The three time estimates used in PERT network are Optimistic time, Pessimistic time and most likely time.

Optimistic time (t_o): this is the shortest time the activity can take to complete.

Most likely time (t_m): this refers to the time that would be expected to occur most often. complete.

Pessimistic time (t_p): this is the longest time the activity can take to complete.

- All the above time estimates follows Beta-distribution (Important for GATE)

5. List out network models:
 - Shortest route / path problem,
 - Minimum spanning tree ,
 - Maximum flow models,
 - transportation model and
 - Assignment model.

Job shop sequencing: in this all jobs does not follow the same sequence of operations.

6. Objective of sequencing models is to find optimum processing order of jobs so that the total time required to complete all Jobs (makespan) and idle times of machines is minimum
7. Write the assumptions in sequencing models..
 - a. Only one operation is carried out on a machine at a particular time
 - b. Each operation once started must be completed.
 - c. Only one machine of each type is available

- d. A job is processed as soon as possible, but only in the order specified
 - e. An operation must be completed before its succeeding operation can start
 - f. Processing times are independent of order of performing the operation
 - g. The transportation time is negligible i.e. the time required to transport jobs from one machine to another is negligible
 - h. Jobs are completely known and are ready for processing when period under consideration starts.
8. When do you use dummy activity?
It is used to satisfy the immediate predecessors and successors relationships among the activities in the network.
9. What is critical path? Critical activities?
The longest path in the network diagram is called critical path. The activities on critical path are called critical activities. If you change the duration of these activities or delay these activities, it affect the project completion time. Hence, they are called critical activities. Float or slack for these critical activities is zero.
10. Applications of CPM / PERT
These methods have been applied to a wide variety of problems in industries and have found acceptance even in government organizations. These include
- Construction of a dam or a canal system in a region
 - Construction of a building or highway
 - Maintenance or overhaul of airplanes or oil refinery
 - Space flight
 - Cost control of a project using PERT / COST
 - Designing a prototype of a machine
 - Development of supersonic planes
11. List the Rules for constructing network diagrams. **Dec 2016.**
- Each defined activity is represented by only one arrow in the network. Therefore, no single activity can be represented more than once in the network.
 - Before an activity can be undertaken all activities preceding it must be completed.
 - The arrow direction indicates the general progression in time.
 - A network should have only one initial and one terminal node.
 - Try to avoid arrows which cross each other
 - Use straight arrows
 - Do not attempt to represent duration of activity by its arrow length

- Use arrows from left to right. Avoid mixing two directions, vertical and standing arrows may be used if necessary.
- Use dummies freely in rough draft but final network should not have any redundant dummies.

12. Define float or total float. Explain different types of floats.

Total float of an activity represents the amount of time by which it can be delayed without delaying the project completion date.

Total float = Latest Finish Time (LFT) – Earliest Finish Time (EFT)

(or)

= Latest Start Time (LST) – Earliest Start Time (EST)

Types of floats:

Interference float:

Utilization of the float of an activity may affect the float times of the other activities. The part of the total float which causes a reduction in the float of the successor activities is called Interference float.

Free Float:

The part of the total float which can be used without affecting the float of the succeeding activities is called free float.

Independent float:

The independent float time of an activity is the amount of float time which can be used without affecting either the head or the tail events.

It represents the amount of float time available for an activity when its preceding activities are completed at their latest and its succeeding activities begin at their earliest time.

Independent float = EST for following activity – LST of preceding activity – duration of present activity

UNIT – 4; Game Theory

1. What is a saddle point? (Dec 2016, May 2017, May 2018)

It is an equilibrium point at which minimax value is equal to maximin value.

Max(Min) = Min(Max)

2. Define the Value of the game? (May 2017)

The final pay-off to the winning player by the losing player is called the value of the game.

3. What is a fair game?

If the value of the game is equal to zero, then it is called fair game.

4. What is strategy? (May 2018)

A strategy refers to the action to be taken by a player in various contingencies (circumstances or possible situations) in playing the game.

5. What is dominance rule or dominance property in game theory? (Dec 2017, May 2016, Dec

2016)

In game theory, one strategy for a player is better than another strategy. We consider better strategy and delete the other one is called dominance rule.

6. What is two persons zero-sum game? (May 2018)

In this game two players involved and gain of one player is equal to loss of another player. The sum of gain of a player and loss of another player is equal to zero.

7. What is non-zero sum game?

In this game gain of one player is not equal to loss of another player.

The sum of gain of a player and loss of another player is not equal to zero.

8. What is the optimum strategy for a player?

It is the strategy at which the player gets maximum pay-off.

9. Define pay-off. (Dec 2016)

Pay-off is the outcome of possible choice of strategies made by the players.

10. Explain maximin and minimax principle or strategy? (Dec 2017, May 2017)

- **Minimax** (sometimes called **MinMax**) is a decision rule used in [game theory](#), for *minimizing* the possible [loss](#) or *maximum* loss)
- "maximin" strategy used to maximize the minimum gain.

11. What are the strategies used to reduce the size of a game?

Dominance principle and graphical methods are used to reduce the game.

12. What are the Assumptions in game theory? (Dec 2016)

- It is assumed that players within the game are rational and will strive to maximize their payoffs in the game.
- The players in the game knows the pay-off matrix
- The number of players (competitors) is finite.
- All players act rationally and intelligently.
- Each player has a definite course of action.
- There is conflict of interest between the players.
- The rules of play are known to all the players.

13. What is game theory?

Game theory is a body of knowledge which is concerned with the study of decision making in situations where two or more rational opponents are involved under conditions of competition and conflicting interests.

- **A game refers a situation in which two or more players are competing.**

14. What are the various applications of game theory ? Dec 2016

The game theory can be applied to decide the best course in conflicting situations. In business decisions it has wider possibilities. With the help of computer large number of independent variables can be considered with mathematical accuracy. The main advantages of this theory are:

- Game theory provides a systematic quantitative approach for deciding the best strategy in competitive situations.
- It provides a framework for competitor's reactions to the firm actions.

- It is helpful in handling the situation of independence of firms.
- Game theory is a management device which helps rational decision-making.

15. What are the various limitation of game theory?

Limitations:

- As the number of players increases in the actual business the game theory becomes more difficult.
- It simply provides a general rule of logic not the winning strategy.
- There is much uncertainty in actual field of business which cannot be considered in game theory.
- Businessmen do not have adequate knowledge for the game theory.

UNIT – 5: QUEING THEORY

1. Explain the terms: Bulk Arrival, Balking, Reneging, Jockeying and patient customer.
(May 2016, May 2017)

Bulk Arrival: the arrival of customers in groups for service is called Bulk arrival or batch arrivals. Examples: families visiting restaurants, ship discharging cargo at dock.

Balking: some customers arrive to the system for service but they will not join the queue for some reason and decide to come at a later time for service. This is known as balking.

Reneging: some customers arrive to the system for service and they join the queue for service but they leave the system without taking service. This is known as Reneging.

Jockeying: when customers come to multiple servers with multiple queues, they look for shortest queues and joins for service. Sometimes customers move from one queue to other queues which are moving fast. This is known as Jockeying.

Patient Customer: Customer who wait in the system still he gets the service is called Patient customer.

2. Define service utilization factor or traffic intensity or clearing ratio in queuing systems. (Dec 2016, Dec 2017)

let λ = arrival rate

μ = service rate, then

service utilization factor, $\rho = \frac{\lambda}{\mu}$ for single server models

$\rho = \frac{\lambda}{C\mu}$ for multiple server models

where C = number of servers

3. List out Various notations used in the Queuing system or characteristics of a queuing system. **May 2017**

n = Number of customers in the system

p_n = Probability of exactly ' n ' customers in the system

W_q - Average time a customer spends waiting in the queue

W_s - Average time a customer spends in the system

L_q - Average number of customers in the queue

L_s - Average number of customers in the system

ρ = Utilisation factor for the service system

μ = Mean number of customers served per time period

λ = Mean number of arrivals per time period

4. Features of a queuing system or elements in queuing system. **Dec 2016**

The essential features of queuing systems are:

1) Calling population – it may be finite or infinite

2) Arrival process – describes the arrival of customers to the system. In Queuing theory, it is assumed that customer arrivals follows poisson distribution

3) queue configuration,

4) Queue discipline: this represents the order in which the customers are picked up from the waiting line for service. The following are the possibilities.

- FCFS – First Come First Served
- LCFS – Last Come First Served
- SIRO – Service In Random Orders
- Priority service

5) Service process - describes the service given to the customers, Structure and speed of the service. In Queuing theory, it is assumed that service process follows exponential distribution.

5. Write the Kendal`s Notation of a Queuing models. **Dec 2017**

Kendals notation of a Queuing model is as follows

A/B/C/D/E

where A represents the distribution of the customer arrivals

B = the distribution of service process

C = number of servers in the systems

D = Queue length (finite or infinite)

E = Population (finite or Infinite)